Asymmetric Lévy flights in the presence of absorbing boundaries[1]

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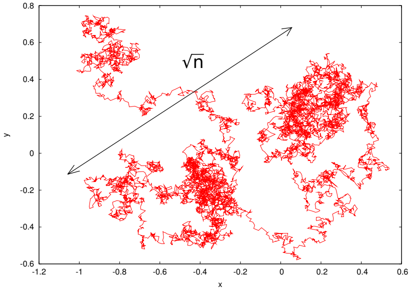
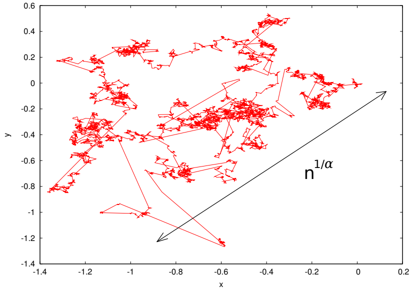
[1] J. Stat. Mech. (2013) P10006

Introduction

$$\begin{cases} x(0) = 0 \\ z(0) = 0 \end{cases} \qquad \begin{cases} x(n) = x(n-1) + \eta_x(n) \\ z(n) = z(n-1) + \eta_z(n) \end{cases}$$





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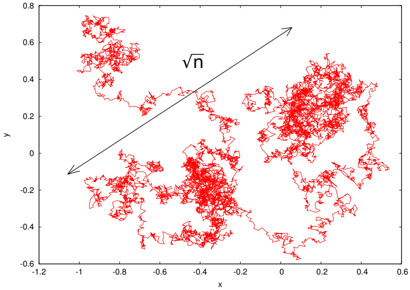

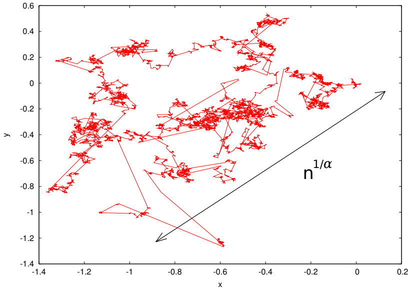

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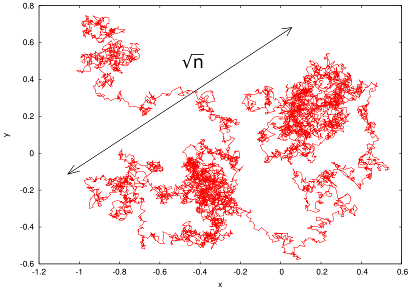

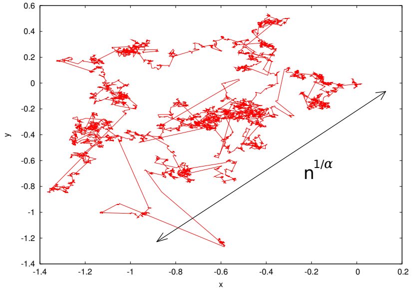

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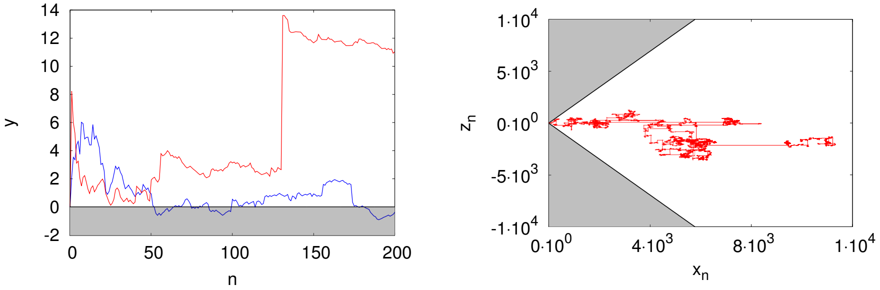
$$\phi(\eta) \sim \begin{cases} \dfrac{c}{\eta^{\alpha+1}}\,, & \text{for } \eta \longrightarrow +\infty \\ \dfrac{c/\gamma}{|\eta|^{\alpha+1}}\,, & \text{for } \eta \longrightarrow -\infty \end{cases}$$

Survival probability



$$q_+(n) = \text{Prob.}[y(n) \geq 0, \cdots, y(1) \geq 0 | y(0) = 0]$$

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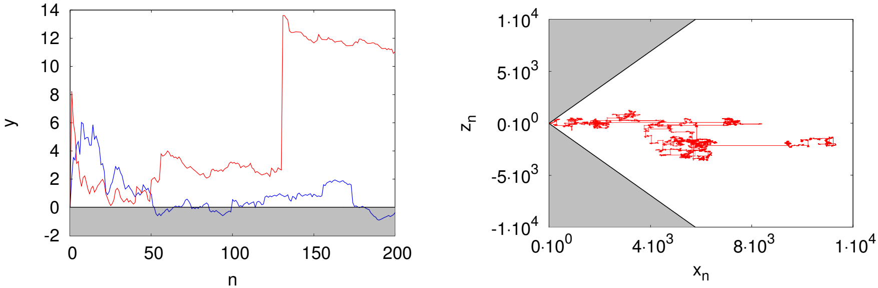
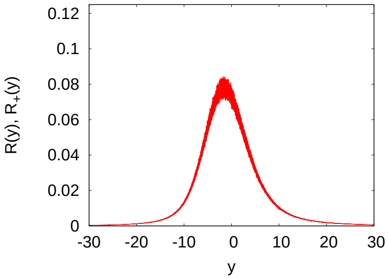


$$q_+(n) = \text{Prob.}[y(n) \geq 0, \cdots, y(1) \geq 0 | y(0) = 0]$$

$$q_+(n) \underset{n\to\infty}{\propto} n^{-\theta_+}, \qquad \text{1D} \quad \theta_+ = \frac{1}{2} - \frac{1}{\pi\alpha} \arctan\left(\beta \tan\left(\frac{\pi\alpha}{2}\right)\right), \quad \alpha \neq 1$$

2D Open problem

Position of a walker after n steps



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$$R(y) \sim \begin{cases} \dfrac{c}{y^{\alpha+1}}, & y \to +\infty \\ \dfrac{c/\gamma}{|y|^{\alpha+1}}, & y \to -\infty \end{cases}$$

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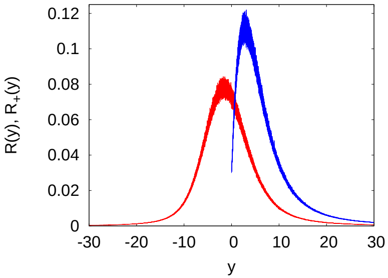

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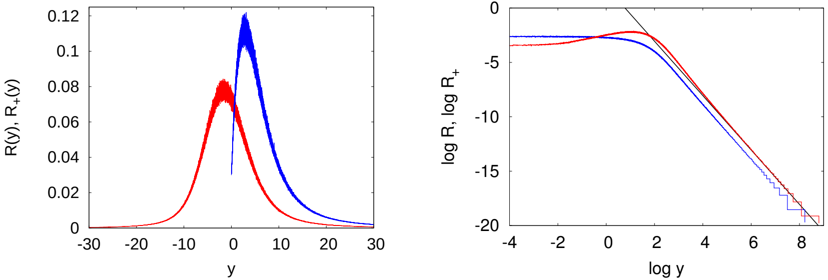

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$$R_+(y) \sim \frac{c_+}{y^{\alpha+1}}, \qquad c_+ = \frac{c}{1-\theta_+}$$