Physics of heavy flavors in QCD

RAPPORT DE STAGE DE RECHERCHE

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Résumé

On considère certains problèmes de la physique du quark lourd. Utilisant le formalisme de la Théorie Effective du Quark Lourd (HQET), on constate que les interactions molles à l'intérieur des états hadroniques sont factorisées en valeur moyenne de la ligne de Wilson et on trouve la correspondance entre la fonction HQET de Isgur-Wise et la boucle de Wilson, moyennée sur des légers constituants du hadron. Il est démontré que la même fonction universelle $\Gamma_{cusp}(\theta, \alpha_s)$ détermine en HQET la dimension anomale dépendante de vélocité et les propriétés de rénormalisation des valeurs moyens des lignes de Wilson avec une rupture, évaluées au long des trajectoires classiques du quark lourd. On calcule la $\Gamma_{cusp}(\theta, \alpha_s)$ rénormalisée et les coefficients de correspondance (matching) QCD-HQET. Les concepts développés sont appliqués à la diffusion méson-méson dans la limite du quark lourd. Il se trouve que l'amplitude de diffusion est contrôlée par les propriétés de rénormalisation des singularités dites de croisement de boucles de Wilson. Utilisant ce fait, on évalue à l'ordre un la matrice 2×2 correspondante de la dimension anomale ; dans la limite $N_c \to \infty$ on exprime également l'amplitude de diffusion en termes des facteurs de forme de Isgur-Wise. Le cas des saveurs identiques est examiné. Le calcul des coefficients de correspondance à une boucle est considéré.

Les résultats présentés ici résultent de la continuation du travail entamé l'année dernière. Pendant le stage de recherche du M2 de Physique Théorique l'auteur s'est concentré sur les problèmes présentés dans la partie 4. Les parties précédentes sont données pour la cohérence du texte et des notations.

Abstract

We consider selected problems of heavy quark physics. Using the formalism of the Heavy Quark Effective Theory (HQET), we perform factorization of soft gluon exchanges inside the hadron state into Wilson lines expectation values and find correspondence between the HQET Isgur-Wise function and Wilson loop, averaged over the hadron's light constituents. It is shown that the same universal function $\Gamma_{cusp}(\theta, \alpha_s)$ determines the velocity-dependent anomalous dimension in HQET and the renormalization properties of the averaged Wilson lines with a cusp, evaluated along the classical trajectories of the heavy quark. We calculate renormalized $\Gamma_{cusp}(\theta, \alpha_s)$ and the QCD-HQET matching coefficient functions at one loop. The developed concepts are applied to elastic meson-meson scattering in the heavy quark limit. It turns out that the scattering amplitude is controlled by the renormalization properties of the so called cross singularities of Wilson loops. Using this fact, we evaluate to one-loop order the corresponding 2×2 matrix of anomalous dimension and express the scattering amplitude in terms of Isgur-Wise form factors in the limit $N_c \to \infty$. The case of identical flavors is discussed. The calculation of the matching coefficient functions is considered at one loop level.

The presented results result from the continuation of the research started last year. During the scientific internship of the M2 in Theoretical Physics the attention of the author has been focused on the problems presented in the part 4. The previous parts are given for the coherence of the text and conventions.

Contents

1 Introduction

Nowadays it is clear that hadronic states are described by Quantum Chromodynamics (QCD), but hadronic properties are at best understood at a qualitative level, the progress in quantative description of the question remaining slow. When studying the hadronic structure, one needs to deal with soft degrees of freedom which are the realm of nonperturbative phenomena. Heavy quarks are among the best probes of the soft component of hadronic states at our service. Understanding the dynamics of hadrons containing heavy quarks is an essential step toward the nonperturbative study of soft degrees of freedom. There is another reason for the growing interest in heavy quark physics. The B-factories at SLAC and KEK produce a high-quality experimental data about weak decays of B-mesons. The understanding of strong-interaction effects in these decays will help us to evaluate fundamental electroweak parameters using the information provided by these experiments. Finally, the investigation of the B-meson, which is the QCD "hydrogen atom", is interesting fundamentally. In this work, we study hadrons with single heavy quark, b or c, and light components including one of the light quarks (u, d or s) and the gluonic cloud. Surprisingly, the top quark t is of no relevance to our discussion, since it is too heavy to form a bound state before it decays into $b + W$.

There are several possible ways to deal with heavy quark problems. We can discuss the heavy quark expansions directly in full QCD in the framework of Wilson operator product expansion (OPE), starting from the full QCD Lagrangian [1]. But we can also accept the viewpoint of the Heavy Quark Effective Theory (HQET), a concept introduced at the beginning of 90's to reproduce the results of QCD for problems involving single heavy quark with mass m_Q, expanded to some order in k/m, where k is the characteristic momentum in the problem. To the leading order in $1/m_Q$, it has symmetries which are not explicit in the original QCD Lagrangian. In what follows, we will use the HQET formalism which provides powerful simplifications and which is sufficient for attaining our aim.

An important observation made in [2] states that the effective currents $\overline{h}_{v_2}\Gamma h_{v_1}$ are characterized by the same universal function, called the Isgur-Wise function $\xi(v_1 \cdot v_2)$, which depends only on the scalar product $w = (v_1 \cdot v_2) = \cosh\theta$. We will see that this function describes both elastic $B + \gamma^* \to B$ and weak $B \to De\nu-$type transitions. The Isgur-Wise form factor can be expressed in terms of the gauge-invariant Wilson loops

$$\langle W_C \rangle = \frac{1}{N_c} Tr \, \langle light, v_2 | \, TP \exp[ig \oint_C dx_\mu A^\mu(x)] \, | light, v_1 \rangle \, , \tag{1}$$

which dynamics is described by the w-dependent anomalous dimension $\gamma_w(\alpha_s)$ [3]. The Wilson loop in (1) is averaged over the light constituents of meson containing heavy quark and appears as eikonal phase of the heavy quark. Thus, the integration contour C corresponds to the classical trajectory of the heavy quark. One should stress that the objects like (1) were considered earlier in other branches of the quantum field theory. First, there was an attempt in 80's to describe the nonperturbative dynamics of QCD in terms of Wilson loops. In the pioneer work [4], Polyakov observed that the vacuum average of a Wilson loop having a cusp, possesses extra ultraviolet divergences. Anomalous dimension $\Gamma_{cusp}(\theta, \alpha_s)$, generated by the cusp, coincides with $\gamma_w(\alpha_s)$. The same loop functions enter into consideration when we study the infrared (IR) behavior of perturbative QCD involving soft gluons. The relevant anomalous dimension $\Gamma_{IR}(\theta, \alpha_s)$ appears to be the same as $\gamma_w(\alpha_s)$ [3]. In general, the path-ordered exponentials appear as eikonal phases of partons in interaction with soft gluons.

The paper is organized in the following way. In part 2 we follow [5, 6, 7, 10] to introduce the basics of the HQET formalism which we use to show the relation of the Isgur-Wise function to the Wilson loop averages (1). Then, in part 3 we calculate the corresponding anomalous dimension in the one-loop approximation in the HQET framework and study the HQET-QCD matching properties. We show that the obtained results are in correspondence with [3, 5, 10]. Finally, in part 4 we generalize the concept developed in previous sections to analyze the cross singularities in case of meson-meson scattering in the heavy quark limit and find the general one-loop expression for the matrix of cross anomalous dimension. In order to verify the obtained matrix, we compare its special case of forward scattering with the earlier results in high-energy scattering [17] and study the large N_c limit of the scattering amplitude. We also consider the calculation of the matching coefficient functions for the meson-meson scattering process at one loop level.

2 Heavy quark limit and the Wilson lines

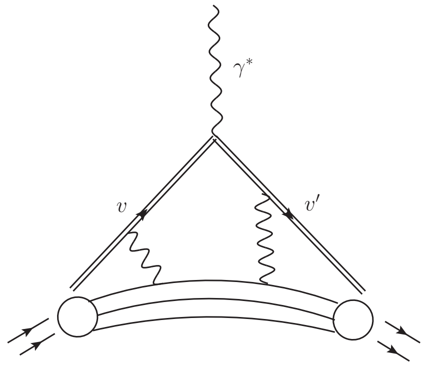
2.1 HQET: The Effective Field Theory

The Heavy Quark Effective Theory (HQET) is the limit of QCD where the heavy quark mass, m_Q, goes to infinity with its four-velocity v^μ held fixed. The strong coupling region leads to nonperturbative phenomena such as the quarks and gluons confinement on the scale of $r_{had} \sim 1/\Lambda_{QCD} \sim 1\, fm$ which determines the typical size of hadrons. In some sense, $\Lambda_{QCD} \sim 0.2\, GeV$, separating the region of nonperturbative effects from the perturbative region of asymptotic freedom, is the only energy scale that we can clearly identify. When the mass of the quark Q is much larger than this scale, $m_Q \gg \Lambda_{QCD}$, Q is called a heavy quark; the effective coupling constant $\alpha_s(m_Q)$ is small, implying that on length scales of the Compton wavelength $\lambda_Q \sim 1/m_Q$ the strong interactions are perturbative and much like electromagnetic ones. That is why the quarkonium states $Q\bar{Q}$ with the size of order $\lambda_Q/\alpha_s(m_Q) \ll r_{had}$ are very much hydrogen-like. For the meson with $Q\bar{q}$ flavor quantum numbers (Q denotes a heavy quark and q a light quark), things are more complicated, because the typical momenta exchanged between heavy and light constituents are of order Λ_{QCD}. The heavy quark velocity change is of order Λ_{QCD}/m_Q, and thus the infinite mass limit is appropriate. The light degrees of freedom experience only electric color field, all the spin involving relativistic effects vanish as $m_Q \to \infty$. Hence, the heavy quark's spin decouples in this limit, giving birth to new spin-flavor symmetries not apparent in QCD. This is not an exact symmetry and the corrections of order Λ_{QCD}/m_Q can be systematically included.

In part 3, we are mainly interested in elastic scattering of B meson by virtual photon $B + \gamma^* \to B$. This process involves B meson which contain a single heavy quark $\bar{b}$ and a photon of great virtuality that changes the B meson's velocity. Even though the overall momentum transfer $q^2 = (m_Q v_1 - m_Q v_2)^2$ is of order m_Q^2, the momentum transfer to the lights degrees of freedom is negligible:

$$q_{light}^2 \sim (\Lambda_{QCD} v_1 - \Lambda_{QCD} v_2)^2 = 2\Lambda_{QCD}^2 (1 - v_1 \cdot v_2) \ll m_Q^2. \qquad (2)$$

Therefore we consider that the photon interacts only with the heavy Q quark:



Being almost on-shell, the heavy quark momentum P_Q is close to the kinetic momentum resulting from the hadron's motion [5]:

$$P_Q^\mu = m_Q v^\mu + k^\mu. \tag{3}$$

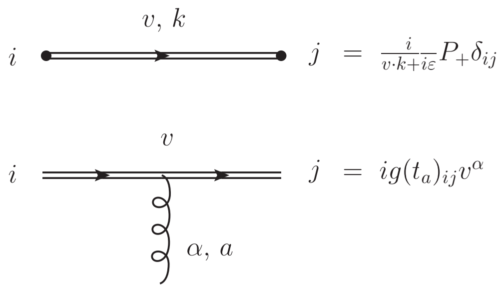
In the limit $m_Q \to \infty$, the Feynman rules of QCD are simplified. In particular, the heavy quark propagator becomes

$$S_Q = \frac{i}{\not{P}_Q - m_Q + i\varepsilon} = \frac{i}{v \cdot k + i\varepsilon}\frac{\not{v}+1}{2} + O(k/m_Q) \to \frac{i}{v \cdot k + i\varepsilon} P_+, \tag{4}$$

where P_+ is a positive-energy projection operator, defined by $P_\pm = (1 \pm \not{v})/2$. Since $P_+ \gamma^\mu P_+ = P_+ v^\mu P_+$, the vertex $igt_a\gamma^\mu$ can be replaced to the leading order in $1/m_Q$ by

$$igt_a v^\mu, \tag{5}$$

where g is the QCD coupling constant, and t_a are the generators of the color $SU(N_c)$ group with $N_c = 3$. The Feynman rules (4) and (5) are depicted as follows:



It has become standard to represent the effective heavy quark propagator by a double line to indicate that it is different from the propagator in QCD. These Feynman rules follow from the effective Lagrangian

$$\mathcal{L}_{eff} = \bar{h}_v iv \cdot D h_v + O(1/m_Q) = \bar{h}_v (iv^\mu \partial_\mu + gt_a v^\mu A_\mu^a) h_v + O(1/m_Q) \tag{6}$$

which is the approximation for the original QCD Lagrangian part for heavy quark

$$\mathcal{L}_{QCD} = \overline{Q}(i\not{D} - m_Q)Q. \tag{7}$$

Indeed, the effective heavy-quark field $h_v(x)$ is related to the original field $Q(x)$ by

$$Q(x) \approx \exp(-im_Q v_\mu x^\mu) h_v(x) \tag{8}$$

and is subject to the on-shell constraint

$$\not{v} h_v(x) = h_v(x) = P_+ h_v(x). \tag{9}$$

Because of this condition, $h_v(x)$ is a two-component field. Putting this into the QCD Lagrangian, one obtains

$$\mathcal{L}_{QCD} \to \bar{h}_v(m_Q \not{v} + i\not{D} - m_Q)h_v = \bar{h}_v i\not{D} h_v =$$

$$= \bar{h}_v P_+ i\not{D} P_+ h_v = \bar{h}_v \left[iv \cdot D + i\not{v} \left(\frac{-\not{v} + 1}{2} \right) \left(\frac{\not{v} + 1}{2} \right) \right] h_v = \bar{h}_v iv \cdot D h_v. \tag{10}$$

We see that HQET is a theory of heavy quark. The field $h_v(x)$ destroys a heavy quark but it does not create the corresponding anti-quark. Thus, there is no pair creation in HQET.

2.1.1 Heavy quark symmetry

There are no Dirac matrices present in the effective Lagrangian, interactions of the heavy quark with gluons leave its spin unchanged. Associated with this is a $SU(2)$ symmetry group under which $\mathcal{L}_{eff}$ is invariant [5]. When there are N_h heavy quarks moving at the same velocity, one can simply extend (6) by writing

$$\mathcal{L}_{eff} = \sum_{i=1}^{N_h} \bar{h}_v^i iv \cdot D h_v^i. \tag{11}$$

This Lagrangian is also invariant under rotations in flavor space, so the symmetry group becomes promoted to $SU(2N_h)$. This is the heavy quark spin-flavor symmetry [2]. Note that this symmetry is a bit unusual since it relates states at the same velocities but different momentum.

The effective Lagrangian (11) is not invariant under Lorentz transformations which change the velocity v. Indeed, in the heavy quark limit, the soft QCD interactions do not change heavy quark velocity, all the kinks in the trajectories must be caused by some external source. Under a Lorentz boost Λ, the color field of the heavy quark is boosted and the light degrees of freedom "see" different color field. Inversely, no matter what the quark flavor is, at a given velocity v the light quark is placed in the same color field. The fact that heavy quarks with different velocities are in no way related to each other is referred to as "velocity super selection rule" [5, 6]. We can imagine an effective Lagrangian which contains heavy quarks of all possible flavors and velocities [6]:

$$\mathcal{L}_{eff} = \sum_{i=1}^{N_h} \sum_{v} \bar{h}_v^i iv \cdot D h_v^i. \tag{12}$$

This Lagrangian suggests $SU(2N_h)$ symmetry for every v, and as far there is an infinite number of velocity intervals and the Lorentz transformations rotate the $SU(2N_h)$'s among themselves (because of the velocity superselection rule, the quarks with different momenta can't rotate into each other under Lorentz boosts), we can denote this symmetry group by

$$SU(2N_h)^\infty \otimes Lorentz = SU(2N_h)^\infty \otimes SO(3,1).$$

2.1.2 $1/m_Q$ Corrections to the HQET Lagrangian

We can take the $1/m_Q$ corrections to the HQET Lagrangian into account as well. Factoring out the phase factor in (8) is not an approximation, but just a field redefinition. The approximation is present in putting (9). To go beyond the leading order in $1/m_Q$ we must consider the general decomposition

$$Q(x) = \exp(-im_Q v_\mu x^\mu) \left[h_v(x) + H_v(x)\right], \tag{13}$$

where

$$h_v(x) = \exp(im_Q v_\mu x^\mu) P_+ Q(x), \quad H_v(x) = \exp(im_Q v_\mu x^\mu) P_- Q(x). \tag{14}$$

We notice that $H_v(x)$ corresponds partly to the anti-quark degrees of freedom. In terms of new fields, the QCD Lagrangian for heavy quarks (7) takes the following form:

$$\mathcal{L} = \left[\bar{h}_v + \bar{H}_v\right] (m_Q(\not{v} - 1) + i\not{D}) \left[h_v + H_v\right] =$$

$$= \bar{h}_v iv \cdot D h_v - \bar{H}_v (iv \cdot D + 2m_Q) H_v + \bar{h}_v i\not{D}_\perp H_v + \bar{H}_v i\not{D}_\perp h_v, \tag{15}$$

where $iD_\perp^\mu = D^\mu - v^\mu v \cdot D$ is orthogonal to the heavy quark velocity, $v \cdot D_\perp = 0$. It is apparent that h_v describes the massless degrees of freedom, whereas H_v corresponds to fluctuations with twice the heavy quark mass. These heavy degrees of freedom can be integrated out of the theory using the equations of motion. Substituting (13) into $(i\not{D} - m_Q)Q = 0$ gives

$$i\not{D} h_v + (i\not{D} - 2m_Q) H_v = 0, \tag{16}$$

and multiplying this by $P_\pm$ one derives

$$H_v = \frac{1}{(iv \cdot D + 2m_Q)} i\not{D}_\perp h_v. \tag{17}$$

Finally, the resulting Lagrangian density to $1/m_Q$ order can be written as [7]

$$\mathcal{L}_{eff} = \bar{h}_v iv \cdot D h_v + \frac{1}{2m_Q} \bar{h}_v i\not{D}_\perp i\not{D}_\perp h_v + O\left(\frac{1}{m_Q^2}\right) \tag{18}$$

with the heavy quark QCD-HQET relation

$$Q = \exp(-im_Q v_\mu x^\mu) \left[1 + \frac{i\not{D}_\perp}{2m_Q}\right] h_v + O\left(\frac{1}{m_Q^2}\right). \tag{19}$$

Introducing $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ and the gluon strength tensor $[iD^\mu, iD^\nu] = igG^{\mu\nu}$, one can express the effective Lagrangian as follows [5]:

$$\mathcal{L}_{eff} = \bar{h}_v iv \cdot D h_v + \frac{1}{2m_Q} \bar{h}_v (iD_\perp)^2 h_v + \frac{g}{4m_Q} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v + O\left(\frac{1}{m_Q^2}\right). \tag{20}$$

The first term in $1/m_Q$ is the heavy quark kinetic energy. It breaks the flavor symmetry but not the spin symmetry. The second term in $1/m_Q$ is a magnetic moment interaction $\mu_Q \cdot B_c$. It breaks both the spin and flavor symmetries [7].

2.1.3 Reparametrization invariance and residual mass

The decomposition (3) of the heavy quark four-momentum is not unique. A small change in v of order Λ_{QCD}/m_Q can be compensated by a change in k of order Λ_{QCD}. Explicitly

$$v \to v + \varepsilon/m_Q, \quad k \to k - \varepsilon, \tag{21}$$

leaves P_Q unchanged. Since the four-velocity satisfies $v^2 = 1$ we must, at linear order in ε, choose $v \cdot \varepsilon = 0$. We also want to preserve the constraint (9). Therefore the theory should be invariant under the reparametrization transform [8]

$$v \to v + \varepsilon/m_Q, \quad h_v \to e^{i\varepsilon\cdot x}\left(1 + \frac{\not{\varepsilon}}{2m_Q}\right) h_v. \tag{22}$$

The HQET Lagrangian is invariant under this transformation. Another ambiguity is related to the choice of the quark mass that should be used in (3). Since quarks cannot appear as free physical states, there is no natural way to define their mass on-shell. Although one can define the "on-shell" mass as the pole of the renormalized propagator to a given order in perturbation theory, such a concept becomes meaningless beyond the perturbative expansion. Again, choosing the residual mass δm of order Λ_{QCD}, we see that P_Q is unchanged if we take

$$m_Q \to m_Q + \delta m, \quad k \to k - \delta m \cdot v. \tag{23}$$

One can easily verify that this leads to a general HQET Lagrangian [5, 10]

$$\mathcal{L}^{\delta m}_{eff} = \bar{h}_v \left(iv \cdot D - \delta m\right) h_v. \tag{24}$$

Physical quantities, of course, don't depend on this choice. The most convenient definition of the heavy quark mass m is one that gives $\delta m = 0$. This corresponds to a pole of the propagator defined in (4).

2.2 Hadronic matrix elements and the Isgur-Wise function

The wave function of a B state in the full theory can be approximated as a product of the free heavy quark wave function and the complicated wave function that describes the light components [6]:

$$|B, s_h, s_l, v\rangle \approx \left|\bar{b}, s_h, v\right\rangle |light, s_l, v\rangle\,, \tag{25}$$

where s_h and s_l are the spins of a heavy and a light components correspondingly and v is the state's velocity. The expression (25) is an exact factorization in the limit that the heavy quark mass goes to infinity. Let us consider the matrix elements of currents between the hadron states. In the heavy quark limit, the current acts only on the free quark wave function. Therefore, the matrix element of the current in QCD is given approximately by the matrix element between free heavy quark states. The only non trivial part of the matrix element of the current between B meson states is an overlap between the corresponding light components. For the states of the same velocity we get:

$$\langle B, s'_h, s'_l, v|\, b\Gamma\bar{b}\, |B, s_h, s_l, v\rangle \approx$$

$$\approx \left\langle \bar{b}, s'_h, v\right|\, b\Gamma\bar{b}\, \left|\bar{b}, s_h, v\right\rangle \langle light, s'_l, v\, |light, s_l, v\rangle = \left\langle \bar{b}, s'_h, v\right|\, b\Gamma\bar{b}\, \left|\bar{b}, s_h, v\right\rangle \delta_{s'_l, s_l}. \tag{26}$$

In this case, the light states are exactly the same, so the overlap integral is trivial:

$$\langle light, s_l', v \,|light, s_l, v\rangle = \delta_{s_l', s_l}. \tag{27}$$

This property is guaranteed by decoupling of the heavy quark spin in the limit $m_Q \to \infty$[1]. If we want to look at the matrix elements between states of different velocities, we will get

$$\langle B, s_h', s_l', v'|\, b\Gamma\bar{b}\, |B, s_h, s_l, v\rangle \approx$$
$$\approx \left\langle \bar{b}, s_h', v'\right|\, b\Gamma\bar{b}\, \left|\bar{b}, s_h, v\right\rangle \langle light, s_l', v' \,|light, s_l, v\rangle = \left\langle \bar{b}, s_h'\right|\, b\Gamma\bar{b}\, \left|\bar{b}, s_h\right\rangle \xi_{s_l', s_l}(v', v). \tag{28}$$

Now the overlap integral $\xi_{s_l', s_l}(v', v)$ becomes more complex and a priori could be a complicated matrix function in the spin space of light components. As shown in [6], because of the decoupling of the heavy quark spins, angular momentum of the light constituents is separately conserved. Together with Lorentz and parity invariance, this gives

$$\xi_{s_l', s_l}(v', v) = \xi(v'v).$$

Hence, the overlap integral depends on only a single function $\xi(v'v)$ which is called the Isgur-Wise function. In the heavy quark limit, it depends only on $v'v = \cosh\theta$ and is the same for flavor-changing processes.

In HQET, the original free heavy quark fields turn to the effective heavy quark fields. We define meson wave functions as follows:

$$B(v) \propto \langle 0|\, \bar{b}_v q\, |B, v\rangle\,. \tag{29}$$

For simplicity, we do not consider excited states B^*, though the corresponding generalization is straightforward. Lorentz invariance, parity and $\not{v}B(v) = B(v)$ imply that we can take [6]

$$B(v) = -\sqrt{m_B}\frac{1+\not{v}}{2}\gamma_5. \tag{30}$$

We are interested in computing matrix elements

$$\langle B, v'|\, \mathcal{O}\, |B, v\rangle\,. \tag{31}$$

In order to stay consistent with the $SU(2N_h)^\infty \otimes SO(3,1)$ symmetry, we replace $|B, v\rangle$ by $B(v)$ and put the wave functions and operators together into invariants under $SU(2)^\infty$, transforming appropriately under Lorentz transformations. Still, for every independent invariant we must include an arbitrary function of the Lorentz invariants. Each of these is fixed by the dynamics. Thus, we contract the heavy quark spin indices for $SU(2)^\infty$ invariance and form the trace over the remaining Dirac indices with all possible Lorentz invariant combinations involving 1, $\not{v}$, $\not{v}'$. But since $\not{v}\bar{b}_v = \bar{b}_v$ and $b_{v'}\not{v}' = -b_{v'}$, all the terms reduce to to a single unknown function $\xi(v'v)$ [6, 7]:

$$\langle B, v'|\, b_{v'}\Gamma\bar{b}_v\, |B, v\rangle =$$
$$= tr\left(\overline{B}(v')\Gamma B(v)\left[-\xi(v'v) + \chi(v'v)\not{v} + \cdots\right]\right) = -\xi(v'v)\cdot tr\left(\overline{B}(v')\Gamma B(v)\right). \tag{32}$$

[1] The heavy quark spin decoupling leads to an interesting observation: in the limit $m_Q \to \infty$, baryons with two heavy quarks, which we can consider as a single object inside the hadron, have the same properties as a meson with a single heavy quark to the lowest order in $1/m_Q$ [10]. We do not consider hadrons with more than one heavy quark in this paper.

Providing this correct normalization, $\xi(v'v)$ is the same Isgur-Wise function that was introduced in (28). It should be conventionally normalized to 1 at the so-called Shifman-Voloshin point, $vv' = 1$, corresponding to a zero recoil momentum:

$$\xi(1) = 1. \tag{33}$$

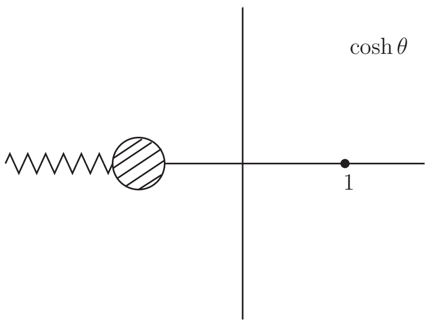
For instance, if we consider the electromagnetic current $\Gamma = \gamma^\mu$, we obtain

$$\langle B, v'|\, b_{v'}\gamma^\mu \bar{b}_v \,|B, v\rangle = \xi(v'v) \cdot \sqrt{m_B m_B} tr \left(\gamma_5 \frac{1+\not{v}}{2}\gamma^\mu \frac{1+\not{v}}{2}\gamma_5\right) = \xi(v'v) m_B \left(v^\mu + v'^\mu\right). \tag{34}$$

For the semileptonic weak decay of the B meson into D, we get in the same way

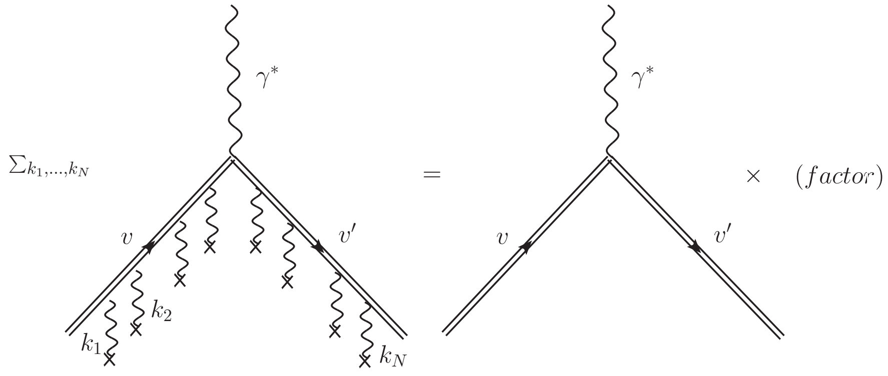
$$\langle D, v'|\, \bar{c}_{v'}\gamma^\mu \bar{b}_v \,|B, v\rangle = \xi(v'v)\sqrt{m_B m_D}\left(v^\mu + v'^\mu\right). \tag{35}$$

The Isgur-Wise form factor has a cut from $\cosh\theta = -1$ to $-\infty$, corresponding to the annihilation channel [10]. Geometrically, $\cosh\theta > 1$ corresponds to Minkowski angles between the world lines of incoming heavy quark and the outgoing one (the scattering channel), for $\cosh\theta = 1$ the world line is straight and there is no transition, $|\cosh\theta| < 1$ correspond to Euclidean angles. When $\cosh\theta < -1$, one of the 4-velocities is directed into the past (the annihilation channel). At the point $\cosh\theta = -1$, the heavy quark returns back along the same world line. The very concept of the Isgur-Wise form factor is inapplicable in vicinity of this point. In HQET, the heavy quarks move along straight world lines, but if their relative velocity in the annihilation channel is $\lesssim \alpha_s$, they rotate around each other instead. Thus we consider that the Isgur-Wise function verify the constraint $|\cosh\theta + 1| \gg \alpha_s$. This region in the complex plane has the following appearance:



2.3 Wilson line factorization, abelian case

Inside a hadronic bound state, the heavy quark Q interacts with light constituents. We study first the abelian case which is more simple and then generalize the results to the non-abelian case in section 2.4. In this part, we suppose that the interaction is governed by exchange of soft photons and wish to prove that all these infrared exchanges can be summarized in one factor corresponding to a Wilson line for the heavy quark. In other words, we have to prove the following identity



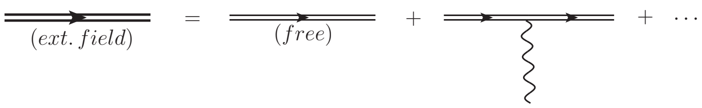
in the limit $k_i \ll m_Q$, where the result for the first diagram has been calculated earlier and is given by (34), and

$$(factor) = \exp[-ie \int_C dx_\mu A^\mu(x)], \tag{36}$$

where the integral contour C corresponds to the quark's world line in the Minkowski space. We will treat $A^\mu(x)$ as an external classical electromagnetic field.

2.3.1 Suggestive considerations.

Let us consider a quark propagator in the external field:



A free fermion propagator $S^0(x-y)$ satisfies the equation

$$(i\partial_\mu \gamma^\mu - m) S^0(x-y) = -\delta^{(4)}(x-y). \tag{37}$$

We see that the propagator with one extra soft photon corresponds to $ie \int dz^4 S^0(x-z)\gamma^\mu S^0(z-y)$. Thus, this solution is given by the first perturbation series member of the free Dirac equation (37). The quark propagator in an external field $S(x,y,A)$ satisfies the equation

$$(iD_\mu \gamma^\mu - m) S(x,y,A) = -\delta^{(4)}(x-y) \tag{38}$$

with a covariant derivative $D_\mu = \partial_\mu + ieA_\mu$, or

$$(i\partial_\mu \gamma^\mu - m) S(x,y,A) = -\delta^{(4)}(x-y) + eA_\mu \gamma^\mu S(x,y,A). \tag{39}$$

We can represent the solution under the form [11]

$$S(x,y,A) = W(x,y,A) \cdot \left[S^0(x-y) + R(x,y,A) \right], \tag{40}$$

where $W(x,y,A) = \exp(-ie \int_y^x A_\mu dz^\mu)$ and $R(x,y,A)$ satisfies the following equation:

$$i\gamma^\mu \partial_\mu R(x,y,A) - e\gamma^\mu (x^\nu - y^\nu) \left[R(x,y,A) + S^0(x-y) \right] \cdot \int_0^1 t dt F_{\mu\nu}((1-t)y + tx). \tag{41}$$

From this equation it is clear that $R(x, y, A)$ depends on the field A_μ only through the gauge invariant $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{i}{e}[D_\mu, D_\nu]$. In the soft photons limit, the Fourier transformed $F_{\mu\nu}$ is given by

$$\tilde{F}_{\mu\nu} \propto k_\mu A_\nu - k_\nu A_\mu \to 0 \tag{42}$$

for small k. Thus the $R(x, y, A)$ term is suppressed at first order in k/m_Q and we obtain

$$S(x, y, A) = W(x, y, A) \cdot S^0(x - y). \tag{43}$$

The structure of $W(x, y, A)$ can be understood from the gauge invariance. Indeed, if we require the gauge invariance under local transformations [9]

$$A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x), \quad \psi(x) \to e^{i\alpha(x)}\psi(x), \tag{44}$$

we are forced to introduce a comparison function $U(y, x)$ transforming

$$U(y, x) \to e^{i\alpha(y)}U(y, x)e^{-i\alpha(x)} \tag{45}$$

in order to define a correct operation of derivation

$$n^\mu D_\mu \psi = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon}\left[\psi(x + \varepsilon n) - U(x + \varepsilon n, x)\psi(x)\right]. \tag{46}$$

The function

$$U_P(y, x) = \exp(-ie \int_P dx^\mu A_\mu(x)), \tag{47}$$

where P is a way from x to y, satisfies (in the abelian case) the expression (45). In our problem, as the heavy quark moves along it's velocity $x^\mu = v^\mu t$ in the limit $m_Q \to \infty$, we can choose a straight line

$$W(-\infty, 0, A) = T \exp(-ie \int_{-\infty}^{0} dt v^\mu A_\mu(vt)). \tag{48}$$

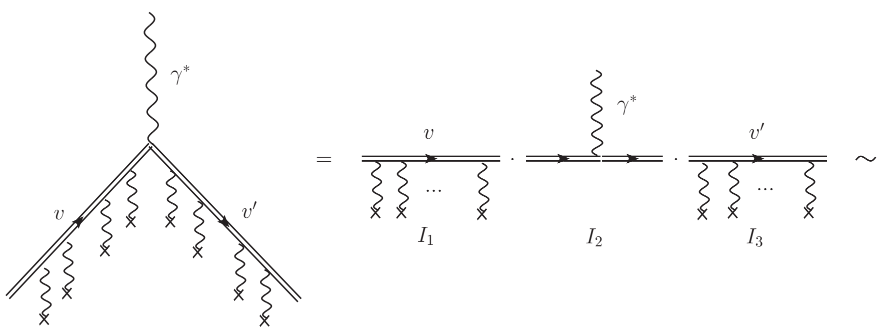
For the overall process in QED approximation, we obtain the total phase

$$W(-\infty, 0, A) \cdot W(0, +\infty, A) \backsimeq \exp[-ie \oint_C dx_\mu A^\mu(x)], \tag{49}$$

where the integration contour C is closed at infinity.

2.3.2 Feynman diagrams summation.

The transition amplitude can be obtained directly by summing up Feynman diagrams (using the HQET Feynman rules (4), (5))



$$\sim \bar{h}_v v^\mu S_Q(v) v^\mu \dots [S_Q(v)\gamma^\mu S_Q(v')] \dots v'^\mu S_Q(v') v'^\mu h_{v'} \tag{50}$$

over all emitted soft photons. Here I_2 gives us the contribution calculated in section 2.2. The calculation of I_1 (constants included) can be performed in the following manner. The sum over all soft photons with momentum k_i for $I_1(N)$

$$I_1(N) = \sum_{perms} \frac{(-ie)^N}{N!} \cdot \frac{iv \cdot \widetilde{A}(k_1) \cdot \ldots \cdot iv \cdot \widetilde{A}(k_N)}{(v \cdot \sum_{i=1}^{N} k_i + i\varepsilon) \cdot \ldots \cdot (v \cdot k_1 + i\varepsilon)} =$$

$$= \frac{(-ie)^N}{N!} \cdot \frac{iv \cdot \widetilde{A}(k_1)}{(v \cdot k_1 + i\varepsilon)} \cdot \ldots \cdot \frac{iv \cdot \widetilde{A}(k_N)}{(v \cdot k_N + i\varepsilon)} \tag{51}$$

becomes[2]

$$\sum_{N \geq 0} I_1(N) = \sum_{N=0}^{+\infty} \frac{e^N}{N!} \prod_{i=0}^{N} \left(\frac{v^\mu \widetilde{A}_\mu(k_i)}{v \cdot k_i + i\varepsilon} \right) = \sum_{perms} \exp\left[\frac{e}{(v \cdot k_i + i\varepsilon)} \cdot \left(v^\mu \widetilde{A}_\mu(k_i) \right) \right]. \tag{52}$$

We can represent this result in other form by considering its Fourier transform. There are two possible ways to proceed in order to evaluate the four-dimensional integral. For instance, we can split $k = k_\| + k_\perp$ $(k \cdot v = k_\| \cdot v)$ to get $d^4k \to d^3k_\perp dk_\|$. The $k_\perp$ terms yield only the delta functions, and the one-dimensional integration in the complex plane over $k_\|$ gives $-2\pi i \theta(\widetilde{x} - \widetilde{y})$. But the simplest way consist in using the alpha-representation

$$\frac{1}{v \cdot k + i\varepsilon} = -i \int_0^{+\infty} d\alpha e^{i\alpha(v \cdot k + i\varepsilon)} = -i \int_0^{+\infty} d\alpha e^{i\alpha(v \cdot k) - \varepsilon\alpha} \tag{53}$$

to get finally

$$\frac{e}{(2\pi)^4} \int d^4k \frac{e^{-ik\widetilde{x}}}{v \cdot k + i\varepsilon} v \cdot \widetilde{A}(k) = \frac{e}{(2\pi)^4} \int d^4\widetilde{y} \int d^4k \frac{e^{ik(\widetilde{y} - \widetilde{x})}}{v \cdot k + i\varepsilon} v \cdot \widetilde{A}(\widetilde{y}) =$$

$$= -ie \frac{1}{(2\pi)^4} \int_0^{+\infty} d\alpha e^{-\varepsilon\alpha} \int d^4\widetilde{y} v \cdot \widetilde{A}(\widetilde{y}) \int d^4k e^{ik(\widetilde{y} - \widetilde{x} + \alpha v)} =$$

$$= -ie \int_0^{+\infty} d\alpha e^{-\varepsilon\alpha} \int d^4\widetilde{y} v \cdot \widetilde{A}(\widetilde{y}) \delta^{(4)}(\widetilde{y} - \widetilde{x} + \alpha v) = -ie \int_0^{+\infty} d\alpha e^{-\varepsilon\alpha} v \cdot A(\widetilde{x} - \alpha v) =$$

$$= -ie \int_{-\infty}^{t_0} dt v \cdot A(vt), \tag{54}$$

where we have put $t = -\alpha$ and $\tilde{x}^\mu = v^\mu t_0$ for the heavy quark, $\|v\| = 1$. Calculations for I_3 can be performed in the same manner; we should note however that in order to describe correctly the physical reality we have to change the sign of the Feynman "$i\varepsilon$" prescription for the outgoing quark propagating from $\tilde{x}$ to $-\infty$, the propagator becoming

$$\frac{i}{v \cdot k - i\varepsilon} \frac{\not{v} + 1}{2}.$$

We obtain finally the required factor (36).

[2]We keep in mind the initial problem where soft photons are supposed to represent interaction with the hadron's light components and where the changes to k^μ in (3) are due to this interaction. Therefore, it is not surprising then that we take the same values of k_i in denominator and in $\widetilde{A}(k_i)$ expression which are of the same order in $\frac{k_i}{m_Q}$.

2.4 Wilson line factorization, nonabelian case

The expression (36) can be directly generalized to the nonabelian case. Replacing soft photons by soft gluons, we introduce the field $A_\mu = A^a_\mu t_a$ of generally non-commuting matrices. The factor (36) becomes

$$(factor) = P \exp[ig \int_C dx_\mu A^\mu(x)], \tag{55}$$

where P is a very useful path-ordered operator (see Appendix A for details). The heavy quark propagates through the gluonic cloud along it's straight world line with a cusp, all the infrared interactions being factorized to the Wilson line (55). Using the HQET result (34), the sum over all soft gluons in (50) can be represented as follows:

$$F_B = m_Q\,(v + v') \cdot C(\frac{m_Q}{\mu_{uv}}) \cdot \xi(v'v, \frac{\mu_{uv}}{\lambda}), \tag{56}$$

where an IR-sensitive factor $\xi(v'v, \frac{\mu_{uv}}{\lambda})$ accumulates all the effects of the heavy quark's interaction with soft gluons and massless quarks. The IR regulator λ is a physical scale of order of the hadron's bound energy $\bar{\Lambda} \sim 0.5\,GeV$. The factorization scale μ_{uv} is working as an ultraviolet cut-off for the new divergences generated by the cusp of the integration path C. As far there is no other dimensional parameters, μ_{uv} and λ should basically appear in the ratio. This fact allows us to identify the λ-dependence of $\xi(v'v, \frac{\mu_{uv}}{\lambda})$ with the cusp anomalous dimension $\Gamma_{cusp}(\alpha_s, \theta)$ [3]:

$$\frac{d \log \xi(v'v, \frac{\mu_{uv}}{\lambda})}{d \log \mu_{uv}} = -\Gamma_{cusp}(\alpha_s, \theta) = -\frac{d \log \xi(v'v, \frac{\mu_{uv}}{\lambda})}{d \log \lambda}. \tag{57}$$

The IR regular QCD-HQET matching factor $C(\frac{m_Q}{\mu_{uv}})$ depends only on the ratio m_Q/μ_{uv} and can be calculated perturbatively. It is clear that in the real QCD the B-meson form factor doesn't possess ultraviolet divergences and thus the final result doesn't depend on μ_{uv}. It is natural to suppose that the final result will depend only on the physical ratio m_Q/λ. It means that $C(\frac{m_Q}{\mu_{uv}})$ obeys the following identity:

$$\frac{d \log C(\frac{m_Q}{\mu_{uv}})}{d \log \mu_{uv}} = +\Gamma_{cusp}(\alpha_s, \theta). \tag{58}$$

We will check this expression in the section 3.2.

In the next section we will calculate $\Gamma_{cusp}(\alpha_s, \theta)$ and thus investigate the dependence of the Isgur-Wise function $\xi(v'v, \frac{\mu_{uv}}{\lambda})$ on λ. Using (25), we have

$$|B, v\rangle = \exp(-i m_Q v_\mu x^\mu) \bar{b}_v(x)\, |light, v\rangle + O(1/m_Q). \tag{59}$$

The explicit expression for the IR-sensitive factor

$$\xi(v'v, \frac{\mu_{uv}}{\lambda}) = \langle light, v'|\, W_C\, |light, v\rangle\,, \tag{60}$$

is given by the Wilson loop (1). It is important to note that the μ_{uv}-dependence of the factor $\langle light, v'|\, W_C\, |light, v\rangle$ is the same as for the vacuum average $\langle 0|\, W_C\, |0\rangle \equiv \langle W_C\rangle$. Indeed, $\langle W_C\rangle$ is the function of μ_{uv}/μ_{IR}, where μ_{IR} is some IR cut-off which can be devoid of any physical sense, in contrast with λ. In what follows we will use this fact and calculate the vacuum average of a Wilson exponential in one-loop approximation to get

$$\frac{d\log \xi(v'v, \frac{\mu_{uv}}{\lambda})}{d\log \mu_{uv}} = \frac{d\log \langle W_C\rangle}{d\log \mu_{uv}} = -\Gamma_{cusp}(\alpha_s, \theta). \tag{61}$$

According to the exponential theorem (174) of the Appendix A, in QED the result is completely determined by the first loop [3]:

$$\langle 0|\, W_C\, |0\rangle_{QED} = \exp\left(\frac{(ie)^2}{2} \oiint_{CC} dz_\mu dz'_\nu \,\langle A^\mu(z) A^\nu(z')\rangle\right) \tag{62}$$

and thus we can get the exact expression for the $\Gamma^{QED}_{cusp}(\alpha, \theta)$. In QCD, however, we get

$$\langle 0|\, W_C\, |0\rangle_{QCD} = 1 + \frac{(ig)^2}{2} \oiint_{CC} dz_\mu dz'_\nu \,\langle A^\mu(z) A^\nu(z')\rangle + O(g^4), \tag{63}$$

as follows from (179), and thus we can calculate $\Gamma_{cusp}(\alpha_s, \theta)$ only at some order in α_s. We will limit ourselves to the one-loop contribution.

3 B-meson form factor

3.1 Vertex renormalization and the cusp anomalous dimension

Our aim is to get one-loop HQET renormalized expressions for heavy quark propagators and for the velocity changing current corresponding to the cusp on the straight Wilson line with minkowskian angle $\cosh\theta = (v \cdot v')$. We will use the minimal renormalization scheme $\overline{MS}$. In this scheme, renormalization constants have the structure [13, 14]

$$Z_i(\alpha) = 1 + \frac{z_1}{\varepsilon}\frac{\alpha}{4\pi} + \left(\frac{z_{22}}{\varepsilon^2} + \frac{z_{21}}{\varepsilon}\right)\left(\frac{\alpha}{4\pi}\right)^2 + \dots \tag{64}$$

The space dimension $D = 4 - 2\varepsilon$. We need allow for the fact that the coupling constant g has a dimension dependent on D. Therefore we rewrite the coupling [14]

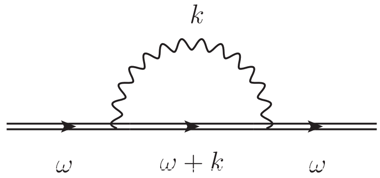
$$g \to \mu_{uv}^{2-D/2} g = \mu_{uv}^{\varepsilon} g \tag{65}$$

to introduce explicitly the corresponding mass scale. In order to renormalize the HQET Lagrangian, we have to express the renormalized quantities via bare fields and couplings. The only new renormalization constant appearing in HQET with regard to QCD is the static quark field renormalization $\widetilde{Q}_b = \widetilde{Q}\widetilde{Z}_Q^{1/2}$ [10]. It can be determined by requirement of finiteness of the renormalized propagator $\widetilde{S}(\omega) = \widetilde{S}_b(\omega)/\widetilde{Z}_Q$ in the limit $\varepsilon \to 0$. We will calculate $\widetilde{Z}_Q$ in the one-loop approximation considering the heavy quark self-energy [12].

3.1.1 Heavy quark self-energy at one-loop

If we denote $-i\Sigma_b(\omega)$ the sum of bare one-particle-irreducible quark diagrams, then the propagator $\widetilde{S}_b(\omega) = \widetilde{S}_0(\omega) + \widetilde{S}_0(\omega)\Sigma_b(\omega)\widetilde{S}_0(\omega) + \dots = (\omega v - \Sigma_b(\omega))^{-1}$. Using the Feynman rules (4) and (5), we obtain[3]

[3] For simplicity, we use the Feynman gauge here. As we will see later, the final result for the vertex renormalization is gauge-invariant.



$$\Sigma_b(\omega) = i\int \frac{d^Dk}{(2\pi)^D}\frac{-ig_{\mu\nu}}{k^2+i\varepsilon}\left[(ig\mu_{uv}^{\varepsilon})t^a v^\mu \frac{i}{(\omega+k)v+i\varepsilon}(ig\mu_{uv}^{\varepsilon})t^a v^\nu\right] =$$

$$= -ig^2\mu_{uv}^{2\varepsilon}C_F\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2(\omega+k)v}, \tag{66}$$

where we have dropped $i\varepsilon$ and $C_F = t^a t^a$ (see Appendix C). In what follows we will use two variants of Feynman parametrization [9, 13]

$$\frac{1}{a_1^{n_1}a_2^{n_2}} = \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)}\int_0^1 \frac{x^{n_1-1}(1-x)^{n_2-1}dx}{[a_1x+a_2(1-x)]^{n_1+n_2}}, \tag{67}$$

$$\frac{1}{a_1^{n_1}a_2^{n_2}} = \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)}\int_0^{+\infty} \frac{x^{n_1-1}dx}{[a_1x+a_2]^{n_1+n_2}}. \tag{68}$$

We will also make use of the standard formula [10]

$$\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2-a^2)^n} = \frac{i(-1)^n\Gamma(n-\frac{D}{2})(a^2)^{D/2-n}}{(n-1)!(4\pi)^{D/2}}. \tag{69}$$

Using first the expression (68) and then (69), we get

$$\Sigma_b(\omega) = -ig^2\mu_{uv}^{2\varepsilon}C_F\int_0^{+\infty}dy\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2+y(\omega+k)v)^2} =$$

$$= -ig^2\mu_{uv}^{2\varepsilon}C_F\int_0^{+\infty}dy\left[\frac{i\Gamma(2-\frac{D}{2})\left(-y\omega v+\left(\frac{yv}{2}\right)^2\right)^{D/2-n}}{(4\pi)^{D/2}}\right] =$$

$$= -ig^2\mu_{uv}^{2\varepsilon}C_F\Gamma(\varepsilon)\int_0^{+\infty}\left[\left(\frac{yv}{2}\right)^2 - y\omega v\right]^{D/2-n}\frac{dy}{(4\pi)^{D/2}}. \tag{70}$$

Using the beta-function variant

$$\int_0^{+\infty} y^\alpha(ay+b)^\beta dy = \frac{b^{\alpha+\beta+1}}{a^{\alpha+1}}\frac{\Gamma(1-\alpha-\beta)\Gamma(1+\alpha)}{\Gamma(-\beta)} \tag{71}$$

we finally get

$$\Sigma_b(\omega) = -\frac{g^2C_F\mu_{uv}^{2\varepsilon}}{(4\pi)^{D/2}}4(\omega v)^{D-3}\Gamma(2\varepsilon-1)\Gamma(1-\varepsilon). \tag{72}$$

As far

$$\Gamma(z) = \frac{1}{z} - \gamma_E + O(z) \tag{73}$$

and

$$\Gamma(z-1) = -\frac{1}{z} + \gamma_E - 1 + O(z), \tag{74}$$

where γ_E is Euler constant, we find by expanding this expression in ε and retaining only $1/\varepsilon$ pole

$$\Sigma_b(\omega)\mid_{pole}= \left(\frac{\mu_{uv}}{\lambda}\right)^{2\varepsilon} C_F\frac{\alpha_s}{2\pi\varepsilon}(\omega v), \tag{75}$$

where $\lambda = \omega v$ and plays the role of an IR regulator. The propagator at one-loop level

$$\widetilde{S}_b(\omega) \simeq \frac{1}{(\omega v)}\left[1+\frac{\Sigma_b(\omega)\mid_{pole}}{\omega v}\right] = \widetilde{S}_0(\omega)\left[1+C_F\frac{\alpha_s}{2\pi\varepsilon}\left(\frac{\mu_{uv}}{\lambda}\right)^{2\varepsilon}\right] \tag{76}$$

and we find for the minimal $\widetilde{Z}_Q$

$$\widetilde{Z}_Q = 1 + C_F\frac{\alpha_s}{2\pi\varepsilon}. \tag{77}$$

In the next paragraph we will find the normalization constant $\widetilde{Z}_\triangle$ for the velocity changing vertex without external momenta. The overall correction for the one-loop approximation will be given by $\widetilde{Z}_V = \widetilde{Z}_Q\widetilde{Z}_\triangle$ [12].

3.1.2 Heavy-heavy vertex at one-loop

In the one-loop approximation, the vertex is

$$\Gamma + \int \frac{d^Dk}{(2\pi)^D}\frac{-ig_{\mu\nu}}{k^2+i\varepsilon}\left[(ig\mu_{uv}^{\varepsilon})t^a v'^{\mu}\frac{i}{kv'+i\varepsilon}\Gamma\frac{i}{kv+i\varepsilon}(ig\mu_{uv}^{\varepsilon})t^a v^{\nu}\right] =$$

$$= \Gamma\left(1 - iC_F g^2\cosh\theta\mu_{uv}^{2\varepsilon}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2(kv)(kv')}\right). \tag{78}$$

We note that this integral possesses not only ultraviolet divergences, but also infrared divergences. Moreover, this integral does not exist because the UV convergence requires $D < 4$ and it converges on the IR limit only if $D > 4$. The appearance of the IR divergences is a penalty for the simplicity of the chosen contour approximation corresponding to one-dimensional fermions. To define this integral in the IR region, we have to use another regularization scheme different from the dimensional one [15]. For example, we can ascribe a fictitious momentum λ (corresponding to quark's external momenta) in quark propagator (4). This gives

$$1 - iC_F g^2\cosh\theta\mu_{uv}^{2\varepsilon}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2(kv+\lambda)(kv'+\lambda)}. \tag{79}$$

We use the Feynman trick (67), (68) to get

$$1 - 2iC_F g^2\cosh\theta\mu_{uv}^{2\varepsilon}\int_0^1 dx\int_0^{+\infty}dy\int\frac{d^Dk}{(2\pi)^D}\frac{y}{\left[k^2+y(xv+(1-x)v')k+y\lambda\right]^3},$$

or, using (69),

$$1 - \frac{C_F g^2}{(4\pi)^{D/2}}\Gamma(1+\varepsilon)\cosh\theta\mu_{uv}^{2\varepsilon}\int_0^1 dx\int_0^{+\infty}dy y^{-\varepsilon}\cdot$$

$$\cdot\left[\frac{1}{4}\left(x^2+(1-x)^2+2x(1-x)\cosh\theta\right)y-\lambda\right]^{-1-\varepsilon} =$$

$$= 1 - \frac{C_F g^2}{(4\pi)^{D/2}} 4\Gamma(2\varepsilon)\Gamma(1+\varepsilon)\cosh\theta \mu_{uv}^{2\varepsilon} \int_0^1 \frac{(-2\lambda)^{-2\varepsilon} dx}{[x^2 + (1-x)^2 + 2x(1-x)\cosh\theta]^{1-\varepsilon}}. \quad (80)$$

Retaining only $1/\varepsilon$ pole and using the substitution $x = \frac{1}{2}(1 + z\coth\frac{\theta}{2})$, we obtain

$$1 - C_F \frac{\alpha_s}{2\pi\varepsilon} \left(\frac{\mu_{uv}}{\lambda}\right)^{2\varepsilon} \coth\theta \int_{-\tanh\frac{\theta}{2}}^{\tanh\frac{\theta}{2}} \frac{dz}{1-z^2} = 1 - C_F \frac{\alpha_s}{2\pi\varepsilon} \theta \coth\theta \left(\frac{\mu_{uv}}{\lambda}\right)^{2\varepsilon}. \quad (81)$$

Finally, in the one-loop approximation,

$$\widetilde{Z}_\triangle = 1 - C_F \frac{\alpha_s}{2\pi\varepsilon} \theta \coth\theta. \quad (82)$$

Using (77), we find

$$\widetilde{Z}_V = \widetilde{Z}_Q \widetilde{Z}_\triangle = 1 - C_F \frac{\alpha_s}{2\pi\varepsilon} (\theta\coth\theta - 1). \quad (83)$$

It is equal to 1 at $\theta = 0$ which corresponds to lack of cusp on the heavy quark world line.

3.1.3 Infrared dependence of Wilson lines in QED and QCD

Using (83) and the $\overline{MS}$ renormalization scheme, we subtract the pole and get the renormalized one-loop vertex correction

$$C_F \frac{\alpha_s}{\pi} (\theta\coth\theta - 1) \log\left(\frac{\mu_{uv}}{\lambda}\right). \quad (84)$$

Now we can use the equation (62) to get the expression of the Wilson line vacuum average $\langle W_C \rangle$. Differentiating it in $\log\mu_{uv}$, we obtain

$$\frac{\partial \log\langle W_C\rangle}{\partial \log\mu_{uv}} = -\frac{\alpha}{\pi}(\theta\coth\theta - 1) = -\Gamma_{cusp}^{QED}(\alpha, \theta). \quad (85)$$

Thus, in QED, the infrared behavior of $\xi(v'v, \frac{\mu_{uv}}{\lambda}) = \langle light, v'|\, W_C \,|light, v\rangle$ is governed by the exact equation

$$\lambda \frac{\partial \xi_{v'v}^{QED}}{\partial \lambda} = \frac{\alpha}{\pi}(\theta\coth\theta - 1)\xi_{v'v}^{QED}, \quad (86)$$

where $\cosh\theta = v\cdot v'$ and we have used the property (61). Providing the solution of this equation

$$\xi_{v'v}^{QED} = \exp\left(-\frac{\alpha}{\pi}(\theta\coth\theta - 1)\log\left(\frac{\mu_{uv}}{\lambda}\right)\right), \quad (87)$$

we conclude that the IR-singularities in QED are exponentialized and (62) is verified for the light-averaging. In QCD, the knowledge of (83) and (63) allows us to write a similar equation up to the one-loop contribution

$$\lambda \frac{\partial \log \xi_{v'v}^{QCD}}{\partial \lambda} = C_F \frac{\alpha_s(\lambda)}{\pi}(\theta\coth\theta - 1) + O(\alpha_s^2(\lambda)) = \Gamma_{cusp}(\alpha_s(\lambda), \theta) + O(\alpha_s^2(\lambda)). \quad (88)$$

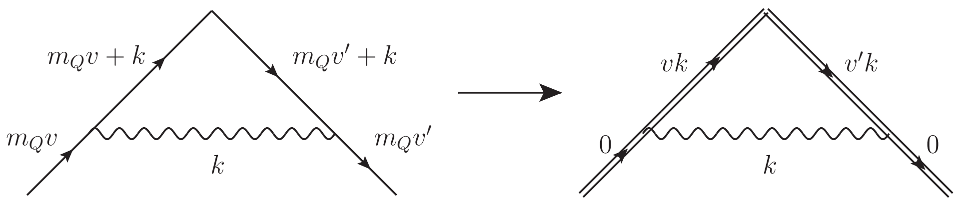
Again, we pay attention that this equation is verified only in the perturbation theory. Thus, it is not verified for the small λ because in this case $\alpha_s(\lambda)$ "explode".

3.2 Matching

Until now we discussed renormalization inside HQET. If we are interested in real processes, we have to discuss the relation of QCD operators to their HQET analogues. We should begin at high energies, where we can make calculations in the full QCD theory and then descend to our low-energy effective theory. We start by matching the two theories on either side of the scale $\mu = m_Q$. There are two approximation involved at this level - $1/m_Q$ and $\alpha_s(m_Q)$ [6]. In leading order, the matching is trivial:

$$|B\rangle \to e^{-im_Q v_\mu x^\mu} |B, v\rangle \,. \tag{89}$$

For $\mu > m_Q$ the studied current doesn't depend on μ; this is not the case if $\mu < m_Q$, and we have calculated earlier the non-zero cusp anomalous dimension for this current. Now our ambition is to perform the matching operation at one-loop level and to verify the expression (58). Therefore, we have to compare two diagrams, the first being calculated in QCD and the second one in HQET:



The QCD matrix element $b\gamma^\mu \bar{b}$ should be equal to the HQET expression

$$b_{v'} \sum_{i=1}^{3} C_i(\frac{m_Q}{\mu}, \alpha_s(\mu), \cosh\theta) \Gamma_i^\mu \bar{b}_v, \tag{90}$$

where $\Gamma_i^\mu = \gamma^\mu$, v^μ, v'^μ, since these three vectors are present in the HQET. At the tree level $C_1 = 1$ and $C_2 = C_3 = 0$. At one-loop level, the matching coefficients are given by (see appendix B for detailed calculation)

$$C_1 = 1 - 2C_F \frac{\alpha_s}{4\pi} \left\{ [wr(w) - 1] \log\left(\frac{m_Q^2}{\mu_{uv}^2}\right) - F(w) \right\},$$

$$C_2 = C_3 = -C_F \frac{\alpha_s}{4\pi} r(w), \tag{91}$$

where $w = v \cdot v'$, $r(w)$ and $F(w)$ are given by (186) and (187). We are interested only in the term containing $\log\left(\frac{m_Q}{\mu_{uv}}\right)$. In order to verify the hypotheses made in the section 2.4, we show that [16]

$$\frac{d \log C\left(\frac{m_Q}{\mu_{uv}}\right)}{d \log \mu_{uv}} = C_F \frac{\alpha_s}{\pi} \left(\frac{w}{\sqrt{w^2 - 1}} \log\left(w + \sqrt{w^2 - 1}\right) - 1 \right) \tag{92}$$

which is equal to the defined above $\Gamma_{cusp}(\alpha_s, \theta)$ for $w = \cosh\theta = v \cdot v'$. Thus, the relation (58) is verified, as it should be.

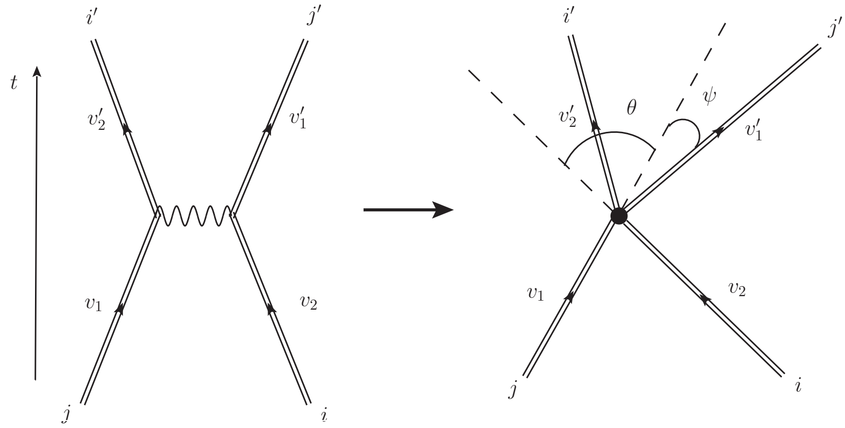
4 Meson-meson scattering in the heavy quark limit

In this part we study the process involving mesons with a single heavy quark, and in particular the elastic scattering $B+D \to B+D$ or $B+B \to B+B$. The concept is somewhat similar to what we have seen in the previous part. In the heavy quark limit, the $\bar{b}$ and c quarks move along their straight world lines, all the interactions with light constituents being factorized to the Wilson lines (55). For the sake of simplicity, we study first the process involving quarks with different flavors, that is to say, $Q_{f_1}(v_1)Q_{f_2}(v_2) \to Q_{f_1}(v_1')Q_{f_2}(v_2')$. The generalization to the identical quarks is straightforward and is achieved by including the Bose symmetric process $Q_{f_1}(v_1)Q_{f_2}(v_2) \to Q_{f_1}(v_2')Q_{f_2}(v_1')$.

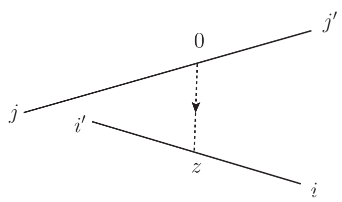
The asymptotics of the process are described by the heavy quark-quark scattering that we study in detail. The light constituents are included later to form the Isgur-Wise form factors. The momentum transferred from one meson to another one is

$$q^2 = (m_Q v_1 - m_Q v_1')^2 = 2m_Q^2(1-\cosh\psi). \tag{93}$$

To the lowest order in coupling constant the scattering goes through exchange of one gluon with the momentum equal to q. In the limit $m_Q \to \infty$, the gluon's virtuality corresponds to an infinitesimal space-time distance. We will reflect this fact by denoting the virtual gluon by the interaction point:



The considered problem can be formulated in terms of quarks separated by the transverse impact vector z.



The scattering amplitude is given by the following expression [21]:

$$T = \langle light, i, j| \, T_{ij}^{i'j'} \, |light, i', j'\rangle =$$

$$= \langle light, i, j| \sinh\theta \int d^2 z e^{-iz\cdot q} \left(W(\theta,\psi,z^2\lambda^2)\right)_{ij}^{i'j'} |light, i', j'\rangle \,, \tag{94}$$

where

$$\left(W(\theta,\psi,z^2\lambda^2)\right)_{ij}^{i'j'} = T\left(W_+^{v_1}(0)W_-^{v_1'}(0)\right)^{ii'}\left(W_+^{v_2}(z)W_-^{v_2'}(z)\right)^{jj'}, \tag{95}$$

and $\{i, i', j', j'\}$ are the color indices. Here, the Wilson lines are evaluated along infinite path along the quark velocities:

$$W_+^{v_1}(0) = P\exp(ig\int_{-\infty}^{0} d\alpha v_1 \cdot A(v_1\alpha)), \quad W_-^{v_1'}(0) = P\exp(ig\int_{0}^{+\infty} d\alpha v_1' \cdot A(v_1'\alpha)), \tag{96}$$

and

$$W_+^{v_2}(z) = P\exp(ig\int_{-\infty}^{0} d\beta v_2 \cdot A(v_2\beta + z)), \quad W_-^{v_2'}(z) = P\exp(ig\int_{0}^{+\infty} d\beta v_2' \cdot A(v_2'\beta + z)). \tag{97}$$

The expression (94) of the amplitude $\langle light, i, j|\, T_{ij}^{i'j'}\, |light, i', j'\rangle$ reproduces the results for the eikonal approximation in the abelian case [22, 23], while the total momentum of gluons propagating in the $t-$channel is restricted to be equal to q because of the integration over z [21]. The Wilson line implies the decomposition into two invariant tensors:

$$W_{ij}^{i'j'} = C_1\delta_{ii'}\delta_{jj'} + C_2\delta_{ij'}\delta_{ji'}, \tag{98}$$

where C_1 and C_2 are some constants. Such defined Wilson line $W_{ij}^{i'j'}$ has ultraviolet cross divergences for $z = 0$ coming from gluons propagating at short distances. But it is easy to notice that $W_{ij}^{i'j'}$ has no UV divergences for nonzero z because z^2 regularizes gluon propagator, having a meaning of an UV cutoff. Thus, the $z-$dependent Wilson line can be represented as the same Wilson line with $z = 0$ but with an UV cutoff which we have to introduce to regularize the cross singularities appearing when the Wilson lines (96) and (97) cross each other. The reason for this property to be valid to all orders of perturbation theory is the following. As far as the line function $W_{ij}^{i'j'}$ depends only on two dimesionfull arguments (z and λ), while the quarks' velocities are dimensionless, and λ plays the role of IR regulator, the transverse shift z becomes the only scale in the original problem that can regularize the cross singularities which the line function $W_{ij}^{i'j'}$ will have for $z = 0$. Considering z as one of possible ways of regularize the cross singularities, we can reformulate the problem in terms of another regularization scale μ_{uv} and impose $z = 0$. This observation suggests the one-to-one correspondence $z = 1/\mu_{uv}$ that we will use later to calculate and express the Wilson lines through the Isgur-Wise form factors.

4.1 B-D cross anomalous dimension, abelian case

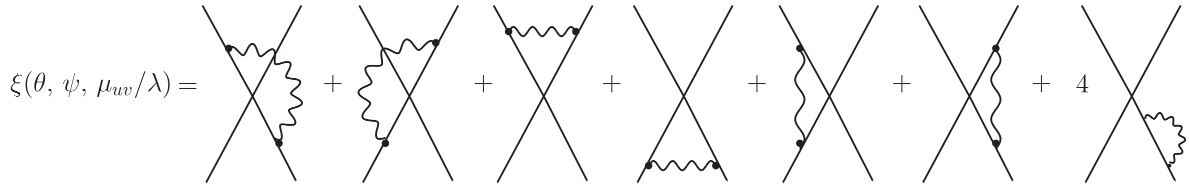
In this section, we consider the QED scattering process which is more simple and will be useful in the next section where we will study the QCD case. The corresponding diagram has 4 external quark propagators, each of them gets a self-energy correction (77) $\sqrt{Z_Q}$, the overall contribution being equal to $Z_Q^2 = 1 + 2\frac{\alpha}{2\pi\varepsilon}$ at one-loop order. We parametrize the heavy quarks' dynamics with the minkowskian angles (θ, α, ψ) defined as follows:

$$\cosh\theta = v_1 \cdot v_2, \quad \cosh\psi = v_1 \cdot v_1'. \tag{99}$$

After an elementary calculation, we get

$$v_1' \cdot v_2' = \cosh\theta, \quad v_2 \cdot v_2' = \cosh\psi,$$
$$v_1 \cdot v_2' = v_2 \cdot v_1' \equiv \cosh\alpha = \cosh\theta - \cosh\psi + 1. \tag{100}$$

The overall QED contribution at one loop



The contribution of the first two Feynman diagrams in these expressions for $\xi(\theta, \alpha, \psi, \mu_{uv}/\lambda)$ is given by (81) with the corresponding angle ψ. The next two diagrams look very similar, but the difference is that on these diagrams one of the quarks has an opposite velocity in regard to (78) and thus $\cosh\theta = v_1 \cdot v_2$ changes it's sign. This affects the result because of the appearance of an additional pole in (80) corresponding to $x^2 + (1-x)^2 - 2x(1-x)\cosh\theta = 0$. In order to to put the axe on the helve, we have to choose a different substitution $x = \frac{1}{2}(1 + z\tanh\frac{\theta}{2})$ and calculate the integral

$$\int_{-\coth\frac{\theta}{2}}^{\coth\frac{\theta}{2}} \frac{dz}{z^2-1} = \int_{-\infty}^{+\infty} \frac{dz}{z^2-1} - \int_{-\infty}^{-\coth\frac{\theta}{2}} \frac{dz}{z^2-1} - \int_{\coth\frac{\theta}{2}}^{\infty} \frac{dz}{z^2-1} = \theta - i\pi.$$

We notice that while (81) is real, this amplitude acquires an imaginary part. This is something that we could predict in advance: the full invariant energy in this case

$$s = (m_Q v_1 + m_Q v_2)^2 = 2m_Q^2(1 + \cosh\theta) > 0 \tag{101}$$

and because of the cut on the complex plane, the amplitude has an imaginary part when $s > 4m_Q^2$ [9]. Taking into account this fact we get

$$\xi(\theta, \alpha, \psi, \mu_{uv}/\lambda) = 1 - 2\frac{\alpha}{2\pi\varepsilon}\left(\frac{\mu_{uv}}{\lambda}\right)^{2\varepsilon}\left[(i\pi - \theta)\coth\theta + \psi\coth\psi + \alpha\coth\alpha - 1\right], \tag{102}$$

and thus, by analogy with (61)

$$\mu_{uv}\frac{d\log\xi(\theta, \alpha, \psi, \mu_{uv}/\lambda)}{d\mu_{uv}} =$$

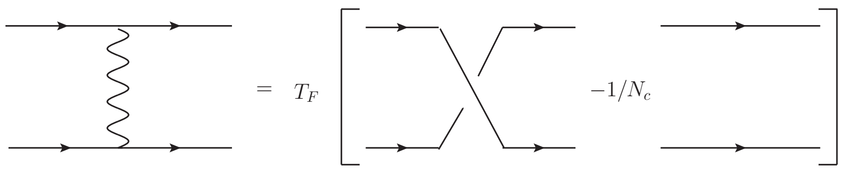
$$= -2\frac{\alpha}{\pi}\left[(i\pi - \theta)\coth\theta + \psi\coth\psi + \alpha\coth\alpha - 1\right] \equiv -2\frac{\alpha}{\pi}\gamma(\theta, \alpha, \psi), \tag{103}$$

where we define

$$\gamma(\theta, \alpha, \psi) = (i\pi - \theta)\coth\theta + \psi\coth\psi + \alpha\coth\alpha - 1. \tag{104}$$

4.2 B-D cross anomalous dimension, nonabelian case

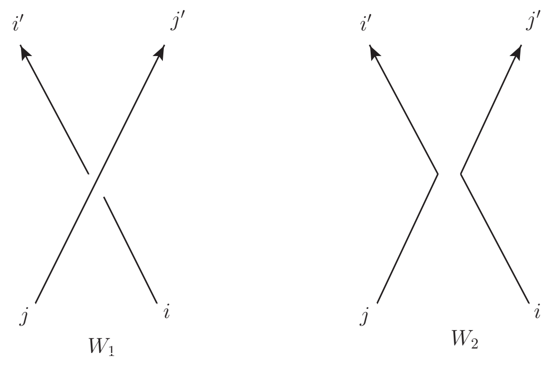
In QCD, the situation is more complicated because of the possible color exchanges:



We proceed by analogy with (56), but now we have to consider two Wilson loops:

$$C_1(\frac{m_Q}{\mu_{uv}},\theta,\alpha,\psi)\cdot W_1(\frac{\mu_{uv}}{\lambda},\theta,\alpha,\psi)+C_2(\frac{m_Q}{\mu_{uv}},\theta,\alpha,\psi)\cdot W_2(\frac{\mu_{uv}}{\lambda},\theta,\alpha,\psi), \tag{105}$$

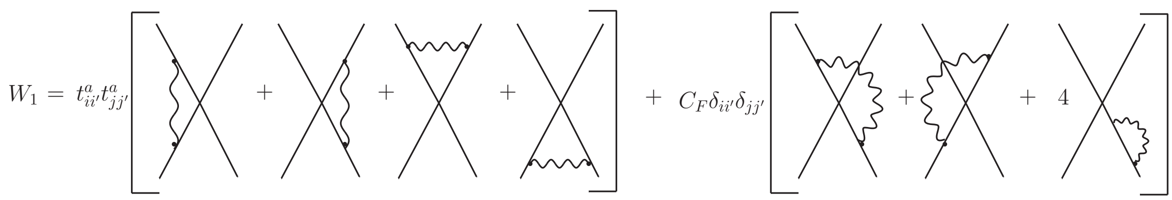
where C_1 and C_2 are two matching coefficients. The integral paths for W_1 and W_2 are defined as follows:

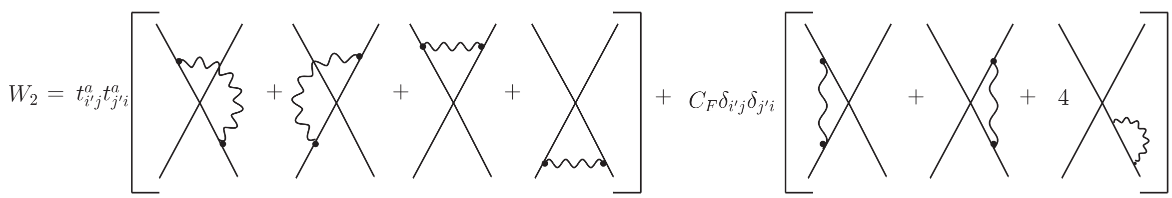


The arrows indicate the color flow. Notice that in the Born approximation,

$$W_1^{(0)}=\delta_{ii'}\delta_{jj'},\quad W_2^{(0)}=\delta_{ij'}\delta_{ji'} \tag{106}$$

in the color space. At one-loop level, the Feynman QED diagrams for W_1 and W_2 are the same, but in QCD their contribution is different because of the the different color weights. Indeed, we obtain for W_1 and W_2:





According to (∗) formula (Appendix C) we have in the fundamental representation

$$t^a_{ij}t^a_{kl}=T_F\delta_{il}\delta_{jk}-\frac{T_F}{N_c}\delta_{ij}\delta_{kl}, \tag{107}$$

where $T_F=\frac{1}{2}$. Thus, taking the results of the section 4.1 and using again the $\overline{MS}$ renormalization scheme, we get the renormalized expressions

$$W_1(\frac{\mu_{uv}}{\lambda},\theta,\alpha,\psi)=-\frac{\alpha_s}{\pi}\log\left(\frac{\mu_{uv}}{\lambda}\right)^2\times$$

$$\left[\left(T_F\delta_{ij'}\delta_{ji'}-\frac{T_F}{N_c}\delta_{ii'}\delta_{jj'}\right)\{(i\pi-\theta)\coth\theta+\alpha\coth\alpha\}+T_F(N_c-\frac{1}{N_c})\delta_{ii'}\delta_{jj'}\{\psi\coth\psi-1\}\right], \tag{108}$$

$$W_2(\frac{\mu_{uv}}{\lambda}, \theta, \alpha, \psi) = -\frac{\alpha_s}{\pi} \log\left(\frac{\mu_{uv}}{\lambda}\right)^2 \times$$

$$\left[\left(T_F \delta_{ii'}\delta_{jj'} - \frac{T_F}{N_c}\delta_{ij'}\delta_{ji'}\right)\{(i\pi - \theta)\coth\theta + \psi\coth\psi\} + T_F(N_c - \frac{1}{N_c})\delta_{ij'}\delta_{ji'}\{\alpha\coth\alpha - 1\}\right]. \tag{109}$$

We notice that one-loop correction (108) to W_1 contains a color structure of W_2 in the Born approximation, equations (106), and vice versa. Therefore, both Wilson lines are mixed under renormalization and the cross anomalous dimension is a matrix:

$$\mu_{uv}\frac{d}{d\mu_{uv}}\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = -\Gamma_{cross}(\theta, \psi, \alpha_s)\begin{pmatrix} W_1 \\ W_2 \end{pmatrix}, \tag{110}$$

where

$$\Gamma_{cross}(\theta, \alpha, \psi, \alpha_s) = \frac{\alpha_s}{\pi}\begin{pmatrix} N_c(\psi\coth\psi - 1) - \frac{1}{N_c}\gamma(\theta, \alpha, \psi) & \alpha\coth\alpha + (i\pi - \theta)\coth\theta \\ \psi\coth\psi + (i\pi - \theta)\coth\theta & N_c(\alpha\coth\alpha - 1) - \frac{1}{N_c}\gamma(\theta, \alpha, \psi) \end{pmatrix} \tag{111}$$

and the definition of $\gamma(\theta, \alpha, \psi)$ is given in (104). In the important special case of the forward scattering ($\psi = 0$, $\alpha = \theta$) we get

$$\Gamma^{forward}_{cross}(\theta, \alpha_s) = \frac{\alpha_s}{\pi}\begin{pmatrix} -\frac{i\pi}{N_c}\coth\theta & i\pi\coth\theta \\ 1 + (i\pi - \theta)\coth\theta & N_c(\theta\coth\theta - 1) - \frac{i\pi}{N_c}\coth\theta \end{pmatrix}, \tag{112}$$

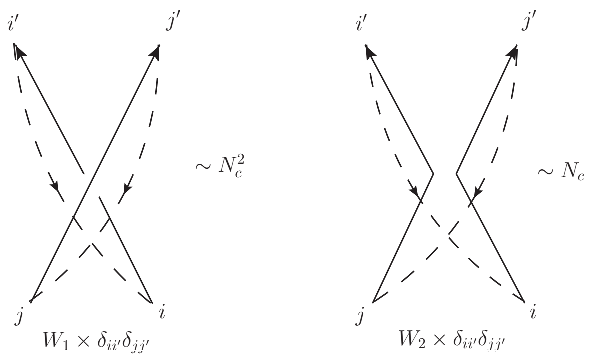
which coincides with the result obtained earlier for the high-energy quarks scattering in [17].

4.2.1 Large N_c limit

We could also study the large N_c limit of the $\Gamma_{cross}(\theta, \psi, \alpha_s)$. We notice that in the limit $N_c \to \infty$ ($\alpha_s N_c$ remain finite) the solution (111) is proportional to the diagonal matrix with corresponding cusp anomalous dimensions $\Gamma_{cusp}(\alpha_s, \psi)$ and $\Gamma_{cusp}(\alpha_s, \alpha)$ as two diagonal elements. This property seems to be general to all orders of perturbation theory [17]. We can give some arguments in favor by contracting the Wilson lines W_1 and W_2 with the color factor $\delta_{ii'}\delta_{jj'}$: we get two closed Wilson loops with the following normalization

$$W_1\delta_{ii'}\delta_{jj'} = N_c^2 w_1, \quad W_2\delta_{ii'}\delta_{jj'} = N_c w_2, \tag{113}$$

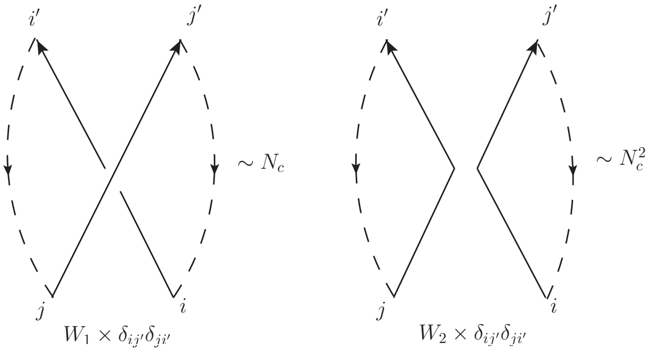
where $w_i = O(N_c^0)$ for $i = 1, 2$.



The projection of the Wilson lines W_1 and W_2 onto the color structure $\delta_{ji'}\delta_{ij'}$ gives

$$W_1\delta_{ji'}\delta_{ij'} = N_c w_1', \quad W_2\delta_{ji'}\delta_{ij'} = N_c^2 w_2', \tag{114}$$

where, again, w_1' and w_2' behave as $O(N_c^0)$.



Such defined w_1 and w_2' are factorized in the large N_c limit into the product of two vacuum averaged Wilson loops evaluated along the paths having cusps with the angles ψ and α correspondingly [18, 20]:

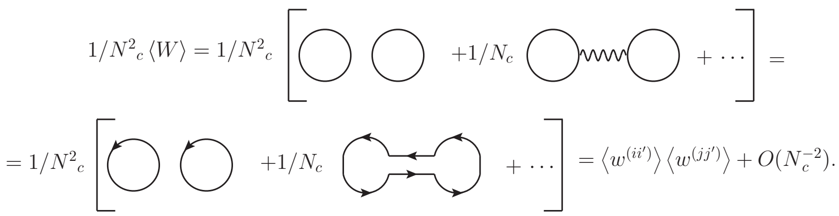
$$\langle w_1 \rangle = \left\langle w^{(ii')} \right\rangle \left\langle w^{(jj')} \right\rangle + O(1/N_c^2),$$

$$\langle w_2' \rangle = \left\langle w^{(ji')} \right\rangle \left\langle w^{(ij')} \right\rangle + O(1/N_c^2), \tag{115}$$

where $w^{(xy)}$ is the Wison loop evaluated along the path xy for $x = i, j$ and $y = i', j'$. This factorization property was first observed by Migdal [19] and can be written as

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle \times \left(1 + O(1/N_c^2)\right) \tag{116}$$

for two arbitrary gauge invariant operators $\mathcal{O}_1$ and $\mathcal{O}_2$. In our case, this feature can be seen directly by applying the "double line" representation [20]:



After the substitution of (115) into equation (110), we obtain

$$\begin{array}{ll} \Gamma_{cross}^{11} = \Gamma_{cusp}(\alpha_s, \psi) + O(N_c^{-2}), & \Gamma_{cross}^{12} = O(N_c^{-1}), \\ \Gamma_{cross}^{21} = O(N_c^{-1}), & \Gamma_{cross}^{22} = \Gamma_{cusp}(\alpha_s, \alpha) + O(N_c^{-2}) \end{array} \tag{117}$$

to all orders of perturbation theory.

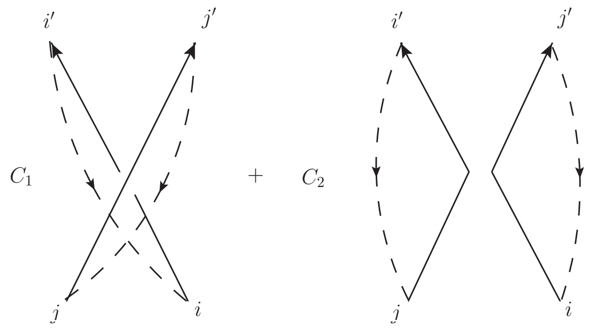
4.2.2 Including light constituents

As far as in the original problem we consider meson-meson scattering, we can ask ourselves what happens to light components in the color space. We remind that in the heavy quark approximation we consider that the velocity-changing interaction occurs only between heavy quarks. In the beginning we deal with two colorless objects (B and D mesons). After an elastic scattering, we must get two colorless objects again. As we saw, the heavy quarks' behavior is governed by the cross anomalous dimension matrix (111) and we had to take in consideration two mixing Wilson lines W_1 and W_2 in order to get a correct description of the heavy quarks' scattering. Now, exactly as in (60), we need to consider the Wilson lines' averages over initial and final states

$$\langle light, i, j| \left(W(\theta,\alpha,\psi,z^2\lambda^2)\right)_{ij}^{i'j'} |light, i', j'\rangle =$$

$$\langle light, v_1', i'|\otimes\langle light, v_2', j'| \left[\sum_{k=1,2} C_k(\frac{m_Q}{\mu_{uv}},\theta,\alpha,\psi)\cdot W_k(\frac{\mu_{uv}}{\lambda},\theta,\alpha,\psi)\right]_{ij}^{i'j'} |light, v_2, j\rangle\otimes|light, v_1, i\rangle\,, \tag{118}$$

where i, j, i', j' are the color quantum numbers. Here, $C_k(\frac{m_Q}{\mu_{uv}},\theta,\alpha,\psi)$ for $k=1$, 2, are the QCD-HQET matching coefficients introduced by analogy with equations (56) and (58). Again, the result of (118) can't depend on the unphysical regularization scale μ_{uv} and should depend only on the physical ratio m_Q/λ, where λ is of order of the B meson's bound energy, $\bar{\Lambda}$. The light components "follow" the heavy quarks to form the colorless mesons in the final state. For instance, we can choose the color structure $\delta_{ii'}\delta_{jj'}$ for the heavy quarks and the light components will also follow this structure. Taking into consideration the Wilson lines' mixing and the projections (113) and (114), we can also observe a transition corresponding to the color factor $\delta_{ji'}\delta_{ij'}$, though this transition will be suppressed by factor $1/N_c$ (117). Using (115), we can express the matrix element (118) in terms of Isgur-Wise form factors:



$$= C_1(\frac{m_Q}{\mu_{uv}},\theta,\alpha,\psi)\xi^2(\cosh\psi,\frac{\mu_{uv}}{\lambda}) + C_2(\frac{m_Q}{\mu_{uv}},\theta,\alpha,\psi)\xi^2(\cosh\alpha,\frac{\mu_{uv}}{\lambda}) + O(1/N_c^2), \tag{119}$$

at order $O(1/N_c^2)$.

4.3 Scattering amplitude and identical particles

4.3.1 Calculation of the meson-meson scattering amplitude

The solution of the RG equation (110) with boundary conditions $W_1(\theta,\alpha,\psi,1) = W_1^{(0)} \equiv \delta_{ii'}\delta_{jj'}$ and $W_2(\theta,\alpha,\psi,1) = W_2^{(0)} \equiv \delta_{ij'}\delta_{ji'}$ is given by

$$\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = T\exp\left(-\int_\lambda^{\mu_{uv}} \frac{d\tau}{\tau}\Gamma_{cross}(\theta,\alpha,\psi,\alpha_s(\tau))\right)\begin{pmatrix} W_1^{(0)} \\ W_2^{(0)} \end{pmatrix}. \tag{120}$$

These boundary conditions mean that when $\lambda = \mu_{uv}$, there is no phase space for gluons and Wilson lines are given by the Born terms. Substituting $\mu_{uv} = 1/z$, we find the following expression for the scattering amplitude [21]

$$T_{ij}^{i'j'} = \sinh\theta \int d^2z e^{-iz\cdot q}\left[C_1\left(A_{11}(\theta,\alpha,\psi,z^2\lambda^2)\delta_{ii'}\delta_{jj'} + A_{12}(\theta,\alpha,\psi,z^2\lambda^2)\delta_{ij'}\delta_{ji'}\right) + \right.$$

$$+C_2\left(A_{21}(\theta,\alpha,\psi,z^2\lambda^2)\delta_{ii'}\delta_{jj'}+A_{22}(\theta,\alpha,\psi,z^2\lambda^2)\delta_{ij'}\delta_{ji'}\right)\Big], \quad (121)$$

where A_{kl} are elements of the 2×2 matrix

$$A(\theta,\alpha,\psi,z^2\lambda^2)=T\exp[-\int_{\lambda}^{1/z}\frac{d\tau}{\tau}\Gamma_{cross}(\theta,\alpha,\psi,\alpha_s(\tau))].$$

One-loop matrices $\Gamma_{cross}(\theta,\alpha,\psi,\alpha_s(\tau))$ commute with each other for different τ and thus we can omit $T-$ordering in the definition of the matrix A. In the leading log-approximation, we may consider that α_s doesn't depend on τ. In order to diagonalize the one-loop matrix $\frac{\pi}{\alpha_s}\Gamma_{cross}(\theta,\alpha,\psi,\alpha_s)$, we need to calculate it's eigenvalues and eigenvectors. They are given by the expressions (194) and (195) in Appendix D. Since the general expressions are very complex, we study its limit when $N_c\to+\infty$. In this case, $\Gamma_{cross}(\theta,\alpha,\psi,\alpha_s(\tau))$ and $A(\theta,\alpha,\psi,z^2\lambda^2)$ take diagonal form and we can write the answer in the form:

$$T_{ij}^{i'j'}\simeq\sinh\theta\int d^2ze^{-iz\cdot q}\left[C_1\cdot A_{11}(\theta,\alpha,\psi,z^2\lambda^2)\delta_{ii'}\delta_{jj'}+C_2\cdot A_{22}(\theta,\alpha,\psi,z^2\lambda^2)\delta_{ij'}\delta_{ji'}\right], \quad (122)$$

where

$$A_{11}(\theta,\alpha,\psi,z^2\lambda^2)=\exp\left(\frac{\alpha_s}{\pi}N_c(\psi\coth\psi-1)\log(z\lambda)\right), \quad (123)$$

$$A_{22}(\theta,\alpha,\psi,z^2\lambda^2)=\exp\left(\frac{\alpha_s}{\pi}N_c(\alpha\coth\alpha-1)\log(z\lambda)\right). \quad (124)$$

Finally, using (119), we get

$$T(BD\to BD)=\sinh\theta\int d^2ze^{-iz\cdot q}\left[C_1(m_Qz,\theta,\alpha,\psi)\xi^2(\cosh\psi,\frac{1}{z\lambda})+\right.$$
$$\left.+C_2(m_Qz,\theta,\alpha,\psi)\xi^2(\cosh\alpha,\frac{1}{z\lambda})\right]+O(1/N_c^2) \quad (125)$$

with $z=1/\mu_{uv}$.We notice that the final expression for the amplitude depends only on the physical ratio m_Q/λ, as it should be.

4.3.2 Identical particles

In the case of the B-B scattering we must take into consideration the quarks' identity. Therefore, we should include another diagram in order to satisfy the requirement of the amplitude exchange symmetry of quantum numbers. For our problem, it means that we should symmetrize the amplitude $T_{ij}^{i'j'}$ with respect to exchanged velocities $v_1'\leftrightarrow v_2'$ (this is equivalent to $\psi\leftrightarrow\alpha$) and colors ($i'\leftrightarrow j'$). After symmetrization we immediately get the expression for the full amplitude:

$$T(BB\to BB)\simeq\sinh\theta\int d^2ze^{-iz\cdot q}\left[\left(C_1(m_Qz,\theta,\alpha,\psi)+C_2(m_Qz,\theta,\psi,\alpha)\right)\xi^2(\cosh\psi,\frac{1}{z\lambda})+\right.$$
$$\left.+\left(C_1(m_Qz,\theta,\psi,\alpha)+C_2(m_Qz,\theta,\alpha,\psi)\right)\xi^2(\cosh\alpha,\frac{1}{z\lambda})\right] \quad (126)$$

in the limit $N_c\to\infty$. The expression (126) is symmetrical with respect to a pair of angles (ψ,α) which are defined through B mesons velocities:

$$\cosh\psi = v_1 \cdot v_1' = v_2 \cdot v_2', \quad \cosh\alpha = v_1 \cdot v_2' = v_2 \cdot v_1'. \tag{127}$$

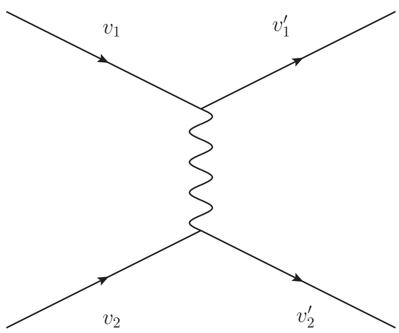
The factorized form of the B-B scattering amplitude (126) expresses it's physical sense. We see that nonperturbative effects are described by the Isgur-Wise form factors that can not be calculated from first principles in QCD, while loop corrections are represented via coefficient functions $C_k(m_Q z, \theta, \alpha, \psi)$ for $k = 1, 2$, that can be computed perturbatively at any order in the coupling constant $\alpha_s(m_Q)$.

4.4 Matching

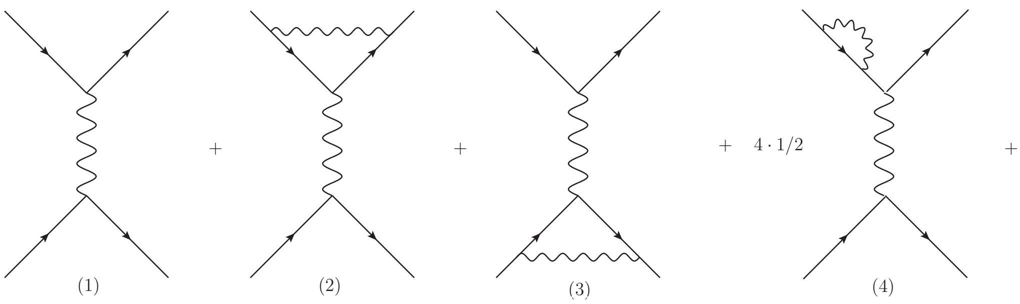
In this section our aim is to show how the calculation of the coefficient functions $C_k(m_Q z, \theta, \alpha, \psi)$ at one loop for the elastic scattering process $BB \to BB$ can be performed. The QCD process matches the one calculated in the effective theory if they give identical physical (on-shell) matrix elements. Thus, using the on-shell renormalization scheme we perform matching for the $\bar{b}\bar{b} \to \bar{b}\bar{b}$ elastic scattering at the mass scale of the b quark. In the on-shell limit the propagators are free. Therefore, we choose all external momenta in HQET to be zero and the corresponding momenta equal to mv in QCD. Then the HQET loops vanish [10]. The matrix elements written in the full theory (QCD) with γ^μ must be expressed in terms of 5 vectors present in the effective theory: γ^μ, v_1^μ, v_2^μ, $v_1^{'\mu}$, $v_2^{'\mu}$. This will give the QCD-HQET matching coefficients for this process.

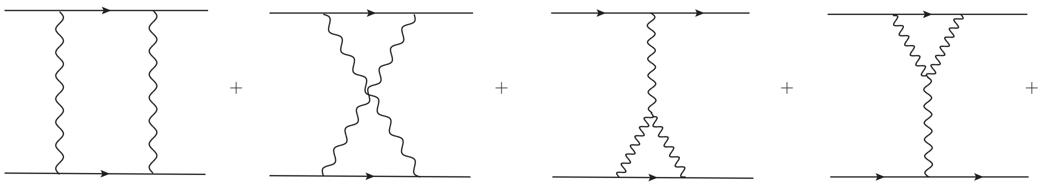
4.4.1 Conventions and notations

The kinematics of the process in QCD is the following:

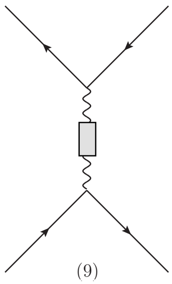


We start with drawing all possible diagrams at one loop:



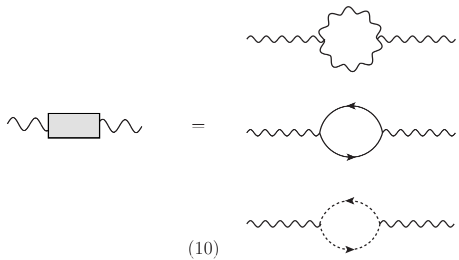


(5) (6) (7) (8)



+ *Bose symmetric terms*

(9)



(10)

Propagators for "massless" $\widetilde{Q}$ (HQET operator) get no corrections because they are no scale. For the diagram (9), the only correction is due to the gluon propagator (10). Because of the heavy quarks decoupling, we have to take into account all the flavors except the top quark which is irrelevant at our energy, $n_f = 5$ [13]. We use the Feynman gauge in our calculation in which the gluon propagator expression is shorter. We used the leading log-approximation to obtain the scattering amplitude (122). Thus, calculating only a few terms in the perturbation series, we should think about the best value for the UV cut-off μ in order to avoid large logarithms.

The calculation of the coefficient functions requires the evaluation of two-, three- and four-point functions. We define the two-point function as

$$B_0(q, m_1, m_2) = \int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 - m_1^2)((k+q)^2 - m_2^2)}, \tag{128}$$

where we omitted the $i\varepsilon$ in the denominator, as usual. Note that our notation differs from the conventions used in the classical 't Hooft-Veltman's work [24], due, inter alia, to the fact that we use the $(+ - - -)$ signature in this paper. The three-point functions are denoted as follows:

$$C_0; C_\alpha(p_1, p_2, m_1, m_2, m_3) = \int \frac{d^D k}{(2\pi)^D} \frac{1; k_\alpha}{(k^2 - m_1^2)((k+p_1)^2 - m_2^2)((k+p_1+p_2)^2 - m_3^2)}. \tag{129}$$

C_α can be written in terms of form factors:

$$C_\alpha = (p_1)_\mu C_{11} + (p_2)_\mu C_{12}. \tag{130}$$

We use the Passarino-Veltman method for the calculation of the corresponding form factors [25].

If one of the factors in the denominator is omitted, then the resulting two-point function will be denoted by indicating the remaining masses. For example, if the second factor in the C_0 integral is omitted, the remaining integral is denoted by $B_0(1, 3)$. Finally, the four-point functions are

$$D_0; D_\alpha(p_1, p_2, p_3, m_1, m_2, m_3, m_4) =$$

$$= \int \frac{d^D k}{(2\pi)^D} \frac{1; k_\alpha}{(k^2 - m_1^2)((k+p_1)^2 - m_2^2)((k+p_1+p_2)^2 - m_3^2)((k+p_1+p_2+p_3)^2 - m_4^2)}. \tag{131}$$

The definitions for the form factors are:

$$D_\alpha = (p_1)_\mu D_{11} + (p_2)_\mu D_{12} + (p_3)_\mu D_{13}. \tag{132}$$

The expressions for the scalar integrals are brought from [26] with some modifications due to different conventions. For instance, every result in [26] must be multiplied by a factor

$$\Omega = \frac{i}{16\pi^2}. \tag{133}$$

4.4.2 Calculation of the coefficient functions in the large N_c limit

The expression for the $BB \to BB$ scattering amplitude (126) is written in the large N_c limit. Thus, we should write down the contribution given by every diagram for the $\bar{b}\bar{b} \to \bar{b}\bar{b}$ process and identify the terms which are relevant in this limit. We will show that the main contribution is given by the planar diagrams. The amplitude for the meson-meson scattering process is obtained by projection onto the corresponding states of the light components $|light, i', j'\rangle$.

Diagram 1. The matching at tree level is trivial: $C_1 = 1$, $C_i = 0$, where $i = 1 \ldots 4$.

Diagram 2. All the divergences in this diagram are given by the vertex corrections, already calculated in the section 3.2. Now we are going to obtain this result using the Passarino-Veltman reduction of loop integrals to scalar s-point functions [25]. First we should factorize the color contribution. We have $t^a t^b t^a = -\frac{T_F}{N_c} t^b = (C_F - C_A/2) t^b$. We see that this diagram is supressed by the color factor $1/N_c$ in the large N_c limit. The Passarino-Veltman method consist in reducing the initial integral to a sum of basic scalar integrals. After some algebra, the vertex contribution can be decomposed as

$$-ig^2 \mu_{uv}^{2\varepsilon} \int \frac{d^D k}{(2\pi)^D} \frac{\langle v_1', s_1'| \gamma_\nu (\not{k} + m\not{v}_1' + m) \gamma^\mu (\not{k} + m\not{v}_1 + m) \gamma^\nu |v_1, s_1\rangle}{k^2 (k^2 + 2mvk)(k^2 + 2mv'k)} =$$

$$= -ig^2 \mu_{uv}^{2\varepsilon} \left[\left(\frac{(D-2)^2}{D} B_0(q, m, m) + 4m^2 \cosh\psi C_0(mv_1, q, 0, m, m) \right) \langle v_1', s_1'| \gamma^\mu |v_1, s_1\rangle + \right.$$
$$\left. + \left(4m^2 v_1^\mu (C_{11} - C_{12}) + 4m^2 (v_1')^\mu C_{12} \right) \langle v_1', s_1' |v_1, s_1\rangle \right], \tag{134}$$

where $q = m(v_1' - v_1)$ and the argument of the form factors is the same as for the $C_0(mv_1, q, 0, m, m)$ integral. The C_{11}, C_{12} form factors (130) can be calculated by contracting $C_\alpha(mv_1, q, 0, m, m)$ with $(mv_1)_\alpha$ and q_α (see appendix E for details). Finally, we find:

$$\begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} = \frac{(-1)}{2m^2 \sinh^2\psi} \begin{pmatrix} (1-\cosh\psi)\left[B_0(1,3) + B_0(1,2) - 2B_0(2,3)\right] \\ B_0(1,2) - \cosh\psi B_0(1,3) - (1-\cosh\psi) B_0(2,3) \end{pmatrix}, \tag{135}$$

where $\cosh\psi = v_1 \cdot v_1'$. Inserting this in (134), we see that the answer for the diagram 2 is expressed in terms of basic scalar integrals [26]:

$$\mu_{uv}^{2\varepsilon} B_0(1,3) = \mu_{uv}^{2\varepsilon} B_0(1,2) = \Omega \left(\frac{\mu_{uv}^2}{m^2} \right)^\varepsilon \left[\frac{1}{\varepsilon} + 2 \right] + O(\varepsilon), \tag{136}$$

$$\mu_{uv}^{2\varepsilon} B_0(2,3) = \mu_{uv}^{2\varepsilon} B_0(q, m, m) = \Omega \left(\frac{\mu_{uv}^2}{m^2} \right)^\varepsilon \left[\frac{1}{\varepsilon} + 2 - \beta \log \frac{\beta+1}{\beta-1} \right] + O(\varepsilon), \tag{137}$$

where

$$\beta = \sqrt{\frac{\cosh\psi + 1}{\cosh\psi - 1}}, \tag{138}$$

and $\mu_{uv}^{2\varepsilon} C_0(mv_1, q, 0, m, m)$ is given by the expression (4.14) in [26], multiplied by Ω.

Diagram 3. The loop involved in this diagram gives the same result as the one calculated for the diagram 2, but with different angle $w = v_2 \cdot v_2'$ instead of $w = v_1 \cdot v_1'$ and v_2^μ, $(v_2')^\mu$ instead of v_1^μ, $(v_1')^\mu$ correspondingly.

Diagram 4. Corrections to the static heavy quark $\widetilde{Q} = \widetilde{Z}_Q^{-1/2}\widetilde{Q}_{os}$ and to the heavy quark in QCD $Q = Z_Q^{-1/2} Q_{os}$ are given by [10]:

$$\widetilde{Z}_Q = 1 + C_F \frac{\alpha_s}{2\pi\varepsilon}, \tag{139}$$

$$Z_Q = 1 + C_F \frac{\alpha_s}{4\pi}\left(\frac{2}{\varepsilon} - 3\log\frac{m^2}{\mu_{uv}^2} + 4\right). \tag{140}$$

The infrared divergence of the static quark propagator is the same as that of the on-shell massive quark propagator.

Diagram 5. The color factor for this diagram is

$$\left(t^a t^b\right)_{i'i}\left(t^a t^b\right)_{j'j} = \left[\left(T_F^2 + \frac{T_F^2}{N_c^2}\right)\delta_{ii'}\delta_{jj'} - 2\frac{T_F^2}{N_c}\delta_{ij'}\delta_{ji'}\right].$$

The remaining integral is

$$-ig^4\mu_{uv}^{4\varepsilon}\int\frac{d^Dk}{(2\pi)^D}\frac{\langle v_1', s_1'|\,\gamma_\nu(m\not{v}_1 - \not{k} + m)\gamma_\mu\,|v_1, s_1\rangle\,\langle v_2', s_2'|\,\gamma^\nu(m\not{v}_2 + \not{k} + m)\gamma^\mu\,|v_2, s_2\rangle}{k^2(q+k)^2(k^2 - 2mv_1k)(k^2 + 2mv_2k)}.$$

Averaging over k directions $\not{k}\Gamma\not{k} \to \gamma^\alpha\Gamma\gamma_\alpha k^2/D$, we get (after some gamma matrices algebra) the following decomposition of the initial integral:

$$\begin{aligned}
-ig^4\mu_{uv}^{4\varepsilon}\Bigg[-&\frac{\langle v_1', s_1'|\,\gamma_\nu\gamma_\lambda\gamma_\mu\,|v_1, s_1\rangle\,\langle v_2', s_2'|\,\gamma^\nu\gamma^\lambda\gamma^\mu\,|v_2, s_2\rangle}{D} C_0(mv_1', mv_2', m, 0, m) + \\
&+2m\,\langle v_1', s_1'|\left\{\gamma_\nu\,|v_1, s_1\rangle\,\langle v_2', s_2'|\,\gamma^\nu\gamma^\alpha\not{v}_1 - \eta^{\alpha\beta}\gamma_\nu\gamma_\beta\not{v}_2\,|v_1, s_1\rangle\,\langle v_2', s_2'|\,\gamma^\nu\right\}|v_2, s_2\rangle\, D_\alpha + \\
&+4m^2\cosh\theta\,\langle v_1', s_1'|\,\gamma_\nu\,|v_1, s_1\rangle\,\langle v_2', s_2'|\,\gamma^\nu\,|v_2, s_2\rangle\, D_0(-mv_1, -mv_1', mv_2', 0, m, 0, m)\Bigg] \equiv \\
&\equiv -ig^4\mu_{uv}^{4\varepsilon}\left[\chi_5^{(1)} C_0 + \left(\chi_5^{(2)}\right)^\alpha D_\alpha + \chi_5^{(3)} D_0\right].
\end{aligned} \tag{141}$$

Again, we use the Passarino-Veltman method to identufy the form factors that appear in the D_α decomposition (132). After a straightforward calculation (see appendix E), we explicitely find the expressions for D_{11}, D_{12}, D_{13} :

$$\begin{pmatrix} D_{11} \\ D_{12} \\ D_{13} \end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} -\sinh^2\theta & \delta & \sigma \\ \delta & -\sinh^2\alpha & \tau \\ \sigma & \tau & -\sinh^2\psi \end{pmatrix}\begin{pmatrix} \xi_5^{(1)} \\ \xi_5^{(2)} \\ \xi_5^{(3)} \end{pmatrix}, \tag{142}$$

where $\xi_5^{(i)}$ are defined by (201) and

$$\Delta = m^2\left(2\cosh\theta\cosh\psi\cosh\alpha - \cosh^2\theta - \cosh^2\psi - \cosh^2\alpha + 1\right), \tag{143}$$

$$\sigma = \cosh\alpha - \cosh\psi\cosh\theta, \tag{144}$$

$$\delta = \cosh\psi - \cosh\alpha\cosh\theta, \tag{145}$$

$$\tau = -\cosh\theta + \cosh\psi\cosh\alpha \tag{146}$$

and

$$\cosh\alpha = \cosh\theta - \cosh\psi + 1. \tag{147}$$

The corresponding scalar integrals are given by [26]:

$$\mu_{uv}^{2\varepsilon} C_0(1,3,4) = \mu_{uv}^{2\varepsilon} C_0(1,2,3) = \frac{\Omega}{m^2}\left(\frac{\mu_{uv}^2}{m^2}\right)^{\varepsilon}\left[-\frac{1}{2\varepsilon}+1\right] + O(\varepsilon), \tag{148}$$

$$\mu_{uv}^2 D_0 = \Omega \frac{\theta}{2m^4(1-\cosh\psi)\sinh\theta}\left[\frac{1}{\varepsilon} + \log\left(\frac{\mu_{uv}^2}{2m^2(\cosh\psi - 1)}\right)\right] + O(\varepsilon), \tag{149}$$

while $\mu_{uv}^{2\varepsilon} C_0(2,3,4) = \mu_{uv}^{2\varepsilon} C_0(mv_1', mv_2', m, 0, m) = \mu_{uv}^{2\varepsilon} C_0(1,2,4)$ are given by (4.14) in [26], multiplied by Ω.

Diagram 6. The color factor for this diagram is

$$\left(t^a t^b\right)_{i'i}\left(t^b t^a\right)_{j'j} = \left[\frac{T_F^2}{N_c^2}\delta_{ii'}\delta_{jj'} + \left(T_F^2 N_c - 2\frac{T_F^2}{N_c}\right)\delta_{ij'}\delta_{ji'}\right].$$

The remaining integral is

$$-ig^4\mu_{uv}^{4\varepsilon}\int \frac{d^D k}{(2\pi)^D}\frac{\langle v_1', s_1'|\,\gamma_\mu(m\not{v}_1 - \not{k} + m)\gamma_\nu\,|v_1, s_1\rangle\,\langle v_2', s_2'|\,\gamma^\nu(m\not{v}_2 - \not{q} - \not{k} + m)\gamma^\mu\,|v_2, s_2\rangle}{k^2(q+k)^2\left[k^2 - 2mv_1 k\right]\left[(q+k)^2 - 2mv_2(q+k)\right]}.$$

Again, averaging over k directions we get the following decomposition:

$$\begin{aligned}
&-ig^4\mu_{uv}^{4\varepsilon}\left[\frac{\langle v_1', s_1'|\,\gamma_\mu\gamma_\lambda\gamma_\nu\,|v_1, s_1\rangle\,\langle v_2', s_2'|\,\gamma^\nu\gamma^\lambda\gamma^\mu\,|v_2, s_2\rangle}{D} C_0(mv_1', -mv_2, m, 0, m) - \right.\\
&-2m\,\langle v_1', s_1'|\left\{\gamma_\mu\,|v_1, s_1\rangle\,\langle v_2', s_2'|\,\not{v}_1\gamma^\alpha\gamma^\mu + \eta^{\alpha\beta}\gamma_\mu\gamma_\beta\not{v}_2\,|v_1, s_1\rangle\,\langle v_2', s_2'|\,\gamma^\mu\right\}|v_2, s_2\rangle\, D_\alpha +\\
&\left.+4m^2\cosh\theta\,\langle v_1', s_1'|\,\gamma_\mu\,|v_1, s_1\rangle\,\langle v_2', s_2'|\,\gamma^\mu\,|v_2, s_2\rangle\, D_0(-mv_1, mv_1', -mv_2, 0, m, 0, m)\right] \equiv\\
&\equiv -ig^4\mu_{uv}^{4\varepsilon}\left[\chi_6^{(1)} C_0 + \left(\chi_6^{(2)}\right)^\alpha D_\alpha + \chi_6^{(3)} D_0\right].
\end{aligned} \tag{150}$$

We use the Passarino-Veltman reduction method to calculate the D_{11}, D_{12}, D_{13} form factors for this case (see appendix E):

$$\begin{pmatrix} D_{11} \\ D_{12} \\ D_{13} \end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} -\sinh^2\theta & \delta & \tau \\ \delta & -\sinh^2\alpha & \sigma \\ \tau & \sigma & -\sinh^2\psi \end{pmatrix}\begin{pmatrix} \xi_6^{(1)} \\ \xi_6^{(2)} \\ \xi_6^{(3)} \end{pmatrix}, \tag{151}$$

where $\xi_6^{(i)}$ are defined by (202) and Δ, δ, σ and τ were defined hereinabove (143)-(146). The appropriate scalar diagrams are [26]:

$$\mu_{uv}^{2\varepsilon} C_0(1,3,4) = \mu_{uv}^{2\varepsilon} C_0(1,2,3) = \frac{\Omega}{m^2}\left(\frac{\mu_{uv}^2}{m^2}\right)^{\varepsilon}\left[-\frac{1}{2\varepsilon}+1\right] + O(\varepsilon), \tag{152}$$

$$\mu_{uv}^2 D_0 = \Omega \frac{\alpha}{2m^4(1-\cosh\psi)\sinh\alpha}\left[\frac{1}{\varepsilon} + \log\left(\frac{\mu_{uv}^2}{2m^2(\cosh\psi - 1)}\right)\right] + O(\varepsilon), \tag{153}$$

$$\mu_{uv}^{2\varepsilon} C_0(2,3,4) = \mu_{uv}^{2\varepsilon} C_0(1,2,4) = \mu_{uv}^{2\varepsilon} C_0(mv_1', -mv_2, m, 0, m) \tag{154}$$

and are given by (4.14) in [26], multiplied by Ω.

Diagram 7. This diagram involves a 3-gluon vertex and we calculate it's contribution. The color factor is given by

$$\frac{C_A}{2} = T_F N_c.$$

The remaining integral in the Feynman gauge is

$$g^2 \mu_{uv}^{2\varepsilon} \int \frac{d^D k}{(2\pi)^D} \frac{\gamma_\mu (m\not{v}_2 - \not{q} + \not{k} + m)\gamma_\nu}{k^2 (k-q)^2 \left[(mv_2 - q + k)^2 - m^2\right]} V^{\alpha\nu\mu}(-q, q-k, k), \tag{155}$$

where

$$V^{\alpha\nu\mu}(-q, q-k, k) = (2k-q)^\alpha \eta^{\nu\mu} + (-q-k)^\nu \eta^{\mu\alpha} + (2q-k)^\mu \eta^{\alpha\nu}. \tag{156}$$

After some gamma matrices algebra and averaging over k directions $\not{k} k^\alpha \to \frac{k^2}{D}\gamma^\alpha$, we get the decomposition

$$\begin{aligned} & g^2 \mu_{uv}^{2\varepsilon} \left[\frac{4(1-D)}{D} \gamma^\alpha B_0(mv_2, 0, m) + \right. \\ & + m \left(4\eta^{\alpha\beta} + (D-6) v_2^\alpha \gamma^\beta + 4 v_2^\beta \gamma^\alpha - (D+2)(v_2')^\alpha \gamma^\beta \right) C_\beta(mv_2', -mv_2, 0, m, 0) + \\ & \left. + m^2 \left(4\gamma^\alpha (\cosh\psi - 1) - 2 v_2^\alpha + 2(v_2')^\alpha\right) C_0(mv_2', -mv_2, 0, m, 0) \right]. \end{aligned} \tag{157}$$

Using the Passarino-Veltman, we obtain the following equation for the C_{11}, C_{12} form factors (see appendix E)

$$-2m^2 \sinh^2\psi \begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} =$$

$$= \begin{pmatrix} (1-\cosh\psi) B_0(1,3) - B_0(2,3) + \cosh\psi B_0(1,2) - \cosh\psi 2m^2 (1-\cosh\psi) C_0 \\ (\cosh\psi - 1) B_0(1,3) - \cosh\psi B_0(2,3) + B_0(1,2) - 2m^2(1-\cosh\psi) C_0 \end{pmatrix}, \tag{158}$$

where $\cosh\psi = v_2 \cdot v_2'$. The scalar integrals are given by the following expressions [26]:

$$\mu_{uv}^{2\varepsilon} B_0(mv_2, 0, m) = \mu_{uv}^{2\varepsilon} B_0(2,3) = \mu_{uv}^{2\varepsilon} B_0(1,2) = \Omega \left(\frac{\mu_{uv}^2}{m^2}\right)^\varepsilon \left[\frac{1}{\varepsilon} + 2\right] + O(\varepsilon), \tag{159}$$

$$\mu_{uv}^{2\varepsilon} B_0(1,3) = \Omega \left(\frac{\mu_{uv}^2}{2m^2(\cosh\psi - 1)}\right)^\varepsilon \left[\frac{1}{\varepsilon} + 2\right] + O(\varepsilon), \tag{160}$$

$$\mu_{uv}^{2\varepsilon} C_0 = \frac{\Omega}{m^2} \left(\frac{\mu_{uv}^2}{m^2}\right)^\varepsilon \left[-\frac{1}{2\varepsilon} + 1\right] + O(\varepsilon). \tag{161}$$

Diagram 8. The loop involved in this diagram gives the same result as the one calculated for the diagram 7, with the remplacements $q \to -q$, $v_2 \to v_1$and $v_2' \to v_1'$.

Diagram 9. All the divergences of this diagram are given by the corrections to gluon propagator. In the Feynman gauge, the expression of the renormalized gluon propagator is given by [13]

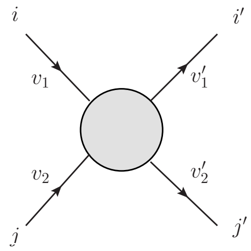
$$D_{\mu\nu}^r = D^r(q^2)\eta_{\mu\nu},$$

with

$$q^2 D^r(q^2) = 1 + \frac{\alpha_s}{4\pi} \left[\left(\frac{5}{3} C_A - \frac{4}{3} T_F n_f\right) \log \frac{-q^2}{\mu_{uv}^2} + \left(\frac{124}{36} C_A - \frac{20}{9} T_F n_f\right) \right]. \tag{162}$$

4.4.3 Analysis

Let us recapitulate the most important results. First, we consider the diagrams (1)-(10) of the section 4.4 without Bose symmetric terms. This contribution can be regarded as a quark-quark scattering with different flavors and give the following color decomposition of the amplitude:



$$= \Psi_1 \, \delta_{ii'} \, \delta_{jj'} + \Psi_2 \, \delta_{ij'} \, \delta_{ji'}$$

In order to reproduce the general hadron-hadron scattering matrix element in the form of Isgur-Wise form factors (119), we should project this amplitude on the hadrons' light constituents $|light, i', j'\rangle$ and take the $N_c \to \infty$ limit. This gives us two matching coefficients behind the Isgur-Wise form factors

$$C_1(\frac{m_Q}{\mu_{uv}}, \theta, \alpha, \psi)\xi^2(\cosh\psi, \frac{\mu_{uv}}{\lambda}) + C_2(\frac{m_Q}{\mu_{uv}}, \theta, \alpha, \psi)\xi^2(\cosh\alpha, \frac{\mu_{uv}}{\lambda}). \tag{163}$$

If we denote $[i]$ the contribution of the corresponding diagram, we obtain in the large N_c limit

$$C_1(\frac{m_Q}{\mu_{uv}}, \theta, \alpha, \psi) = \Psi_1 N_c^2 + \Psi_2 N_c = T_F^2 \left(N_c^2 - 1\right) \times \{[5] + [6]\}, \tag{164}$$

$$C_2(\frac{m_Q}{\mu_{uv}}, \theta, \alpha, \psi) = \Psi_1 N_c + \Psi_2 N_c^2 = T_F^2 N_c^3 \times \{[6] + [7] + [8]\} + T_F N_c^2 \times \{[9] + 2 \cdot [4]\}. \tag{165}$$

Using the results of the diagram calculations obtained above, we can write down the full (and bulky) expressions for the matching coefficients at one loop in the large N_c limit. In the case of identical particles (126) we should also include the Bose symmetric terms. The physical amplitudes (125) and (126) should depend only on the physical ratio m_Q/λ and not on the artificial parameter μ_{uv}. Thus, the conditions

$$\frac{d \log \xi^2(\cosh\psi, \frac{\mu_{uv}}{\lambda})}{d \log \mu_{uv}} = -\frac{d \log C_1(\frac{m_Q}{\mu_{uv}}, \theta, \alpha, \psi)}{d \log \mu_{uv}}, \tag{166}$$

$$\frac{d \log \xi^2(\cosh\alpha, \frac{\mu_{uv}}{\lambda})}{d \log \mu_{uv}} = -\frac{d \log C_2(\frac{m_Q}{\mu_{uv}}, \theta, \alpha, \psi)}{d \log \mu_{uv}} \tag{167}$$

must be verified.

5 Conclusions

In this paper we considered selected problems of heavy quark physics. Adopting the viewpoint of the Heavy Quark Effective Theory, we derived the factor (55) which summarize the interaction of static heavy quark with the hadron's light constituents. Working with the B-meson form factor, we combined the eikonal approximation together with the HQET formalism and found the relation (60) between the HQET Isgur-Wise function and light-components expectation value of the Wilson loop, evaluated along the classical trajectories of the heavy quark. The calculated one-loop anomalous dimension (88) controlling the IR behavior due to soft gluon emissions coincides with the cusp-generated anomalous dimension $\Gamma_{cusp}(\theta, \alpha_s)$. These concepts are generalized in the part 4, where we considered the elastic meson-meson scattering. The scattering amplitude appears to be a Fourier transformed expectation value of a product of four Wilson lines. In the point where these Wilson lines intersect each other, they get very specific cross divergences, which are governed by the 2×2 matrix of cross anomalous dimension $\Gamma_{cross}(\theta, \psi, \alpha_s)$. We calculated it's general one-loop expression in (111). There is a close relation between the renormalization properties of cross divergences and the Wilson lines' dependence on the impact vector z. Using this relation it is possible to find the general expression for the scattering amplitude, providing the eigenvectors of $\Gamma_{cross}(\theta, \psi, \alpha_s)$ given in (194) and (195). We analyzed the scattering amplitude and the matching coefficients in the large N_c limit and in the case of identical mesons.

The concepts considered in this work and a part of results obtained in the framework of HQET are not completely new. Velocity dependent cusp and cross anomalous dimensions that appear in problems involving heavy quarks coincide with the expressions obtained earlier under renormalization of singularities in the problems of the nonperturbative dynamics of QCD or in hadron-hadron scattering in the high energy limit. The renormalization properties of the contour averages are discussed in [27]. This correspondence permitted us to verify, in particular, the special case of the forward scattering anomalous dimension $\Gamma_{cross}^{forward}(\theta, \alpha_s)$ that matches the result obtained in [21], where the high energy parton-parton forward scattering was considered.

It is important to notice that quantitative results derived in this work were obtained at one-loop level and using a number of approximations (large m_Q limit, eikonal approximation, large N_c limit, etc.). Although these results are sufficient to analyze the physical properties of different processes involving heavy quarks, they can be generalized in an obvious way. Exactly as in the section 3.2, we can perform the QCD-HQET matching calculation for the meson-meson scattering. Another possible improvement consist in taking next $1/m_Q$ and $1/N_c$ terms. Furthermore, nowadays the cusp anomalous dimensions and the QCD-HQET matching coefficients are known up to the second and even to the third order of perturbation theory [3, 28]. Similarly, the $\Gamma_{cusp}(\theta, \alpha_s)$ and $\Gamma_{cross}(\theta, \psi, \alpha_s)$ expressions can also be calculated at multiloop level, but the calculations rapidly become very complex [13].

Finally, we must note that although hadron-hadron elastic scattering processes involving heavy quarks are interesting from the theoretical point of view, the experimental investigation of the hadron-hadron interaction is hardly probable in the years to come because of the evident experimental difficulties. Nevertheless, we believe that a detailed study of heavy quarks interacting with soft components is an important step toward understanding of hadronic properties and nonperturbative phenomena.

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A Path-ordered exponential

We introduce here a very useful concept of the path-ordered exponential. Let us investigate the solution $f(t)$ of the ODE

$$[\frac{d}{dt} - M(t)]f(t) = 0, \tag{168}$$

where $M(t) = ig\dot{z}_\mu A^\mu(z(t))$ and $z_\mu(t)$ is the line between points y and $z_\mu(t) : z_\mu(0) = y_\mu,\ z_\mu(t) = z_\mu(t)$. In general $[M(t), M(s)] \neq 0$ and the solution is given by

$$\begin{aligned} f(t) = P\exp[\int_0^t dsM(s)] = \\ = \sum_{m=0}^{+\infty} \int_0^t ds_m \int_0^{s_m} ds_{m-1} \dots \int_0^{s_2} ds_1 M(s_m)M(s_{m-1})\dots M(s_1) = \\ = 1 + \int_0^t ds_1 M(s_1) + \int_0^t ds_2 \int_0^{s_2} ds_1 M(s_2)M(s_1) + \dots \end{aligned} \tag{169}$$

Indeed, we verify that

$$\frac{d}{dt}f(t) = \sum_{m=0}^{+\infty} M(t) \iint \dots = M(t)f(t). \tag{170}$$

This solution shares the following properties:

$$f(t) \mid_{t\epsilon[0,1]} = f(t) \mid_{t\epsilon[s,1]} f(t) \mid_{t\epsilon[0,s]}, \quad f(t) \mid_{t\epsilon[0,s]} = f(t)^{-1} \mid_{t\epsilon[s,0]} . \tag{171}$$

Thus, the path-ordered exponential $f(t)$ can be simply defined as

$$f(t) = P\exp[ig\int_y^{z(t)} dz \cdot A(z)] = \lim_{M\to+\infty} P\prod_{j=0}^{M} (1 + ig\Delta z A(z_j)), \tag{172}$$

where $\Delta z = \frac{1}{M}$, $z_j = j\Delta z$. It is easy to demonstrate that $f(t)$ transforms as follows:

$$f^V(t) = V(x)f(t)V^\dagger(y) \tag{173}$$

for V corresponding to the transformation $A_\mu \to V^\dagger A_\mu V + \frac{i}{g}V^\dagger\partial_\mu V$. Note that $tr\, f(t)$ is a gauge-invariant quantity and $f(t)$ is not.

In QED, the expression

$$\langle 0|\, f(t)\, |0\rangle_{QED} = \exp\left(\frac{(ie)^2}{2} \oiint_{CC} dz_\mu dz'_\nu \left\langle A^\mu(z)A^\nu(z')\right\rangle\right) \tag{174}$$

is verified, so the vacuum average of the time-ordered Wilson line (55) is the exponential of the vacuum average of photon propagator $D_{\mu\nu}$. This can be shown directly by applying the functional integral formalism with a generating function. Indeed, if we introduce

$$J(x) = \sum_{i=1}^{M} \delta^{(4)}(x - z_i)\frac{1}{M}, \tag{175}$$

where $M = \Delta z$, we obtain, using (172)

$$\langle 0|\, TP\exp[ig\int_y^{z(t)} dz\cdot A(z)]\, |0\rangle = \lim_{M\to+\infty} \langle 0|\, TP\prod_{j=0}^{M} e^{ig\Delta z A(z_j)}\, |0\rangle =$$

$$= \frac{1}{Z}\int DAe^{iS_0[A]+ig\int d^4xJ(x)A(x)}. \qquad (176)$$

Now, if we make a change of variable $A' = A - i\int d^4yD(x-y)J(y)$, then

$$Z[J] = Z_0\exp\left(\frac{1}{2}\int d^4xd^4yJ(x)D(x-y)J(y)\right) = Z_0\exp\left(\frac{1}{2}D(z_i)\right) \qquad (177)$$

and, finally

$$\langle 0|\, TP\exp[ig\int_y^{z(t)} dz\cdot A(z)]\, |0\rangle = \exp\left(\frac{(ie)^2}{2}\oiint_{CC} dz_\mu dz'_\nu\, \langle 0|\, TA^\mu(z)A^\nu(z')\, |0\rangle\right). \qquad (178)$$

In QCD we have in the lower order of g the expression for gluon propagator

$$\langle 0|\, TA^a_\mu(x)A^b_\nu(y)\, |0\rangle = D_{\mu\nu}(x-y)\delta^{ab}$$

and thus

$$\langle 0|\, f(t)\, |0\rangle_{QCD} = 1 + \frac{(ig)^2}{2}\oiint_{CC} dz_\mu dz'_\nu\, \langle A^\mu(z)A^\nu(z')\rangle + O(g^4). \qquad (179)$$

B Matching coefficients

We present here the details of the matching coefficients (90) calculation. In QCD, the vertex is

$$-iC_Fg^2\mu_{uv}^{2\varepsilon}\int \frac{d^Dk}{(2\pi)^D}\frac{\gamma_\nu(\not k + m\not v' + m)\gamma^\mu(\not k + m\not v + m)\gamma^\nu}{k^2(k^2+2mvk)(k^2+2mv'k)} =$$

$$= -2iC_Fg^2\mu_{uv}^{2\varepsilon}\int_0^1 dy\int_0^1 dx\int_0^1 dx'\int\frac{d^Dk'}{(2\pi)^D}\delta(x+x'+y-1)\times$$

$$\times\frac{\gamma_\nu(\not k' + m(1-x')\not v' - mx\not v + m)\gamma^\mu(\not k' + m(1-x)\not v - mx'\not v' + m)\gamma^\nu}{(k'^2-a^2)^3}, \qquad (180)$$

where we have used the Feynman parametrization (67), $k' = k + m(xv + x'v')$, $a^2 = m^2(xv+x'v')^2 = m^2(x^2+x'^2+2xx'\cosh\theta)$. If we calculate the loop integral using (69) and substitute $x = \xi(1+z)/2$, $x' = \xi(1-z)/2$ [10], we immediately get

$$C_F\frac{g^2}{(4\pi)^{D/2}}\left(\frac{\mu_{uv}}{m_Q}\right)^{2\varepsilon}\Gamma(1+\varepsilon)\int_{-1}^1 dz\frac{1}{(a_+a_-)^{1+\varepsilon}}\int_0^1 d\xi\times$$

$$\times\gamma_\nu\left(\left(1-\frac{\xi(1-z)}{2}\right)\not v' - \frac{\xi(1+z)}{2}\not v + 1\right)\gamma^\mu\times$$

$$\times\left(\left(1-\frac{\xi(1+z)}{2}\right)\not v - \frac{\xi(1-z)}{2}\not v' + 1\right)\gamma^\nu. \qquad (181)$$

Here $a^2 = m^2\xi^2 a_+ a_-$, $a_\pm = \cosh(\theta/2) \pm z \sinh(\theta/2)$. We use the following expressions

$$\begin{aligned} \gamma_\nu \not{a} \gamma^\nu &= -(D-2)\not{a}, & \{\gamma^\mu \gamma^\nu\} &= 2g^{\mu\nu}, \\ \gamma_\nu \not{a}\not{b} \gamma^\nu &= 4a\cdot b + (D-4)\not{a}\not{b}, & \gamma^\mu \not{v} &= 2v^\mu - \not{v}\gamma^\mu \\ \gamma_\nu \not{a}\not{b}\not{c} \gamma^\nu &= -2\not{c}\not{b}\not{a} - (D-4)\not{a}\not{b}\not{c}, \end{aligned} \tag{182}$$

to simplify the integral and take the ξ integral, using the analytical properties of the dimensional regularization to obtain

$$C_F \frac{g^2}{(4\pi)^{D/2}} \left(\frac{\mu_{uv}}{m_Q}\right)^{2\varepsilon} \Gamma(1+\varepsilon) \times \int_{-1}^{1} \frac{dz}{(a_+ a_-)^{1+\varepsilon}} \left[H_1' \gamma^\mu + H_2'(\not{v}\gamma^\mu + \gamma^\mu \not{v}') + H_3' \not{v}\gamma^\mu \not{v}'\right], \tag{183}$$

where

$$H_1' = \left(\frac{a_+ a_- h^2}{2\varepsilon(1-\varepsilon)} - \frac{(1-z^2)h}{8(1-\varepsilon)} + \frac{\cosh\theta}{\varepsilon(1-2\varepsilon)} + \frac{1}{(1-2\varepsilon)}\right),$$

$$H_2' = -\frac{1}{2}\left(\frac{(1+z^2)h}{4(1-\varepsilon)} + \frac{1}{(1-2\varepsilon)}\right), \quad H_3' = -\frac{h}{8(1-\varepsilon)}(1-z^2)$$

and $h = 1 - D/2$. The z integrals can be calculated for the needed accuracy [10]:

$$\int_{-1}^{1} \frac{dz}{(a_+ a_-)^{\varepsilon}} = 2 - 2\varepsilon\left(\frac{\theta(\cosh\theta + 1)}{\sinh\theta} - 2\right) + O(\varepsilon^2),$$

$$\int_{-1}^{1} \frac{dz}{(a_+ a_-)^{1+\varepsilon}} = 2\frac{\theta + \varepsilon F(\theta)}{\sinh\theta} + O(\varepsilon^2),$$

$$F(\theta) = Li_2(-e^{-\theta}) - Li_2(-e^{\theta}) - \theta\log(2(\cosh\theta+1)) = -\frac{\theta^2}{2} - \frac{\pi^2}{6} - \theta\log(2(\cosh\theta+1)),$$

$$\int_{-1}^{1} \frac{z^2 dz}{(a_+ a_-)^{1+\varepsilon}} = \frac{4}{\cosh\theta - 1}\left(\frac{\theta}{\sinh\theta} - 1\right) + \frac{2\theta}{\sinh\theta} + O(\varepsilon). \tag{184}$$

If we define $H_i = C_F \frac{g^2}{(4\pi)^{D/2}} \left(\frac{\mu_{uv}}{m_Q}\right)^{2\varepsilon} \Gamma(1+\varepsilon) \times \int_{-1}^{1} \frac{dz}{(a_+ a_-)^{1+\varepsilon}} H_i'$ and use the D-dimensional Clifford algebra (182), we can explicitly calculate the required coefficients [10, 16]

$$C_1 = H_1 - 2H_2 - (2\cosh\theta + 1)H_3 = 1 - 2C_F \frac{\alpha_s}{4\pi}\left\{[wr(w) - 1]\log\left(\frac{m_Q^2}{\mu_{uv}^2}\right) - F(w)\right\},$$

$$C_2 = C_3 = 2(H_2 + H_3) = -C_F \frac{\alpha_s}{4\pi} r(w), \tag{185}$$

where $w = v \cdot v'$ and

$$r(w) = \frac{1}{\sqrt{w^2-1}}\log(w + \sqrt{w^2-1}), \tag{186}$$

$$F(w) = \frac{w}{\sqrt{w^2-1}}\{2Li_2\left(\sqrt{w^2-1} - w\right) + \frac{1}{6}\pi^2 +$$

$$+\frac{1}{2}(w^2-1)r(w) - wr(w)\log[2(w+1)] + \frac{3}{2}(w+1)r(w) - 2\}. \tag{187}$$

C $SU(N_c)$ color group

In this appendix we follow [13] to derive some formulae for the $SU(N_c)$ color group diagrams used in the text. Elements of this group are complex $N_c \times N_c$ unitary matrices with $\det U = 1$. Complex N_c column vectors transform as

$$q \to Uq, \quad q^+ \to q^+U^+. \tag{188}$$

Three invariant tensors δ^i_j, $e_{i_1 \dots i_{N_c}}$ and $e^{i_1 \dots i_{N_c}}$ correspond to the color structure of mesons and (anti)baryons. Infinitesimal transformations are given by

$$U = 1 + i\alpha^a t^a, \tag{189}$$

where α^a are infinitesimal real parameters and t^a are called generators of the fundamental representation. They have the following properties:

$$(t^a)^+ = t^a, \quad Tr(t^a) = 0, \quad Tr(t^a t^b) = T_F \delta^{ab}, \quad [t^a, t^b] = i f^{abc} t^c, \tag{190}$$

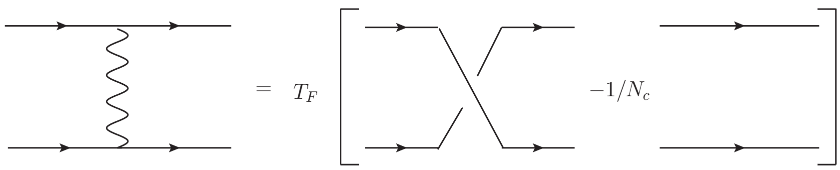
where $T_F = 1/2$ and f^{abc} are real constants. The Casimir operator in the fundamental representation

$$C_F = t^a t^a = T_F \left(N_c - \frac{1}{N_c} \right) = \frac{N_c^2 - 1}{2N_c}. \tag{191}$$

A very useful identity

$$t^a_{ij} t^a_{kl} = T_F \delta_{il} \delta_{jk} - \frac{T_F}{N_c} \delta_{ij} \delta_{kl} \tag{192}$$

can be derived graphically using the Cvitanovic algorithm [13]. It can be represented graphically as



We use this formula in section 4.2.

D $\Gamma_{cross}(\theta, \alpha, \psi, \alpha_s)$ eigenvalues and eigenvectors

If we denote for compactness $a = (i\pi - \theta)\coth\theta$, $b = \psi \coth\psi$, $c = \alpha \coth\alpha$ and $N = N_c$, we obtain the following form of $\Gamma_{cross}(\theta, \alpha, \psi, g)$:

$\Gamma_{cross}(\theta, \alpha, \psi, \alpha_s) = \frac{\alpha_s}{\pi} \Gamma(a, b, c) =$

$$= \frac{\alpha_s}{\pi} \begin{pmatrix} N(b-1) - \frac{1}{N}(a+b+c-1) & c+a \\ b+a & N(c-1) - \frac{1}{N}(a+b+c-1) \end{pmatrix}. \tag{193}$$

After a straightforward calculation, we obtain the eigenvalues and the corresponding eigenvectors of $\Gamma(a, b)$:

$$\Gamma_+ = \frac{1/2\,N^2c - N^2 - a - b - c + 1 + 1/2\,N^2b + 1/2\,\sqrt{4\,N^2ac + 4\,N^2bc + 4\,N^2ba - 2\,N^4cb + N^4c^2 + 4\,N^2a^2 + N^4b^2}}{N},$$

$$E_+ = \begin{pmatrix} \frac{(c+a)N}{1/2\,N^2c-1/2\,N^2b+1/2\,\sqrt{4\,N^2ac+4\,N^2bc+4\,N^2ba-2\,N^4cb+N^4c^2+4\,N^2a^2+N^4b^2}} \\ \frac{(c+a)N}{1/2\,N^2c-1/2\,N^2b-1/2\,\sqrt{4\,N^2ac+4\,N^2bc+4\,N^2ba-2\,N^4cb+N^4c^2+4\,N^2a^2+N^4b^2}} \end{pmatrix} \tag{194}$$

and

$$\Gamma_- = \frac{1/2\,N^2c-N^2-a-b-c+1+1/2\,N^2b-1/2\,\sqrt{4\,N^2ac+4\,N^2bc+4\,N^2ba-2\,N^4cb+N^4c^2+4\,N^2a^2+N^4b^2}}{N},$$

$$E_- = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{195}$$

In the limit $N \to \infty$, we verify that

$$\Gamma_+ = N(b-1), \quad \Gamma_- = N(c-1), \tag{196}$$

as it should be.

E The Passarino-Veltman method of reduction to the scalar integrals

In this appendix, we give some details on the calculation of the integral form factors (130) and (132) for the corresponding diagrams from the section 4.4.

Diagram 2. Contracting $C_\alpha(mv_1, q, 0, m, m)$ with $(mv_1)_\alpha$ and using the identity

$$mk \cdot v_1 = \frac{1}{2}\left[(k+mv_1)^2 - m^2 - k^2\right],$$

we get the first equation for C_{11}, C_{12} :

$$C_{11}m^2 + C_{12}m(v \cdot q) = \frac{1}{2}\left[B_0(1,3) - B_0(2,3)\right] \equiv \xi_2^{(1)}. \tag{197}$$

Likewise, contracting $C_\alpha(mv_1, q, 0, m, m)$ with q_α and using

$$k \cdot q = \frac{1}{2}\left[\left((k+mv_1+q)^2 - m^2\right) - \left((k+mv_1)^2 - m^2\right) - (mv_1+q)^2 + (mv_1)^2\right],$$

we get the second equation for C_{11}, C_{12} :

$$C_{11}m(v_1 \cdot q) + C_{12}q^2 = \frac{1}{2}\left[B_0(1,2) - B_0(1,3) - (2m(v_1\cdot q) + q^2)C_0\right] \equiv \xi_2^{(2)}. \tag{198}$$

In the matrix form

$$\begin{pmatrix} \xi_2^{(1)} \\ \xi_2^{(2)} \end{pmatrix} = \begin{pmatrix} m^2 & m(v_1 \cdot q) \\ m(v_1 \cdot q) & q^2 \end{pmatrix} \begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix}. \tag{199}$$

In order to find the expression for C_{11}, C_{12}, we should invert the matrix. Finally, we find:

$$\begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} = \frac{(-1)}{2m^2\sinh^2\psi} \begin{pmatrix} (1-\cosh\psi)\left[B_0(1,3) + B_0(1,2) - 2B_0(2,3)\right] \\ B_0(1,2) - \cosh\psi B_0(1,3) - (1-\cosh\psi)B_0(2,3) \end{pmatrix}, \tag{200}$$

where $\cosh\psi = v_1 \cdot v_1'$.

Diagram 5. Contracting D_α with $-mv_1$, $-mv_1'$ and mv_2', we get three equations for $D_{11}, D_{12,} D_{13}$:

$$\begin{pmatrix} \xi_5^{(1)} \\ \xi_5^{(2)} \\ \xi_5^{(3)} \end{pmatrix} = \begin{pmatrix} m^2 & -m^2\cosh\psi & -m^2\cosh\alpha \\ -m^2\cosh\psi & m^2 & m^2\cosh\theta \\ -m^2\cosh\alpha & m^2\cosh\theta & m^2 \end{pmatrix} \begin{pmatrix} D_{11} \\ D_{12} \\ D_{13} \end{pmatrix}, \tag{201}$$

where

$$\xi_4^{(1)} = \frac{1}{2}\left[C_0(1,3,4) - C_0(2,3,4)\right],$$

$$\xi_4^{(2)} = \frac{1}{2}\left[C_0(1,2,4) - C_0(1,3,4) - q^2 D_0\right],$$

$$\xi_4^{(3)} = \frac{1}{2}\left[C_0(1,2,3) - C_0(1,2,4) + q^2 D_0\right].$$

Inverting the matrix, we explicitely find the expressions for $D_{11}, D_{12,} D_{13}$ (142).

Diagram 6. The equation for the $D_{11}, D_{12,} D_{13}$ form factors in this case is:

$$\begin{pmatrix} \xi_6^{(1)} \\ \xi_6^{(2)} \\ \xi_6^{(3)} \end{pmatrix} = \begin{pmatrix} m^2 & -m^2\cosh\psi & m^2\cosh\theta \\ -m^2\cosh\psi & m^2 & -m^2\cosh\alpha \\ m^2\cosh\theta & -m^2\cosh\alpha & m^2 \end{pmatrix} \begin{pmatrix} D_{11} \\ D_{12} \\ D_{13} \end{pmatrix}, \tag{202}$$

where

$$\xi_5^{(1)} = \frac{1}{2}\left[C_0(1,3,4) - C_0(2,3,4)\right],$$

$$\xi_5^{(2)} = \frac{1}{2}\left[C_0(1,2,4) - C_0(1,3,4) - q^2 D_0\right],$$

$$\xi_5^{(3)} = \frac{1}{2}\left[C_0(1,2,3) - C_0(1,2,4) + q^2 D_0\right].$$

Inverting the matrix, we explicitely find the expressions for $D_{11}, D_{12,} D_{13}$ (151).

Diagram 7. The equation for the C_{11}, C_{12} form factors in this case is:

$$\begin{pmatrix} \xi_7^{(1)} \\ \xi_7^{(2)} \end{pmatrix} = \begin{pmatrix} m^2 & -m^2\cosh\psi \\ -m^2\cosh\psi & m^2 \end{pmatrix} \begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix}, \tag{203}$$

where

$$\xi_7^{(1)} = \frac{1}{2}\left[B_0(1,3) - B_0(2,3)\right],$$

$$\xi_7^{(2)} = \frac{1}{2}\left[B_0(1,2) - B_0(1,3) - q^2 C_0\right].$$

Inverting the corresponding matrix, we finally obtain the answer (158).

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