Fragmented-condensate solid of dipolar excitons

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Plan:

1. Introduction: dipolar excitons in semiconductor quantum wells
2. 2002 discovery of the MOES
3. Theory of dipolar 1D BEC (Part I): stable dilute supersolid
4. Theory of dipolar 1D BEC (Part II): fragmented condensate
5. Comparison with the experiment
Wannier-Mott excitons in direct band-gap semiconductors

\[ \psi(\vec{r}_e, \vec{r}_h) = e^{iK\vec{R}} \phi(\vec{r}) \]

\[ \Psi_{+1}(\vec{r}_e, \vec{r}_h, t) = \psi(\vec{r}_e, \vec{r}_h)|t\rangle \]
\[ |t\rangle = |0\rangle + ce^{-i\omega t}|+1\rangle \]
\[ |+1\rangle = S(\vec{r}_e)|-1/2\rangle_e \otimes \frac{X(\vec{r}_h) + iY(\vec{r}_h)}{\sqrt{2}}|+1/2\rangle_h \]

Exciton-polaritons in bulk GaAs crystals

Dipolar excitons in QW's

Polariton-polariton interaction:

At low excitation power \( na_B \ll 1 \) \( \rightarrow \) dilute gas

Exciton-exciton interaction

Exciton-polaritons in bulk GaAs crystals

Dipolar excitons in QW's
Exciton BEC:

- An exciton Bose-Einstein condensate is a source of the spatially coherent PL
- Coherence of the PL is suppressed in a biexciton BEC
- The condensate is dilute in typical experimental conditions. However, dense regime is also possible. Example: dipolar excitons.
- As in atomic BEC’s, the two-body interactions play an important role.
- The dipolar interaction is long-range
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Macroscopically ordered exciton state (MOES)

Cold dipolar excitons at the ring:

$\psi(\mathbf{r}_e, \mathbf{r}_h) = e^{i K \mathbf{R} \phi(\mathbf{r})}$


[radial electric field polarizes and traps the excitons, see S. V. Andreev, PRB 94, 165308 (2016)]

Repulsive interaction in the MOES:

$\mu_X = \mu_e + \mu_h$

Calculated exciton-exciton interaction potentials:

PRB 75, 033311 (2007)


$E(\text{meV})$

PRB 78, 045313 (2008)

PL Intensity (arb. units)

$1.547$

$1.545$

Energy (eV)

Position (µm)

Valley

Sample A (d=12nm)

Sample B (d=14nm)

$d = 14 \text{ nm}$

$d = 12 \text{ nm}$

$400 \text{ mkm}$

$d_c = 8 \text{ nm}$
Experimental facts about the MOES:

- The ring consists of independent quasi-1D segments separated by defects.
- There is a critical temperature for the transition.
- Coherence length is on the order of the size of one bead and much less than the length of a segment.
- Coherence is suppressed in the centres of the beads.
- Repulsive interaction.
- The external ring has been observed by several experimental groups in the samples with $d<d_c$ and $d>d_c$.

The fragmented state of the ring has been observed only by the Butov group in the sample with $d$ approaching $d_c$ from above.

Hypothesis:

- One can use thermodynamics to describe cold exciton gases in the segments of the ring.
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Stable dilute supersolid of quasi-1D dipolar bosons

\[
\hat{H}^{1D} = \sum_{k_x} \left( \frac{\hbar^2 k_x^2}{2m} + \frac{\hbar \omega_y}{2} \right) \hat{c}^+_k \hat{c}_k + \frac{1}{2L} \sum_{k_x, p_x, q_x} V(p_x - q_x) \hat{c}^+_k \hat{c}_{k+p_x} \hat{c}^+_{k-p_x} \hat{c}_{k+q_x} \hat{c}_{k-q_x}
\]

Beliaev diagrammatic approach:
S. T. Beliaev,
Sov. Phys. JETP 34, 289 (1958)

\[ T = 0 \]

Assumptions:
- \( a_y \gg r_* \)
- \( mg^{2D} \frac{\hbar^2}{\hbar^2} < 1 \)
- \( mg^{2D} \frac{\hbar^2}{\hbar^2} \ll n_1a_y \ll \frac{\hbar^2}{mg^{2D}} \)
- \( n_1r_* \ll 1 \)

\[ f^{1D}(p_x, q_x) = \frac{g^{2D}}{\sqrt{2\pi a_y}} + \frac{\hbar^2}{2mr_*} (|p_x - q_x| r_*)^2 \ln(|p_x - q_x| r_*) \]

S. V. Andreev, PRB 92, 041117(R) (2015)

Two-body scattering problem

Elementary excitation spectrum:

Roton-maxon structure

\[ \xi = \frac{\hbar}{\sqrt{2mg^{2D} n}} \]

PRB 95, 184519 (2017)
Challenge for the theory:

\[ n_1 r_* \rightarrow \frac{1}{2 \ln(\hbar^2 a_y / mg^{2D} r_*)} \quad \leftrightarrow \quad \frac{mg^{2D}}{\hbar^2} \rightarrow \frac{a_y}{r_*} e^{\frac{1}{2n_1 r_*}} \quad n_1 r_* \ll 1 \]

Coupling constant exponentially small in the diluteness parameter - an almost non-interacting gas!

Solution: a resonantly paired binary mixture

\[ V_{\uparrow\uparrow}(x - x') \quad \text{and} \quad V_{\uparrow\downarrow}(x - x') \]

\[ d < d_c \]

\[ d \geq d_c \]

\[ \beta \quad \text{and} \quad \epsilon > 0 \]

Beliaev approach for a binary mixture:
M. Baglai, O. Utesov and S. V. Andreev paper under preparation

S. V. Andreev, PRB 94, 140501(R) (2016)
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Fragmentation of the supersolid

\[ a_y = \sqrt{\frac{\hbar}{m \omega_y}} \]

\[ m g^{2D} \ll n_y a_y \ll \frac{\hbar^2}{m g^{2D}} \]

2D Thomas-Fermi cigar

\[ R_c = a_y \left( \frac{3 m g^{2D} a_y}{2 \hbar^2} \right)^{1/3} \]

\[ \lambda \]

\[ \xi \]

\[ f^{1D}(k) = \frac{g^{2D}}{\sqrt{2 \pi a_y}} + \frac{\hbar^2}{m r_*} \left( k r_* \right)^2 \ln(k r_*) \]

\[ V^{TF}(k) \]

\[ V^{TF}_0(2k_0) \]

\[ 2k_0 R_c^{-1} \sqrt{n} \]

\[ 2 \mu \]

\[ F = E^{TF} + E_{\text{kin}} - k_B T \ln Z_\Phi \]

\[ \frac{dF}{dJ} = 0 \Rightarrow \]

\[ \Rightarrow \lambda = \frac{L}{J} = \frac{g}{R_c k_B T} \sqrt{\frac{\pi m g}{6 \hbar^2}} \frac{e}{2 \pi} \exp \left( \frac{24 x \ h^2 n}{\pi \ m k_B T} \right) \]

PRL 110, 146401 (2013)
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Comparison with the 2015 experiment

The two-body interaction constant $g$ is known from the calculations and measurements of the blueshift.

\[
\frac{dF}{dJ} = 0 \Rightarrow \lambda = \frac{L}{J} = \frac{g}{R_c k_B T} \sqrt{\frac{\pi}{6} \frac{m g}{\hbar^2} \frac{e}{2\pi} \exp \left( \frac{24 \pi \hbar^2 n}{\pi m k_B T} \right)}
\]

\[
n(V_g) = n(V_g^*) + \frac{dn(V_g^*)}{dV_g}(V - V_g^*)
\]

\[
k_B T_c^0 = \hbar \sqrt{\frac{6}{\pi^2 N_0 \omega_x \omega_y}}
\]

\[
k_B T_c^0 = 2 \sqrt{\frac{6 m g}{\pi \hbar^2} \frac{\hbar^2 n}{m}}
\]
Summary:

• The theory identifies the MOES with a form of the supersolid state of matter

• By simultaneously reducing the density and the temperature one can try to observe a transition from the fragmented to the coherent (true) supersolid state.

• The solid forms due to resonant pairing of excitons as the temperature $T$ approaches $T_c$.

At $T<< T_c$ a biexciton BEC forms in the cores of the beads.
Polarization textures in the MOES.
Spin-orbit coupled BEC of excitons at the edges of the beads.


Theory:
Acknowledgements:

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Polar molecules in the bilayer geometry

\[ V_+ (r) = \frac{\hbar^2 r_s}{m} \frac{r^2 - 2l^2}{(r^2 + l^2)^{5/2}} \]

Example: NaK molecules at \( T \sim 1 \text{ nK} \)

D. Petrov, PRL 112, 103201