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GENERALISATION OF OPACITY FORMULAS FOR NEUTRON TRANSPORT

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INTRODUCTION

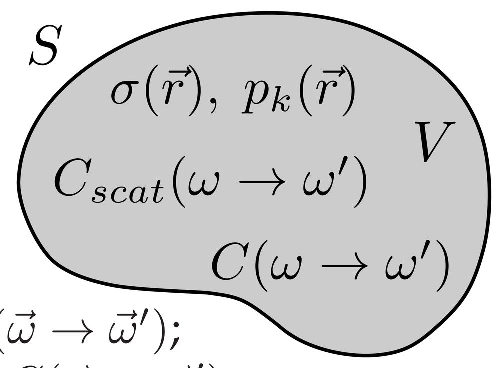
In the context of reactor physics, one is often called to relate the physical properties of a medium to the statistics of the random trajectories of the neutrons flowing through it. To this aim, we focus on the **Cauchy's formula**, which establishes a link between the average length of the neutron paths (proportional to the medium opacity) and the volume-to-surface ratio of the traversed medium. Originally established for random straight lines, this formula was recently shown to apply also to Pearson random flights [1–3], and to branching exponential flights [4,5]. In this work, thanks to a **Feynman-Kac approach** [4], we consider some extensions of such results for neutrons undergoing scattering, absorption and branching, in heterogeneous and anisotropic media. A validation of the proposed formulas via **Monte Carlo** simulations is discussed.

The simplicity of some of these opacity formulas might provide a useful tool for the validation of a Monte Carlo code.

MEDIUM AND HYPOTHESIS

Properties of the medium :

- heterogeneous,
  cross section $\sigma(\vec r)$
  probabilities $p_k(\vec r)$;
- anisotropic:
  scattering kernel $C_{scat}(\vec\omega \to \vec\omega')$;
  isotropic fission kernel $C(\vec\omega \to \vec\omega') = cst.$

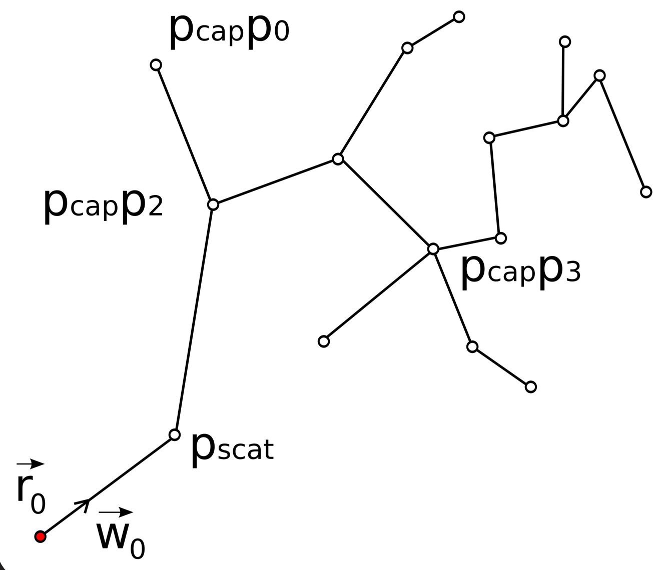


Hypotheses:

- the velocity v of the neutron is assumed constant along the trajectory;
- media have vacuum boundary conditions.

BRANCHING EXPONENTIAL FLIGHTS

The paths performed by neutrons are random in nature, and can be modelled by resorting to a stochastic process: the **branching exponential flights**.



At each time step dt, the neutron :

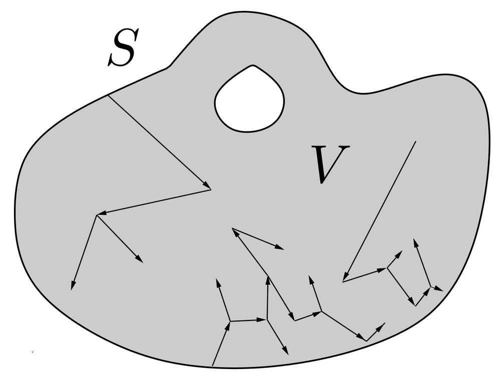
- collides with a nucleus of the medium, with a probability $\sigma\, v\, dt$
- has no interaction with the medium and goes straight ahead, with a probability $(1 - \sigma\, v\, dt)$

When neutrons interact with the medium, they

- are scattered on a nucleus with a probability p_{scat} and a kernel C_{scat}
- are captured by a nucleus with p_{capt} and then $k_{,k\geq 0}$ neutrons are emitted by the nucleus with a probability p_k in a direction given by C.

INTERESTING QUANTITIES

The **opacity** Ω is proportional to the average length travelled in the volume V by the particles that are injected uniformly at its surface S : $\langle L\rangle_S$.



Two other observables are also relevant,

- total **number of** occured **collisions** in V,
- the **survival probability** S_t, which is the probability that at time t at least one particle is still in V.

BACKWARD EQUATION FOR THE m-TH MOMENT OF THE TRACE L^m

We followed the lines of the Feynman-Kac formalism and found the backward equation for $L^m(\vec r_0, \vec\omega_0, t)$:

$$\frac{1}{v}\frac{\partial L^m}{\partial t} = \mathcal{L}^* L^m + \underbrace{m\, V(\vec r_0, \vec\omega_0)\, L^{m-1}}_{past\ history} + \underbrace{\sigma(\vec r_0)\, p_{cap}(\vec r_0) \sum_{j=2}^{m} \nu_j(\vec r_0)\, B_{m,j}[C^*\{L^i\}]}_{purely\ branching\ term}, \tag{1}$$

where $$\mathcal{L}^* = \underbrace{\vec\omega_0.\vec\nabla_{\vec r_0} - \sigma(\vec r_0)}_{streaming} + \underbrace{\sigma(\vec r_0)\, p_{cap}(\vec r_0)\, \nu_1(\vec r_0)\, C^*\{.\}}_{emission\ after\ capture} + \underbrace{\sigma(\vec r_0)\, p_{scat}(\vec r_0)\, C^*_{scat}\{.\}}_{scattering}.$$

$B_{m,j}[z_i] = B_{m,j}[z_1, \cdots, z_{m-j+1}]$ are the Bell's polynomials.

$\nu_j(\vec r_0) = \langle k(k-1)\cdots(k-j+1)\rangle$ are the falling factorial moments.

$\nu_1(\vec r_0) = \sum_k k\, p_k(\vec r_0)$ is the average number of descendants per generation.

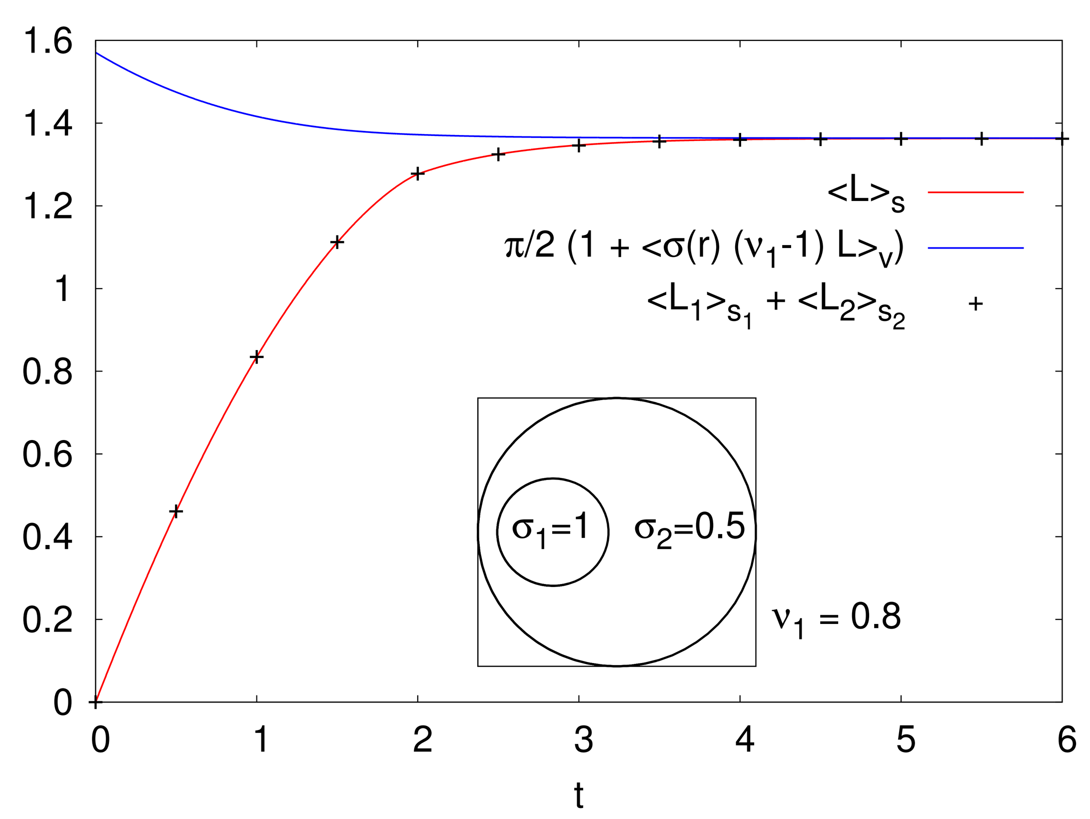
STATIONARY MOMENTS AND AVERAGES: TRAVELLED LENGTH $\langle L^m\rangle$

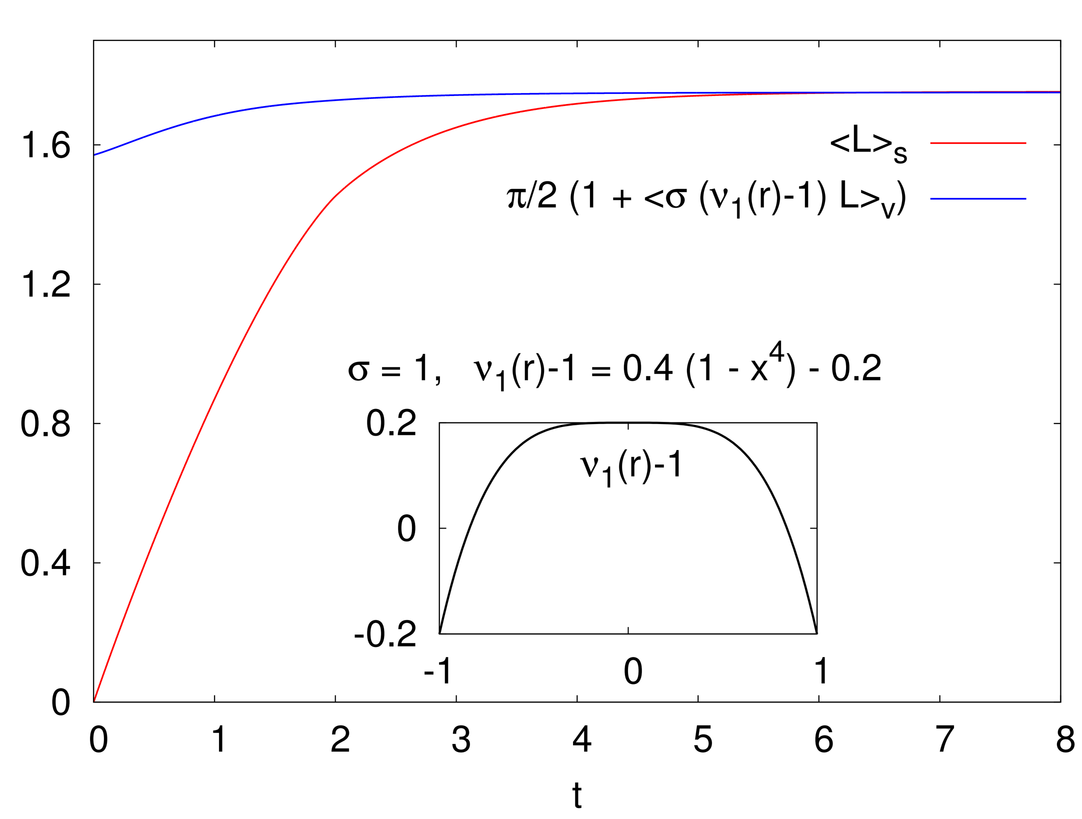
If the particle losses due to absorptions in V and leakages from the boudaries are larger than the gain due to population growth, then the limit $\lim_{t\to+\infty} L^m(\vec r_0, \vec\omega_0, t)$ exists. In this case, we obtain the equation for the **stationary moments** of the travelled lenght L^m. By integrating this equation uniformly over all initial positions and directions in the volume V, we get:

$$\underbrace{\langle L^m\rangle_S}_{entering\ flux} = \eta_d \frac{V}{S}\Bigg[\underbrace{m\,\langle L^{m-1}\rangle_V}_{past\ history} - \underbrace{\langle\sigma(\vec r_0) L^m\rangle_V}_{loss\ by\ collisions} + \underbrace{\langle\sigma(\vec r_0)\, p_{scat}(\vec r_0)\, C^*_{scat}\{L^m\}\rangle_V}_{scattering} + \underbrace{\langle\sigma(\vec r_0) p_{cap}(\vec r_0)\nu_1(\vec r_0) L^m\rangle_V}_{emission\ after\ capture} + \underbrace{\sum_{j=2}^{m}\langle\sigma(\vec r_0)\, p_{scat}(\vec r_0)\, \nu_j(\vec r_0)\, B_{m,j}[C^*\{L^i\}]\rangle_V}_{purely\ branching\ term}\Bigg]. \tag{2}$$

AVERAGE LENGTH $\langle L\rangle$ - GENERAL CASE

$$\langle L\rangle_S = \eta_d \frac{V}{S}\Big(1 + \langle\sigma(\vec r_0)\,(\mathcal{V}-1)\, L\rangle_N\Big), \qquad \text{where } \mathcal{V} = p_{cap}(\vec r_0)\,\nu_1(\vec r_0) + p_{scat}(\vec r_0)\, C^*_{scat}\{.\}. \tag{3}$$





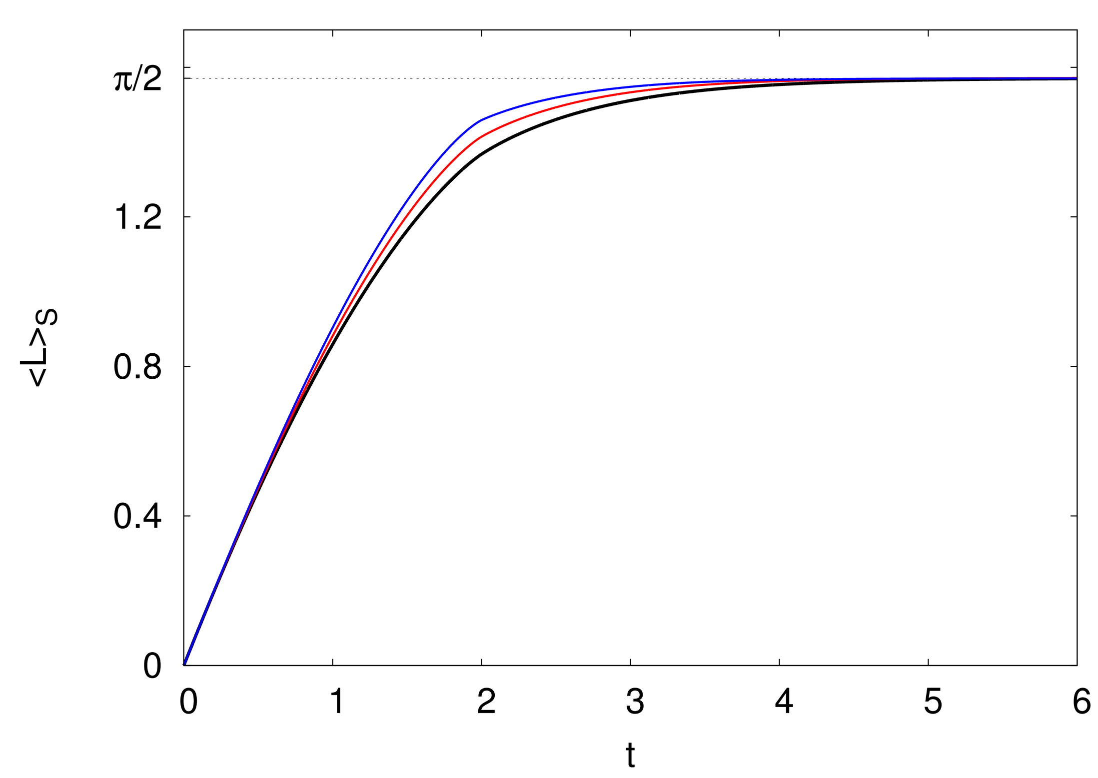
CRITICAL CASE, $\nu_1 = 1$

In this case, the formula (3) can be simplified. Then we recover the Cauchy formula's:

$$\langle L\rangle_S = \eta_d \frac{V}{S}.$$

- Homogeneous medium (black): $\sigma = 1$
- Heterogeneous medium (red): $\sigma_{1|2} = 1|0.5$
- Anisotropic medium (blue): $\sigma = 1$

$$\omega' = \begin{cases} \text{uniform}(0, 2\pi) & \text{with proba } 0.5 \\ \omega & \text{else} \end{cases}$$



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