



# Charge density wave sliding under the Hall voltage

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- CDW sliding in a few words
- Can the CDW slide under a transverse current?

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- Can the CDW slide under a Hall voltage?

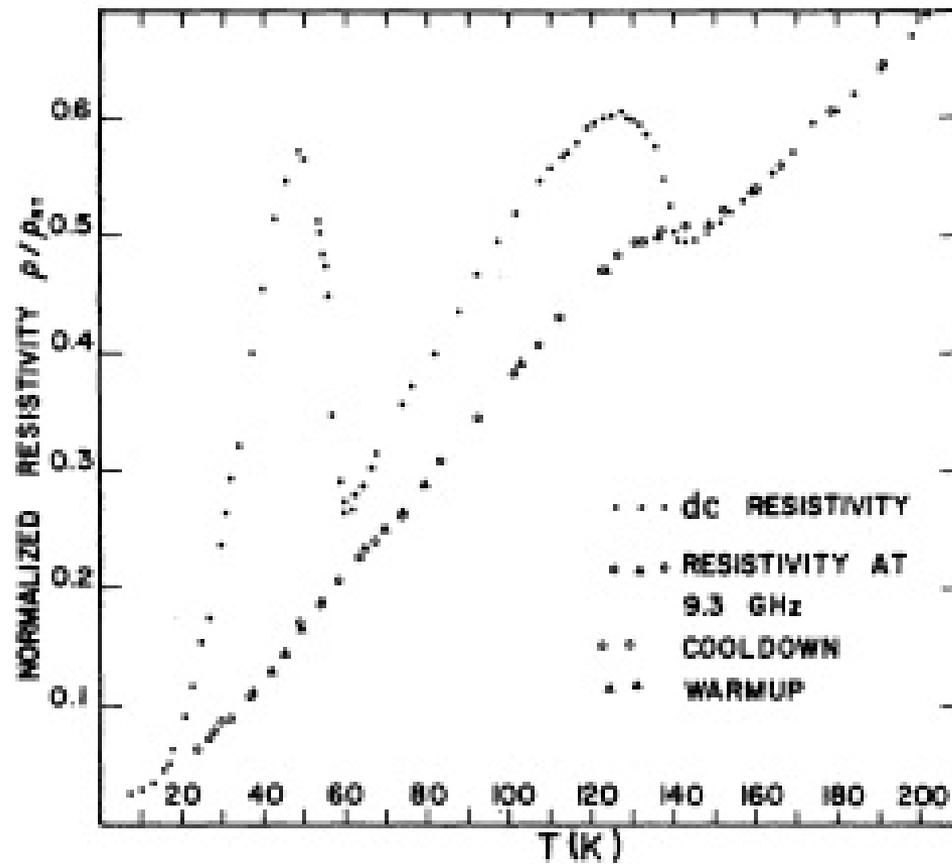
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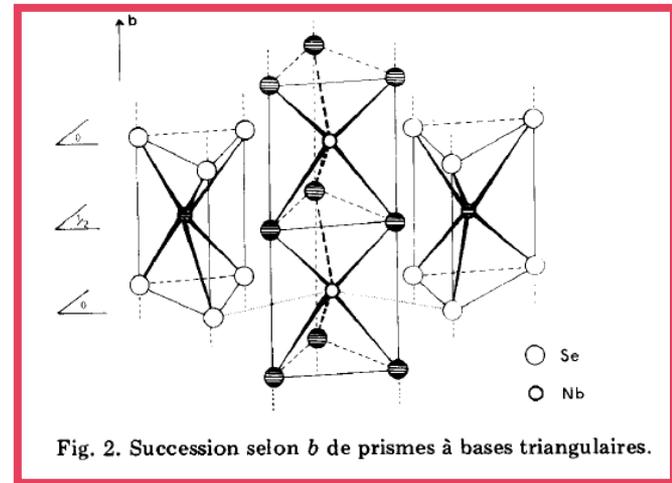
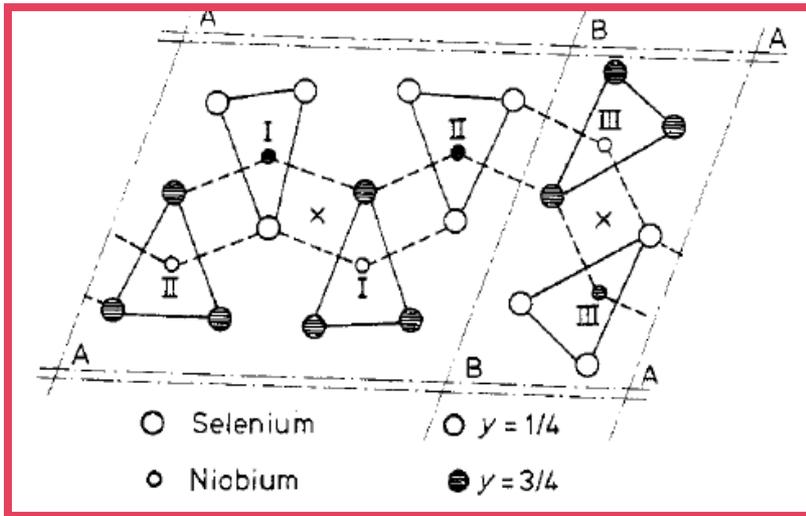
- Can the CDW slide under a Hall voltage?

Yes

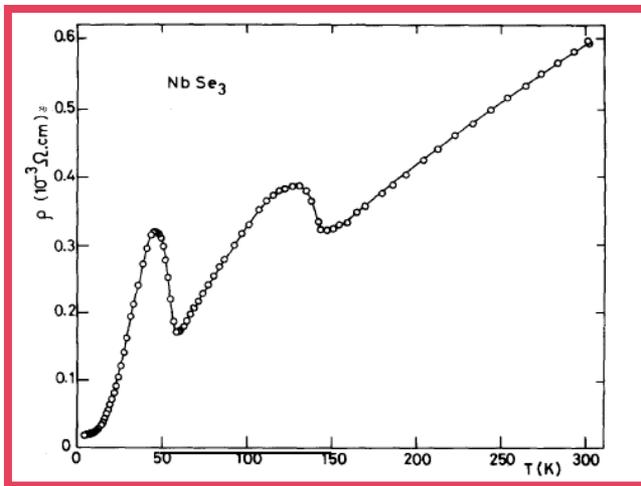


*Phys. Rev. B16 (1977) 3443*

# NbSe<sub>3</sub>

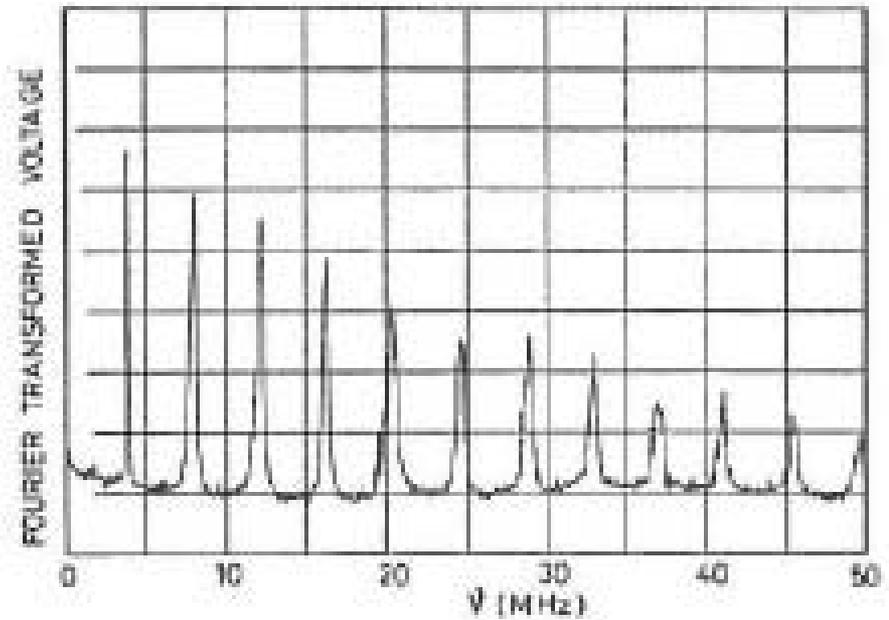
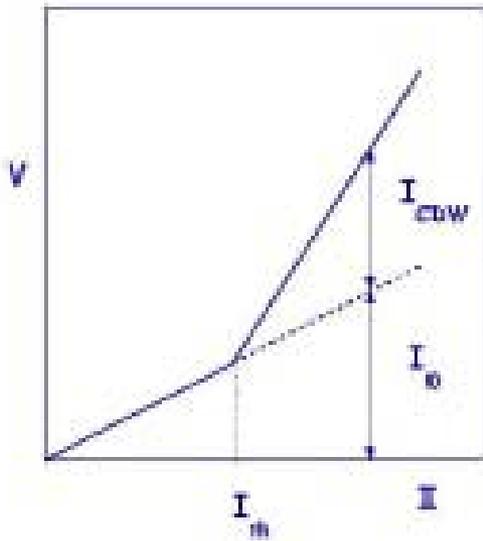


Meerschaut and Rouxel  
*Journal of Less-Common Metals* 39 (1975)197

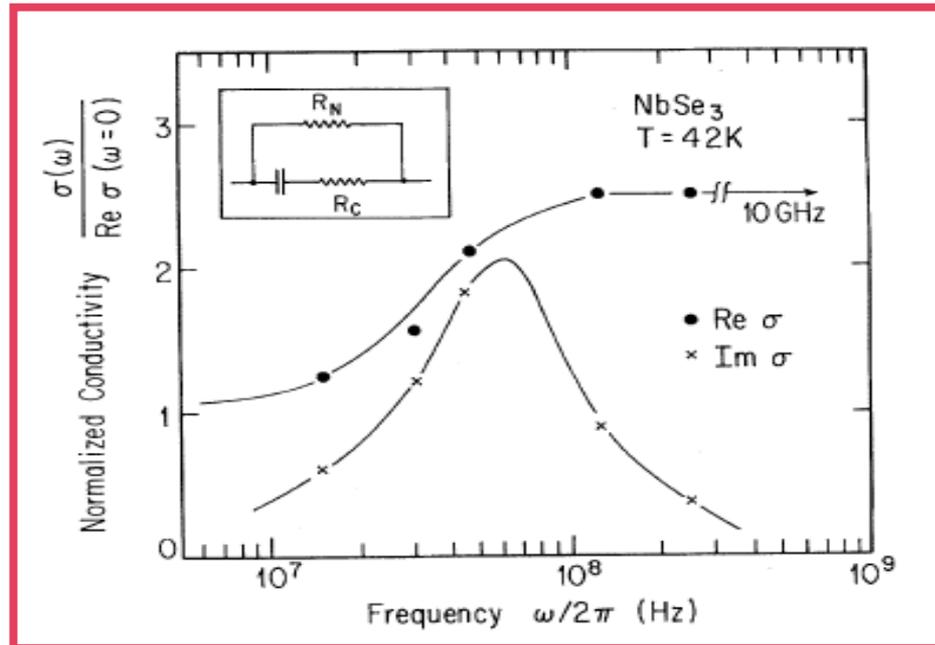


*Solid State Communications* 20(1976) 759

# Non-linearity

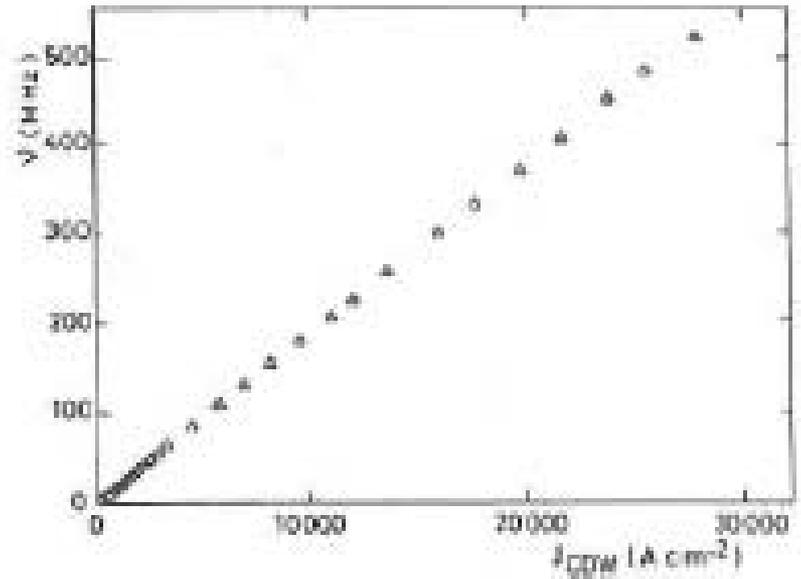
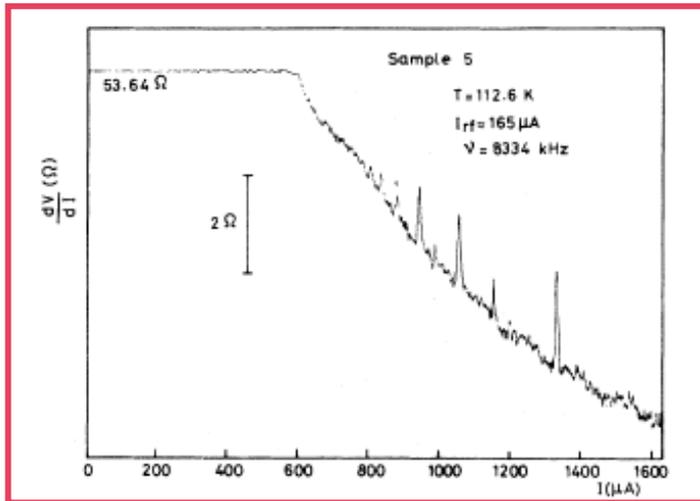


# Frequency-dependent conductivity in NbSe<sub>3</sub>



CDW pinned mode

# Interference effects of the charge-density-wave motion in NbSe<sub>3</sub>

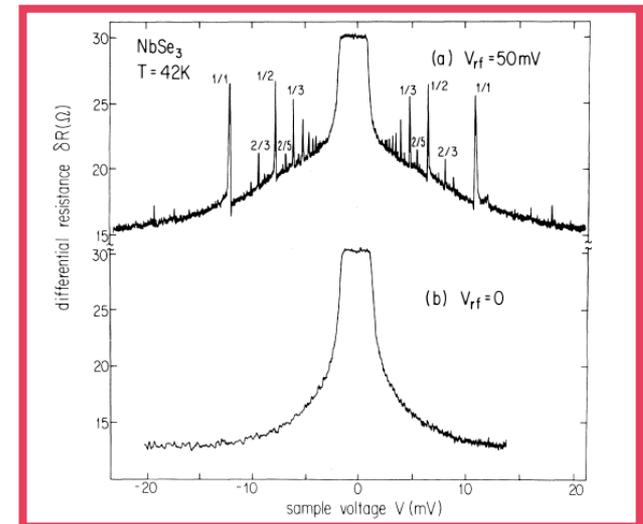


$$J_{\text{CDW}} = ne v_{\text{drift}}$$

Assuming the pinning force to be periodic with the CDW phase

$$\nu = (Q/2\pi)v_{\text{drift}}$$

$$J_{\text{CDW}} = ne \frac{2\pi}{Q} \nu = ne \lambda_{\text{CDW}} \nu.$$

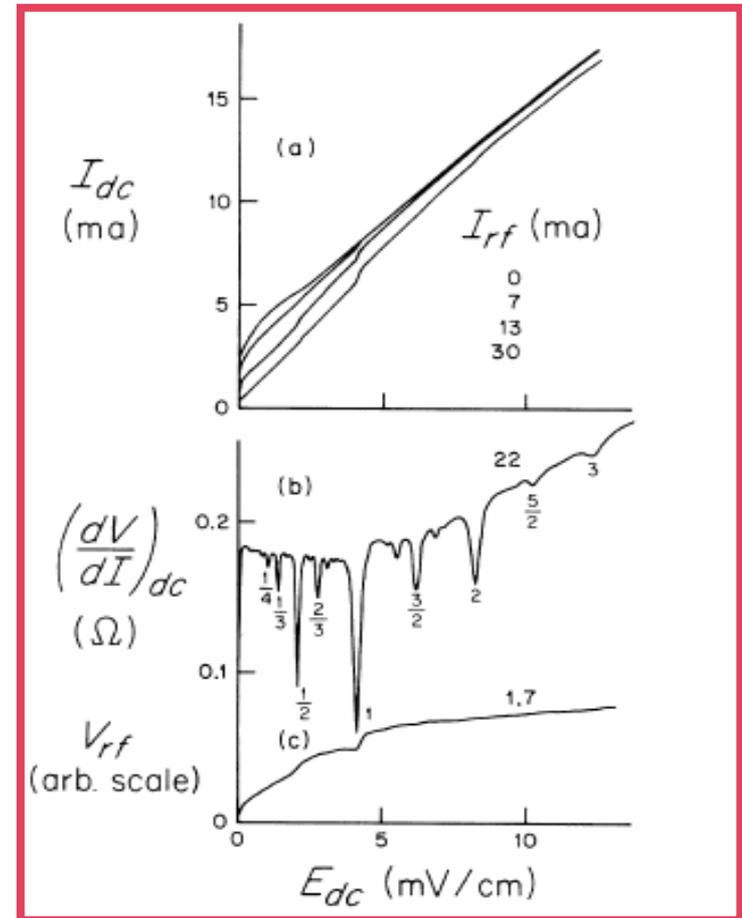


# Quantum interference of a moving vortex lattice in Al films

An experimental study is reported of steps induced in the flux-flow  $I$ - $V$  characteristics of superconducting aluminum films by a superimposed rf current, whose frequency is a harmonic or subharmonic of the ratio of the vortex velocity and the lattice parameter.

$$f = nv/\lambda$$

where  $f$  is the frequency of the rf current,  $v$  is the average vortex velocity,  $\lambda$  is the magnitude of a two-dimensional lattice vector of the vortex structure, and  $n$  is an integer. Values of  $\lambda$  which fit the data are appropriate to the triangular lattice of singly quantized vortices.<sup>1</sup>



A.T. Fiory, PRL 27 (1971) 501

## Weak versus strong coupling

**Table 1.** Qualitative comparison of weak-coupling and strong-coupling CDWs.

	Weak-coupling CDW	Strong-coupling CDW
PLD/CDW amplitude	Small	Large
Energy gap	Small ( $\Delta/E_F \ll 1$ )	Large ( $\Delta/E_F \lesssim 1$ )
Coherence length	Large ( $\xi/a \gg 1$ )	Small ( $\xi/a \gtrsim 1$ )
Electronic energy gain	Arising mostly near $k_F$ ( $\propto \Delta^2 \ln \Delta$ )	Spread over Brillouin zone ( $\propto \Delta$ )
CDW periodicity w.r.t. original lattice	Incommensurate ( $\lambda_0 = \pi/k_F$ )	Tends to be commensurate
Thermal disordering	Due to electronic entropy	Due to lattice entropy
Electron–hole pairing above $T_0$	No	Yes, but pairs are incoherent
Qualitative picture	Fermi surface instability	Local chemical bonding

*K. Rossnagel, J. Phys.:Condens.Matter 23(2011) 213001*

- CDW motion in 1D systems is observed with the applied electric field parallel to the incommensurate component of the modulation  $Q$  vector
- What about the effect of a transverse current?.

# Transversely Driven Charge-Density Waves and Striped Phases of High-Tc Superconductors: The Current-Effect Transistor

We show that a normal (single particle) current density  $J$  *transverse* to the ordering wave vector  $2k_z \hat{z}$  of a charge-density wave (CDW) has dramatic effects both above and *below* the CDW depinning transition.

We explore these in a transverse geometry in which the charge current  $\mathbf{J}$  flows *transversely* to the CDW ordering wave vector  $2k_F \hat{z}$ , i.e.,  $\mathbf{J} = J_x \hat{x}$ . The hydrodynamics of a CDW in the presence of such transverse current  $J_x$  is described by

$$\gamma(\partial_t + \tilde{\mathbf{v}} \cdot \nabla)u(\mathbf{r}, t) - K\nabla^2 u + F(\mathbf{r}, u) + e\rho_{\text{CDW}}E_z$$

The essential difference between our model Eq. (5) and the commonly used Fukuyama-Lee-Rice (FLR) [1] model is the convective  $\tilde{v}_x \partial_x u$  term, which physically arises because a tipped ( $\partial_x u \neq 0$ ) “layer” of the CDW deflects the transverse normal current downward by an angle  $\theta \sim \partial_x u$ , leading to a reaction force back on the CDW, proportional to  $\partial_x u$  and the normal current  $J_x$ .

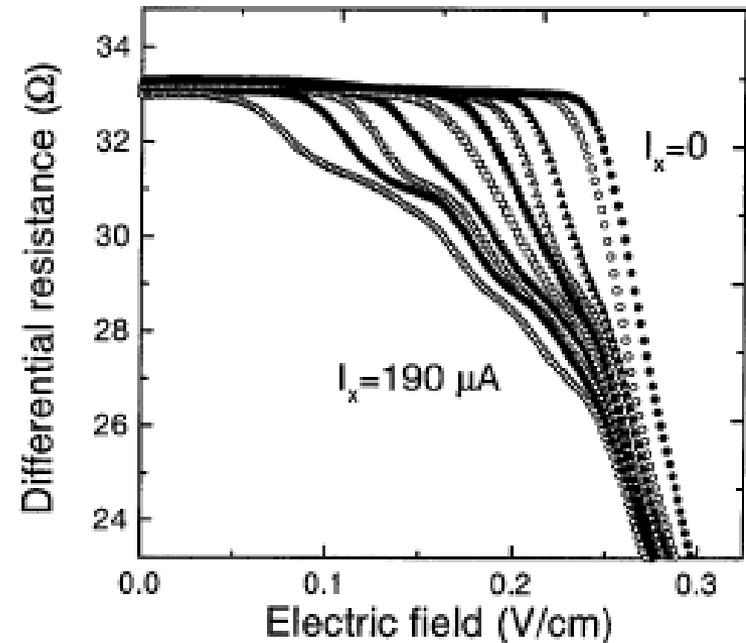
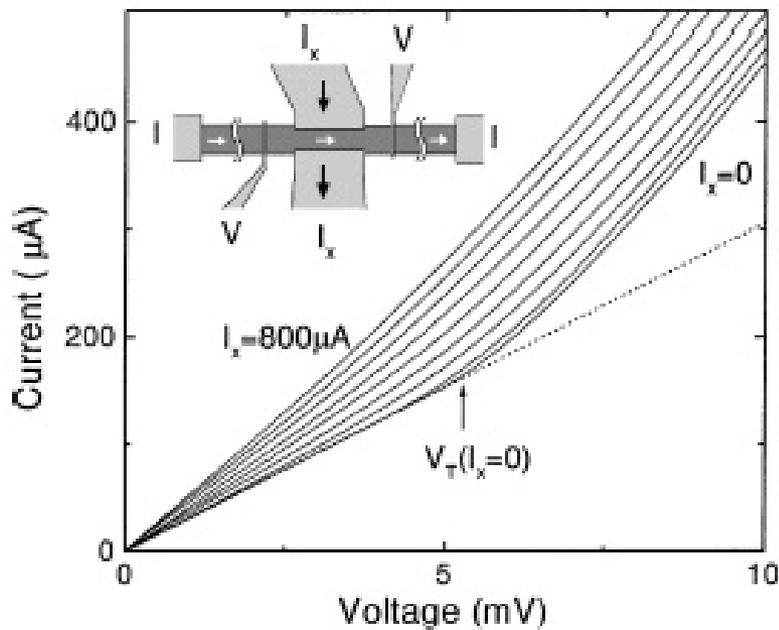
this transverse current  $J_x$  exceeds a *crossover* value  $J_c$ , it makes the CDW much more ordered. The correlation length of a three dimensional (3D) CDW along the direction ( $\hat{x}$ ) of the transverse current obeys

$$\xi_x(J_x) \approx \begin{cases} \xi_L, & \text{for } J_x < J_c, \\ \xi_L \frac{J_c}{J_x} e^{2(J_x - J_c)/J_c}, & \text{for } J_x > J_c, \end{cases} \quad (1)$$

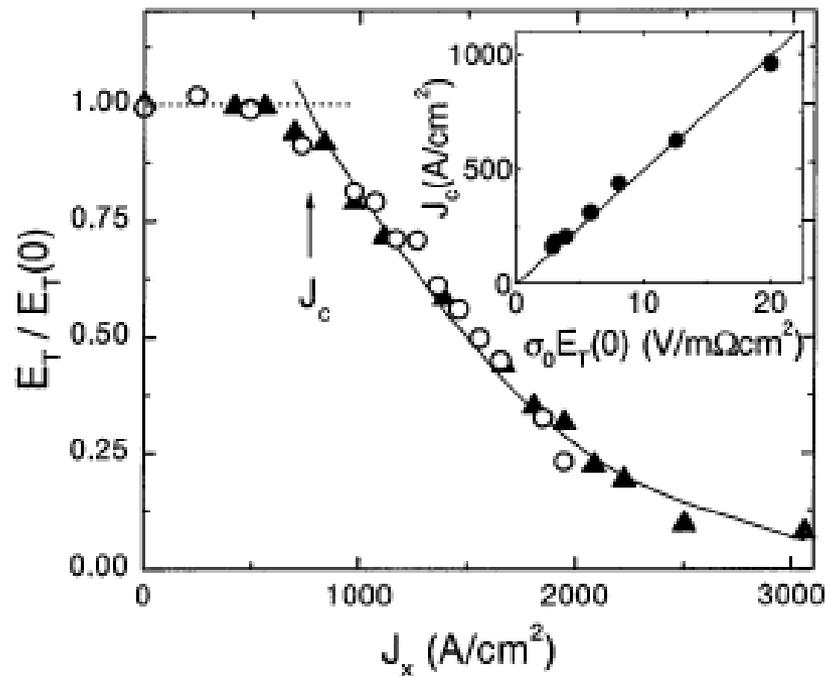
where  $\xi_L$  is the “Larkin length” (see below) of the CDW in zero transverse current, which is finite due to the random pinning of the CDW by impurities.

# Tunable Charge-Density Wave Transport in a Current-Effect Transistor

The collective charge-density wave (CDW) conduction is modulated by a transverse single-particle current in a transistorlike device. Nonequilibrium conditions in this geometry lead to an exponential reduction of the depinning threshold, allowing the CDWs to slide at much lower bias fields. The results are in excellent agreement with a recently proposed dynamical model in which “wrinkles” in the CDW wave fronts are “ironed” by the transverse current. The experiment might have important implications for other driven periodic media, such as moving vortex lattices or “striped phases” in high- $T_c$  superconductors.



Markocic et al. PRL84, 534 (2000)

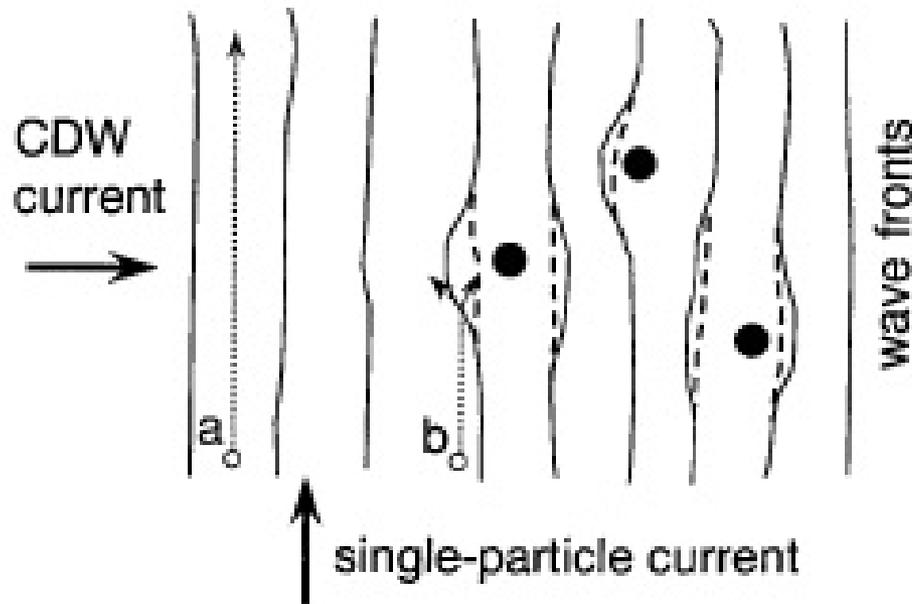


$$E_T(J_x) = E_T(0) \frac{J_x}{J_c} \exp\left(1 - \frac{J_x}{J_c}\right)$$

$$J_c \propto \sigma_0 E_T(0) (\xi_L k_F) (\rho_n / \rho_{CDW}), \quad (2)$$

where  $\sigma_0$  is the conductivity at very high bias fields,  $k_F$  the Fermi wave vector, and  $\rho_n$  and  $\rho_{CDW}$  are normal and CDW electron densities, respectively. The correlation length  $\xi_L$  [8,30] is a measure for the coherence in the sample and decreases with increasing disorder.

## Convective term



Markocic et al. PRL84, 534 (2000)

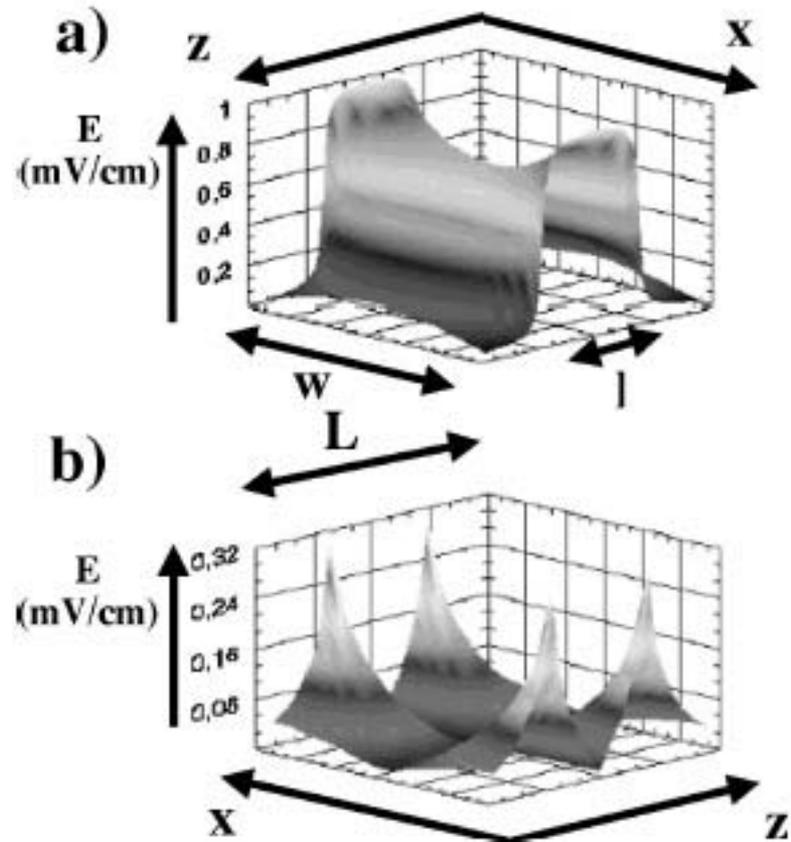
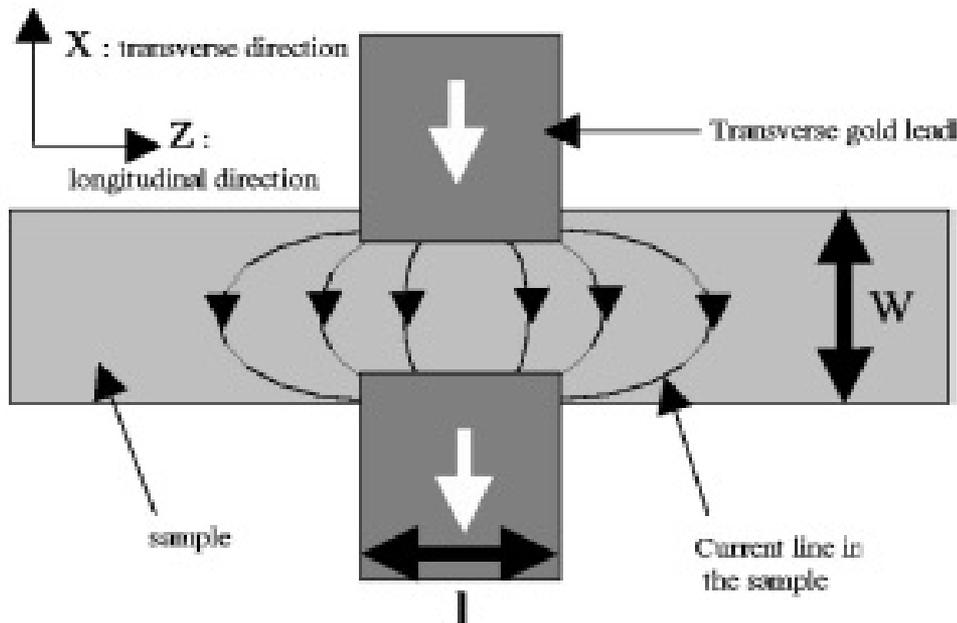
The physical origin of this effect may be that CDWs become more ordered due to momentum transfer with transversely moving normal carriers [25]. This mechanism is illustrated in Fig. 4. In the absence of defects, the charge-density wave fronts are straight and parallel to each other (the left side of the picture). The single-particle transverse current, marked by “a” on Fig. 4, can flow with little or no interaction with the CDW. In the presence of defects or impurities in the crystal, the CDW deforms to lower its energy and the wave fronts are “wrinkled” (the right side of the picture). In this case, the transversely moving electrons (“b”) are more likely to be deflected. The conservation of linear momentum results in a reaction force back on the CDW. This way the CDW roughness is reduced as the CDW wave fronts are straightened out or “ironed” by the transverse current. The CDW transport across the sample is therefore more coherent and less susceptible to pinning. The lower pinning strength then leads to a lower threshold field.

Calculation of the damping coefficient from the microscopic theory show that the effect is orders of magnitude smaller (contribution of holes and electrons nearly annihilate)

The effect can also be caused by geometric effects due to the high anisotropy of conductivity

Artemenko et al., PRL22 (2000) 5184

# Transverse injection inhomogeneity and reduction of the threshold field



A. Ayari and PM, PRB 66 (2002) 235119

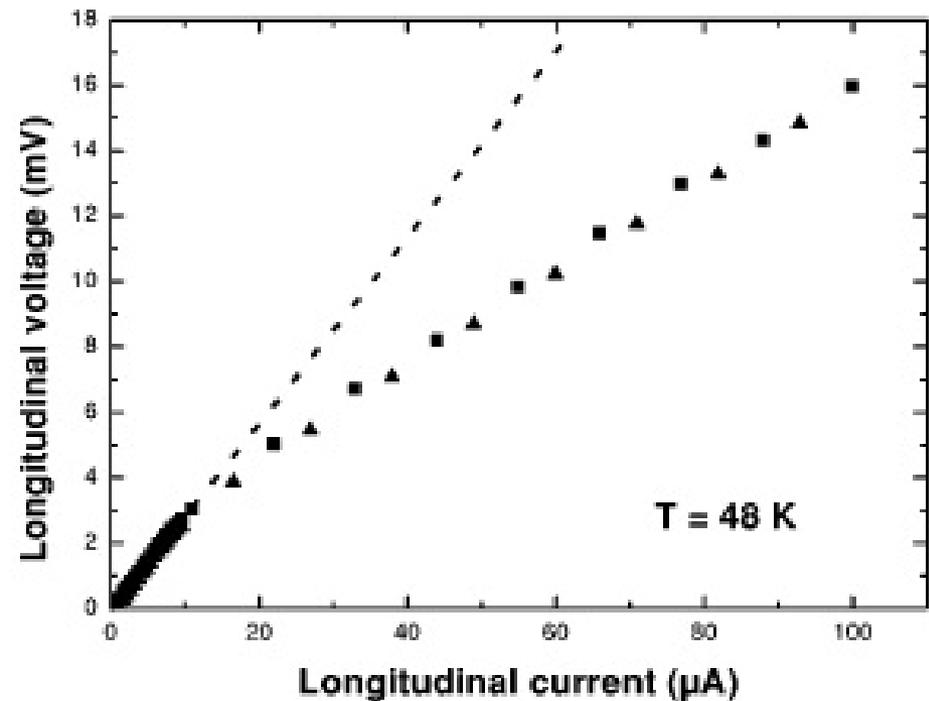
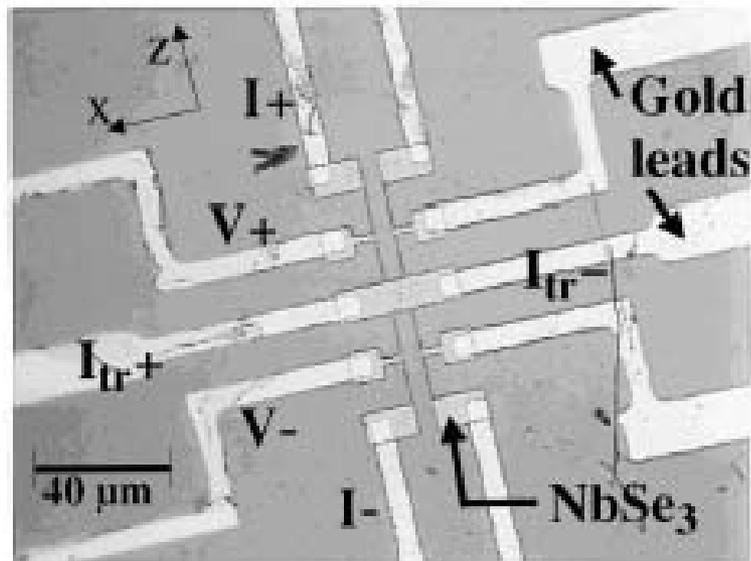
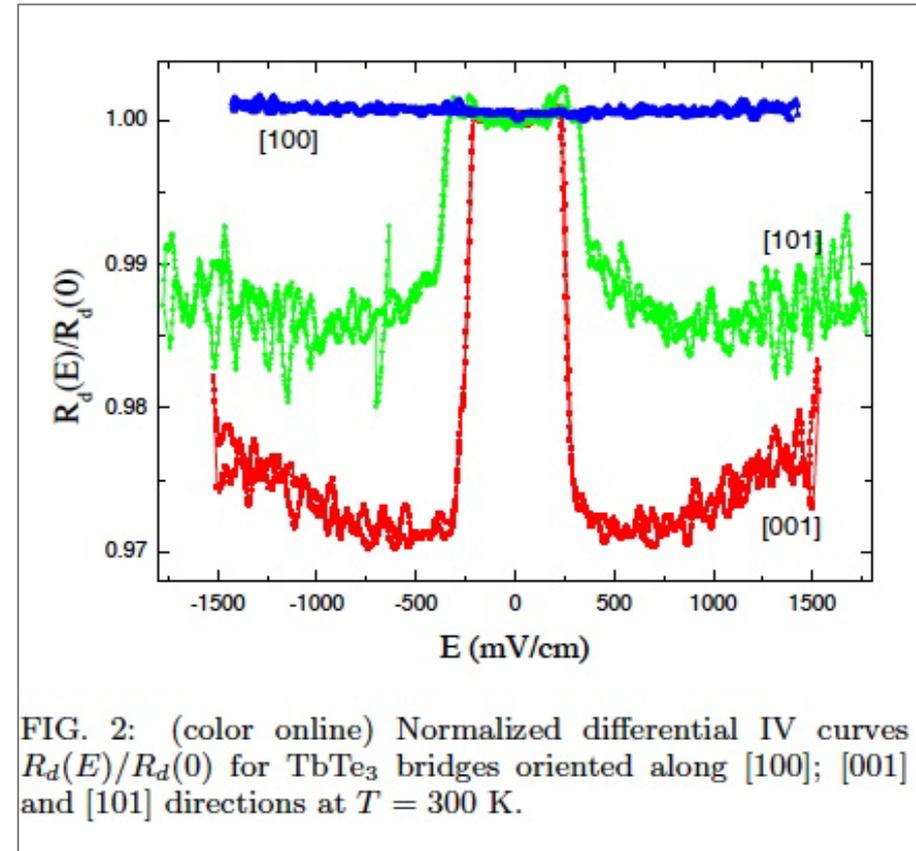
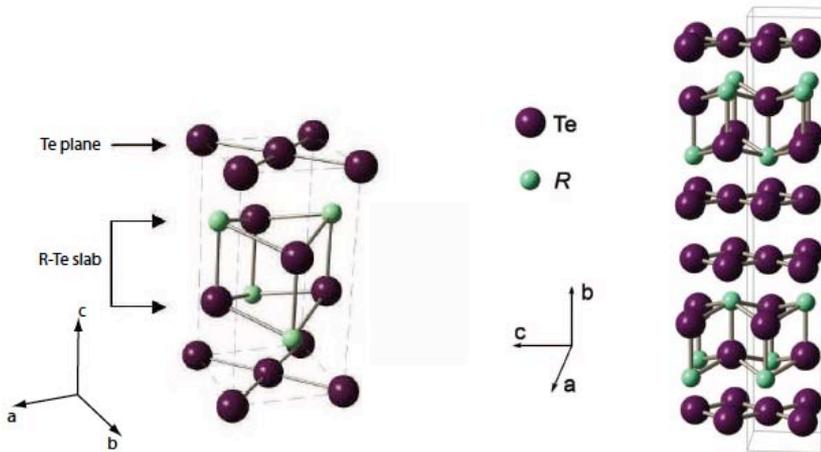


FIG. 6. Longitudinal  $I$ - $V$  characteristic at 48 K. Squares:  $J_L = 0$  A/cm<sup>2</sup>; triangles:  $J_L = 2 \times 10^4$  A/cm<sup>2</sup>. The dashed line shows the  $I$ - $V$  characteristic of normal electrons alone.

$$J_c = \sigma_{\infty} E_T(0) (k_F \xi_L) (\rho_N / \rho_{CDW}),$$

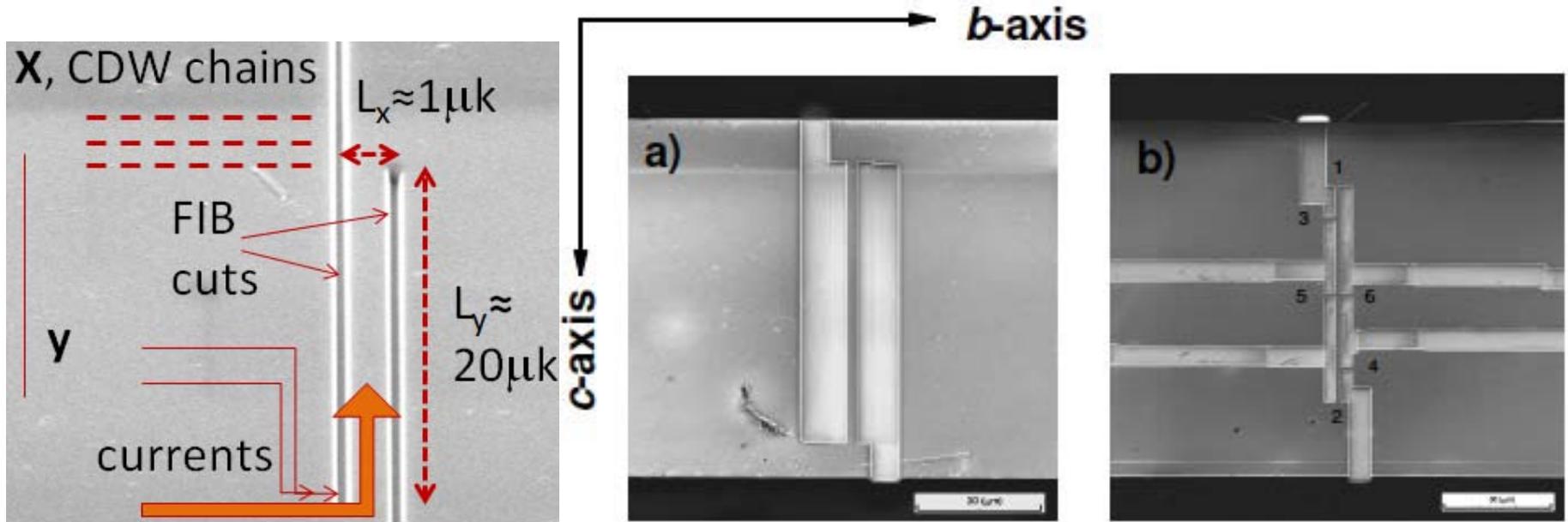
A. Ayari and PM, PRB 66 (2002) 235119

# Sliding of the unidirectional CDW in 2D TbTe<sub>3</sub>



Next talk in this session: A.A. Sinchenko et al.

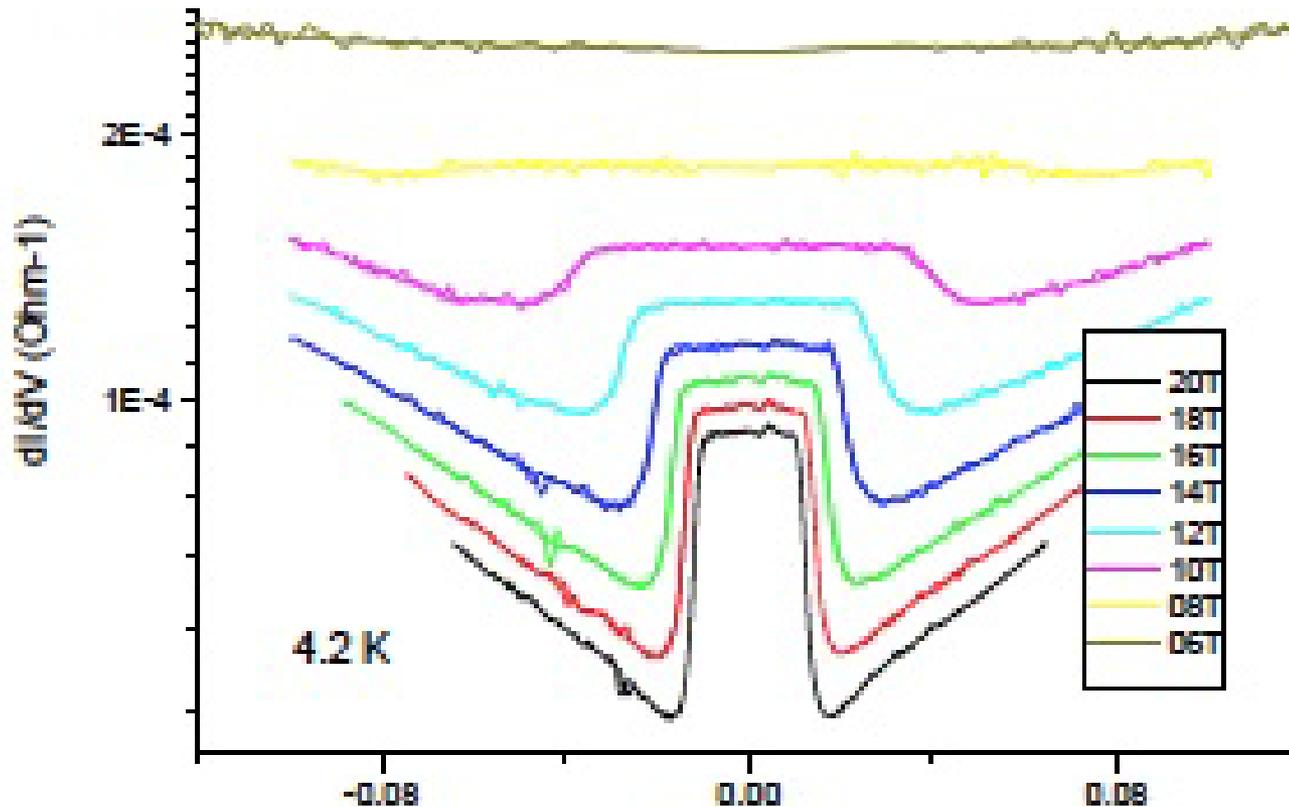
# Hall bar geometry



The current is forced into the transverse channel.  
Resulting Hall voltage in chains direction  $x$  reaches  $3\text{mV}$ .  
Electric field  $E_x$  created to oppose the Lorentz force upon moving normal electrons can exceed the depinning threshold for the host CDW.

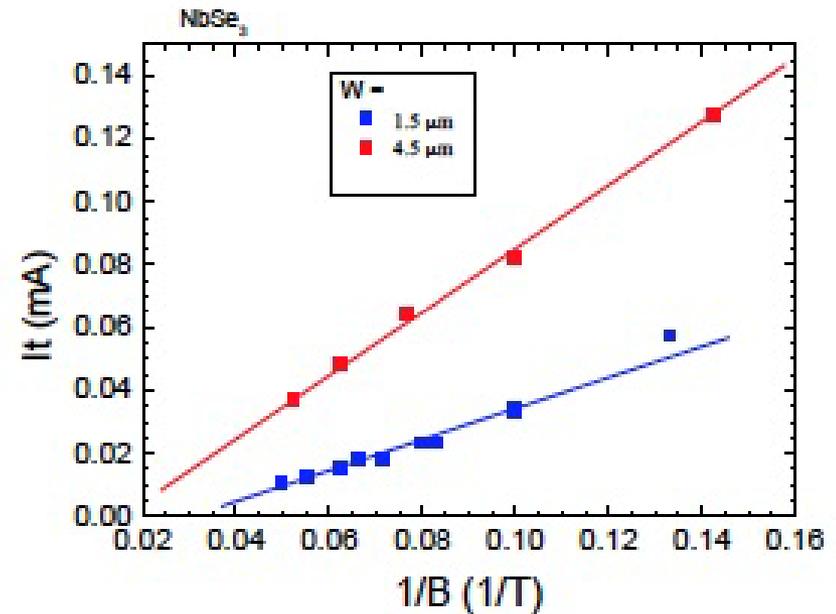
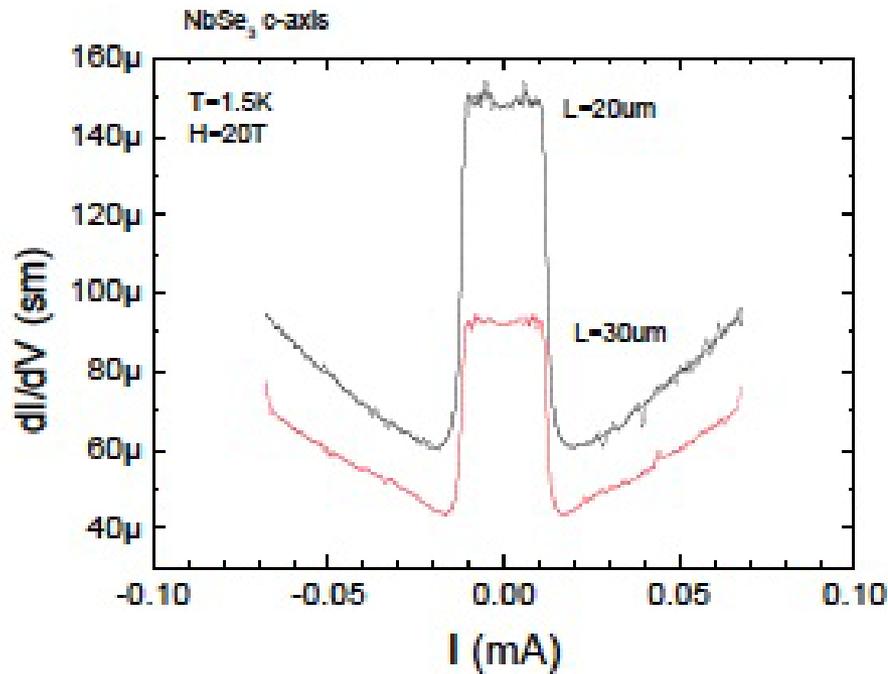
<b>sample</b>	<b><math>L</math> (<math>\mu\text{m}</math>)</b>	<b><math>W</math> (<math>\mu\text{m}</math>)</b>	<b><math>D</math> (<math>\mu\text{m}</math>)</b>
<b>1</b>	<b>23</b>	<b>1.4</b>	<b>0.3</b>
<b>2</b>	<b>43</b>	<b>1.5</b>	<b>0.8</b>
<b>3</b>	<b>22</b>	<b>1.2</b>	<b>0.5</b>
<b>4</b>	<b>30</b>	<b>1.08</b>	<b>0.8</b>
<b>5</b>	<b>20</b>	<b>1.08</b>	<b>0.8</b>
<b>6</b>	<b>30</b>	<b>5.3</b>	<b>0.9</b>
<b>7</b>	<b>30</b>	<b>3.2</b>	<b>0.66</b>
<b>8</b>	<b>24</b>	<b>4.5</b>	<b>0.90</b>

## Non-linearity in the I-V along the channel



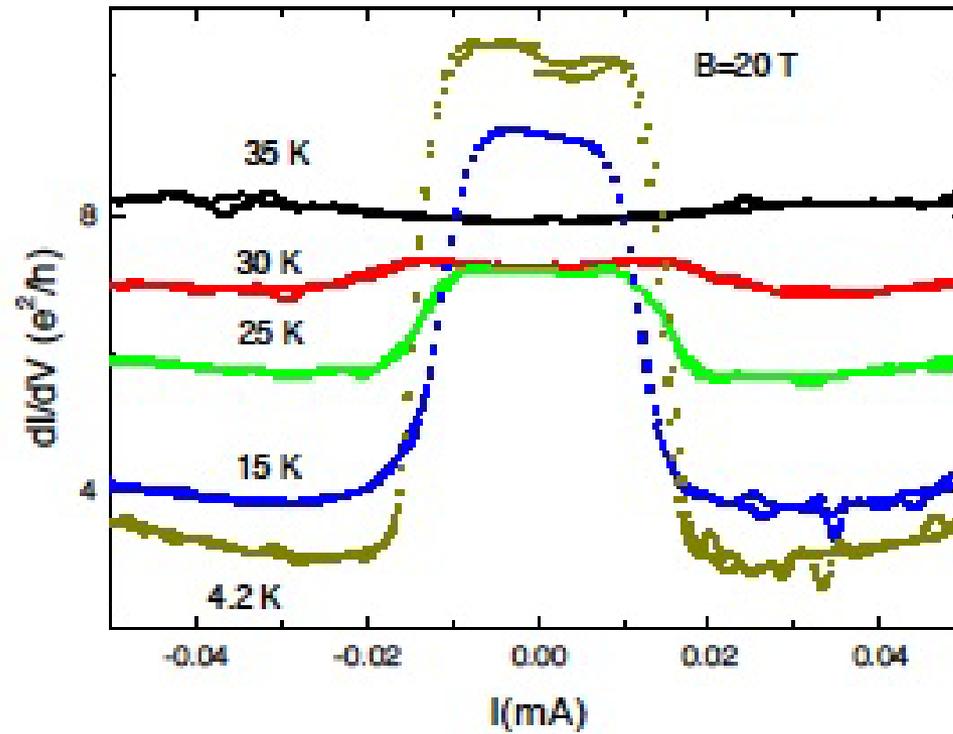
Sharp threshold in I-V like for the best sliding but with the opposite sign: the drop of conductivity

Threshold appears only at high  $B > 8\text{T}$  and low  $T < 25\text{-}30\text{ K}$



Threshold current  $I_t \propto L_x$  - the channel width,  
but does not depend on the channel length  $L_y$ .

$I_t \propto 1/B \rightarrow$  threshold is originated by the  
Hall voltage  $V_H \propto IB$  in chains' direction



## Depinning and sliding of the CDW

The mutual orthogonal magnetic field and the bias current, both lying in transverse interchain directions, produce the Hall voltage  $V_x$  in the chain direction.

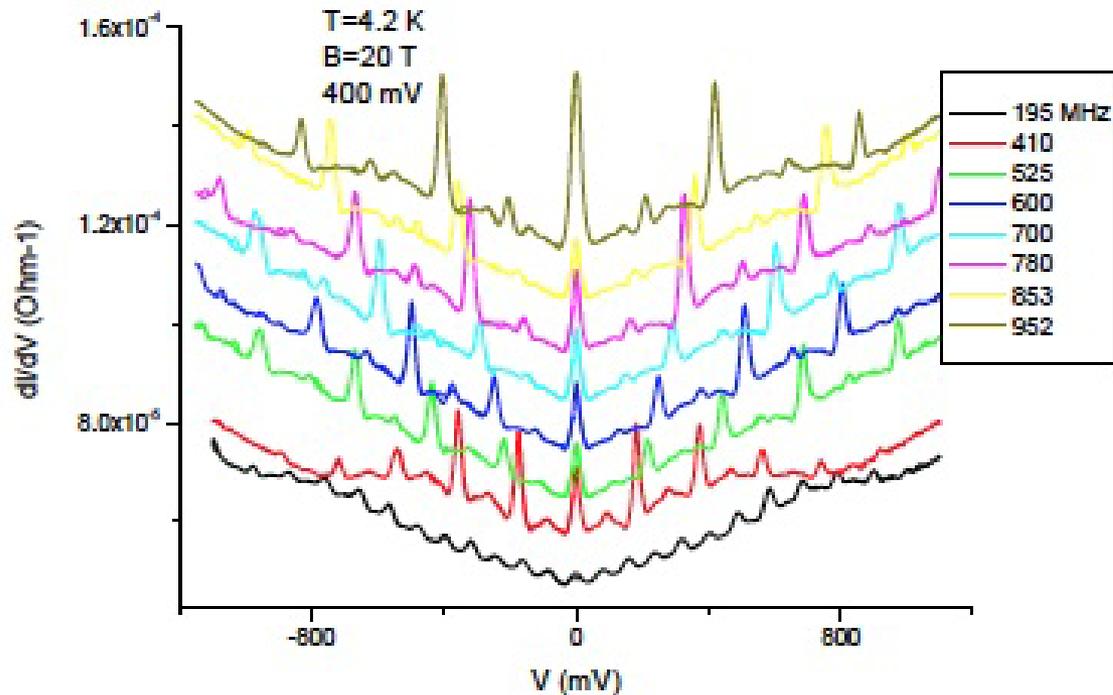
The Hall electric field is  $E_H = V_x / W = R_H IB / (DW)$  where  $R_H$  is the Hall constant,  $D$  the sample thickness.

The threshold current  $I_t$  corresponding to the critical Hall electric field  $E_{ht}$  is:

$$I_t = E_{ht} WD / (R_H B) \propto W / B \text{ as experimentally observed.}$$

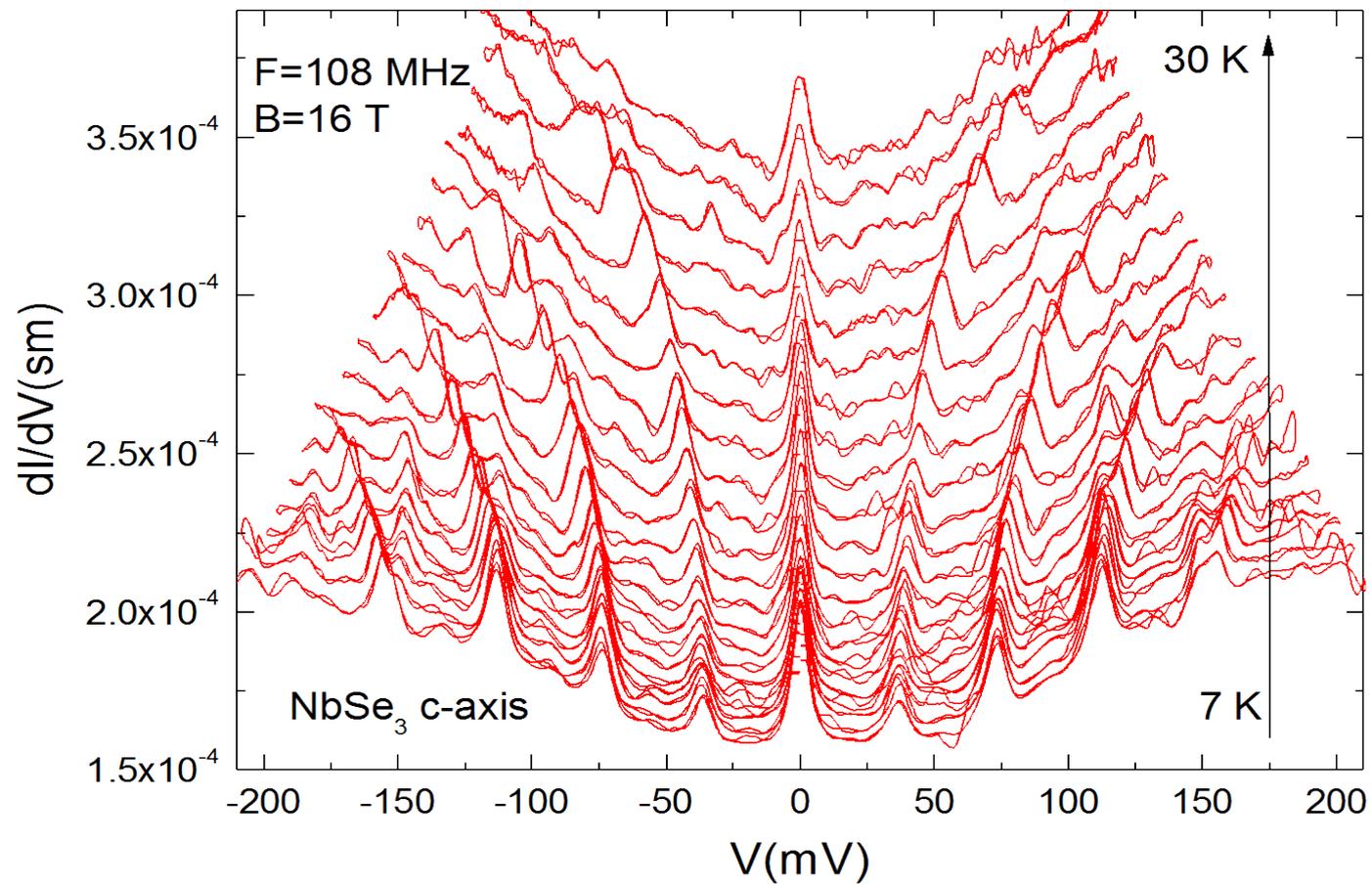
The value of  $V_x$  is directly measured using Hall probes yields  $E_{ht} = 0.3 \text{ V/cm}$  at  $B=20\text{T}$  and  $T=4.2\text{K}$  right scale of threshold fields at low  $T$

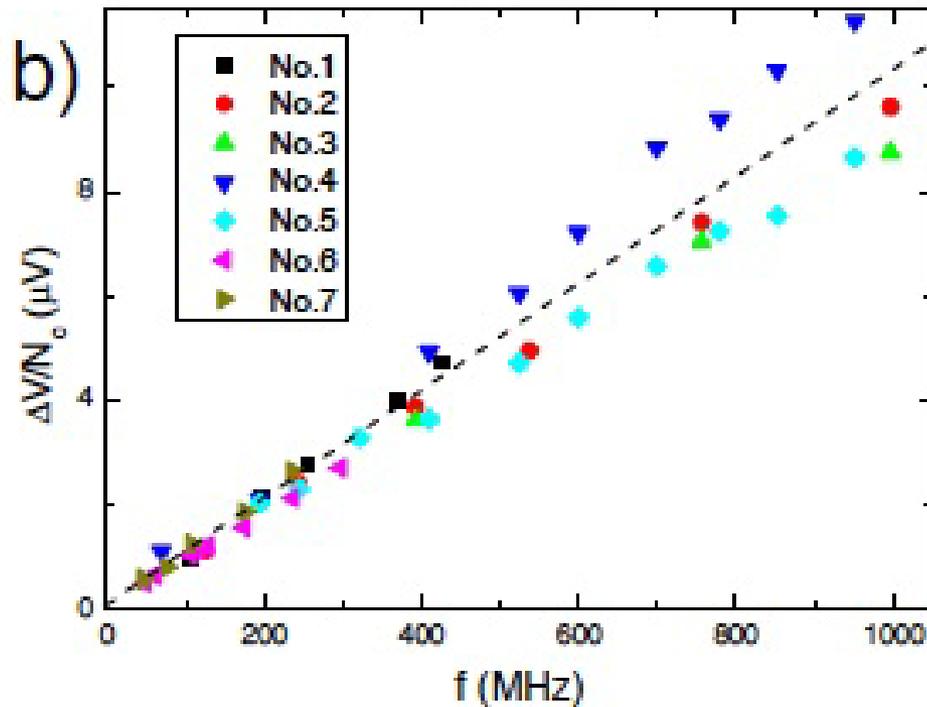
# Shapiro steps



Shapiro steps are seen for frequencies  $f$  up to **3GHz**. They are quantized in  $\Delta V$  (superconductors) rather than in  $\Delta I$  - CDW

The position of fundamental Shapiro step is proportional to the length of the bridge and also is proportional to the external frequency, but not on the thickness or the width of the bridges





$\Delta V/f = 12.5\text{mV/GHz}$  for  $1\mu\text{m}$  of the structure length

From the structure the number of chains for a length of  $1\mu\text{m}$ :

$$N_c(1\mu\text{m}) = 641 \times 2 \text{ chains}$$

then

$$\Delta V/(N_c f) = 10\mu\text{V/GHz}$$

to be compared to the superconducting Josephson relation  $\Delta V/f = 2.5\mu\text{V/GHz}$

The non-linearity found for structures oriented across the chains is completely different from the non-linearity that has been observed many years ago along the chains related with CDW depinning and sliding.

The non-linearity along the chains corresponds to an increase of conductivity above the threshold voltage, while the conductivity across the chains drops down above the threshold current

How the depinning works since there are no electrodes along the chain axis and then the total current is zero in x direction

The collective current  $J_c$  carried by the CDW sliding is compensated by the counter-current of remnant carriers driven by the Hall voltage.

# Interpretation

The interpretation and the modeling consider the deformable CDW in a restricted geometry for the quantum limit of remnant carriers.

Under the HMF but at no applied current, the carriers are distributed homogeneously forming a fractionally ( $\nu < 1$ ) filled quantized state. (The value of  $\nu < 1$  is estimated as  $\nu \approx 0.5 - 0.1$  at  $B = 20T$ . When the transverse current is  $J_y$  applied, the Lorentz force in  $x$  direction appears pushing the carriers orbits towards one end (chosen as  $x = 0$ ) until the charge redistribution gives rise to the compensating Hall field  $E_x$ .

The first key peculiarity with respect to conventional Hall setup is the presence of the CDW background with its gigantic polarizability, estimated here as  $\epsilon \propto 10^6$ . The screening by displacements of the CDW phase  $\phi \propto \epsilon E_x$  allows for a strong redistribution of the electronic density up to (in the  $T = 0$  limit) the complete charge segregation when all carriers occupy, with the maximum filling 1, a segment  $l = \nu L_x$  of the chain length  $L_x$ , thus forming here the IQH state while leaving the segment  $(1 - \nu) L_x$  unoccupied.

The electric field accompanying such a charge separation easily exceeds the pinning threshold for the CDW sliding.

The modeling shows that the depinning propagates into the nominally pinned central region via sharp walls.

The resulting picture is that of compensated collective  $j_c$  and normal  $j_{nx}$  pulsing counter-currents driven by the Hall voltage.

The appearance of  $j_{nx}$  gives rise to the secondary Hall voltage, now in the  $y$  direction, which persistent part is registered as a sharp threshold in the I-V characteristics while the periodic part gives rise to Shapiro steps.

Allowance for the CDW sliding current  $J_c$  brings about the normal counter current  $J_x = -J_c$ . The Lorentz force component appears in  $y$  direction giving rise to the additional voltage  $V_y$ .

The current per chain is (written for low T):  $j_x = e^2/h \Delta v_y(x)$  where  $\Delta v_y(x)$  is the main interchain voltage drop

The CDW counter-current per chain is  $j_c = -ed_t\phi / \pi$  and requires phase slips to annihilate with the normal current  $j_x$  near the boundary. If each phase slip absorbs/releases  $M$  electrons per chain, then the repetition frequency is

$$f = \frac{|\dot{\phi}|}{2\pi M} = \frac{|j_x|}{2Me} \approx \frac{e|\Delta V_y|}{2Mh}, \quad \frac{|\Delta V_y|}{f} = \frac{2Mh}{e}$$

Josephson type relation with the additional factor  $M$  approximately 4

Phase slip appears as a spacio-temporal supervortex with the  $M$ -fold circulation

# Modeling

There are three different regimes which will be modeled by two sets of equations:

1. Formation of the inhomogeneous distribution in presence of the pinned CDW;
2. Depinning and its propagation;
3. Sliding regime with periodic phase slip process.

The first two regimes can be considered at a nominal constant amplitude  $A$  of the CDW while the third one requires to allow for amplitude nodes to be formed in the centers of the phase ' vortices.

# Numerical modeling

The numerical modeling is performed on the bases of the time-dependent GL approach

Numerical results for the regime 1 (as defined in the main text) were obtained from the following equations. The potential  $\Phi$  and the total charge density per chain  $n_{tot} = (\partial_x \varphi / \pi + n - \bar{n}_1)$  are related by the Poisson eq. which actually is reduced to the constraint of the local electroneutrality

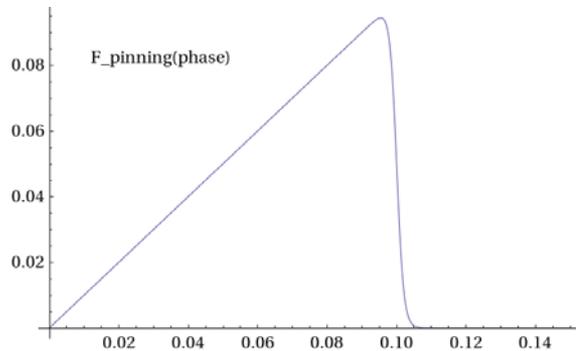
$$\pi \bar{n}_1 [f_F(\Phi/T)/\nu - 1] + \partial_x \varphi = 0 \quad (3)$$

The equation for the phase is the balance of forces: the friction  $\propto \partial_t \varphi$ , the stress - the gradient of the chemical potential  $\mu_e$  of condensed electrons, and the pinning force  $F_p$ :

$$\gamma \partial_t \varphi - \partial_x \mu_e + F_p(\varphi) = 0, \quad \mu_e = \Phi + \partial_x \varphi / (\pi N_F) \quad (4)$$

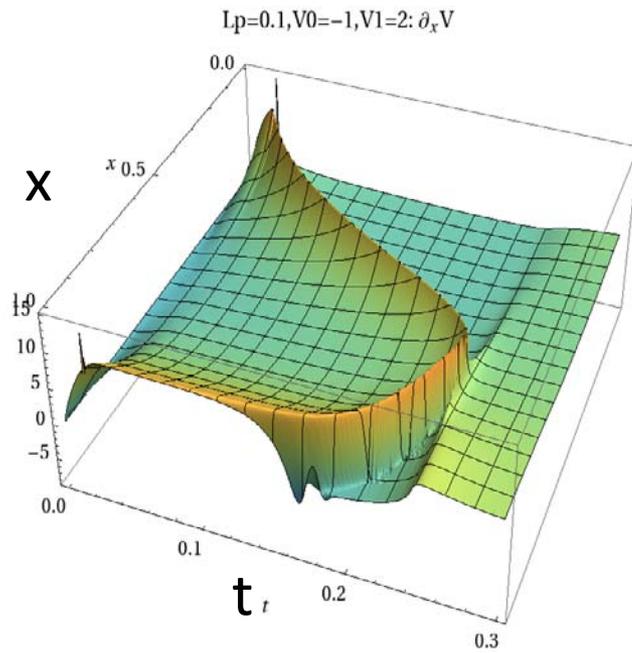
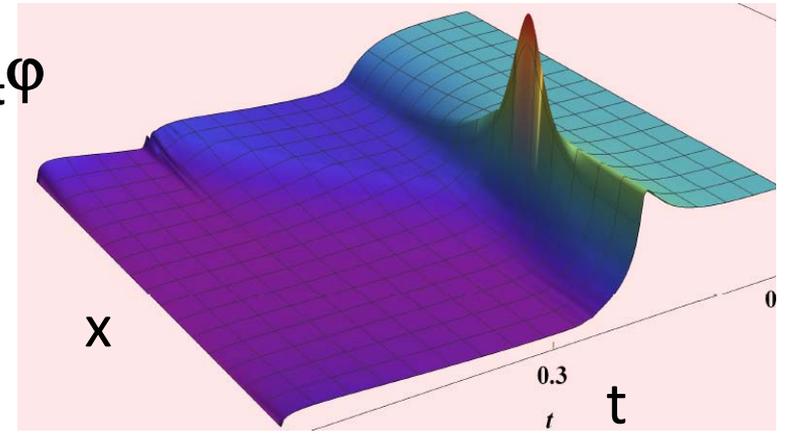
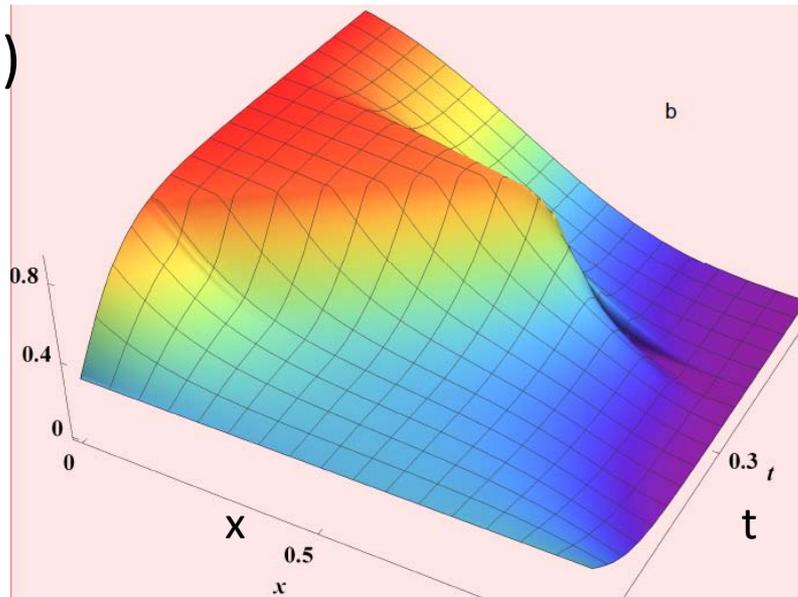
We model the function  $F_p(\varphi)$  in such a way that it starts with the linear law  $F_p(\varphi) \propto \varphi$  corresponding to  $\epsilon = const$  and then sharply drops to zero at  $\varphi > \varphi_p$  - the threshold phase displacement  $\varphi_p$  of depinning:

$$F_p(\varphi) = \frac{\varphi / (\pi N_F L_p^2)}{1 + \exp(k(\varphi - \varphi_p))} \quad (5)$$



Guess for the pinning force  $F_p(\varphi)$  as a function of the phase  $\varphi$  displacement

Eqs. (3,4) have been solved numerically with separate boundary conditions for potentials which turn on the boundary voltage values gradually, over a short time  $\tau$ .  $\Phi(0, t) = \Phi_0 \tanh(t/\tau) < 0$ ,  $\Phi(L_x) = \Phi_1 \tanh(t/\tau) > 0$ .

**E** $\partial_t \phi$  $f(x,t)$ 

Two fronts of the propagating depinning collide giving rise at higher  $t$  to the stationary moving depinned state with essential but smooth variations over the length.

Phase velocity shows a strong, very narrow in time peak at the collision.

**Interpretation:** electric fields (created by electrons' redistributions near the boundaries) exceed the threshold field while the bulk is still below that.

The depinning penetrates into the pinned region by propagating sharp fronts of the electric field.

# Generation of periodic phase slips (1)

Allowing for variations of  $A(x, t)$ , the basic relations are generalized as

$$n_c(x, t) = A^2 \partial_x \varphi(x, t) / \pi, \quad j_c = -A^2 \partial_t \varphi(x, t) / \pi \quad (6)$$

$$\Phi(x, t) = T \ln \left( \frac{n_1 / \nu - n_1 + n_c(x, t)}{n_1 - n_c(x, t)} \right) \quad (7)$$

Here the relations (6) give the commonly used definitions of the collective density and the current, the dependence (7) is the resolution of the eq. (3) for  $\Phi$  as a function of  $n_c$ .

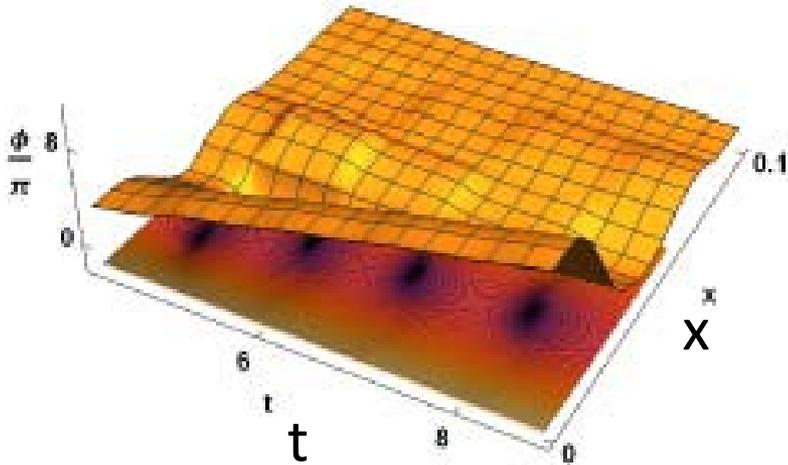
We need two eqs. to describe the entangled evolution of  $\varphi(x, t)$  and  $A(x, t)$ .

$$\gamma j_c(x, t) + \partial_x n_c(x, t) - N_F \partial_x \Phi(x, t) = 0 \quad (8)$$

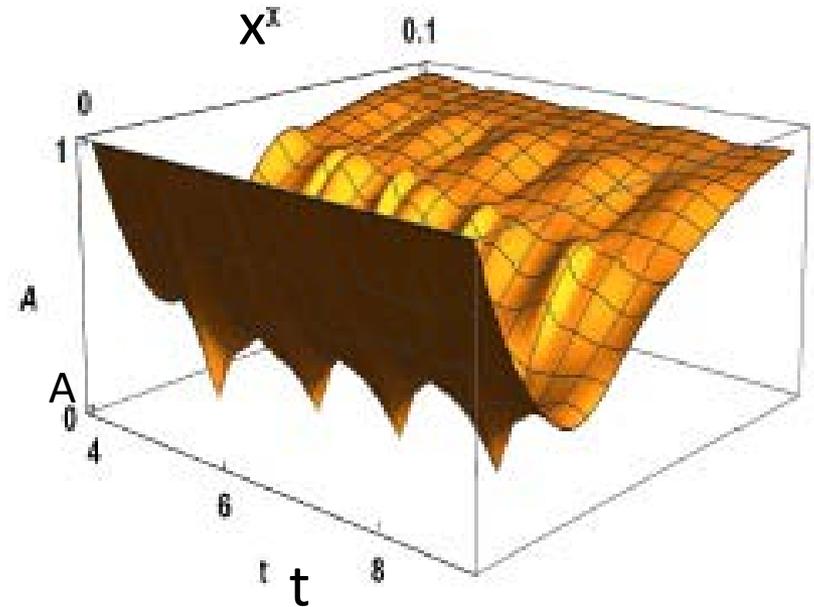
$$\begin{aligned} \gamma_A \partial_t A(x, t) - \xi^2 \partial_x^2 A(x, t) + \xi^2 (\partial_x \varphi(x, t))^2 A(x, t) \\ - A(x, t) + A(x, t)^3 = 0 \end{aligned} \quad (9)$$

Eq.(8) is the generalization of Eq.(4) allowing for deviations of  $A(x, t)$  from 1 while neglecting the pinning force. Eq.(9) describes the dissipative evolution of  $A(x, t)$  when it deviates from the equilibrium at  $A = 1$  which was enforced at the boundaries:  $A(0, t) = A(1, t) = 1$ . Boundary values of the potential were turned on, in view of the unique relation (7), via  $n_c(0, t)$  and  $n_c(1, t)$ , following the smooth switching  $\propto \tanh(20t)$ . The parameters can be estimated as  $N_F = 3/(mV\mu m)$ ,  $\xi = 0.1\mu m$ , and we have chosen  $\gamma_A = 0.001\gamma$  assuming a fast relaxation of the amplitude.

## Generation of phase slips (2)



phase  $\varphi(t, x)/\pi$



amplitude  $A(t, x)$

Spatio-temporal evolution with periodic phase slips

# Conclusions

- Bias current  $J_y$  flowing through the sample generates the Hall voltage  $V_x$  that causes the redistribution of electron density in x direction
- When the resultant electric field  $E_x$  overcome the threshold, the CDW along the chain axis starts sliding. The collective current  $J_c$  carried by the CDW sliding is compensated by the counter-current of remnant carriers
- The Lorentz force for the counter current causes the inter-chain voltage drop in y direction, leading to the nonlinear behavior of conductance (sharp decrease of conductance at the threshold)
- The counter current disappears at the boundary of the sample. The annihilation of the current requires the phase slip of the CDW, which causes the periodic absorption and release of carriers (this is the origin of the Shapiro-steps-like response).

The combination of the FIB fabrication of microscopic devices, application of a high magnetic field, and the modeling has allowed to identify the never seen regime of two-fluid quantum magneto-hydro-dynamics with compensated collective and normal conduction in the frame of the QH regime.

The basic ingredient of this highly nonlinear and nonstationary regime is a sequence of periodic phase slip processes providing the annihilation of the normal and the collective currents.