Instability and periodic reconstruction of the vapor-liquid interface

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Outline.

1. Conventional instabilities
2. Self-limited instabilities
3. Resume
Kelvin-Helmholtz instability

\[ \omega = k \frac{\rho_1 V}{(\rho + \rho_1)} \pm \left[ \frac{k g (\rho - \rho_1)}{(\rho + \rho_1)} - \frac{k^2 \rho \rho_1 V^2}{(\rho + \rho_1)^2} + \frac{\alpha k^3}{(\rho + \rho_1)} \right]^{1/2} \]

\[ V^4 \leq \frac{4 \alpha g (\rho - \rho_1)(\rho + \rho_1)^2}{\rho^2 \rho_1^2} \]
Benard instability

\[ R_{\alpha L} = \frac{g \beta}{\nu \gamma} (T_b - T_u) L^3 \]
Title: Bénard cells convection.ogv
Author: WikiRigaou
Date: 2005
Self-limited instability for charged helium surface.
Surface states

\[ V(z) = -\frac{\Lambda e^2}{z} + eE_\perp z \]

\[ \Lambda = \frac{\epsilon - 1}{4(\epsilon + 1)} \]
Wave functions

\[
\left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} - \frac{\Lambda e^2}{z} + eE_\perp z \right] \psi_n(z) = \epsilon_n \psi_n(z)
\]

\[E_\perp = 95 \text{ V/cm}\]
Spectroscopy of electrons in image-potential–induced surface states outside liquid helium

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(Received 4 August 1975)
Magnetoplasmons

Graph showing mode frequency vs. $\omega_c$ (MHz) for various $l$ values.
Phonon-Ripplon Coupling and the Two-Dimensional Electron Solid on a Liquid-Helium Surface

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FIG. 1. Schematic of the dispersion relation of the longitudinal coupled modes (solid curves). The dashed lines show the uncoupled mode spectra and the vertical lines represent the wave vectors excited in the experiment. The resonances are labeled as in Ref. 3.
Evidence for a Liquid-to-Crystal Phase Transition in a Classical, Two-Dimensional Sheet of Electrons

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FIG. 1. Experimental traces displaying the sudden appearance with decreasing temperature of coupled plasmon-ripplon resonances. The resonances only appear below 0.457 K where the sheet of electrons has crystallized into a triangular lattice.
FIG. 2. Portion of the solid-liquid phase boundary for a classical, two-dimensional sheet of electrons. The data points denote the melting temperatures measured at various values of the electron areal density, $N_s$. Along the line, the quantity $\Gamma$, which is a measure of the ratio of potential energy to kinetic energy per electron, is 137.
Dispersion law and exp data for a charged helium surface

\[ \omega^2 = \frac{k}{\rho} \left[ \rho g + \alpha k^2 - k \frac{E_+^2 + E_-^2}{4\pi} \right] \]

\[ \omega(k) = 0, \quad \frac{\partial \omega^2}{\partial k} = 0. \]

\[ \kappa^2 = \frac{\rho g}{\alpha} \]

\[ (n_s^{\text{max}})^2 = \frac{\kappa \alpha}{2\pi e^2} \]

\[ E_+^{\text{max}} = \frac{4\pi e n_s^{\text{max}}}{\kappa} \]

Закон дисперсии риплонов на поверхности раздела фаз Не$^3$-Не$^4$; пунктирные линии - теория, эксперимент - данные для $E/E_- = 0.12, 0.71, 0.99$. 
Dispersion law minimum versus pressing electric field
Interferogram of charged helium surface just below and above critical field
Individual multi electron dimple
Total charged dimple energy

\[ W = \int d^2 \tau \left[ \frac{\alpha}{2} (\nabla \xi)^2 + \kappa^2 \xi^2 \right] + eE_{\perp} n(r) \xi + e^2 \int d^2 r' \frac{n(r)n(r')}{|r - r'|} \]

\[ W = \frac{Q^2 E_{\perp}^2}{8\pi \alpha} \exp \left( \frac{\kappa^2 R^2}{2} \right) Ei \left( -\frac{\kappa^2 R^2}{2} \right) + V_c(R). \]

\[ n(r) = \frac{N}{\pi R^2} \exp \left( -\frac{r^2}{R^2} \right) \quad V_c(R) = C_0 Q^2 / R, \]
Total multi electron dimple energy $W$ versus of core radius $R$ for different pressing electric fields
Single multielectron dimple profile
Dimple depth versus $E_{\perp}$
Time evolution of corrugated state
Dimple “company” versus filling factor
Общая фазовая диаграмма
Resume.

1. Instability of the charged helium surface is really self-limited.
2. There are some additional quite impressive examples of self-limited instability.
Single electron bubble wave function
Single electron bubble geometry

\[ R_e \approx 17 \text{A} \]

\[ M_{\text{ass}} = \frac{2}{3} \pi R_e^3 \rho_l \approx 200m_4 \]
Free electron mobility versus of helium gas density
Individual multi electron bubble

\[ W = \frac{Q^2}{R} + 4\pi R^2 \sigma \]

\[ \frac{\partial W}{\partial R} = 0 \quad \text{или} \quad R^3 = \frac{Q^2}{8\pi \sigma} \]

Такой bubble неустойчив?
Multi-bubble in presence of Coulomb crystallisation
Set-up for multi bubbles preparation

Fig. 1. – Schematic set-up of the experiment.
Multi bubble visible picture
Anode current peaks from the bubbles
Instability phenomena
Charged vortexes rings
Drift velocity versus driving electric field
Vortex ring velocity versus energy of this cluster
FIG. 3. 2DEG edge at magnetic field corresponding to the bulk filling factor $\nu_0 = 1.5$. The dashed line is the electron density at zero magnetic field. The solid line is the electron density, and the thick line is the electrostatic energy at $\nu_0 = 1.5$. 
Electrostatics for 1D channel

\[ E_x(\zeta) = \text{Re} \left[ \left( \frac{\zeta^2 - b^2}{d^2 - \zeta^2} \right)^{1/2} \right] \frac{2\pi en_s}{\kappa} \]

\[ E_z(\zeta) = \text{Im} \left[ \left( \frac{\zeta^2 - b^2}{d^2 - \zeta^2} \right)^{1/2} - i \right] \frac{2\pi en_s}{\kappa} \]

\[ \zeta = x + iz \]
H=1.02 .................. H=1.05
Ethylene-water binary solution
Spinodale instability
Binodal and spinodal definitions

\[ \mu_1 = \mu_2 \]  
Binodal line

\[ \frac{\partial \delta \mu}{\partial n} = 0 \]  
Spinodal line

\[ \frac{\partial n}{\partial t} + \text{div } j = 0 \]

\[ j = -D \nabla \mu \]

\[ \delta \mu = 0 \]

Diffusion coef. \( D \) has the sign-inversion.