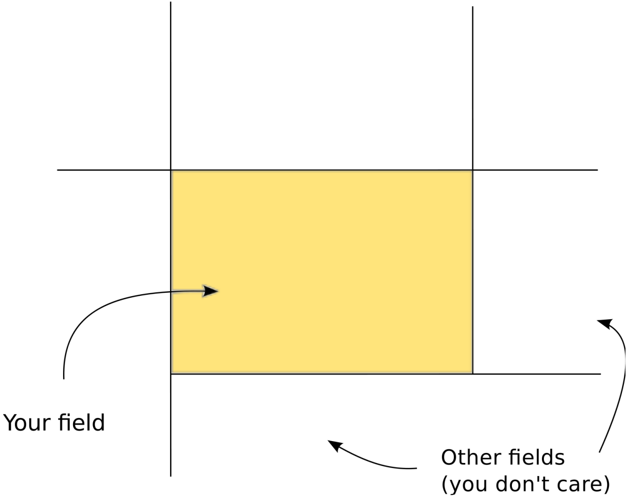
Universal properties of branching random walks in confined geometries

Idea of the problem...



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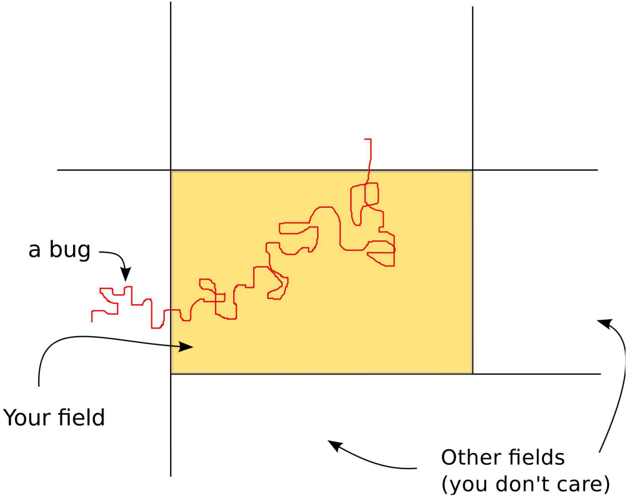

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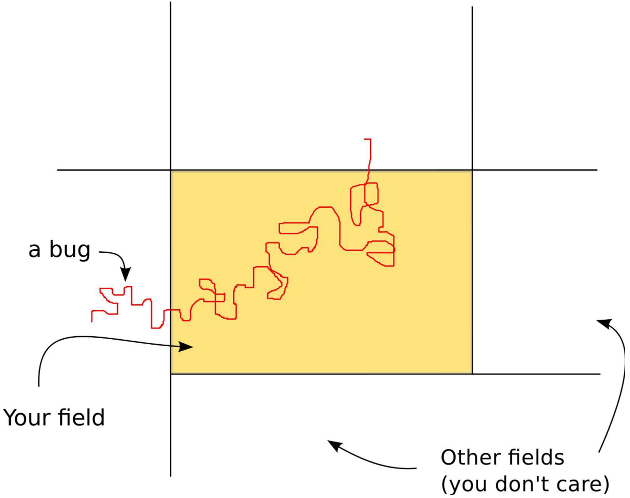
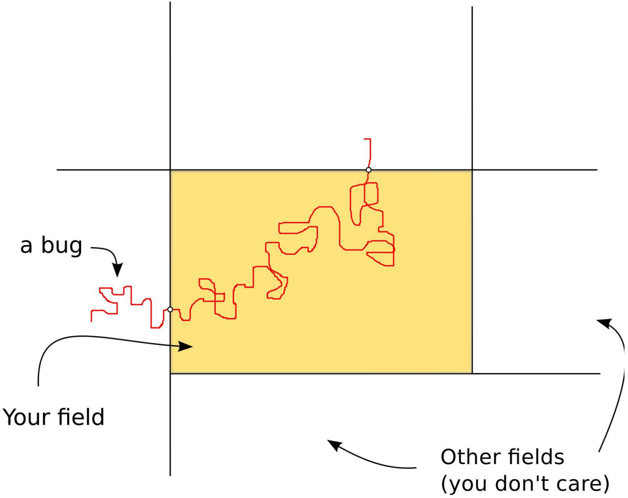


The quantity of corn that the bug eats is proportional to the lenght ℓ that it travels in the field.

Idea of the problem...

What is the mean lenght that a bug can travel inside the field ?



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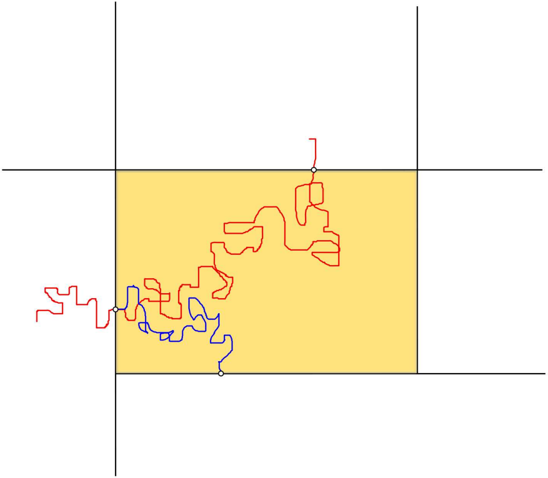

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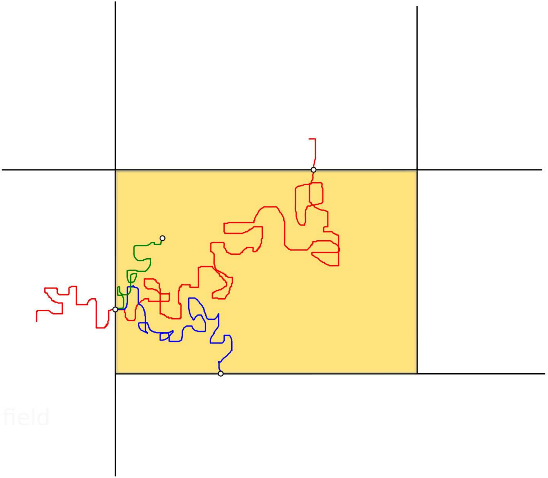

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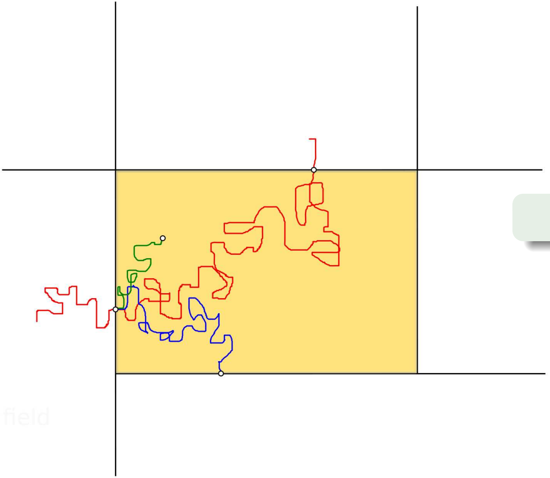

$$L = \langle \ell \rangle_{trajectories}$$

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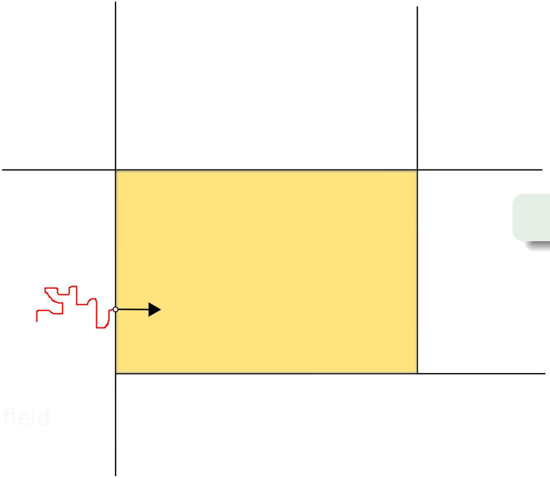

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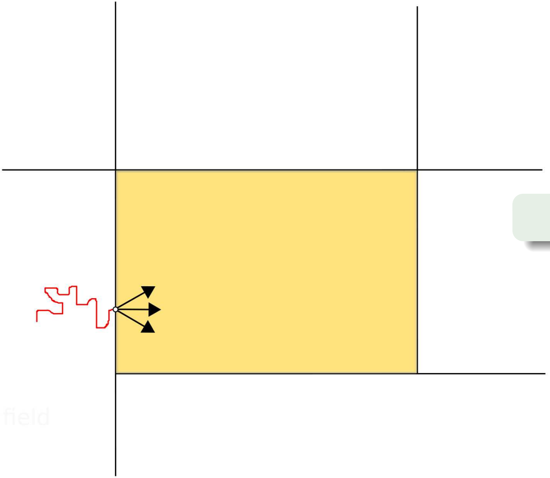

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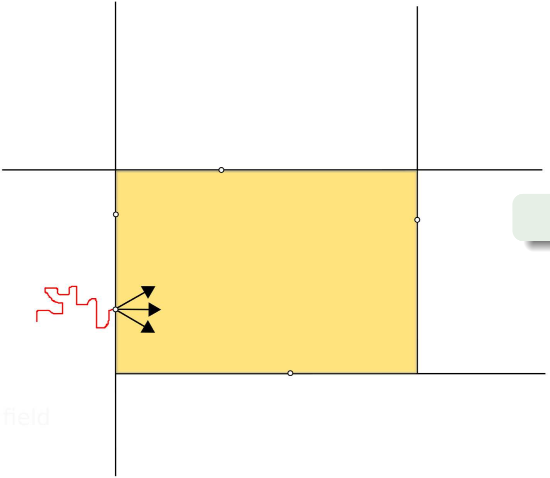

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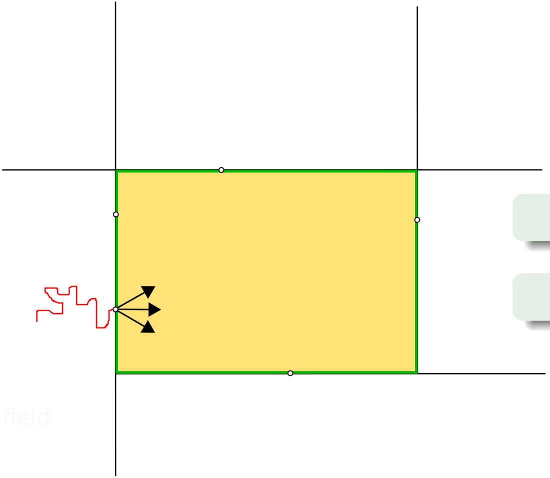

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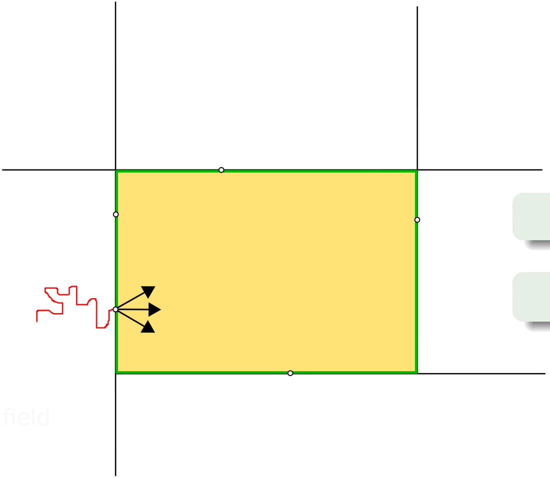

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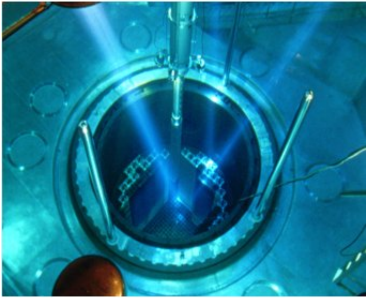
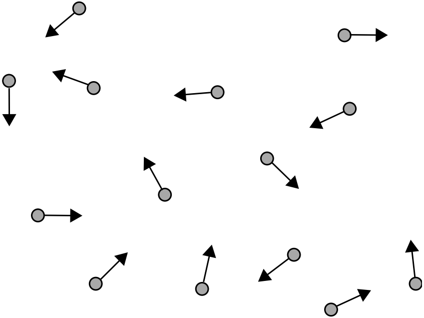
... in the field of nuclear reactors





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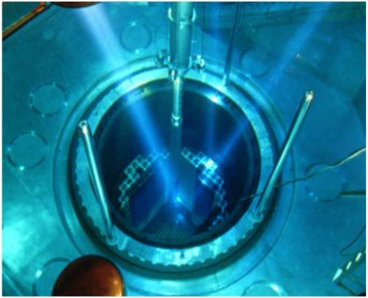

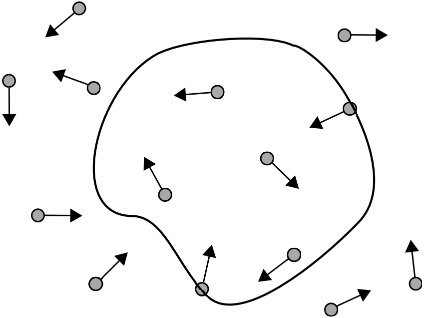

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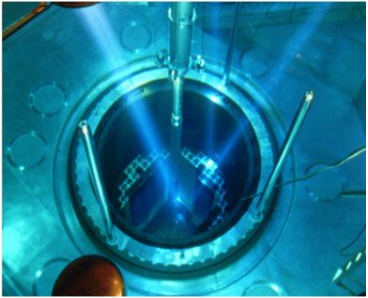
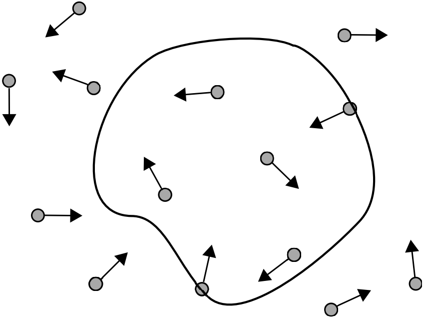




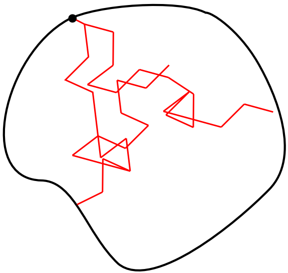
The power delivered by a certain region of the reactor is proportional to the mean lenght travelled by neutrons inside.

... in the field of nuclear reactors

At each collision with a nucleus of the medium, the neutron

- can be scattered by the nucleus
- can be absorbed. Then in case of a fission, this gives rise to 1, 2, or 3 or more neutrons, with different probabilities

$\longrightarrow$ Branching random walks



Describe the walk...

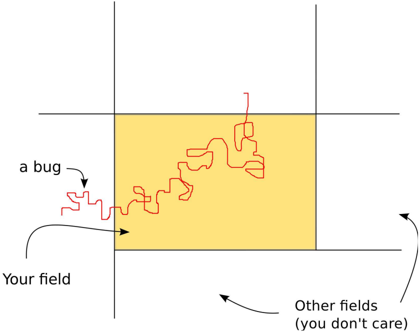
- At every scattering, the new direction is chosen uniformly
- the lenght r travelled between two collisions is given by $p(r)$

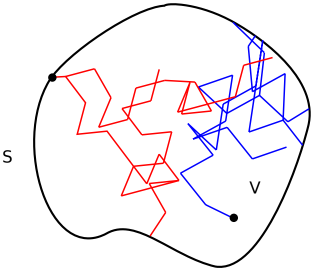
Most of the time we consider that the process is markovian (the bug/neutron has "no memory"), which gives, in a homogeneous medium, an exponential form for p :

$$p(r) = \sigma\, \mathrm{e}^{-\sigma r} ,$$

where $1/\sigma$ is the mean of p.
σ can be seen as the cross section of the medium.

... and first result, known since a "long" time





Purely diffuse problem

$$\begin{cases} \langle L \rangle_S & = \eta_d \dfrac{V}{S} \\ \langle N \rangle_S & = \sigma \langle L \rangle_S \end{cases}$$

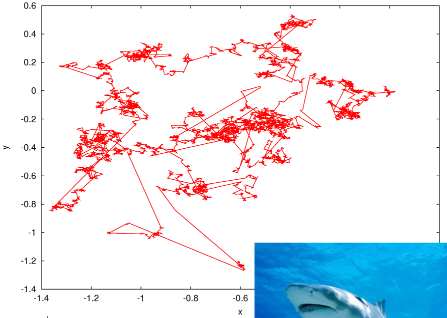
Bénichou et al., *EPL*, **70** 42 (2005)
Blanco et Fournier, *PRL*, **97** 230604 (2006)

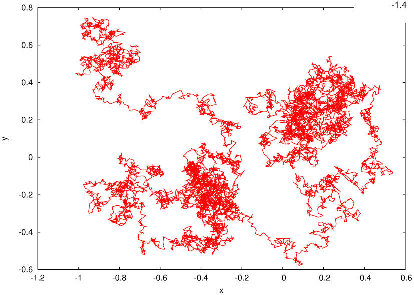
with absorption and branching

$$\begin{cases} \langle L \rangle_S & = \eta_d \dfrac{V}{S} \left[1 + \sigma(\nu - 1) \langle L \rangle_V \right] \\ \langle N \rangle_V & = \sigma \langle L \rangle_V \end{cases}$$

Zoia, Dumonteil, Mazzolo, *EPL*, **100** 40002 (2012)

But...


0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
y
-1.4
-1.2
-1
-0.8
-0.6
x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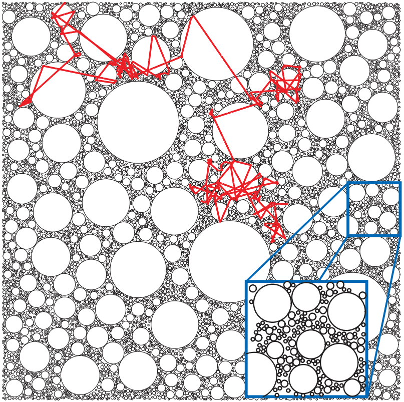
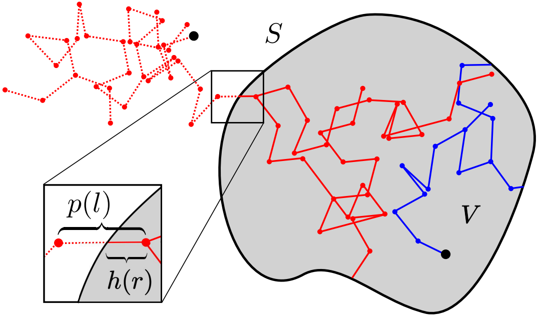


Svensson T., Vynck K., Adolfsson E., Farina A., Pifferi A. and Wiersma D. S., *PRE*, **89** 022141 (2014)

Solution...

The processus is not markovian anymore. We can not stop the walker at the border and restart it as if he actually starts from the border !

Thus the first step is not given by $p(r)$ anymore.



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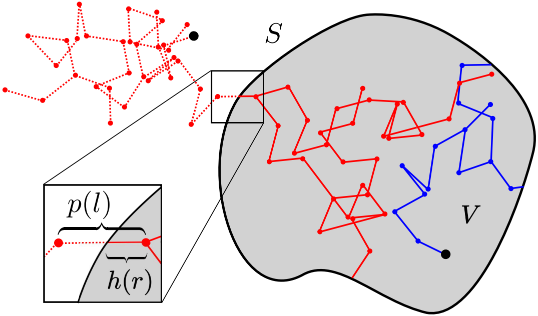

Real first step

$$h(r) = \sigma \int_r^{+\infty} p(\ell)\, \mathrm{d}\ell$$

Last frame

Purely diffusive process

$$\begin{cases} \langle L \rangle_S = \eta_d \dfrac{V}{S} \\ \langle N \rangle_S = \sigma \langle L \rangle_S \end{cases}$$

with absorption and branching

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