Multiferroic skyrmions

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Cargese

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Oleg Sushkov

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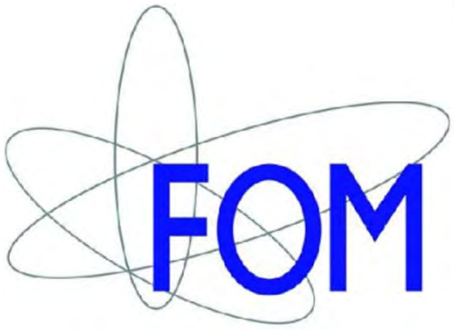
U of New Hampshire
Jiadong Zang



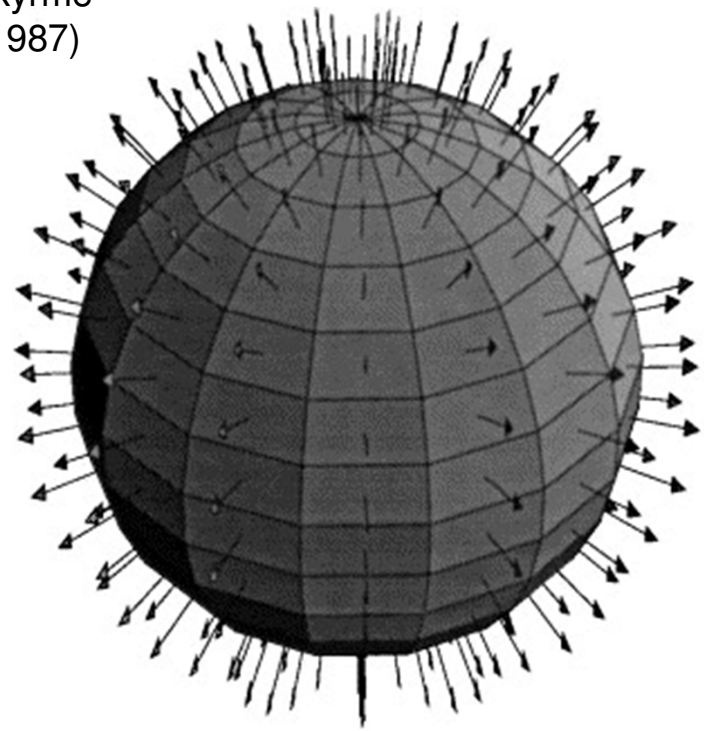
Outline

- **Skyrmions and skyrmion materials**
- **Physical meaning of topological charge**
  Topological transport
- **Skyrmion dynamics**
- **Skyrmions in frustrated magnets**
- **Electric field excitation of skyrmions**

Tony Skyrme
(1922-1987)

Skyrmion

T. H. R. Skyrme, A Nonlinear field theory,
*Proc. Roy. Soc. London **A 260**, 127 (1961)*



$$\pi_1^2 + \pi_2^2 + \pi_3^2 + \sigma^2 = 1$$

$$S_3 \rightarrow S_3$$

Topological charge = Baryon number

Skyrme model

$$E = \int d^3x \left[\frac{F_\pi^2}{8} \sum_{\alpha,i} (\partial_i n^\alpha)^2 + \frac{1}{4e^2} \sum_{\alpha,\beta,i,j} (\partial_i n^\alpha \partial_j n^\beta - \partial_i n^\beta \partial_j n^\alpha)^2 \right]$$

$$\mathbf{n} = (\pi_1, \pi_2, \pi_3, \sigma) \qquad \mathbf{n}^2 = 1$$

Nuclei as multi-Q skyrmions

^{1}H ^{2}H ^{3}H, ^{3}He ^{4}He

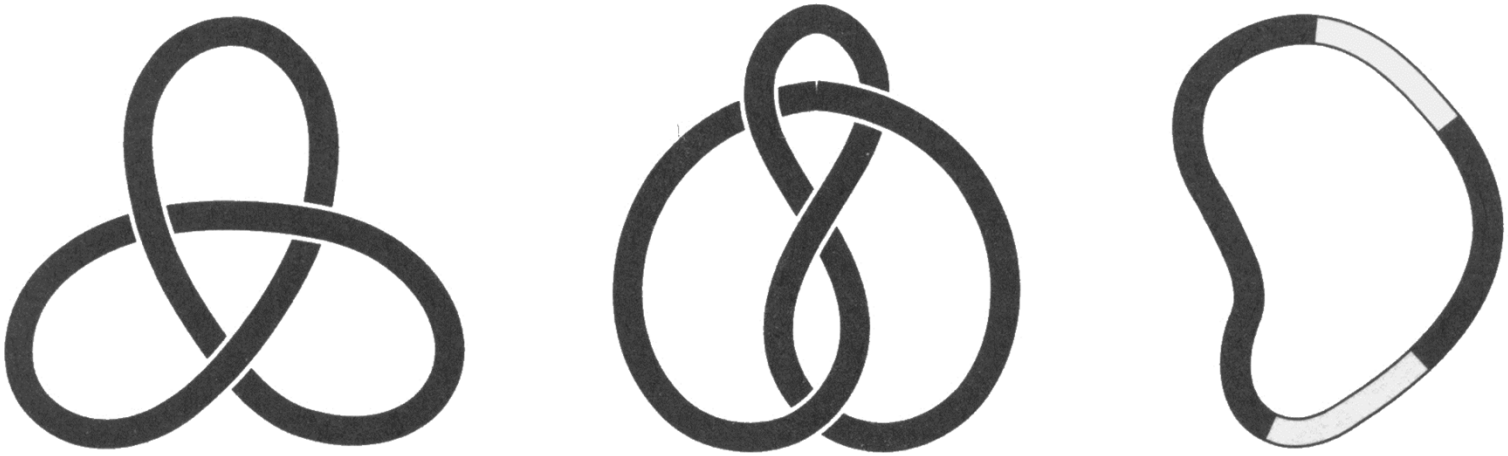
Q = 1 **Q = 2** **Q = 3** **Q = 4**

Q = 5 **Q = 6** **Q = 7** **Q = 8**

*R.A. Battye & P.M. Sutcliffe, PRL **79** 363 (1997)*

Lord Kelvin
(1824-1907)

Atoms as vortices

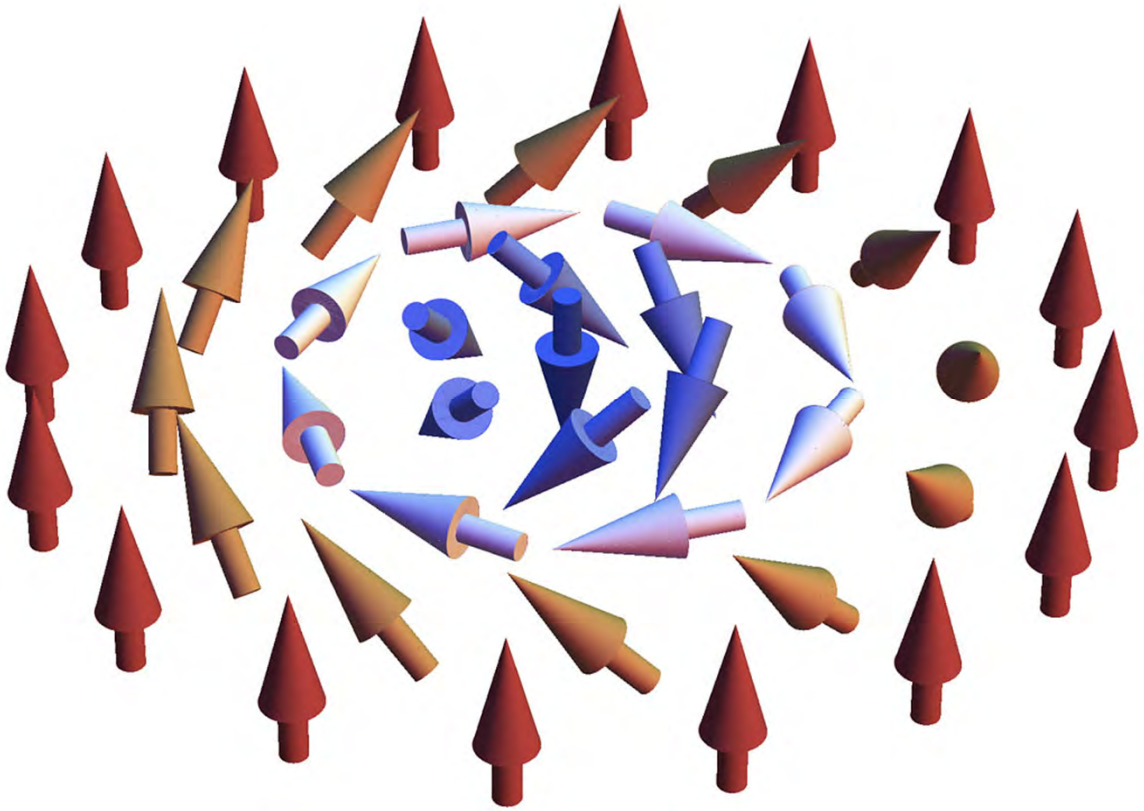


Carbon Oxygen Hydrogen

Magnetic skyrmions

Magnetic skyrmion

$$\boldsymbol{n}_x^2 + \boldsymbol{n}_y^2 + \boldsymbol{n}_z^2 = 1$$

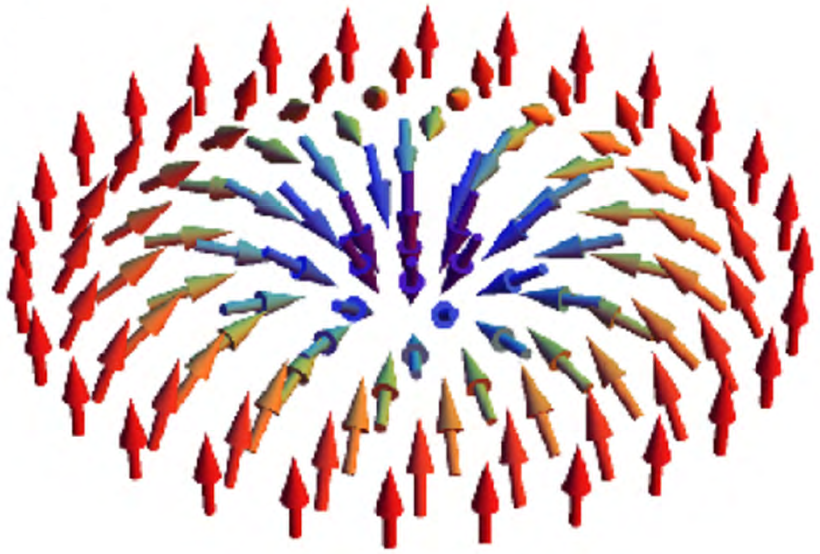


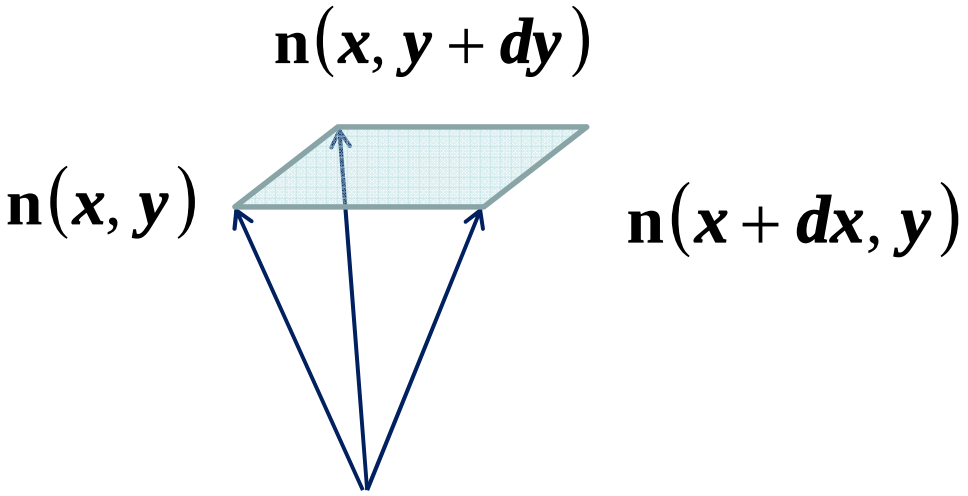
$$\boldsymbol{S}_2 \to \boldsymbol{S}_2$$

Topological charge

Pontryagin number:

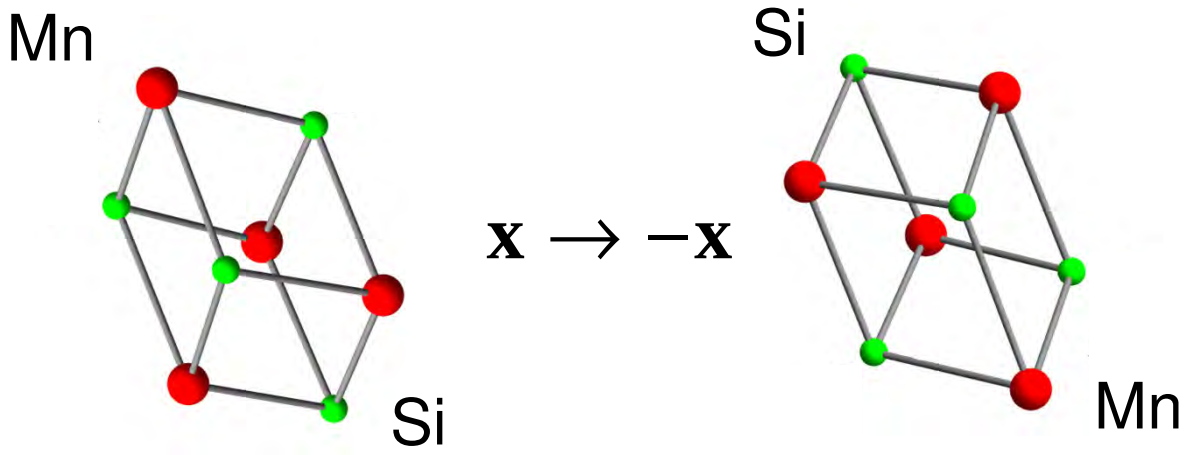
$$Q = \frac{1}{4\pi}\int d^2x\left(\mathbf{n}\cdot\left[\partial_x\mathbf{n}\times\partial_y\mathbf{n}\right]\right) = \frac{1}{4\pi}\int d\Omega = \text{integer } \#$$



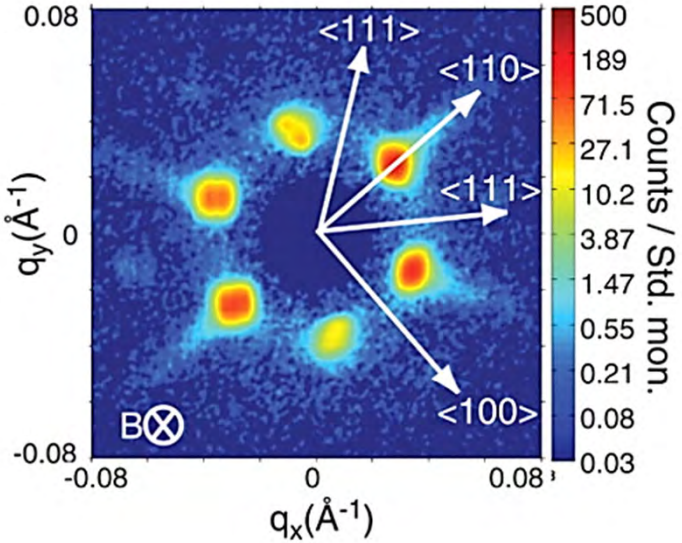


Skyrmions in chiral magnets

*A.N. Bogdanov & D.A. Yablonskii, Sov. Phys. JETP **95**, 178 (1989)*

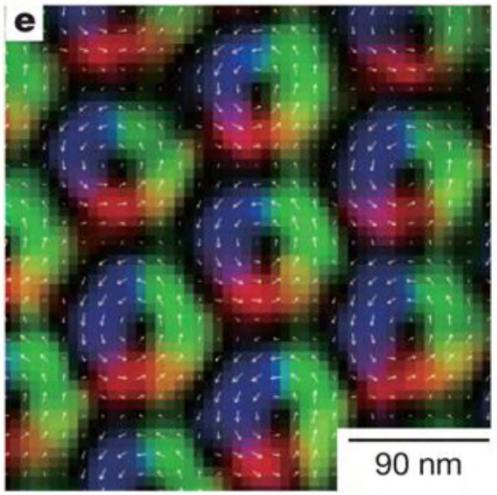


Neutron scattering



Mühlbauer et al, Science (2009)

Lorentz microscopy



X. Yu et al. Nature (2010)

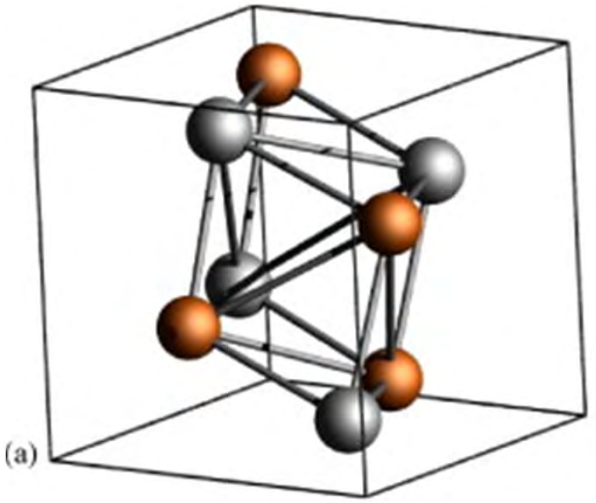
Non-centrosymmetric magnets

*I. Dzyaloshinskii, Sov. Phys. JETP **19**, 960 (1964)*

*P. Bak & M.H. Jensen, J. Phys. C: Solid State **13**, L881 (1980)*

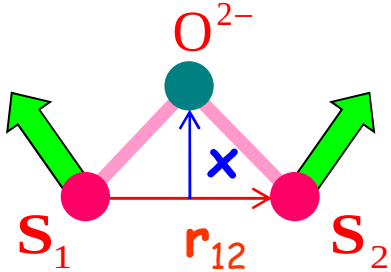
Cubic B20 structure $P32_1$

MnSi, $Fe_{1-x}Co_xSi$, FeGe



$$f = \frac{J}{2}(\nabla \mathbf{n})^2 + \boldsymbol{D}\,\mathbf{n}\cdot[\nabla \times \mathbf{n}]$$

Exchange energy



DM energy

$$\boldsymbol{E}_{DM} = \mathbf{D}_{12}\cdot[\mathbf{S}_1 \times \mathbf{S}_2]$$

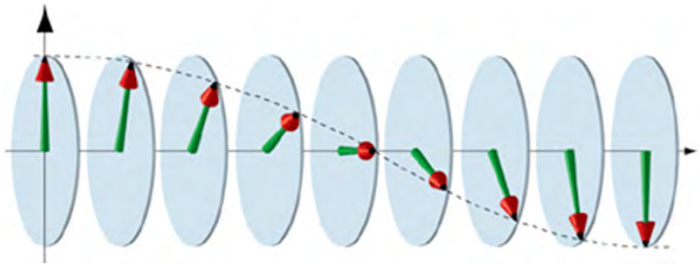
Helical spiral state

$$\mathbf{n} = \mathbf{e}_1 \cos(\mathbf{q}\cdot\mathbf{x} + \varphi) + \mathbf{e}_2 \sin(\mathbf{q}\cdot\mathbf{x} + \varphi)$$

$\mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2$ is the spin rotation axis

$$f = \frac{J}{2} q^2 - D(\mathbf{q}\cdot\mathbf{e}_3)$$

$$\begin{cases} \mathbf{e}_3 \uparrow\uparrow \mathbf{q}, \text{ for } D > 0 \\ \mathbf{e}_3 \uparrow\downarrow \mathbf{q}, \text{ for } D < 0 \end{cases}$$



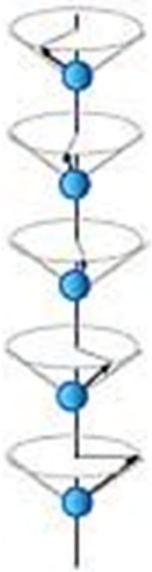
$\mathbf{e}_3 \parallel \mathbf{q}$

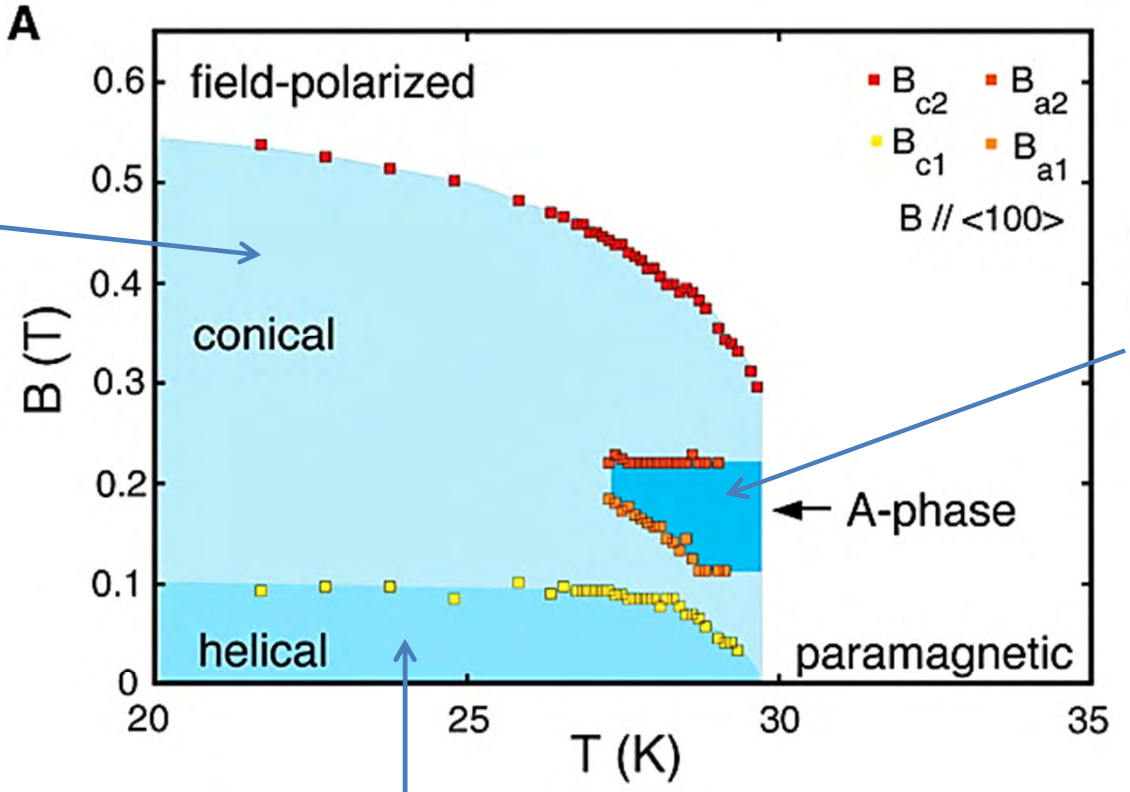
Spiral wave vector $q = D/J$ **Weak crystallization**

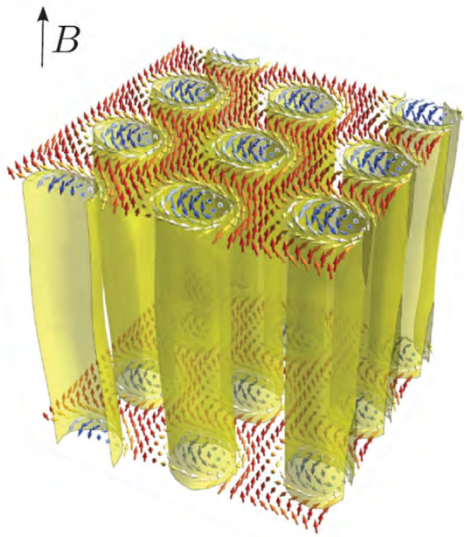
Phase Diagram of MnSi

*Mühlbauer et al, Science **323**, 915 (2009)*

$\mathbf{q} \| \mathbf{H}$

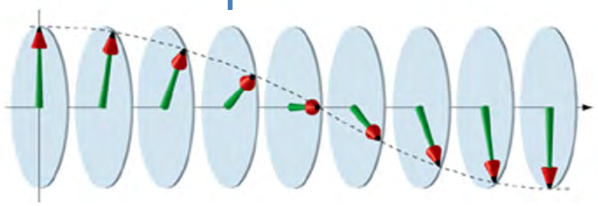


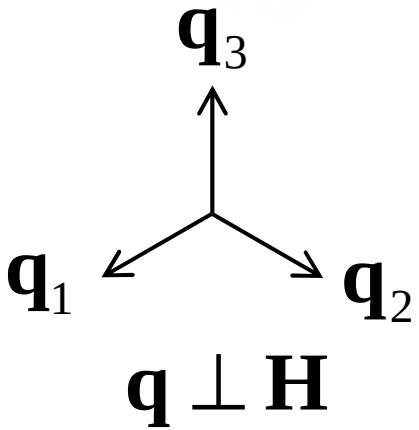




$\mathbf{q} \| [111]$

$\lambda = 18\,\mathrm{nm}$

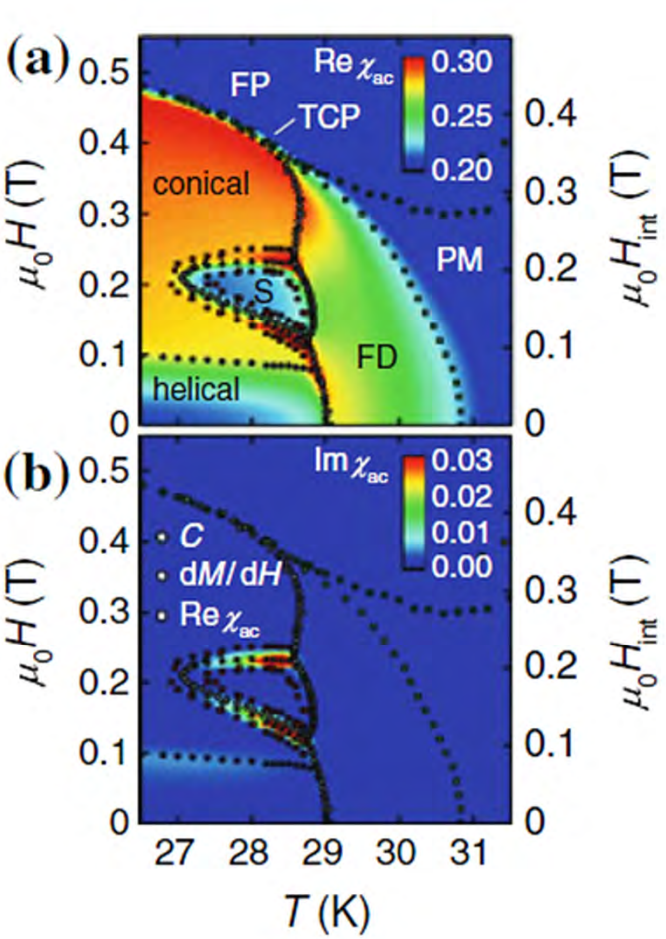


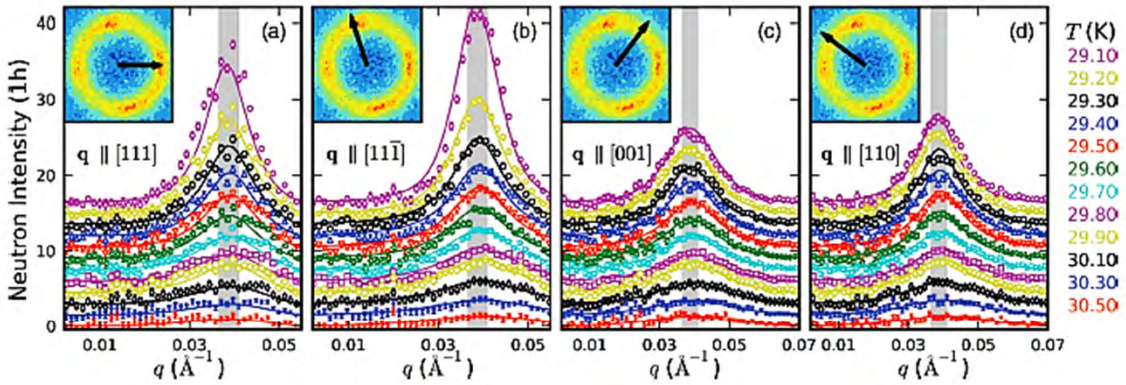


$\mathbf{q} \perp \mathbf{H}$

Fluctuations

*M. Janoschek et al., PRB **87**, 134407 (2013)*





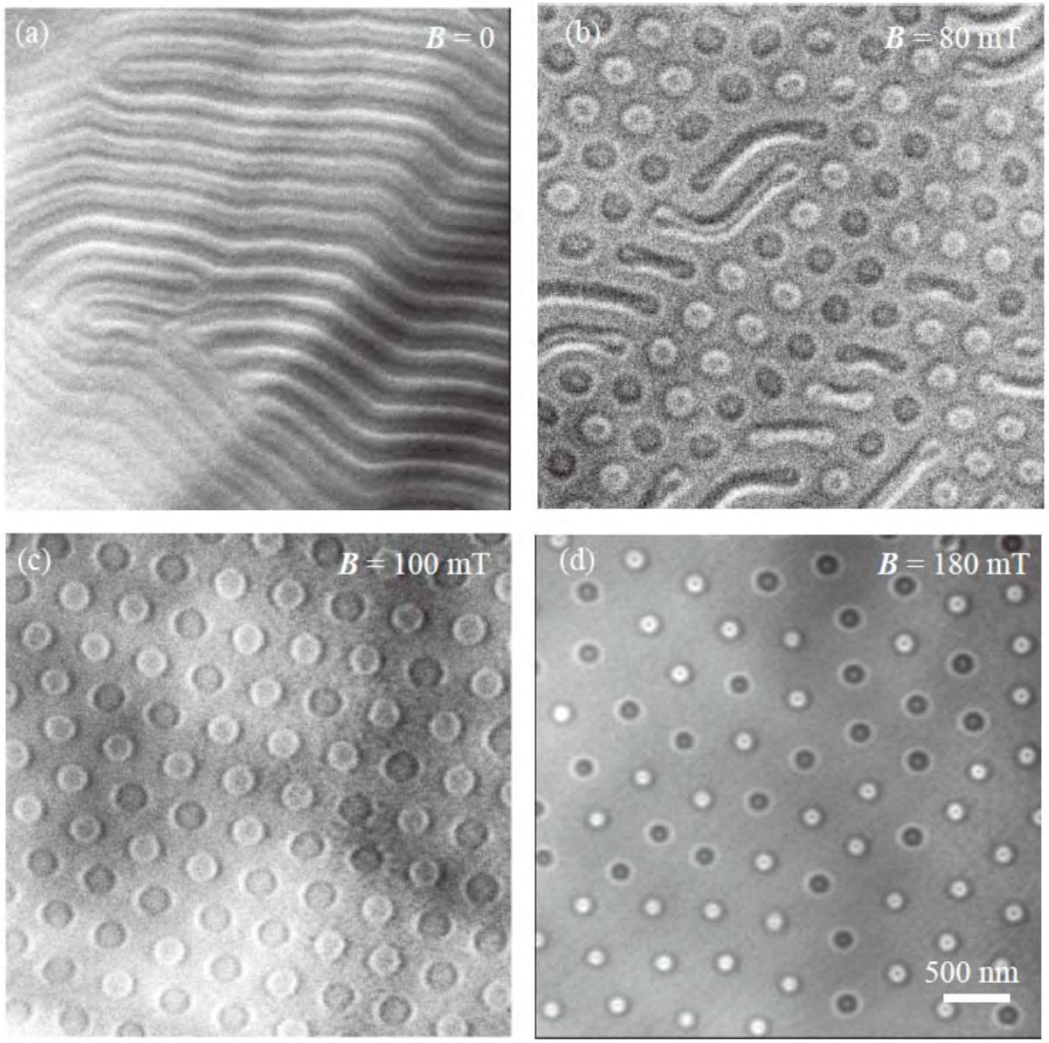
Inverse correlation length

$$\kappa^2 = \kappa_{\mathrm{MF}}^2 + \frac{\kappa_{\mathrm{Gi}}^3}{\kappa}$$

*S. A. Brazovskii, Sov. Phys. JETP **41**, 85 (1975)*

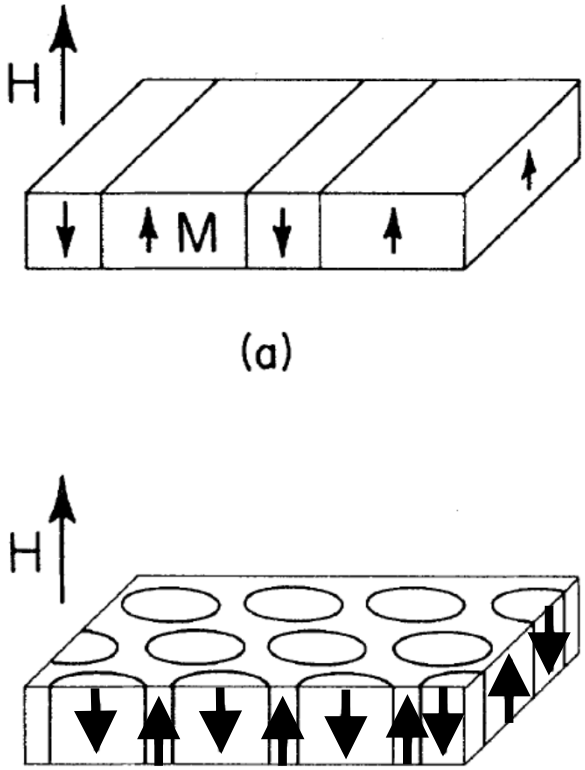
Skyrmion Crystal State

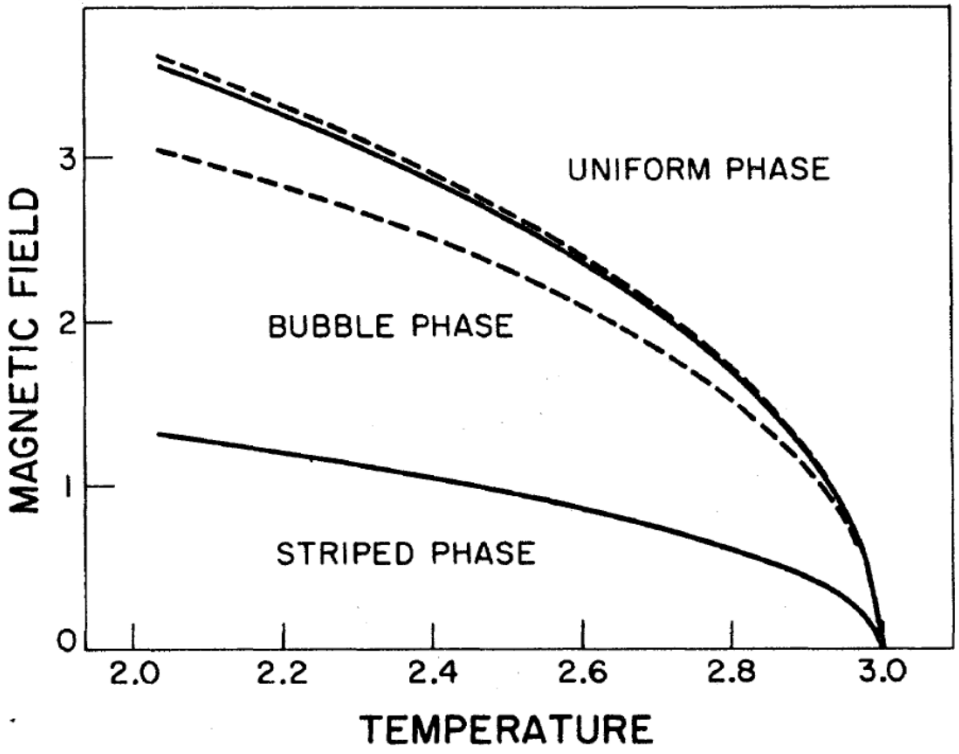
Stripes & magnetic bubbles



X. Yu et al. PNAS 109, 8856 (2012)

Bubble arrays in ferromagnetic films





Stabilization of bubble array

*T. Garel & S. Doniach, PRB **26** , 325 (1982)*

Free energy of Ising ferromagnet

$$F = \int d^3x \left(c(\nabla M)^2 + aM^2 + bM^4 - MH \right) + F_{MD}$$

Magnetodipolar energy favors non-uniform state

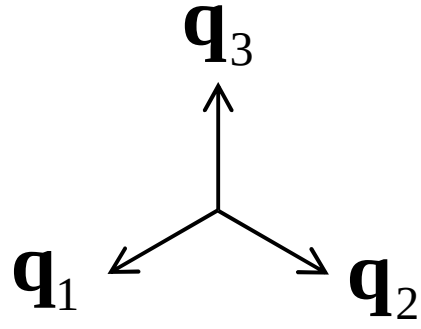
$$F_{MD} = 2\pi d \sum_{\mathbf{q}} f(qd) \left| M_{\mathbf{q}} \right|^2 \qquad f(x) = \frac{1 - e^{-x}}{x}$$

$$f(qd) \approx 1 - \frac{qd}{2}, \quad qd << 1$$ F_{MD} linearly decreases with q

4th-order term stabilizes 3q-state in magnetic field

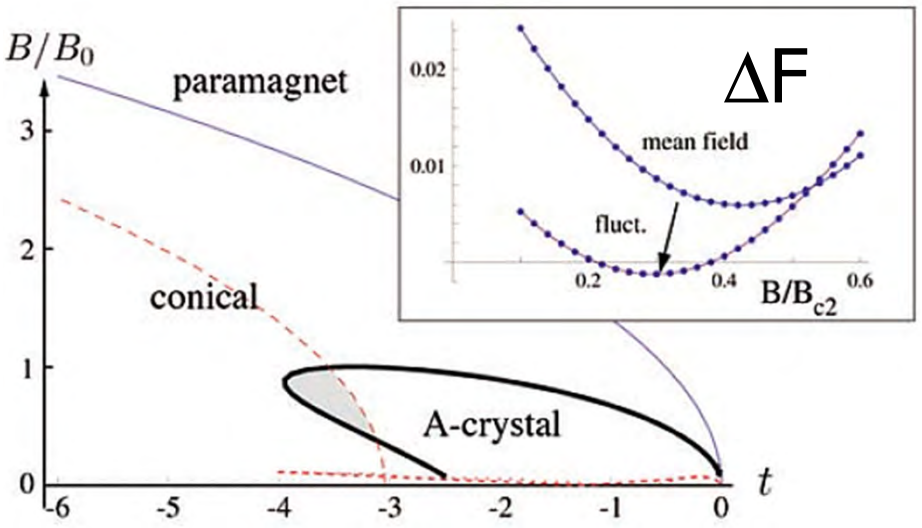
$$\int d^3x M^4 = \sum M_{\mathbf{q}_1} M_{\mathbf{q}_2} M_{\mathbf{q}_3} M_{-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3}$$

$$H \neq 0 \qquad \sum_{\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3=0} M_{\mathbf{q}_1} M_{\mathbf{q}_2} M_{\mathbf{q}_3} M_0$$



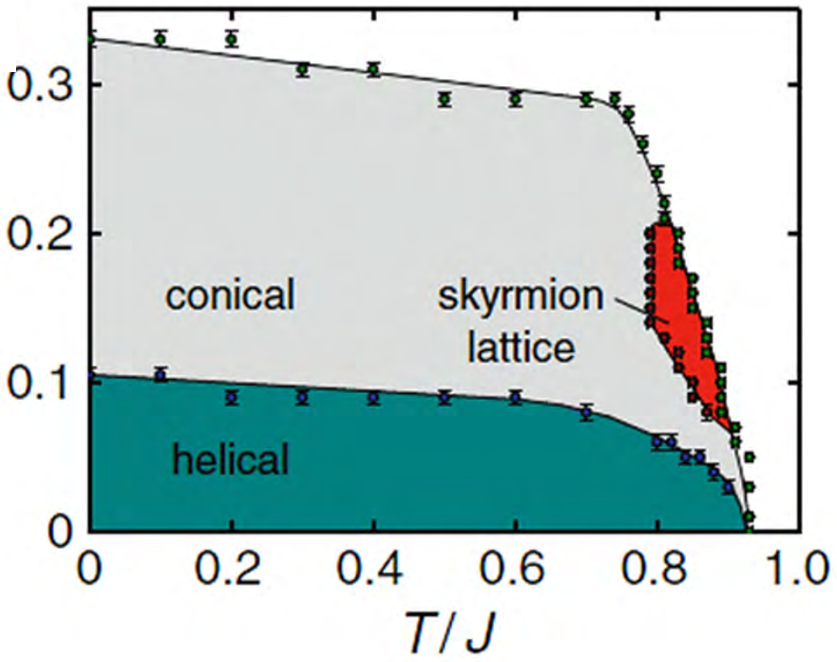
Order from disorder

$$F = F_{MF} + F_{fluct}$$



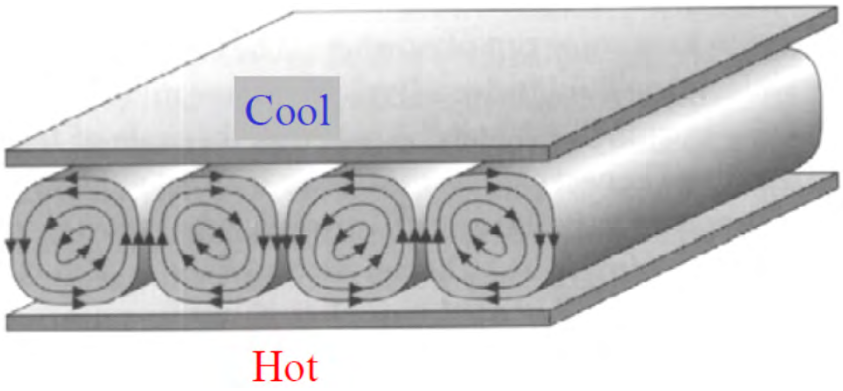
Mühlbauer et al, Science **323**, 915 (2009)

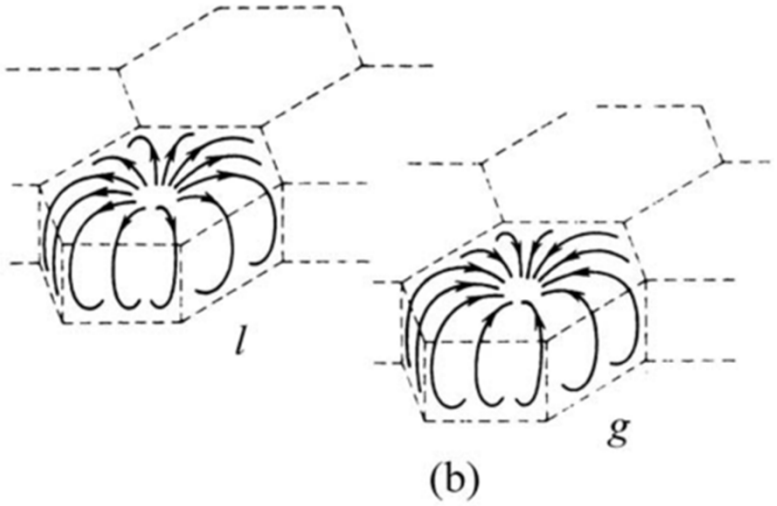
Monte Carlo

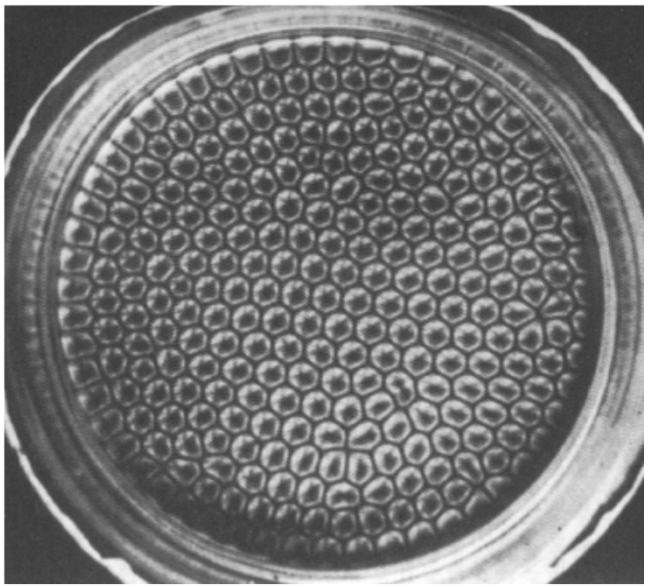


S. Buhrandt & L. Fritz,
PRB **88** 195137 (2013)

Rayleigh-Bénard convection

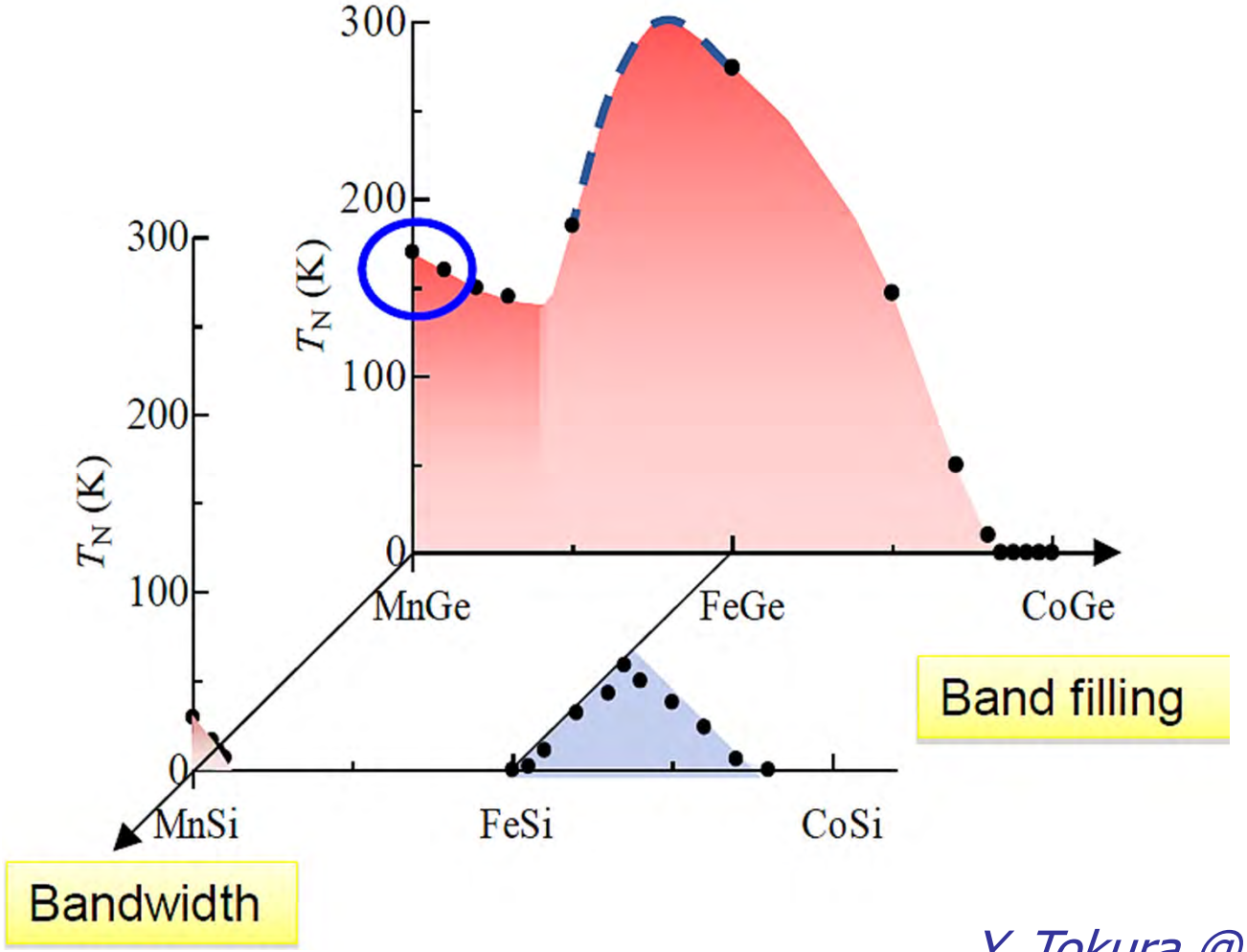






More skyrmion materials

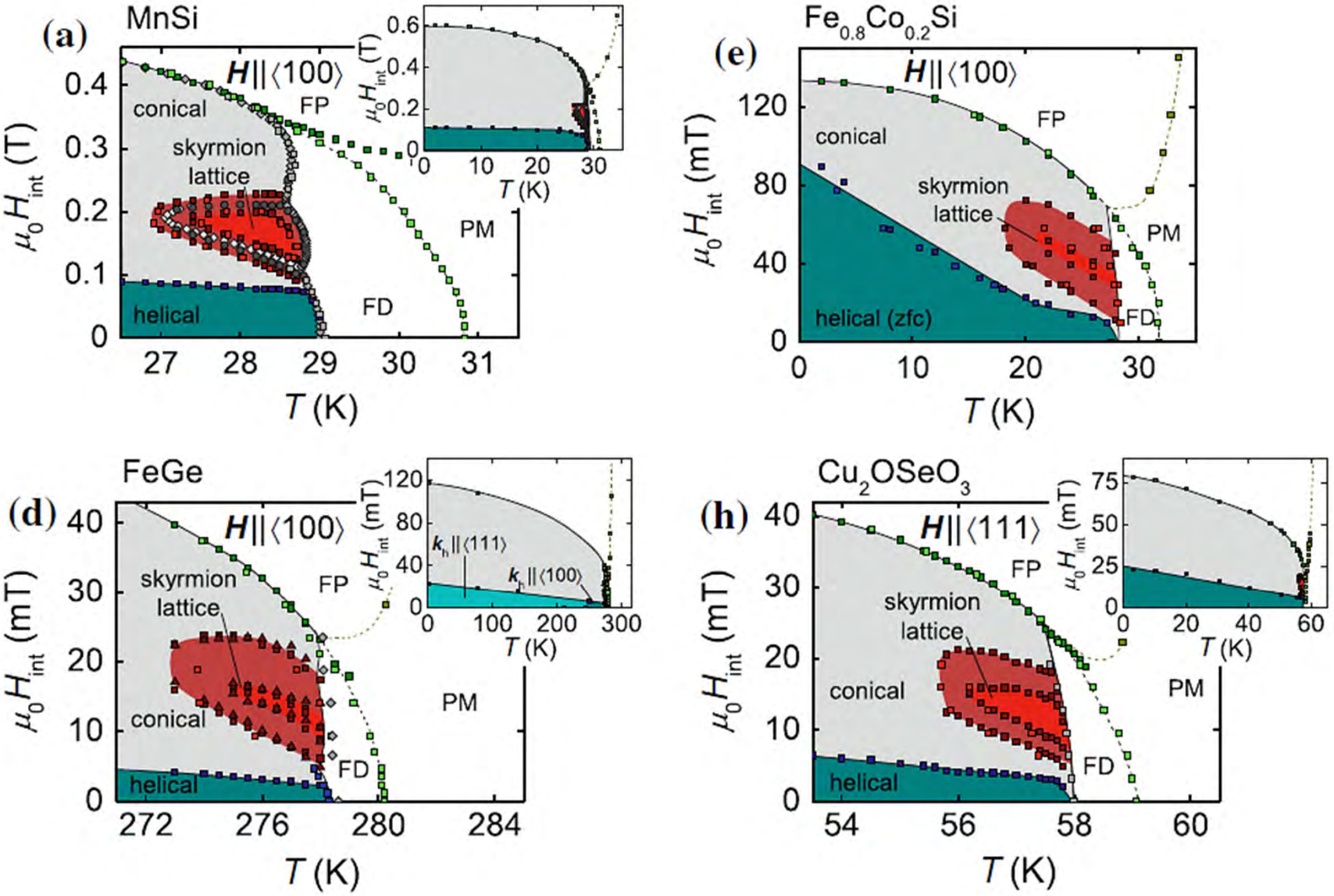
B20-compounds



Y. Tokura @ KITP

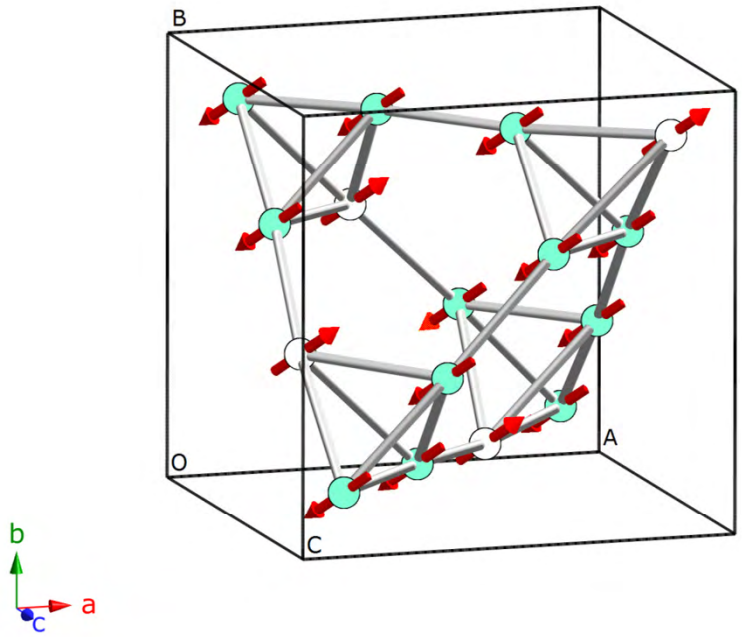
Universal phase diagram

A. Bauer & C. Pfleiderer (2016)

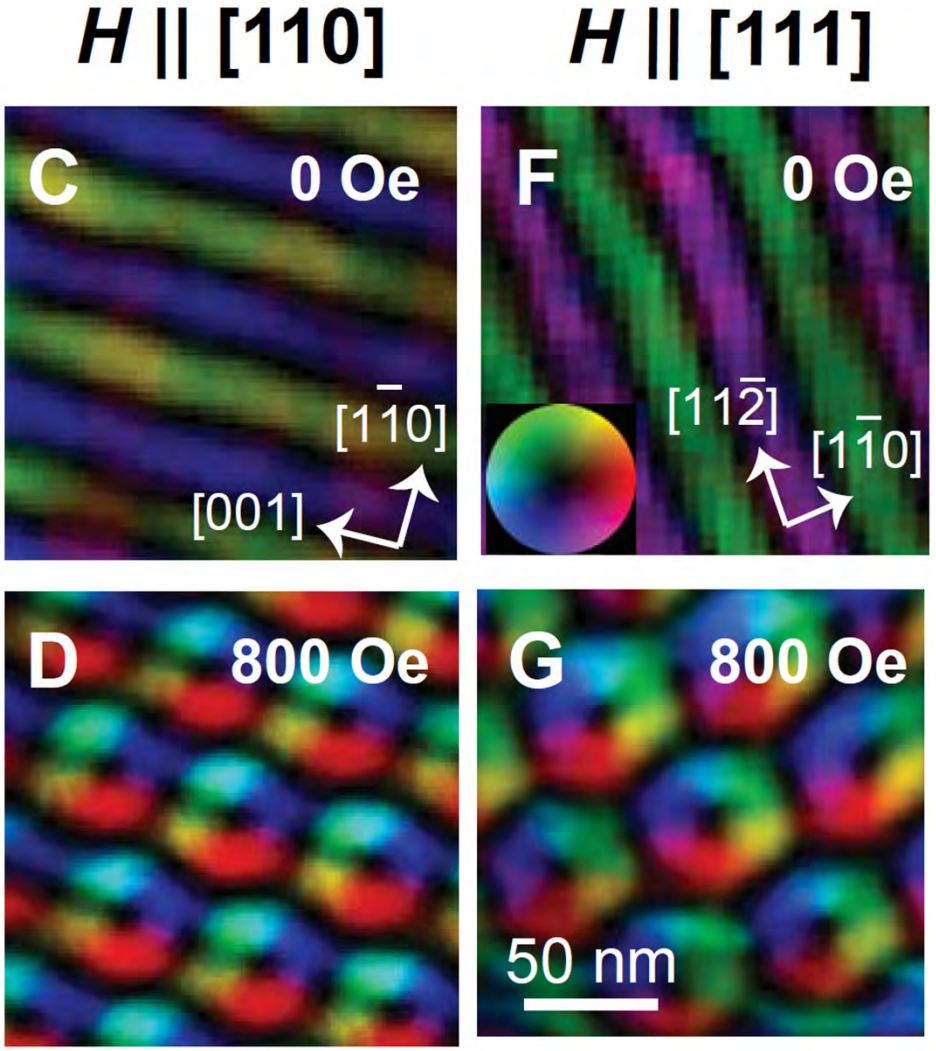


Skyrmions in Cu_2OSeO_3

$P32_1$ $\lambda = 60$ nm

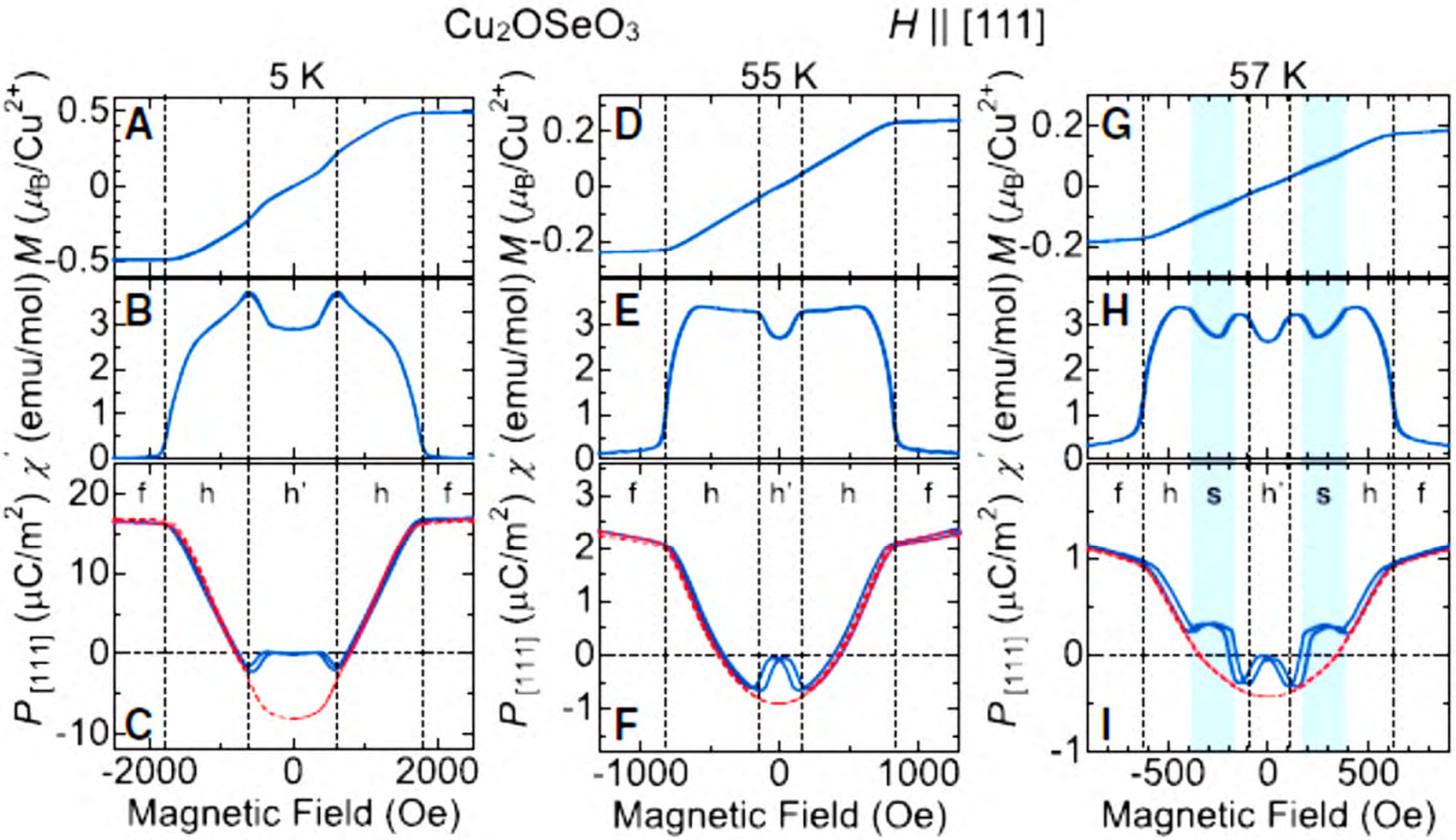


*J-W. G. Bos et al PRB **78,** 094416 (2008)*



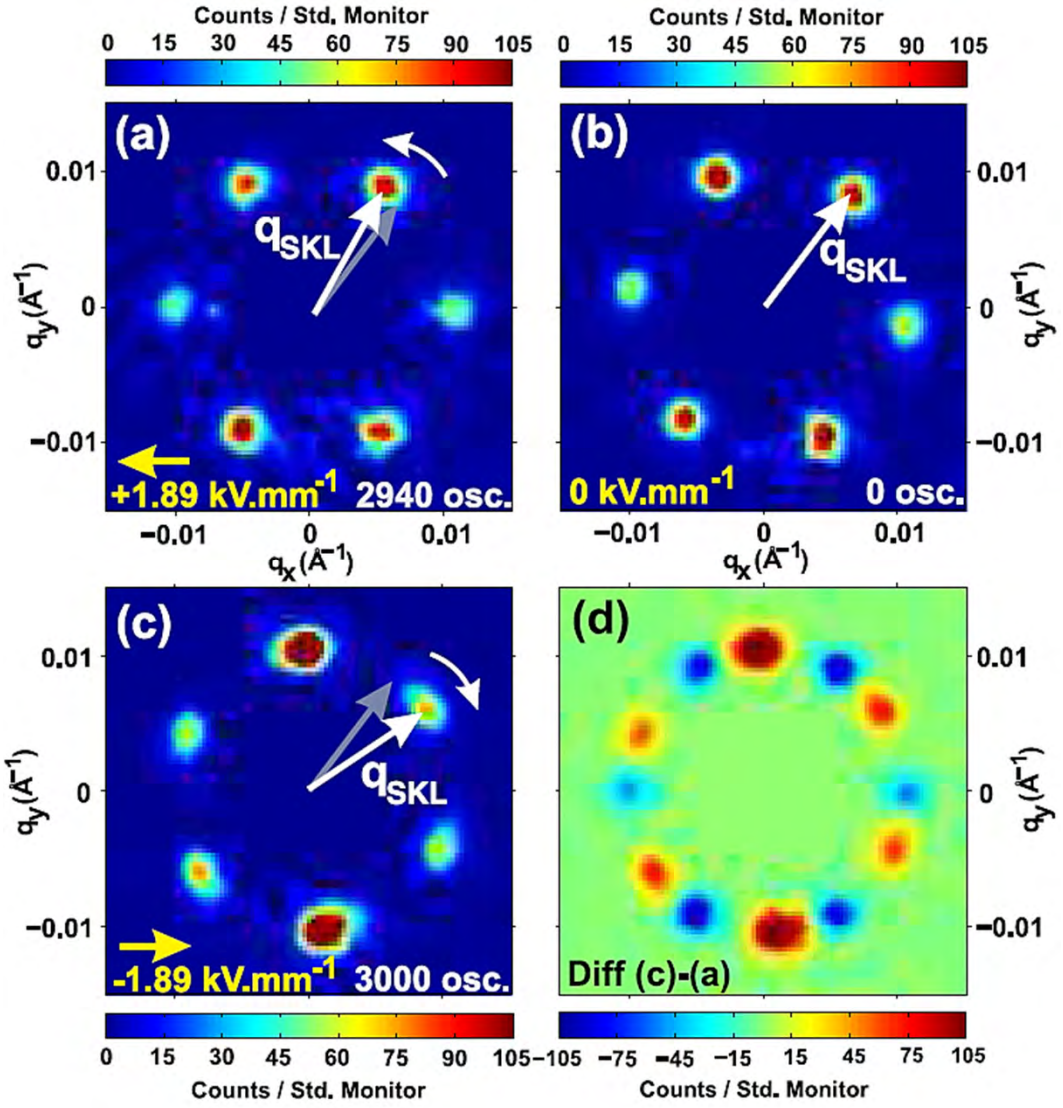
*S. Seki et al Science **336**, 198 (2012)*

Multiferroic behavior



*S. Seki et al Science **336**, 198 (2012)*

Rotation of SkX in electric field

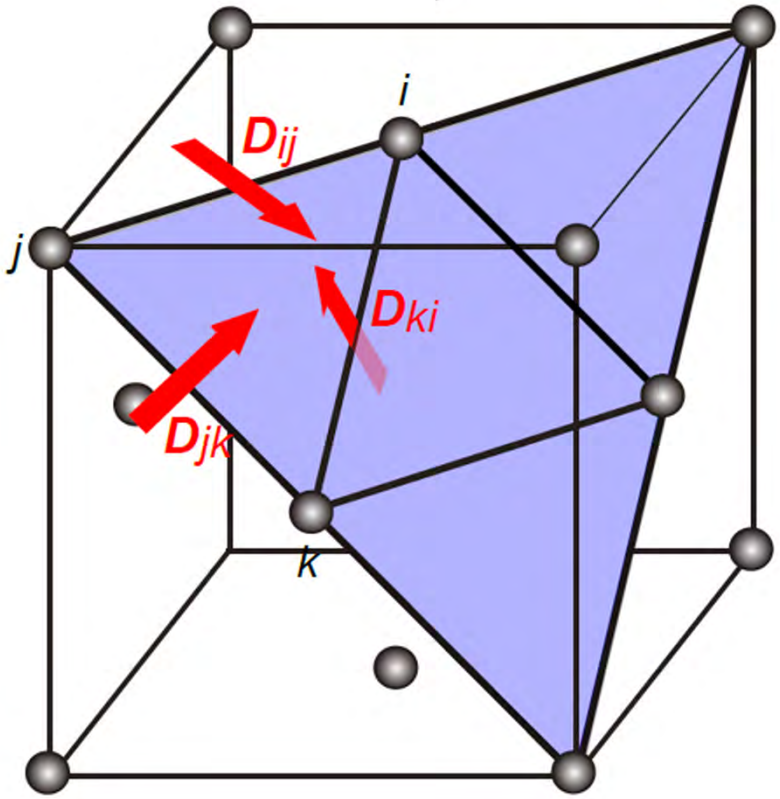


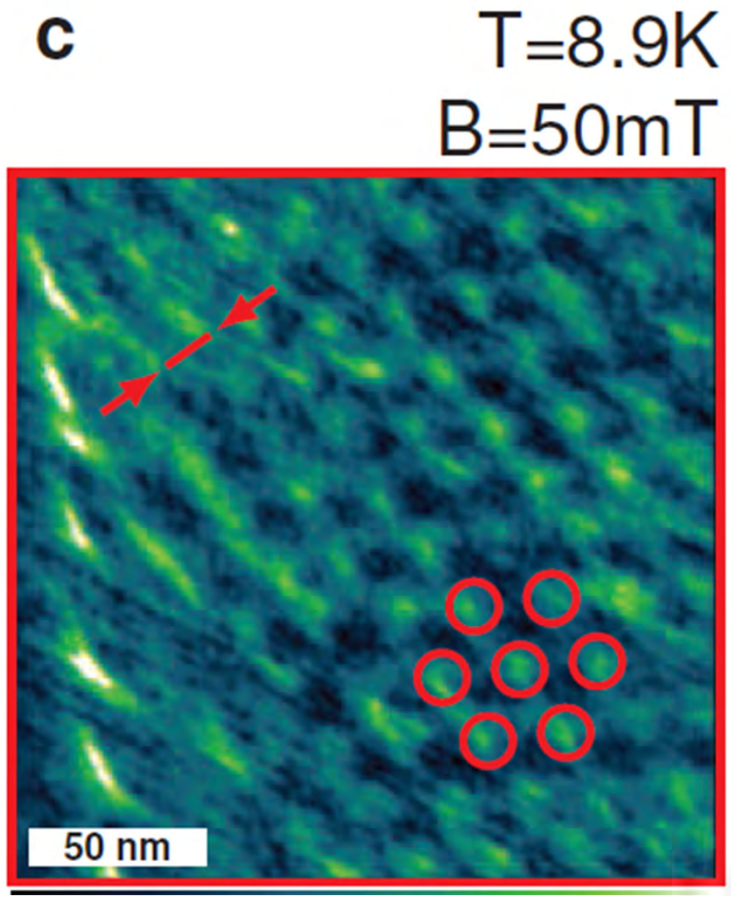
*J. S. White, PRL **113** 107203 (2014)*

GaV_4S_8

I. Kézsmárki et al. Nat. Mater. ***14****, 1116 (2015)*

$\boldsymbol{F\bar{4}3m} \rightarrow \boldsymbol{R3m}$ at 42K



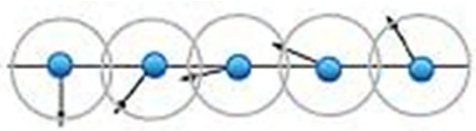


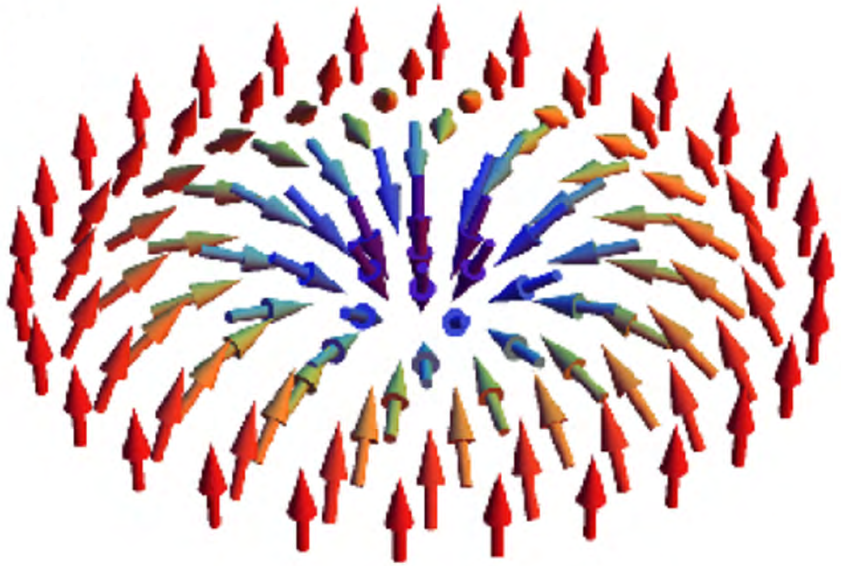
Néel skyrmion

Lifshitz Invariant R3m

$$(n_z \partial_x n_x - n_x \partial_x n_z) + (n_z \partial_y n_y - n_y \partial_y n_z)$$

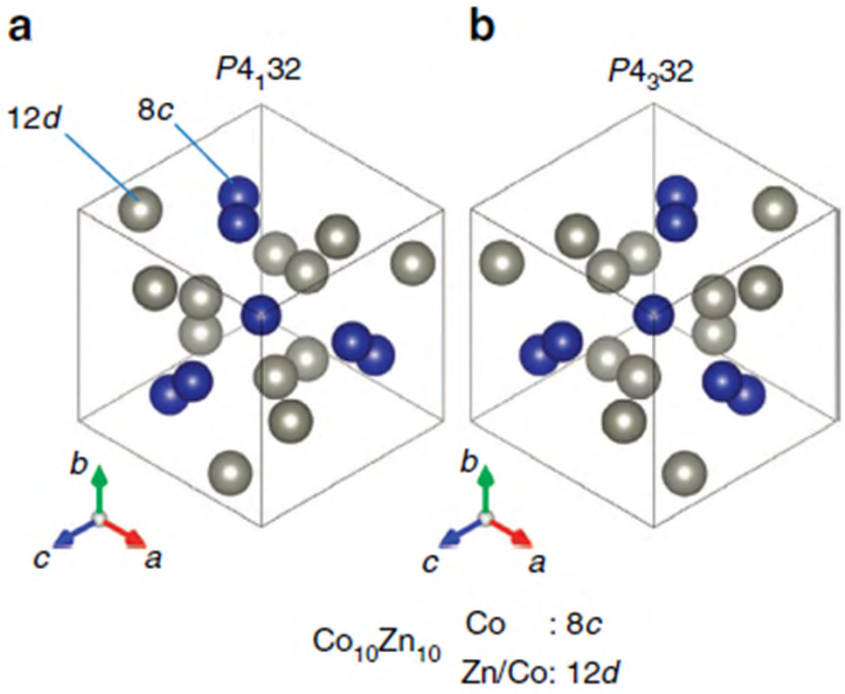
Cycloidal spiral

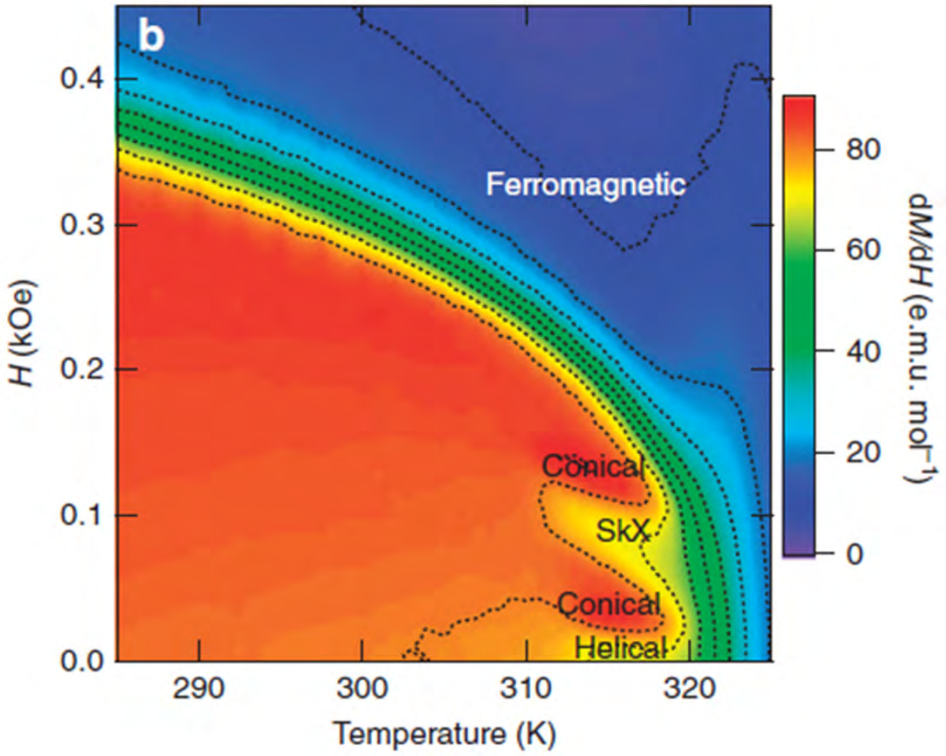




Room-temperature skyrmions

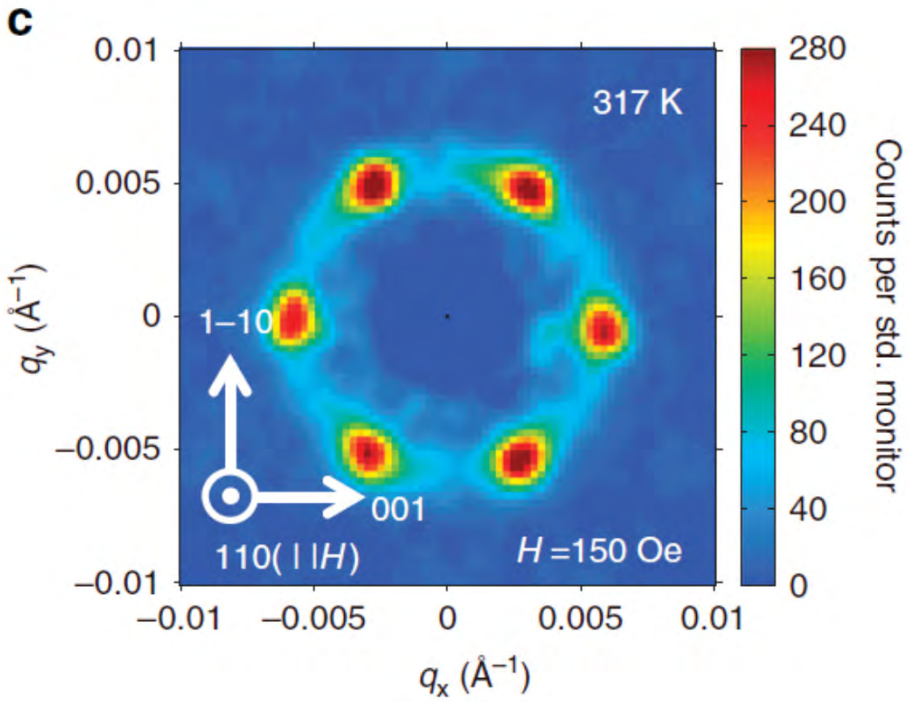
Y. Tokunaga et al. Nature Communications (2015)

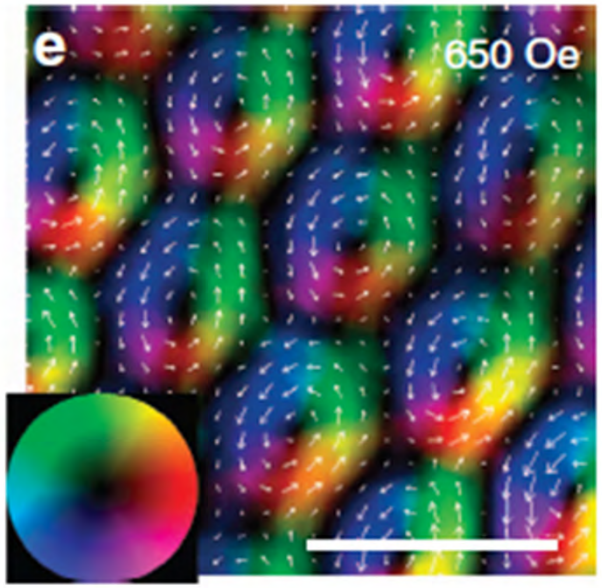




$Co_8Zn_9Mn_3$

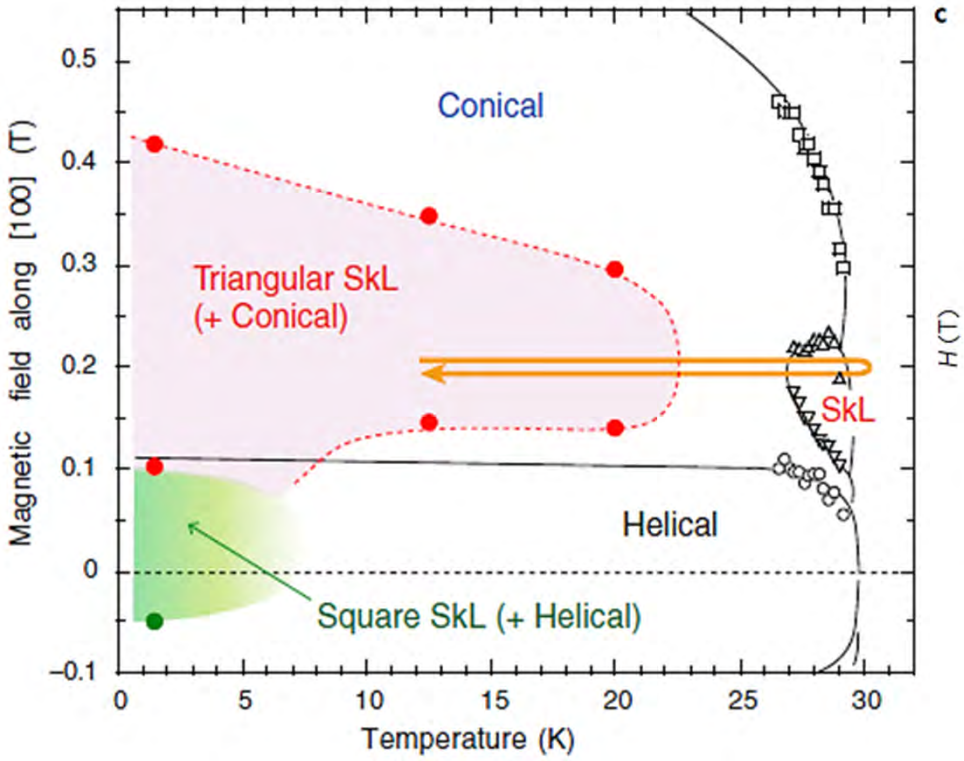
SkX in $Co_8Zn_8Mn_4$



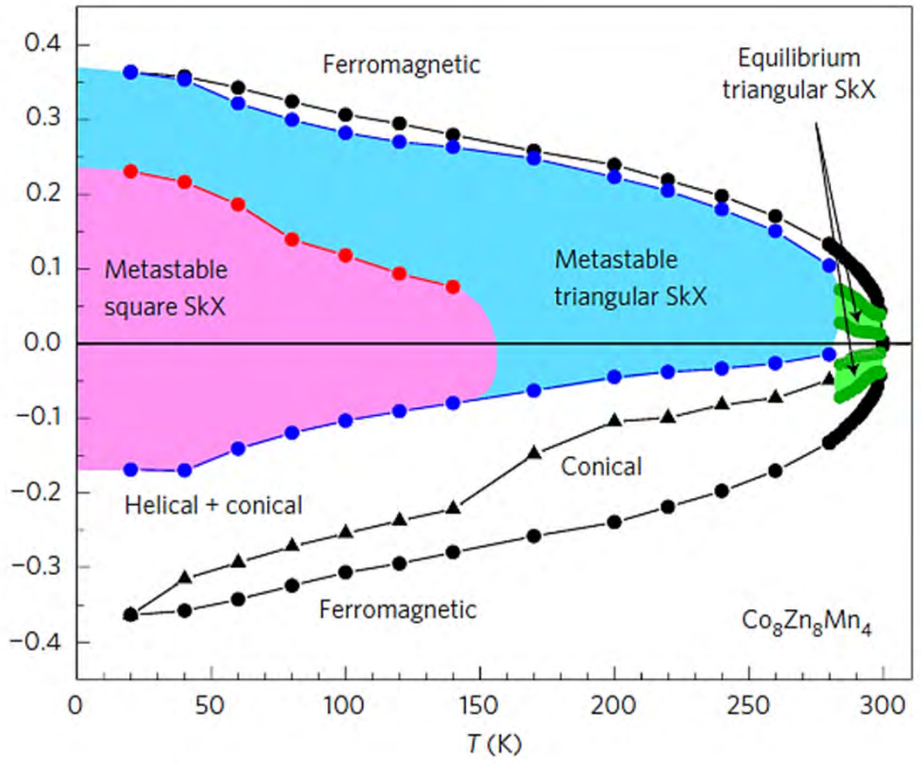


Metastable SkX with square lattice

MnSi



$Co_8Zn_8Mn_4$



Nakajima et al., Sci. Adv. 2017 e1602562

K. Karube et al Nat. Mater. 15, 1237 (2016)

Physical meaning of topological charge

Adiabatic motion of spin-polarized electrons

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\triangle - J\boldsymbol{\sigma}\cdot\boldsymbol{n}\right)\psi \qquad \boldsymbol{n} = \boldsymbol{n}(\boldsymbol{x}, t)$$

Adiabatic projection

$$|\psi\rangle = \chi|\boldsymbol{n}\rangle \qquad \boldsymbol{\sigma}\cdot\boldsymbol{n}|\boldsymbol{n}\rangle = |\boldsymbol{n}\rangle$$

$$i\hbar\frac{\partial\chi}{\partial t} = \left(-ea_0 + \frac{1}{2m}\left(\frac{\hbar}{i}\boldsymbol{\nabla} + \frac{e}{c}\boldsymbol{a}\right)^2 - J + \frac{\hbar^2}{8m}\partial_i\boldsymbol{n}\cdot\partial_i\boldsymbol{n}\right)\chi$$

Effective gauge potentials

$$a_0 = \frac{i\hbar}{e}\langle\boldsymbol{n}|\frac{\partial}{\partial t}|\boldsymbol{n}\rangle \qquad \boldsymbol{a} = -\frac{i\hbar c}{e}\langle\boldsymbol{n}|\boldsymbol{\nabla}|\boldsymbol{n}\rangle$$

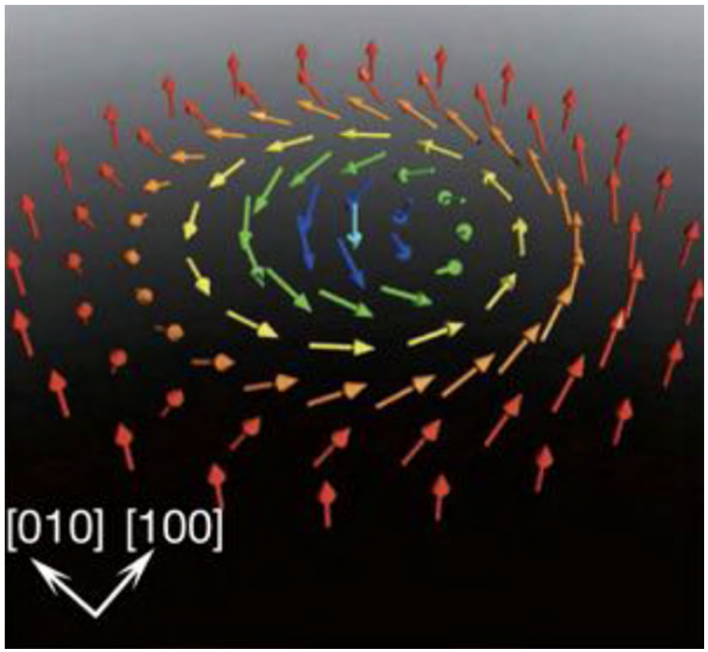
Effective gauge fields

$$\begin{cases} \boldsymbol{e}_i = -\partial_i \boldsymbol{a}_0 - \dfrac{1}{\boldsymbol{c}} \partial_t \boldsymbol{a}_i = \dfrac{\hbar}{2\boldsymbol{e}} \left(\mathbf{n} \cdot \left[\partial_i \mathbf{n} \times \partial_t \mathbf{n} \right] \right) \\ \boldsymbol{h}_i = \left[\nabla \times \mathbf{a} \right]_i = \dfrac{\hbar \boldsymbol{c}}{4\boldsymbol{e}} \varepsilon_{ijk} \left(\mathbf{n} \cdot \left[\partial_j \mathbf{n} \times \partial_k \mathbf{n} \right] \right) \end{cases}$$

Lorentz force acting on spin-polarized electron

$$\Delta \mathbf{F} = -e\,\mathbf{e} - \frac{e}{c}[\mathbf{v} \times \mathbf{h}]$$

Physical meaning of Skyrmion charge



Topological charge:

$$Q = \frac{1}{4\pi}\int d^2x\left(\mathbf{n}\cdot\left[\partial_x\mathbf{n}\times\partial_y\mathbf{n}\right]\right)$$

$$Q = \frac{e}{hc}\int d^2x h_z$$

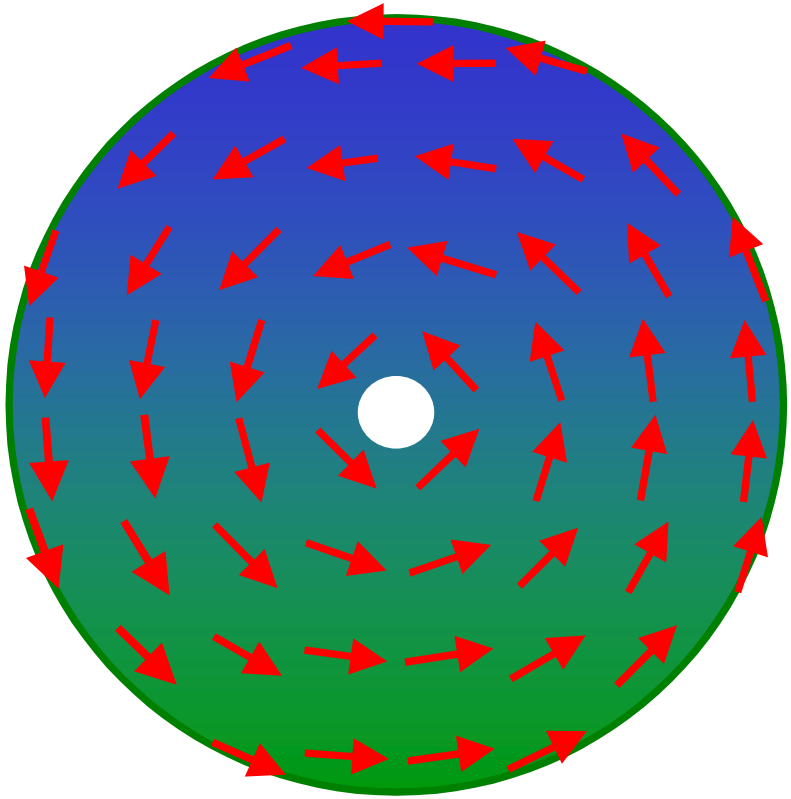
Flux of internal magnetic field:

$$\Phi = \int d^2x h_z = Q\Phi_0 \qquad \Phi_0 = \frac{hc}{e}$$

Topological charge of vortex

Vortex in superconductor

$$\Psi = |\Psi| e^{i\phi}$$



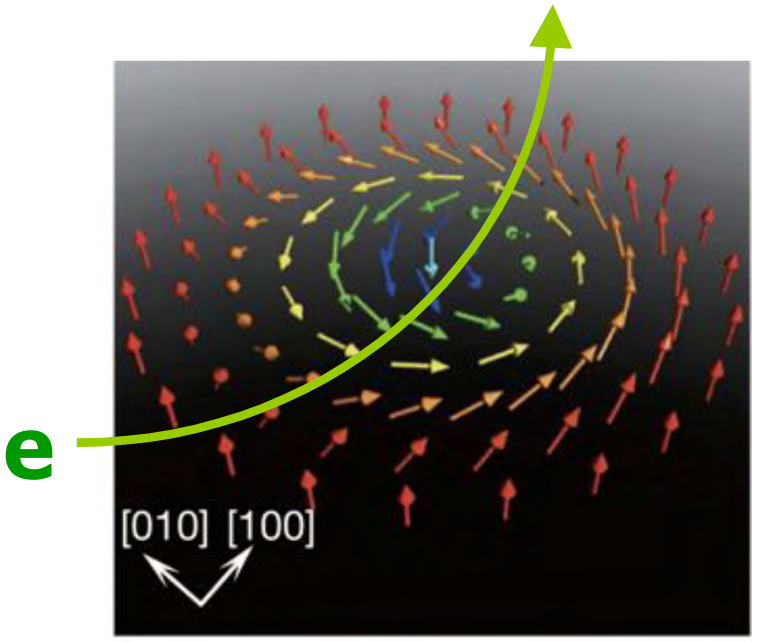
Winding number

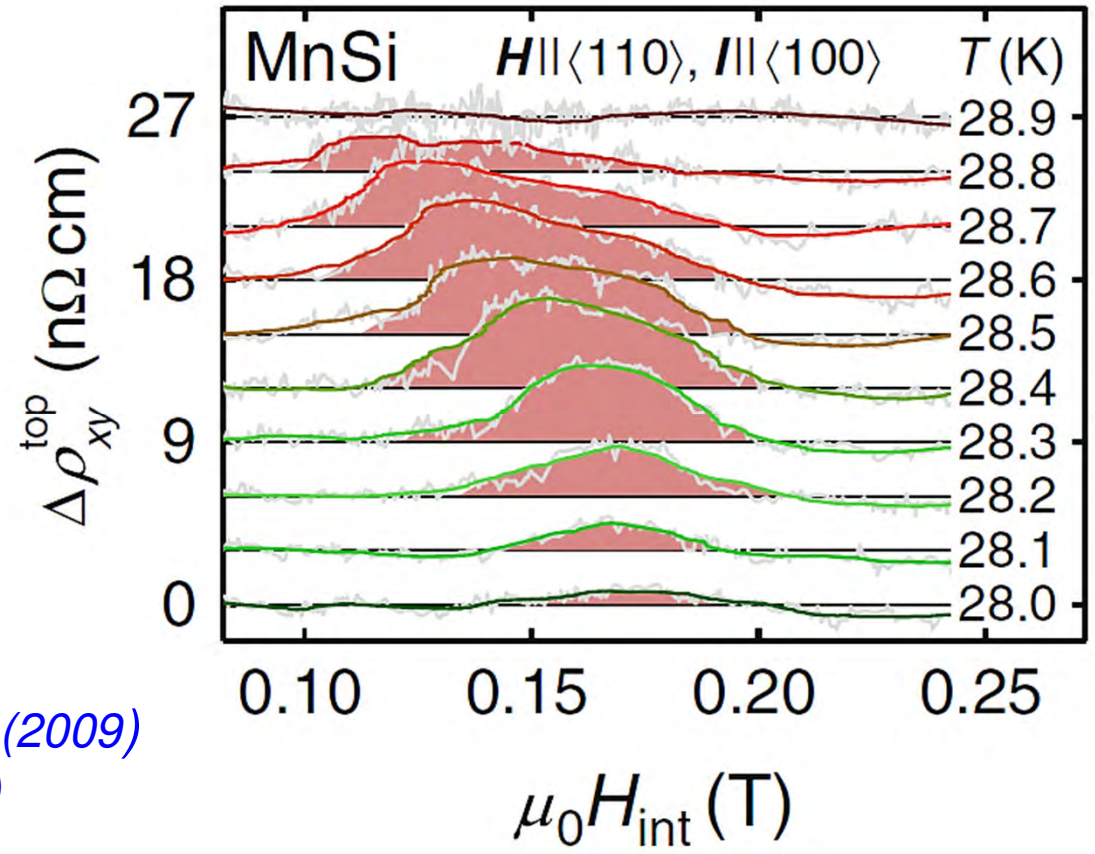
$$Q = \frac{1}{2\pi} \oint d\mathbf{x} \cdot \nabla \phi$$

$$\Phi = \oint d\mathbf{x} \cdot \mathbf{A} = -\Phi_0 Q$$

$$\Phi_0 = \frac{hc}{2e}$$

Topological Hall Effect





*A. Neubauer et al PRL **102**, 186602 (2009)*
*M. Lee et al PRL **102**, 186601 (2009)*

Emergent electric field

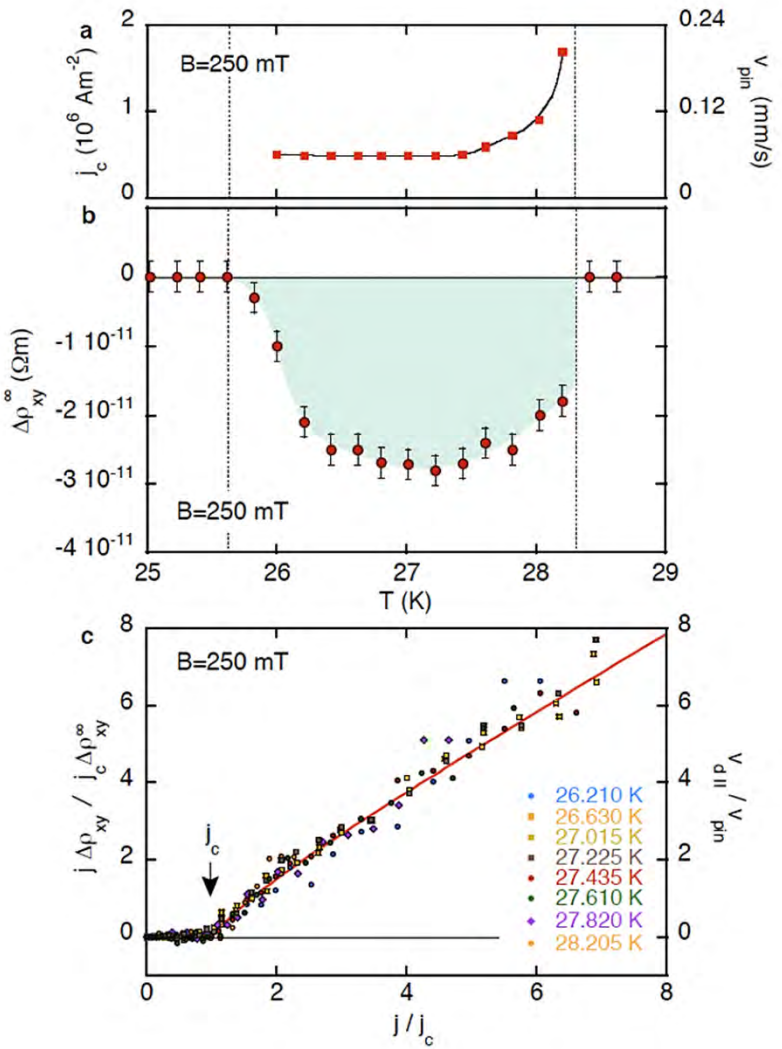
Electric field of moving skyrmion

$$\mathbf{E} = -\frac{1}{c}[\mathbf{V} \times \mathbf{H}]$$

Lorentz force

$$\mathbf{F} = \frac{(-e)}{c}[(\mathbf{v} - \mathbf{V}) \times \mathbf{H}]$$

*J. Zang, MM et al PRL **107**, 136804 (2011)*

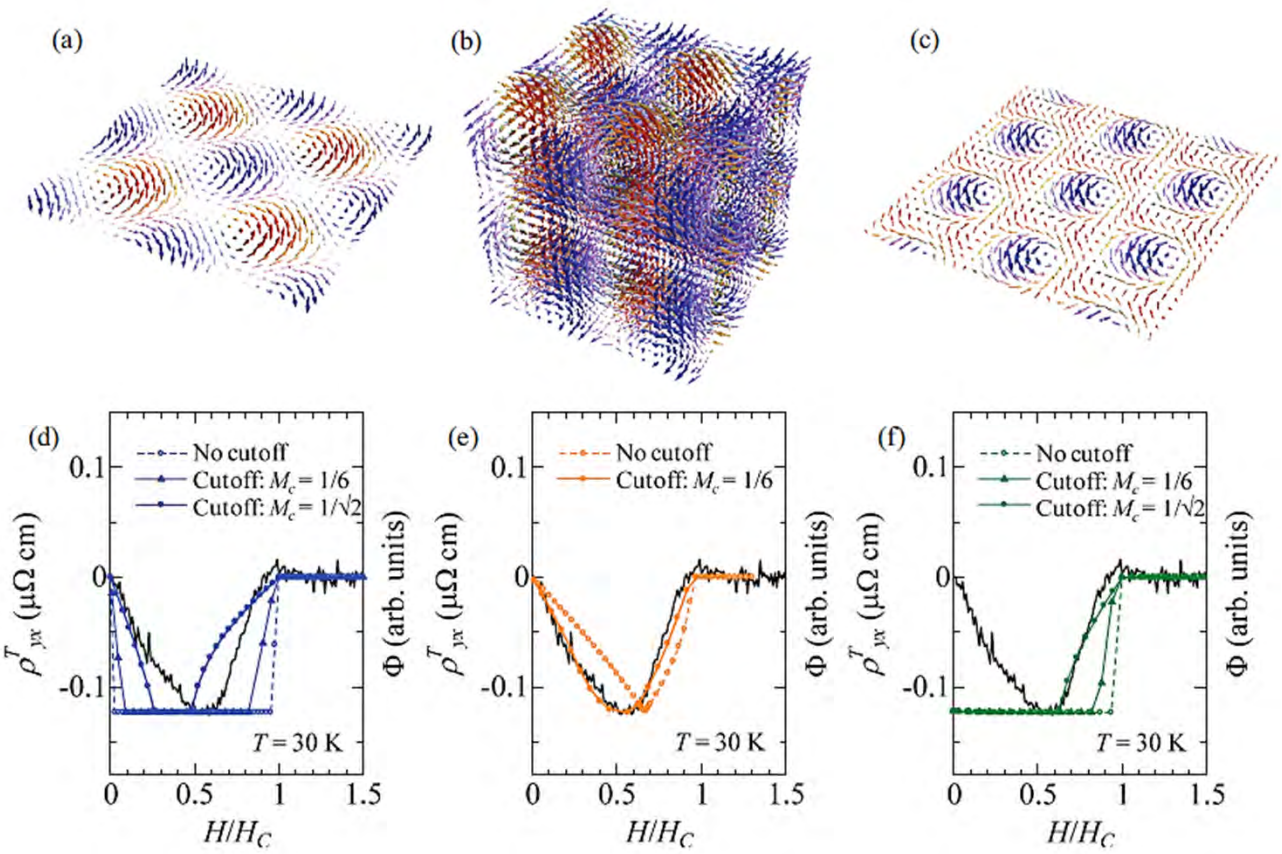


*T. Schultz et al Nat. Phys. **8**, 301 (2012)*

THE in MnGe

$\lambda = 3\,\mathrm{nm}$

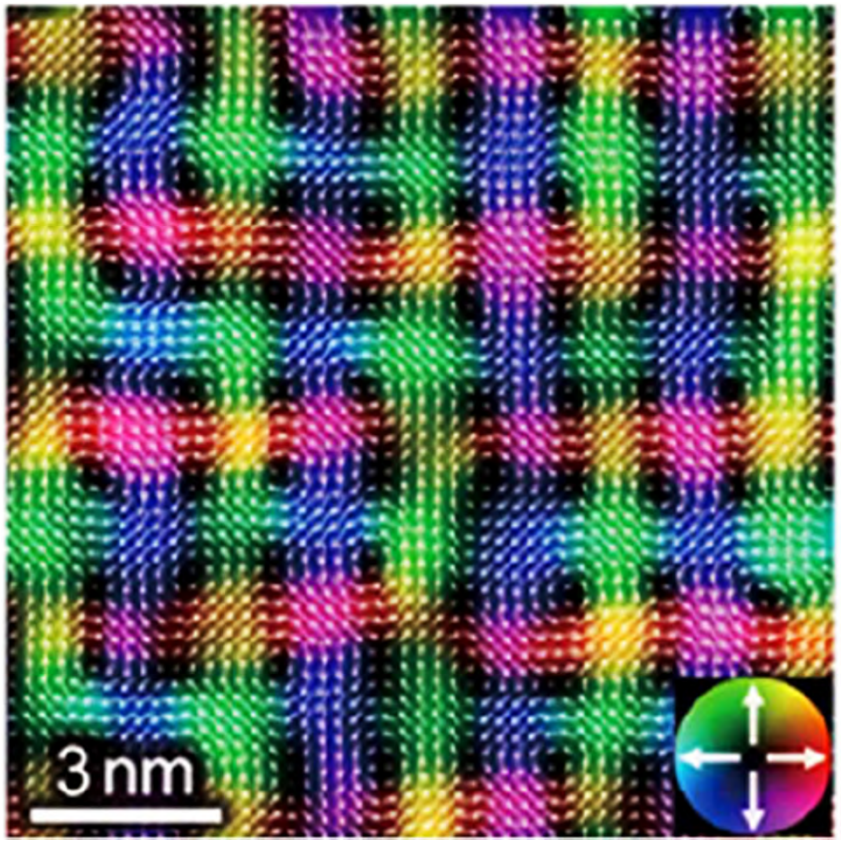
*N. Kanazawa et al. PRB **86**, 134425 (2012)*

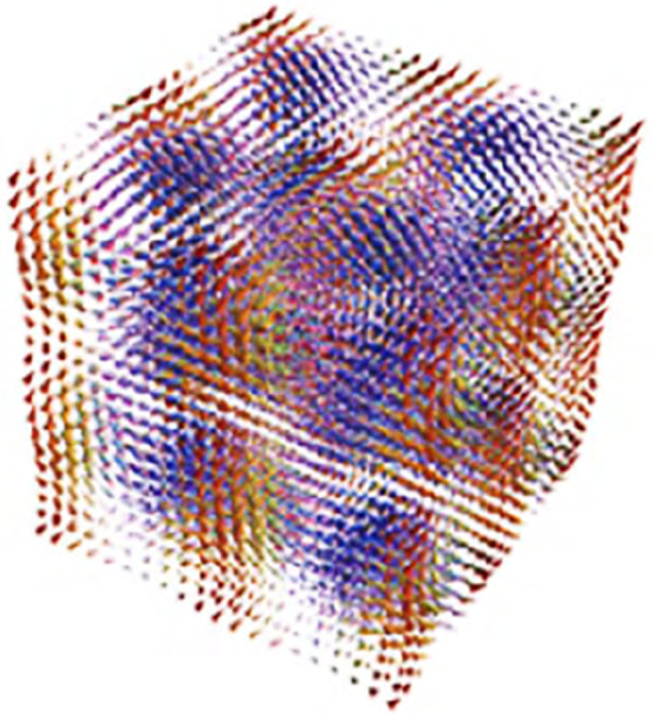


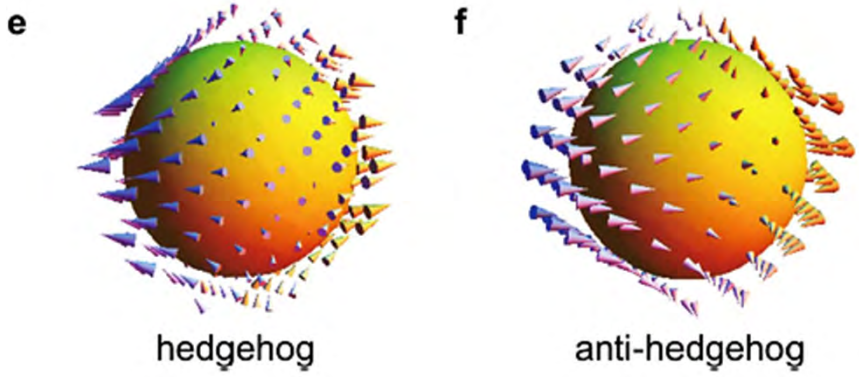
$$\mathbf{M} = (0, \cos \boldsymbol{qx}, -\sin \boldsymbol{qx}) + (-\sin \boldsymbol{qy}, 0, \cos \boldsymbol{qy})$$

$$+ (\cos \boldsymbol{qz}, -\sin \boldsymbol{qz}, 0) + (0, 0, \boldsymbol{m_z})$$

Lorentz TEM images of MnGe

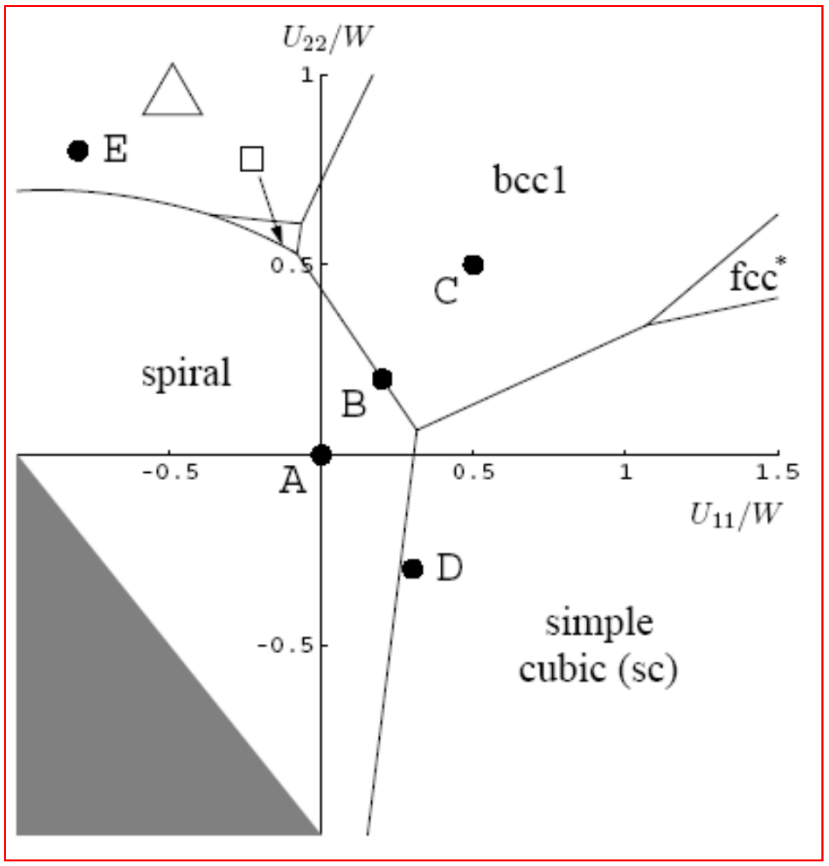




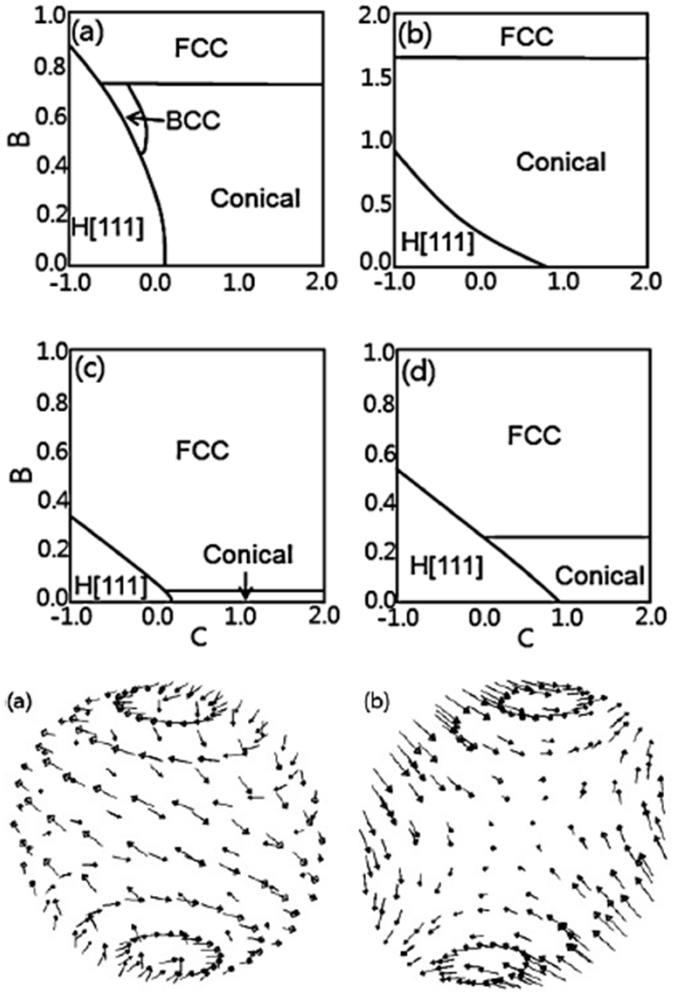


Y. Tonigaki et al.
Nano Lett. ***15****, 5438 (2015)*

Magnetic crystals



B. Binz et al., PRL **96**, 207202 (2006)

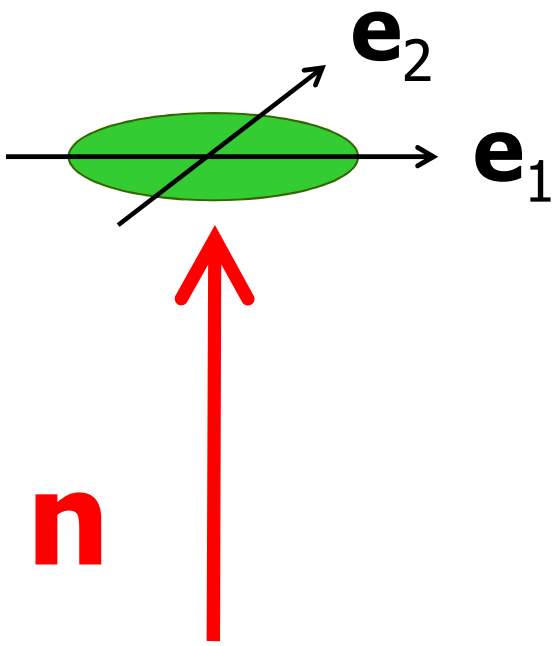


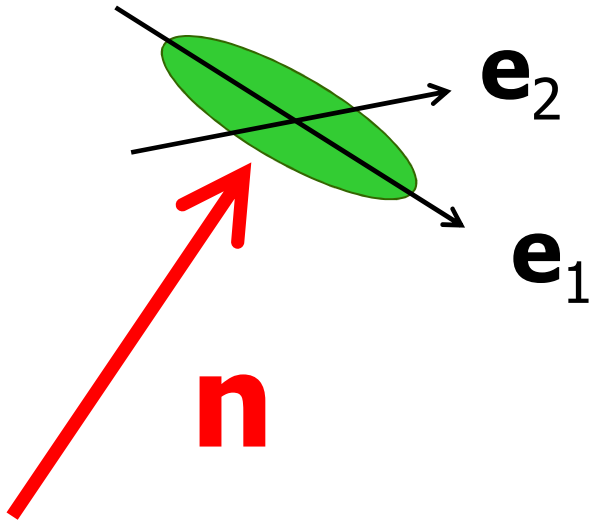
J-H Park and J-H Han PRB **83**, 184406 (2011)

Topological Magnon Hall Effect

Magnon effective gauge field

$$\delta \mathbf{n}(\mathbf{x},t) = \psi_1 \mathbf{e}_1 + \psi_2 \mathbf{e}_2 \qquad \mathbf{e}_1, \mathbf{e}_2 \perp \mathbf{n}$$

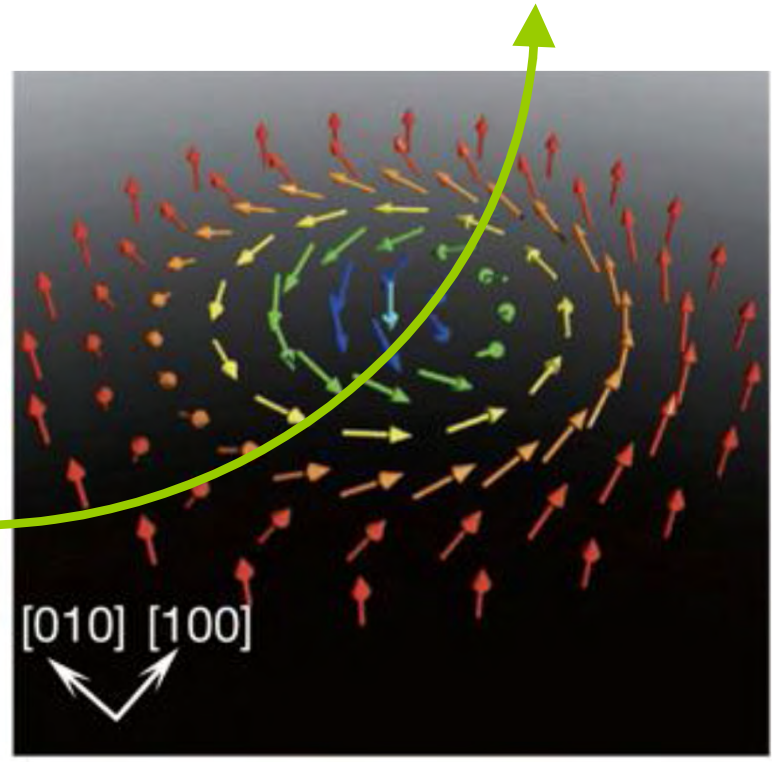




Effective gauge potential

$$a_i = \left(\mathbf{e}_2 \cdot \partial_i \mathbf{e}_1\right)$$

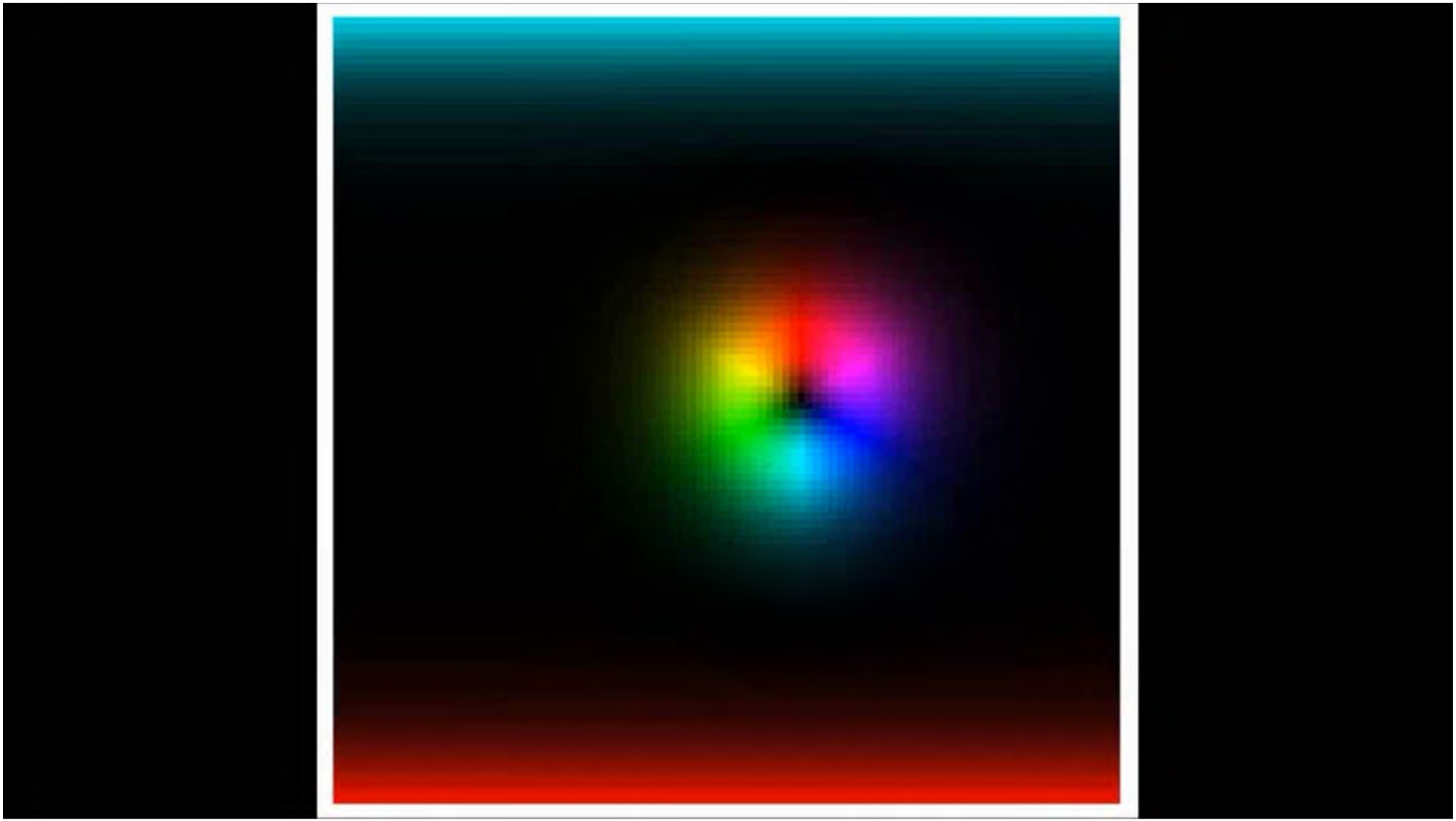
Topological Magnon Hall effect



For magnons, skyrmion is a flux of effective magnetic field

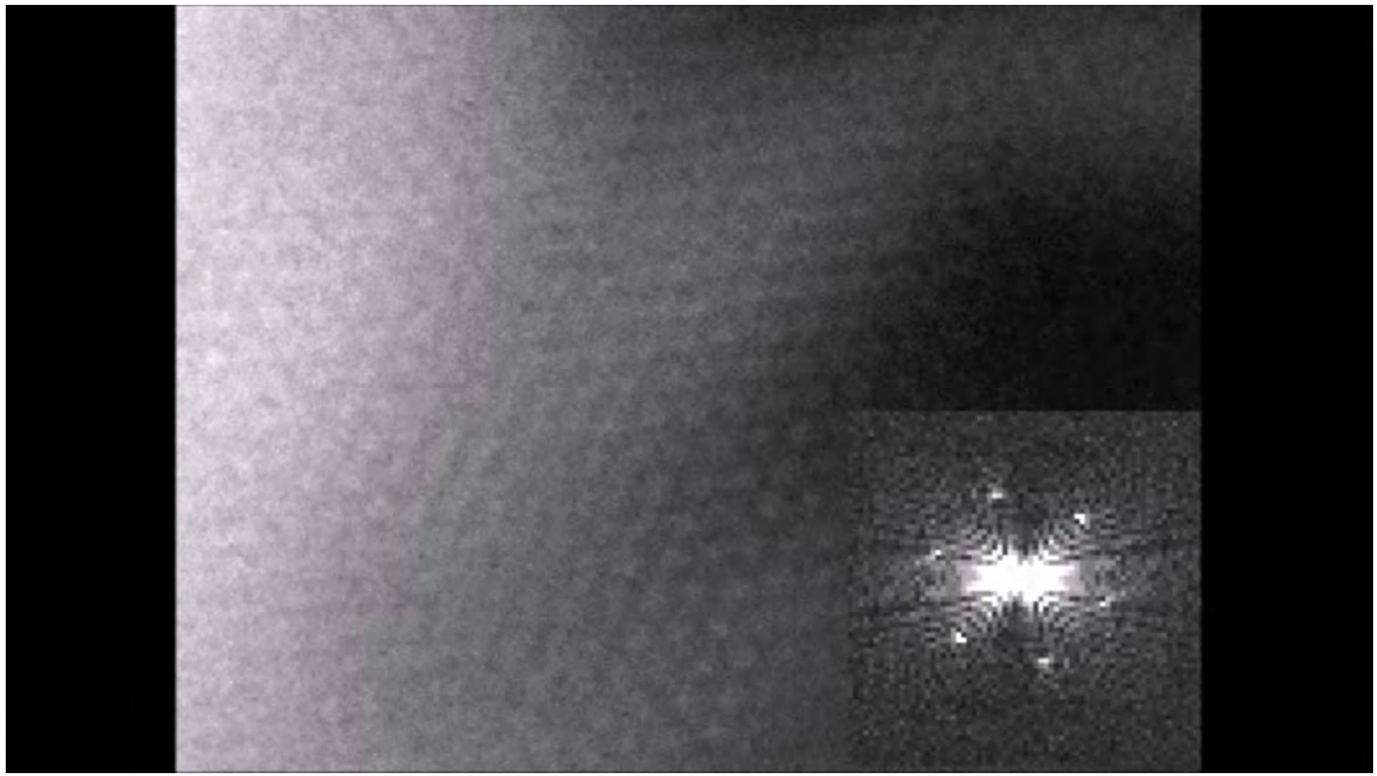
$\Phi = \pm 2\Phi_0$

Magnon-skyrmion scattering



*J. Iwasaki et al. PRB **89**, 064412 (2014)*

Rotation of SkX

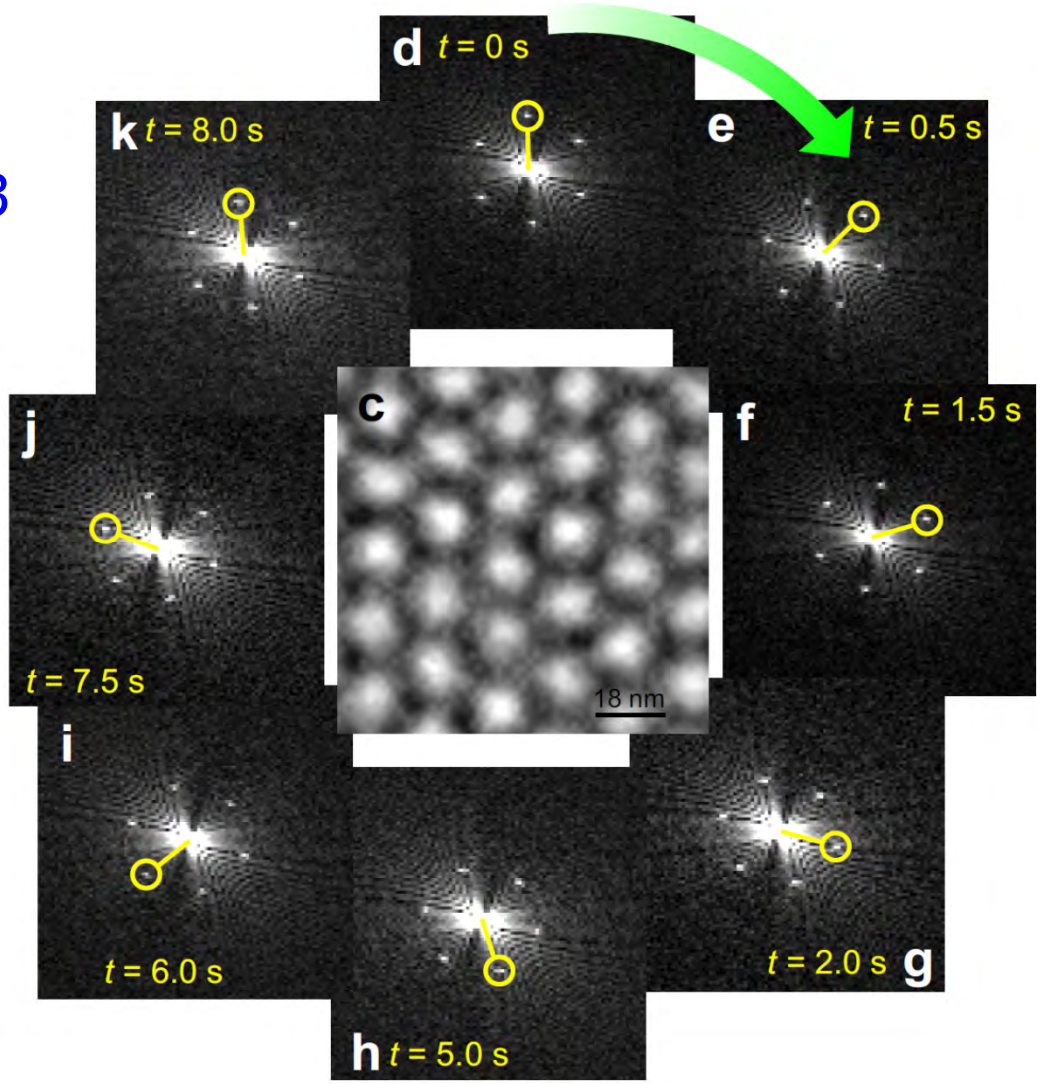


M. Mochizuki et al, Nat. Mater. 13, 241 (2014)

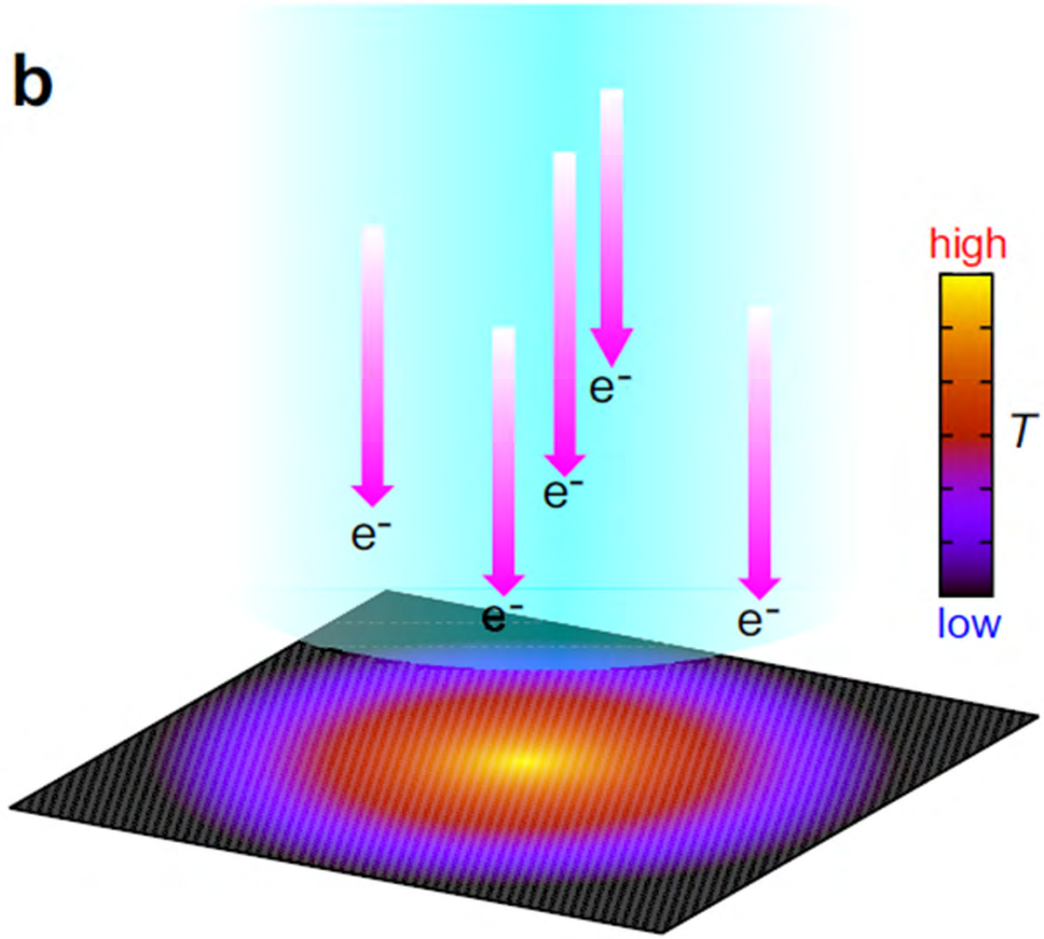
Rotation of skyrmion crystal

Cu_2OSeO_3

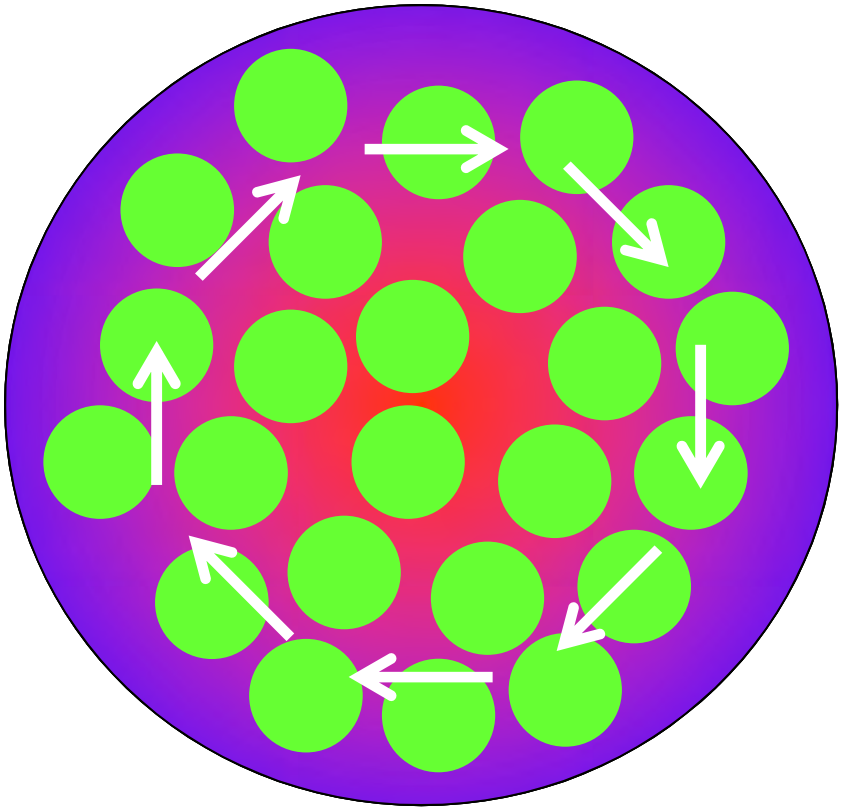
MnSi



Heating by electron beam



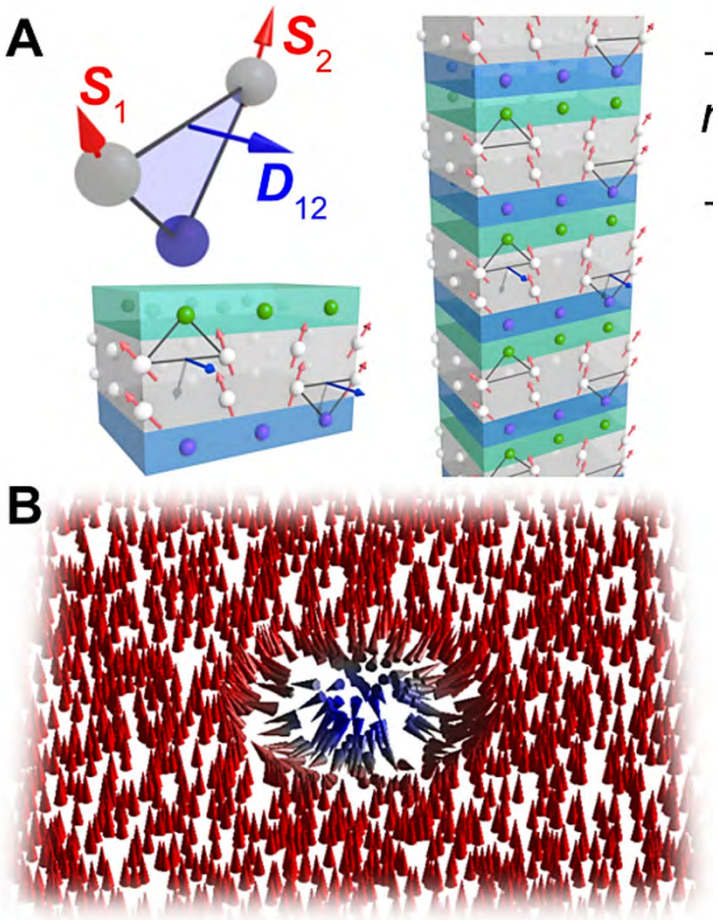
Magnon-driven rotation of SkX



Room-temperature skyrmions in mulilayers

*C. Moreau-Luchaire et al Nat. Nano **11**, 444 (2016)*

*S. Woo et al Nat. Mater. **15**, 501 (2016)*

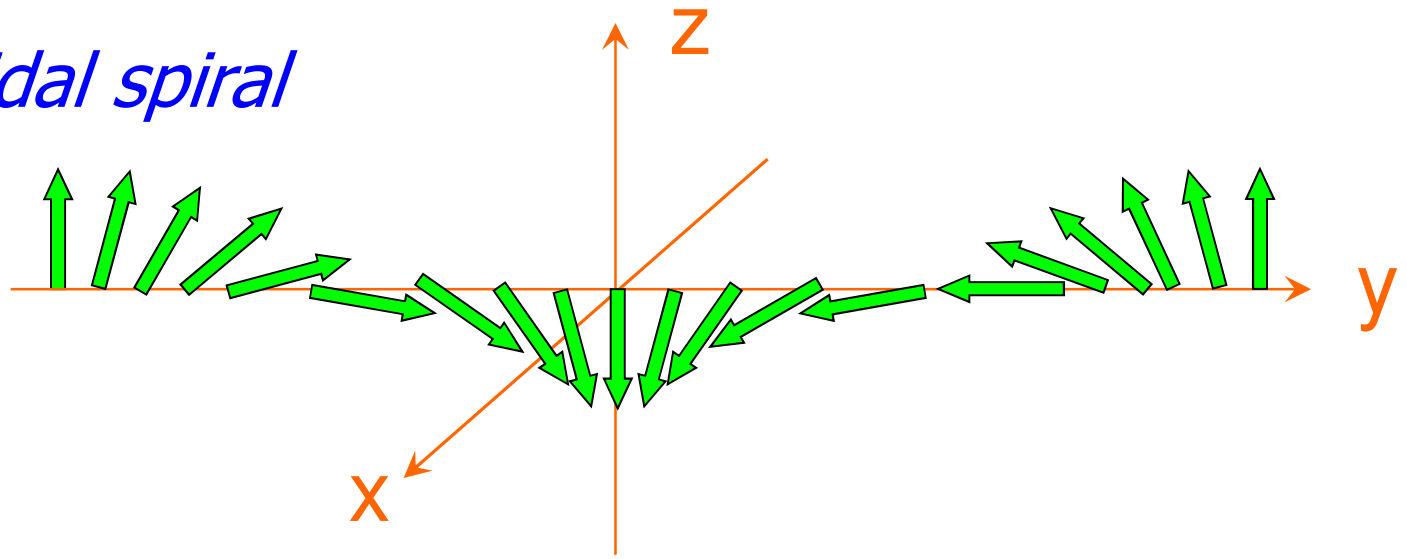


Interfacial DM interaction

Lifshitz invariant for broken m_z

$$\boldsymbol{M}_z\partial_x\boldsymbol{M}_x-\boldsymbol{M}_x\partial_x\boldsymbol{M}_z+\boldsymbol{M}_z\partial_y\boldsymbol{M}_y-\boldsymbol{M}_y\partial_y\boldsymbol{M}_z$$

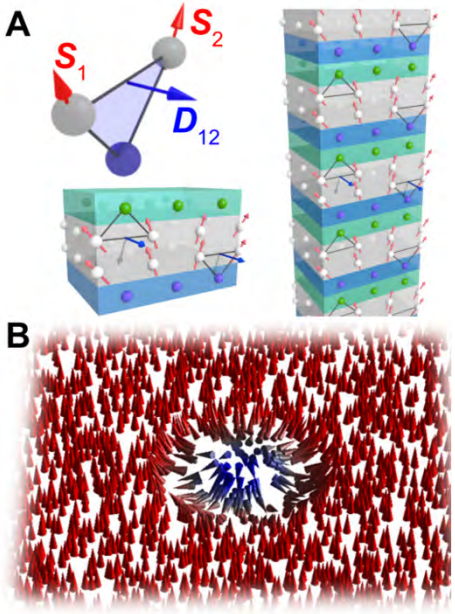
Cycloidal spiral



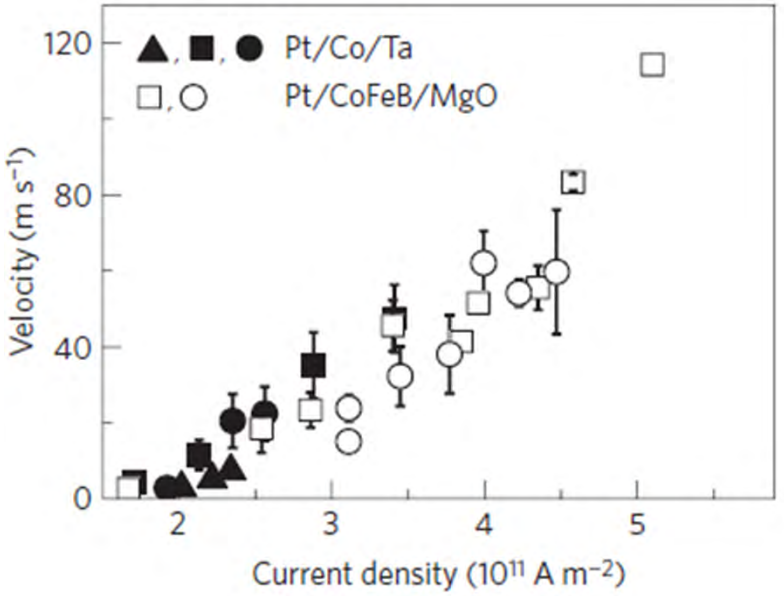
Skyrmionics

- **Nanometer size**
- **Topological protection**
- **Low critical currents**

Albert Fert et al Nature Nano (2013)



C. Moreau-Luchaire et al Nature Nano 11, 444 (2016)

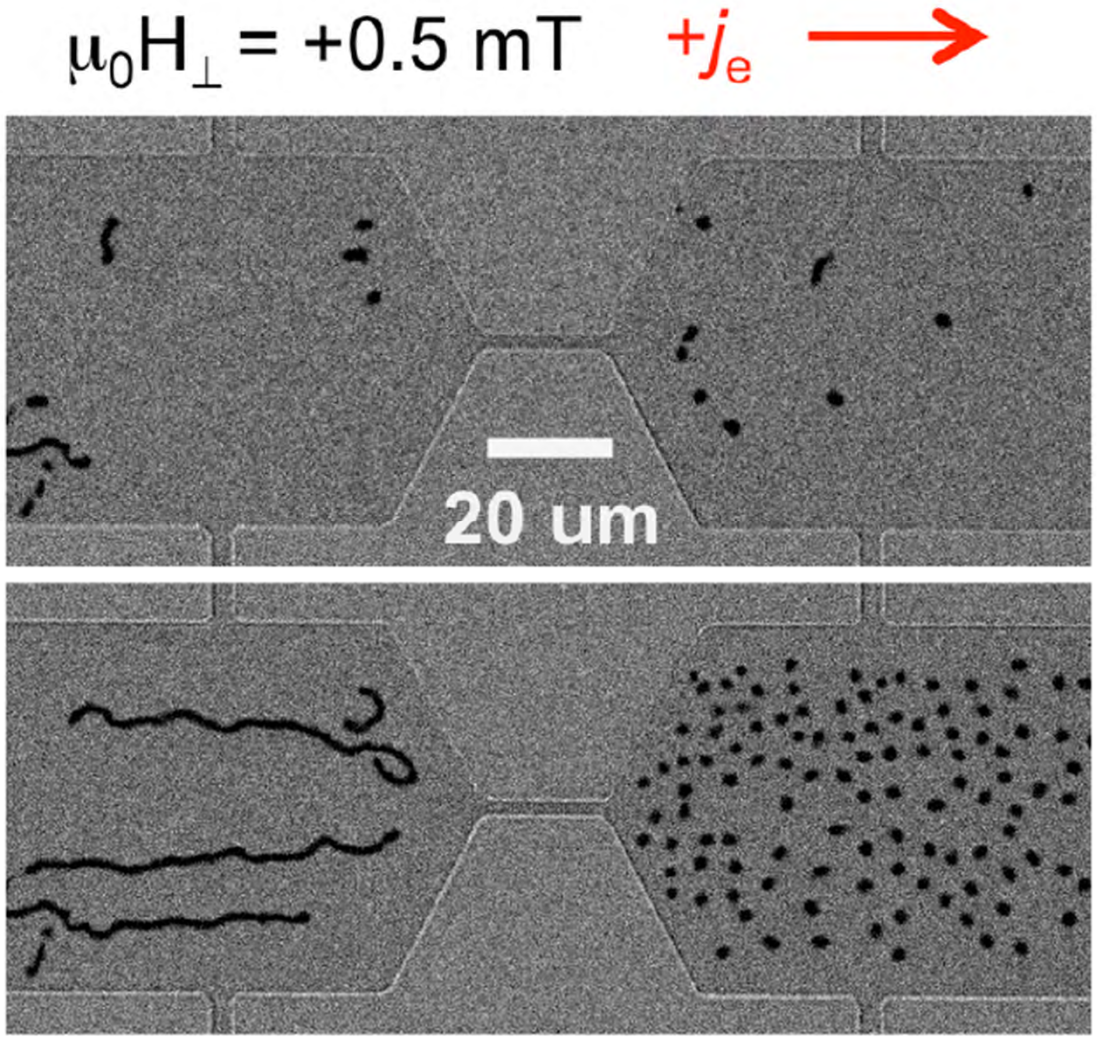


S. Woo et al, Nature Materials (2016)

Magnetic bubble memory upgrade

Blowing magnetic bubbles

W. Jiang, Science. ***349****, 283 (2015)*



Skyrmion dynamics

Landau-Lifshitz-Gilbert equation

Landau-Lifshitz-Gilbert equation:

$$\dot{\mathbf{m}} = -\gamma \mathbf{m} \times \mathbf{H}_{eff} + \alpha \, \mathbf{m} \times \dot{\mathbf{m}}$$

$$\mathbf{m} = \frac{\mathbf{M}}{M_0} \uparrow\downarrow \mathbf{S} \qquad \mathbf{H}_{eff} = -\frac{\delta H}{\delta \mathbf{M}} \qquad \gamma = \frac{g \mu_B}{\hbar}$$

Dimensionless LLG equation: $t \to t \dfrac{M_0}{\gamma}$

$$\dot{\mathbf{m}} = \mathbf{m} \times \frac{\delta H}{\delta \mathbf{m}} + \alpha \, \mathbf{m} \times \dot{\mathbf{m}}$$

Thiele equations for Skyrmion

Center-of-mass coordinates:

$$\mathbf{m}(\mathbf{x},t)=\mathbf{m}(\mathbf{x}-\mathbf{R}(t)) \qquad \mathbf{R}=(X,Y)$$

$$\begin{aligned}\alpha\Gamma\dot{\mathrm{X}}-G\,\dot{\mathrm{Y}}&=F_X\\ G\,\dot{\mathrm{X}}+\alpha\Gamma\,\dot{\mathrm{Y}}&=F_Y\end{aligned}$$

$$G=\sum_{\mathbf{x}}\mathbf{m}\cdot\frac{\partial\mathbf{m}}{\partial x}\times\frac{\partial\mathbf{m}}{\partial y}=4\pi Q$$

$$\Gamma=\sum_{\mathbf{x}}\frac{\partial\mathbf{m}}{\partial x}\cdot\frac{\partial\mathbf{m}}{\partial x}=\sum_{\mathbf{x}}\frac{\partial\mathbf{m}}{\partial y}\cdot\frac{\partial\mathbf{m}}{\partial y}$$

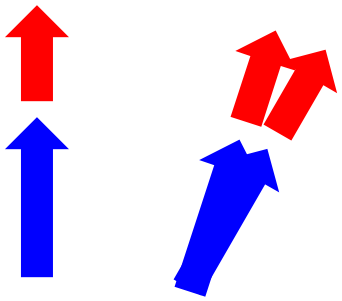
Skyrmion in inhomogeneous magnetic field

Current-driven dynamics of spin textures

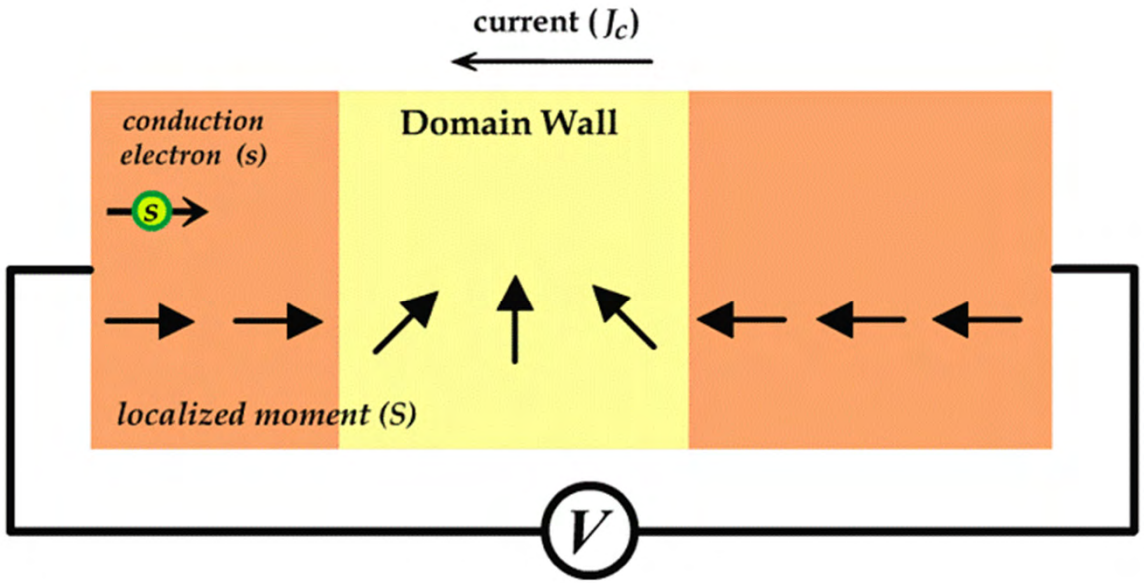
LLG equation in presence of electric current

$$\dot{\mathbf{m}} = \mathbf{m} \times \frac{\partial \boldsymbol{H}}{\partial \mathbf{m}} + \alpha\, \mathbf{m} \times \dot{\mathbf{m}} + (\mathbf{j} \cdot \nabla)\mathbf{m} - \beta\, \mathbf{m} \times (\mathbf{j} \cdot \nabla)\mathbf{m}$$

spin-transfer torque



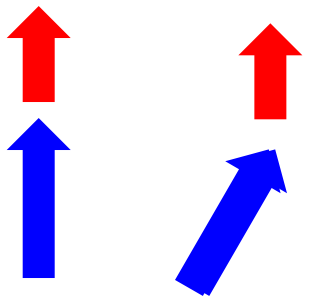
Spin transfer torque



LLG equation in presence of electric currents

$$\dot{\mathbf{m}} = \mathbf{m} \times \frac{\partial \boldsymbol{H}}{\partial \mathbf{m}} + \alpha\, \mathbf{m} \times \dot{\mathbf{m}} + (\mathbf{j} \cdot \nabla)\mathbf{m} - \beta\, \mathbf{m} \times (\mathbf{j} \cdot \nabla)\mathbf{m}$$

β-term



Current-driven Skyrmion dynamics

Thiele equations :

$$\begin{cases} \alpha\Gamma\dot{\boldsymbol{X}} - \boldsymbol{G}\dot{\boldsymbol{Y}} = -\beta\Gamma\boldsymbol{j}_x \\ \boldsymbol{G}\dot{\boldsymbol{X}} + \alpha\Gamma\dot{\boldsymbol{Y}} = -\boldsymbol{G}\,\boldsymbol{j}_x \end{cases}$$

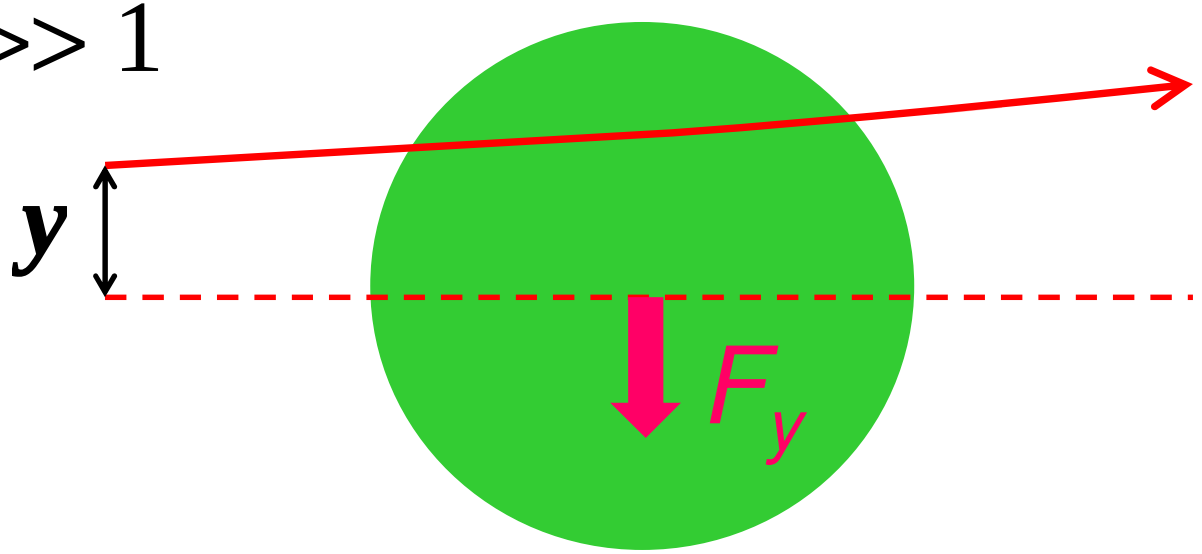
$$\dot{\boldsymbol{X}} = -\frac{\boldsymbol{G}^2 + \alpha\beta\Gamma^2}{\boldsymbol{G}^2 + (\alpha\Gamma)^2}\,\boldsymbol{j}_x \approx -\boldsymbol{j}_x \qquad \dot{\boldsymbol{Y}} = \frac{(\beta-\alpha)\Gamma\boldsymbol{G}}{\boldsymbol{G}^2 + (\alpha\Gamma)^2}\,\boldsymbol{j}_x$$

Skyrmion Hall Effect :

$$\frac{\boldsymbol{V}_y}{\boldsymbol{V}_x} \approx (\alpha-\beta)\frac{\Gamma}{\boldsymbol{G}}$$

Relation between the THE and current-driven dynamics

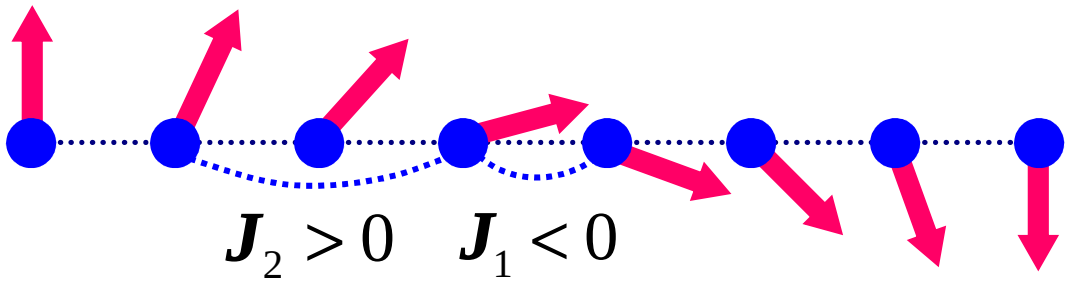
$kR_s >> 1$



$$\Delta p_y = -\frac{e}{c}\int dx h_z(x, y)$$

$$F_y = -n_e v_x \int dy \Delta p_x = \frac{Q\Phi_0}{c} j_x$$

Skyrmions in frustrated magnets



NANO LETTERS

Letter

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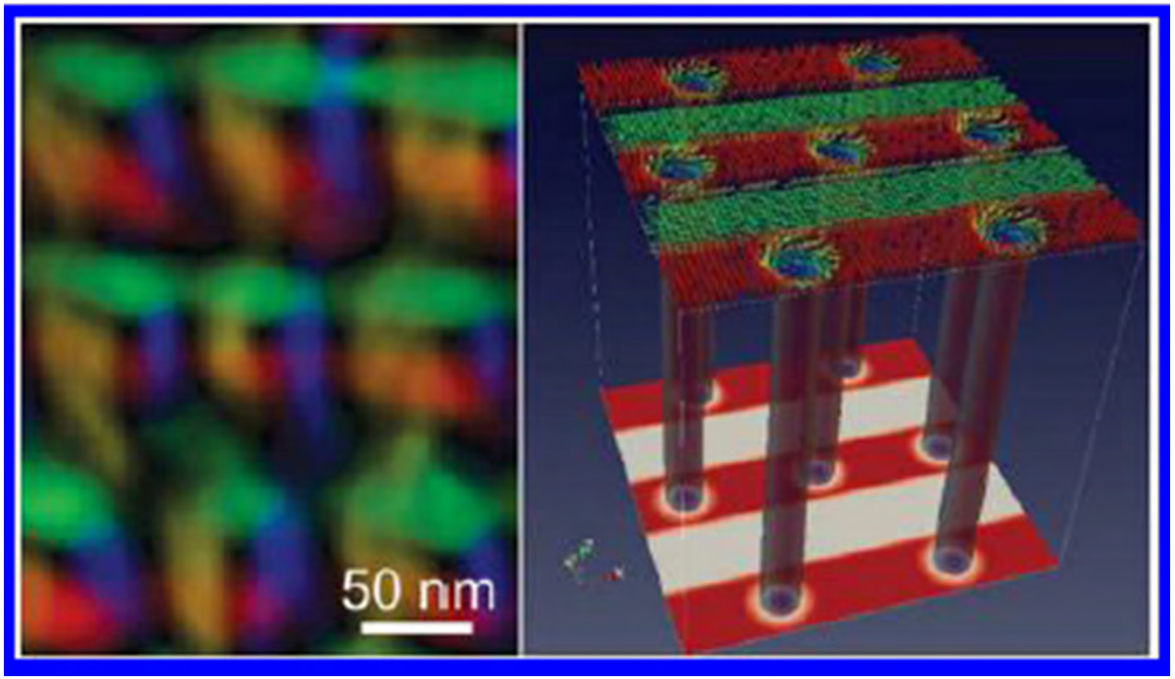
Nanoscale Skyrmions in a Nonchiral Metallic Multiferroic: Ni_2MnGa

Charudatta Phatak,*[,†] Olle Heinonen,[†] Marc De Graef,[‡] and Amanda Petford-Long[†,§]

[†]Materials Science Division, Argonne National Laboratory, Lemont, Illinois 60439, United States

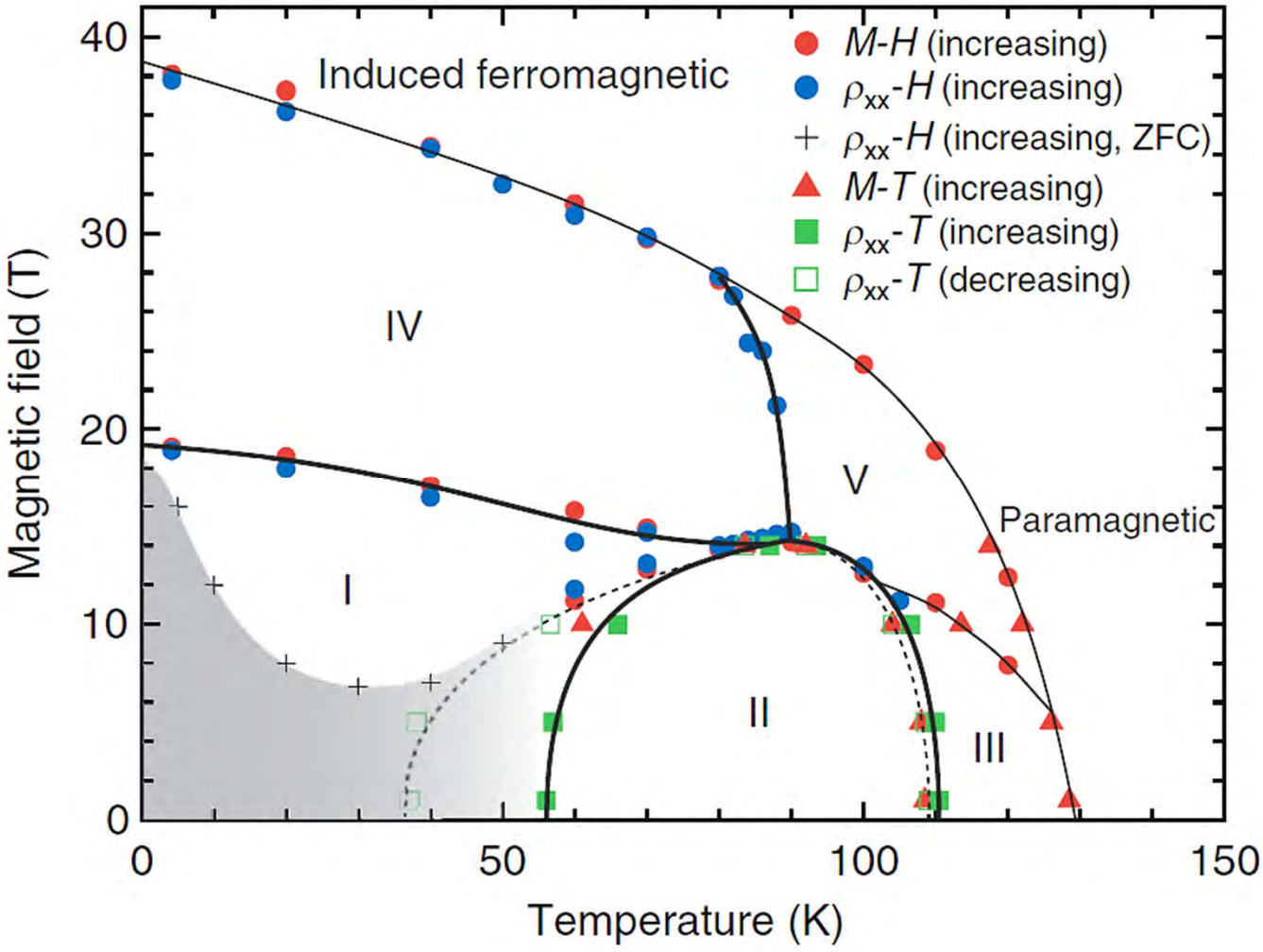
[‡]Department of Materials Science and Engineering, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, United States

[§]Department of Materials Science and Engineering, Northwestern University, Evanston, Illinois 60208, United States



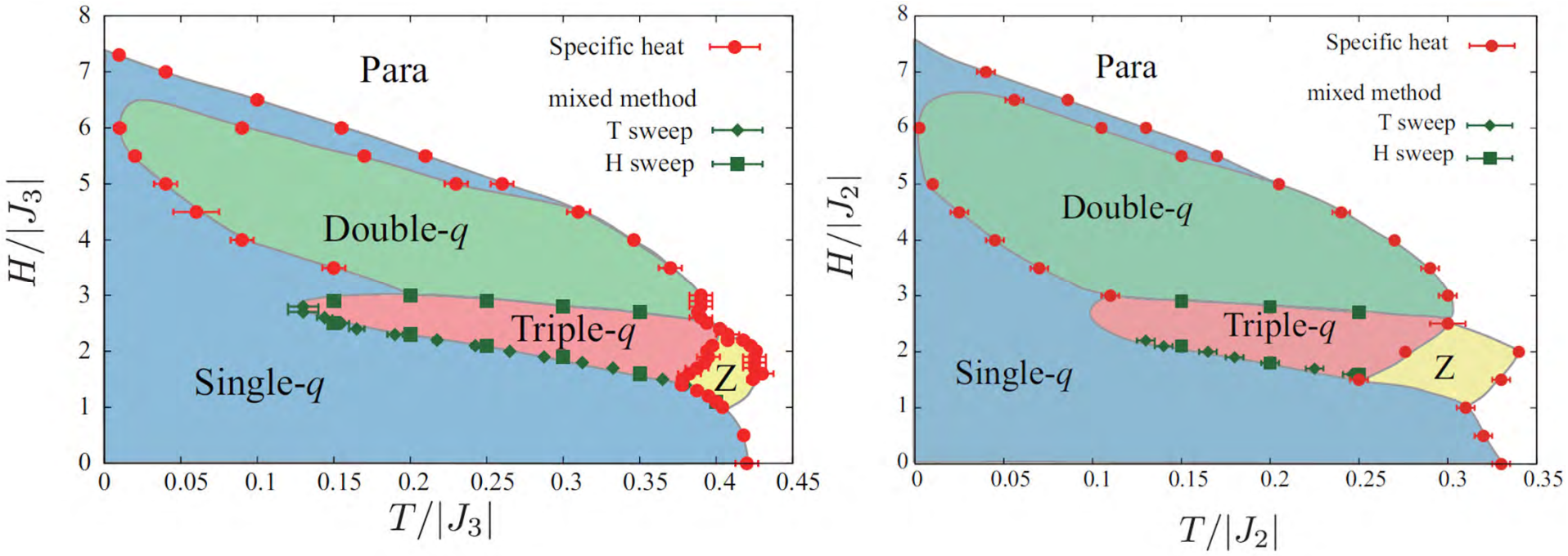
Spiral ordering in $SrFeO_3$

*T. Takeda JPSJ **33**, 967 (1972); H. Oda JPSJ **42**, 101 (1977); A. Lebon PRL **92**, 037202 (2004)*



*S. Ishiwata et al. PRB **84**, 054427 (2011)*

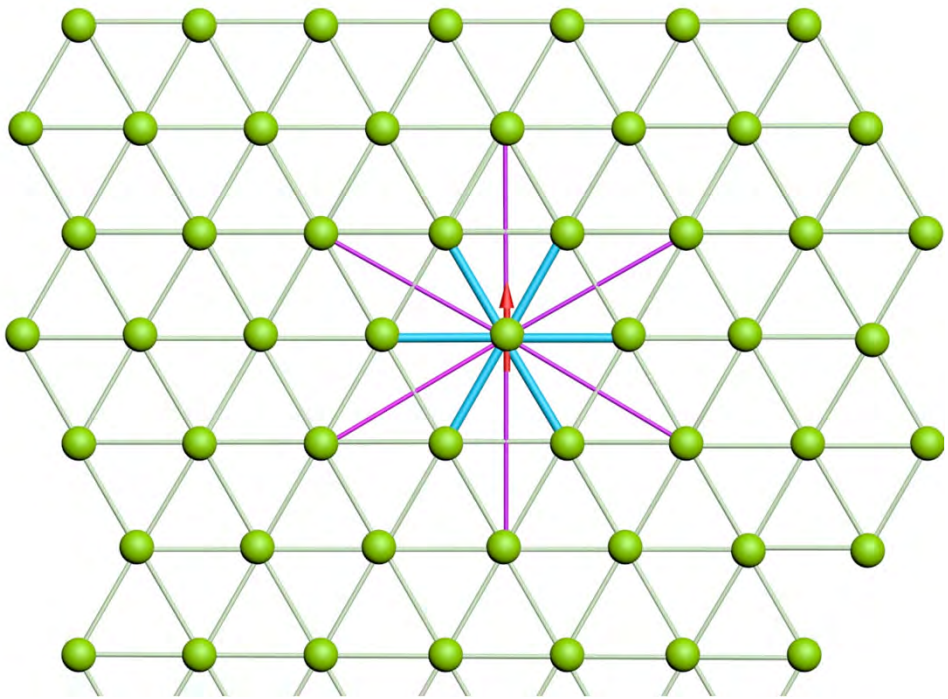
Skyrmions in frustrated triangular magnets



T. Okubo et al. Phys. Rev. Lett. ***108****, 017206 (2012)*

Frustrated triangular magnet with anisotropy

$$E = -J_1 \sum_{<i,j>} \boldsymbol{S}_i \boldsymbol{S}_j + J_2 \sum_{\ll i,j \gg} \boldsymbol{S}_i \boldsymbol{S}_j - h \sum_i S_{iz} - \frac{K}{2} \sum_i (S_{iz})^2$$

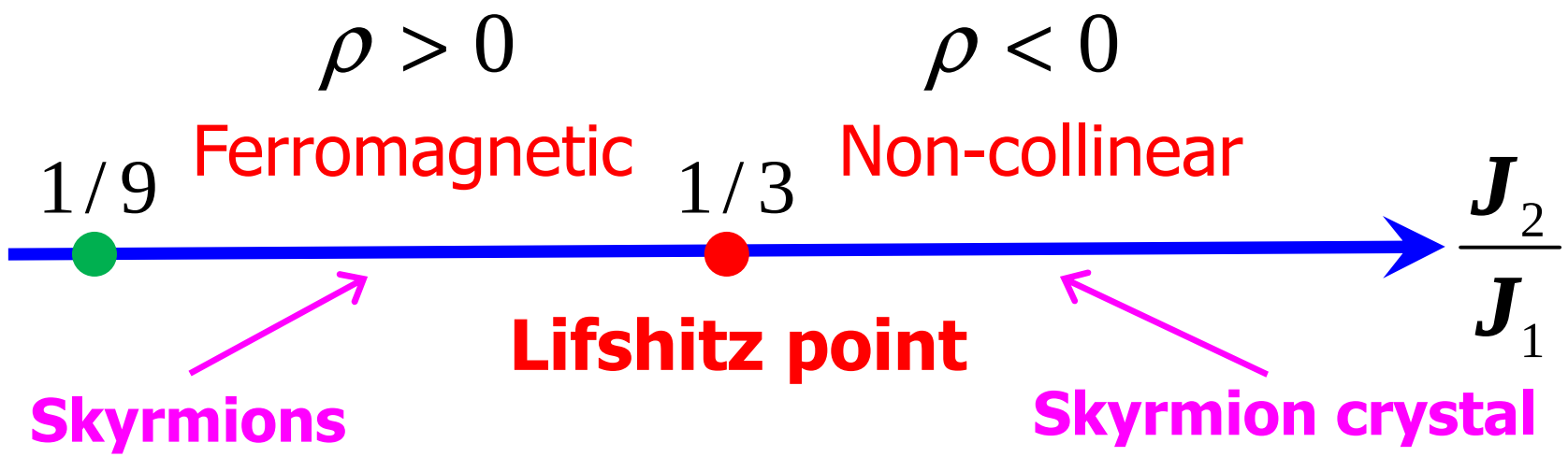


Continuum model

$$E = \frac{1}{2}\int d^2\mathbf{x}\left(\rho\,\partial_i\mathbf{n}\cdot\partial_i\mathbf{n} + \mu\,\Delta\mathbf{n}\cdot\Delta\mathbf{n} - 2hn_z - Kn_z^2\right)$$

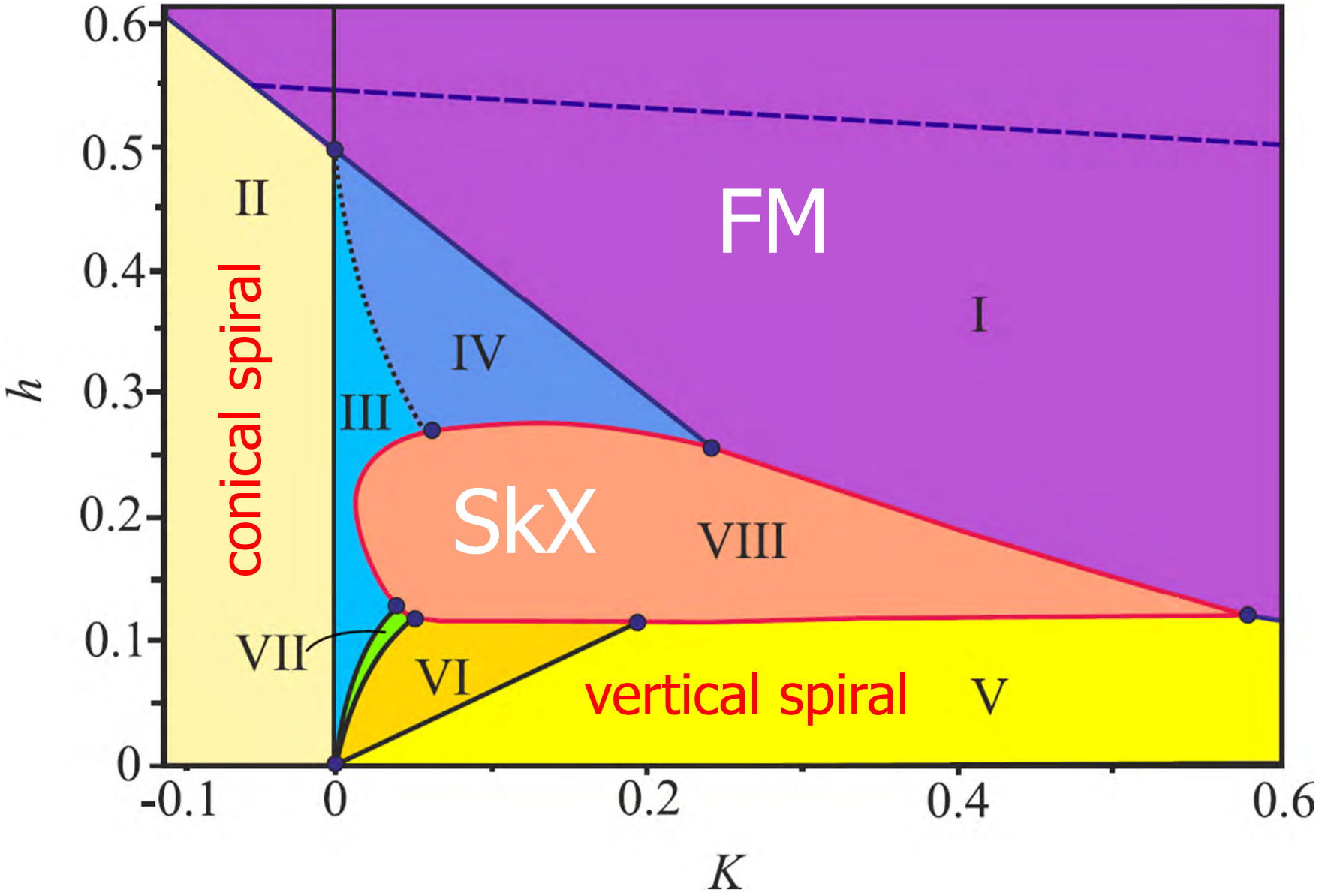
Spin stiffness

$$\rho = \frac{3}{2}(J_1 - 3J_2) \qquad \mu = \frac{3}{32}(9J_2 - J_1)$$

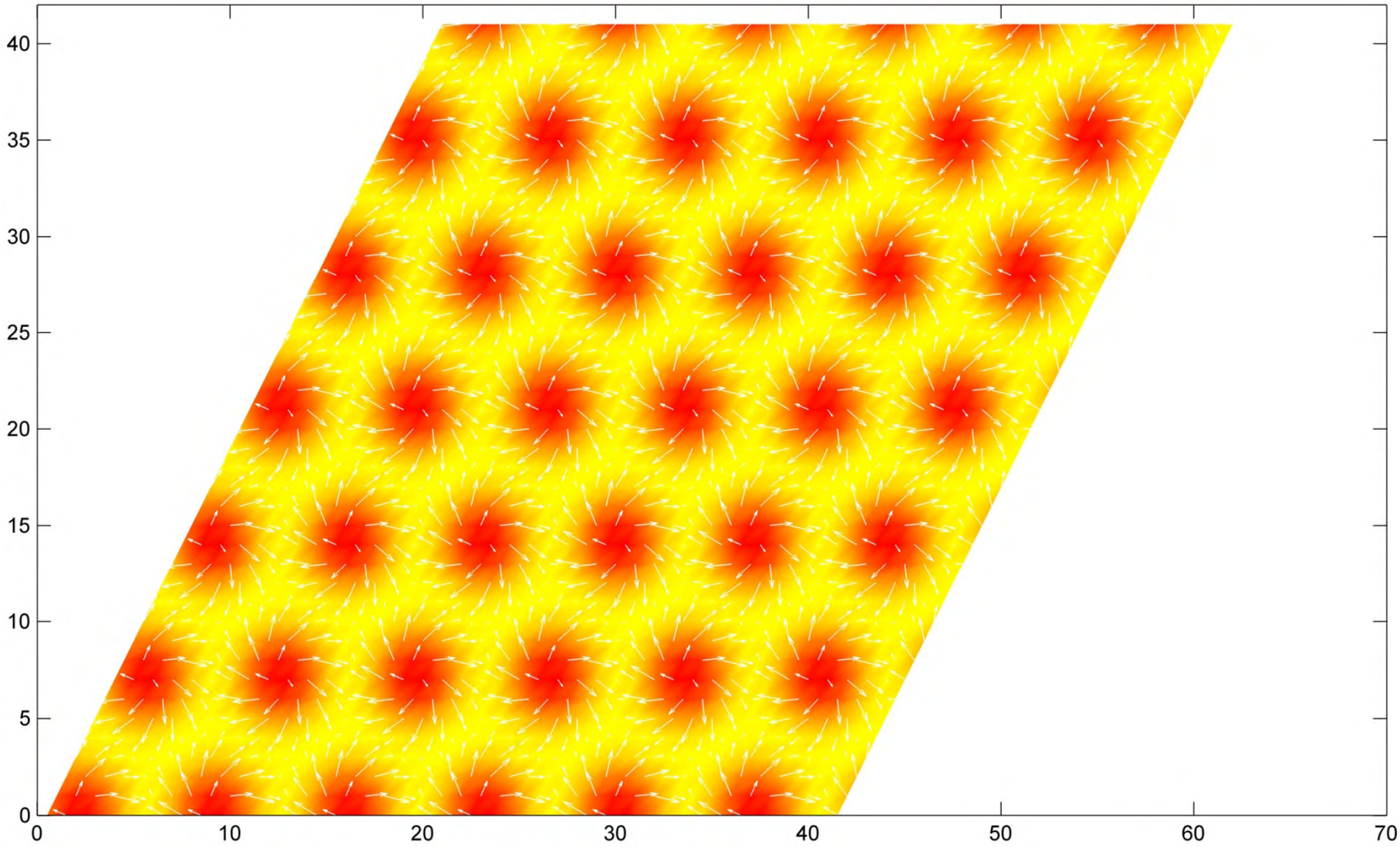


Phase Diagram ($\rho < 0$)

A. Leonov and MM, Nature Commun. 6, 8275 (2015)



Skyrmion crystal

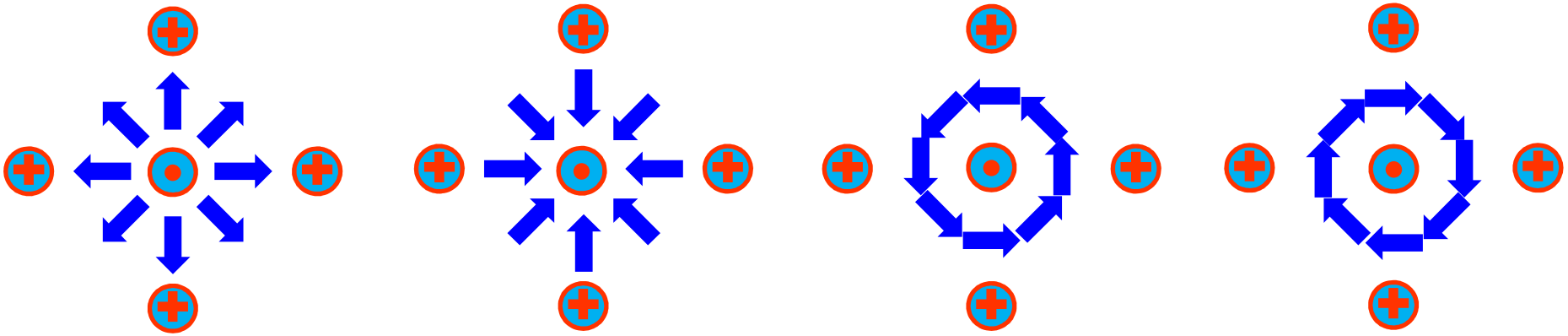


Skyrmion charge, vorticity & helicity

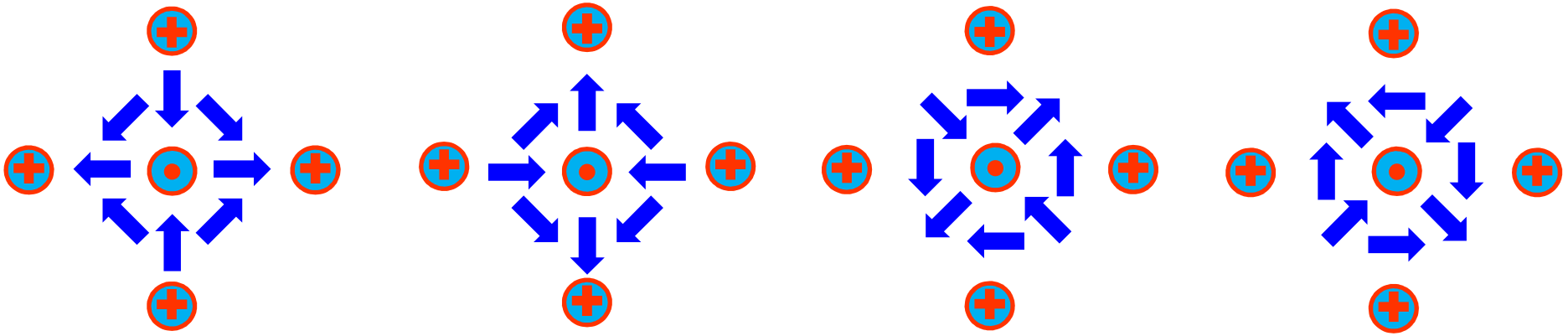
N. Nagaosa and Y. Tokura, Nature Nano ***8****, 899 (2013)*

Q = +1

$\chi = 0$ $\chi = \pi$ $\chi = +\pi/2$ $\chi = -\pi/2$



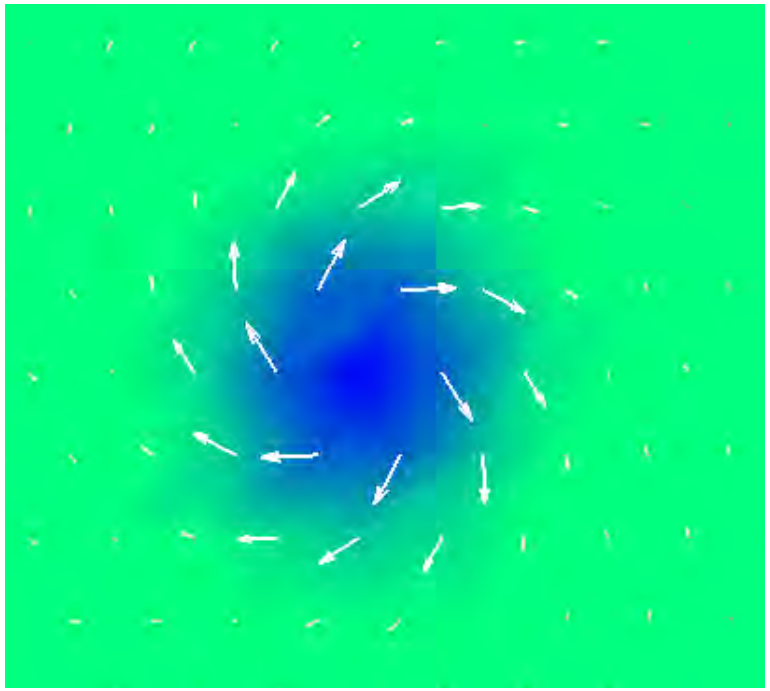
Q = -1

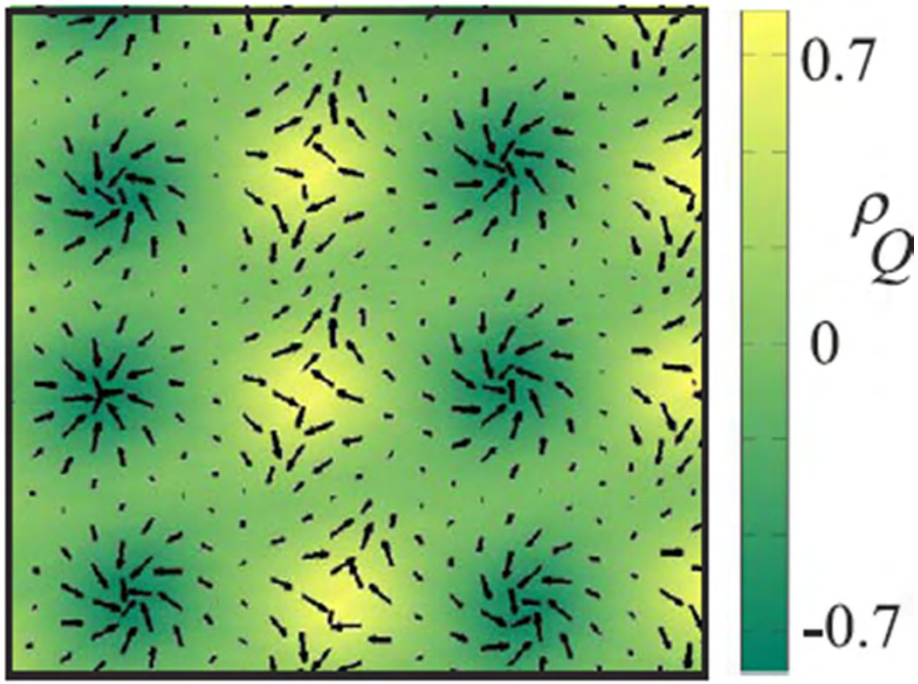


Arbitrary helicity and vorticity

$$\Theta = \Theta(\rho) \qquad \Phi = \mathrm{v}\,\varphi + \chi$$

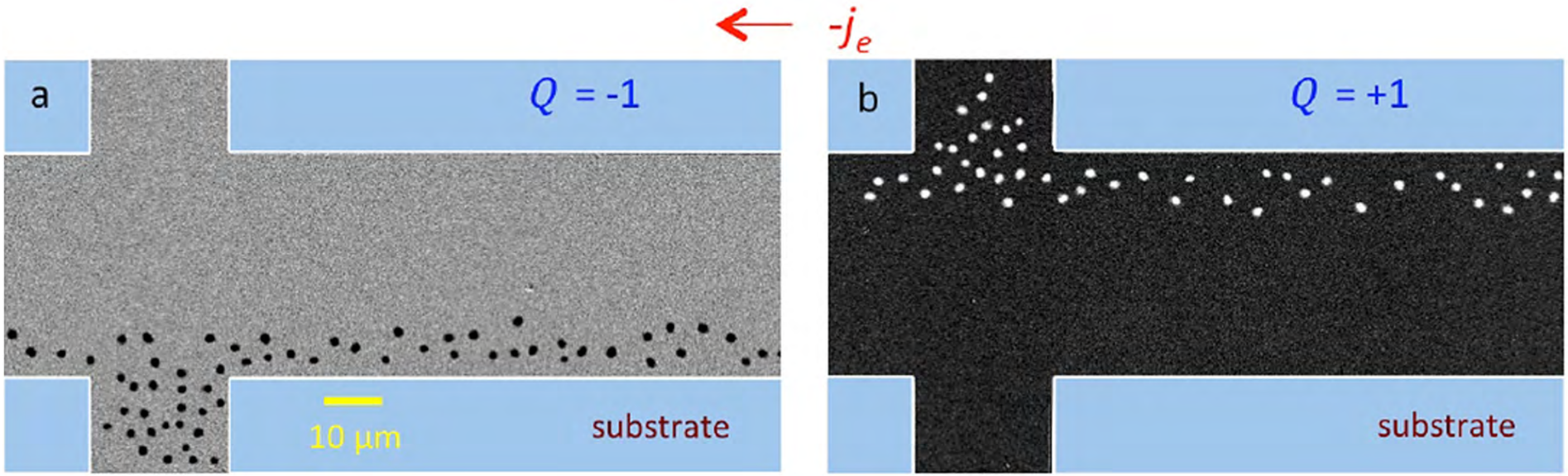
$$\boldsymbol{Q} = \mathrm{v}\,\frac{\left(\boldsymbol{n}_z(0) - \boldsymbol{n}_z(\infty)\right)}{2}$$





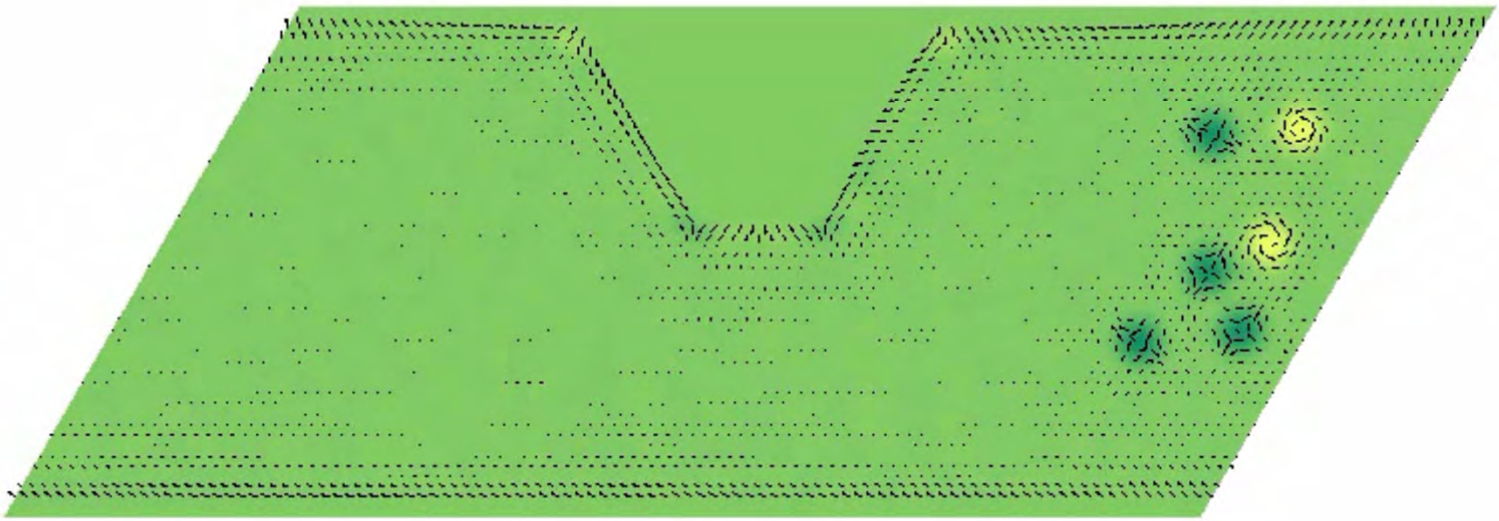
Skyrmion Hall effect

W. Jiang at al. Nature Physics ***13***, *162 (2017)*



Separation of skyrmions from

A. Leonov & MM Nature Commun. ***8****, 14394 (2017)*

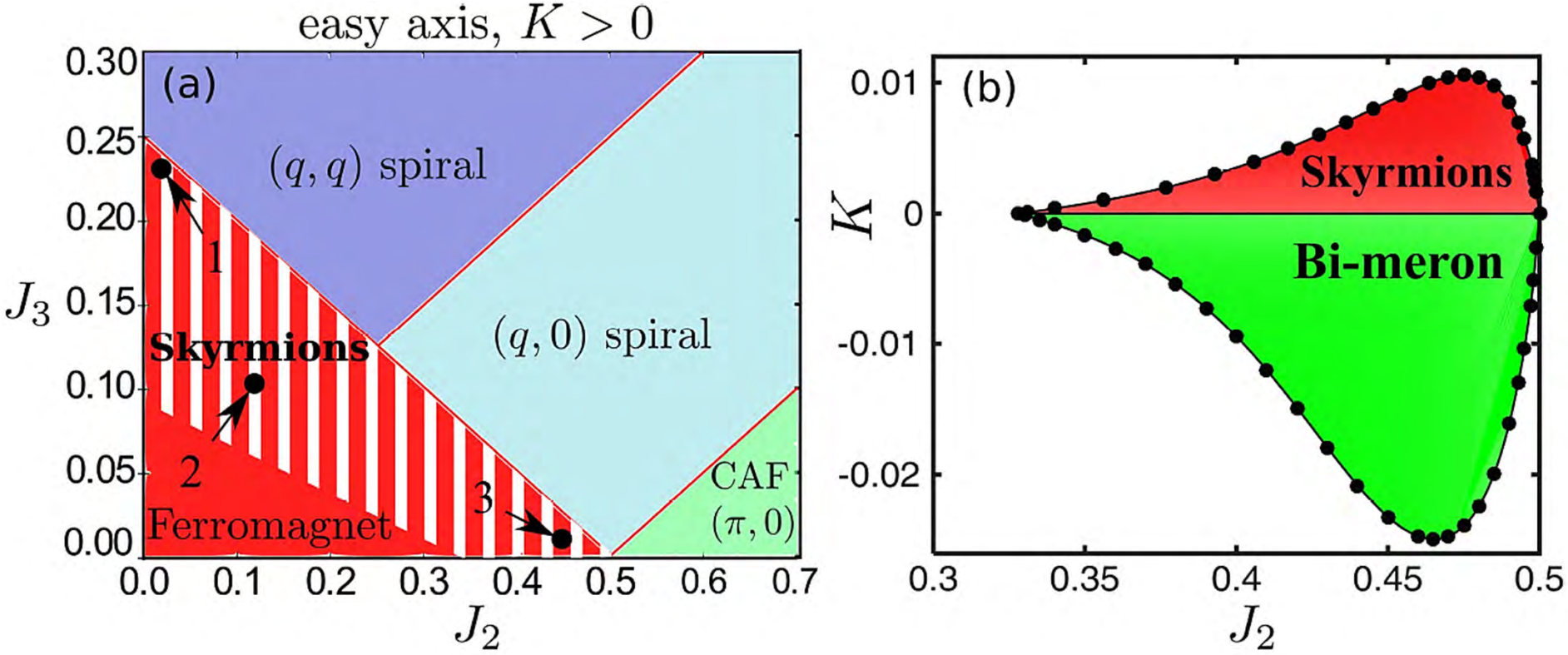


Multi-Q skyrmion states in underfrustrated magnets $\rho > 0, K > 0$

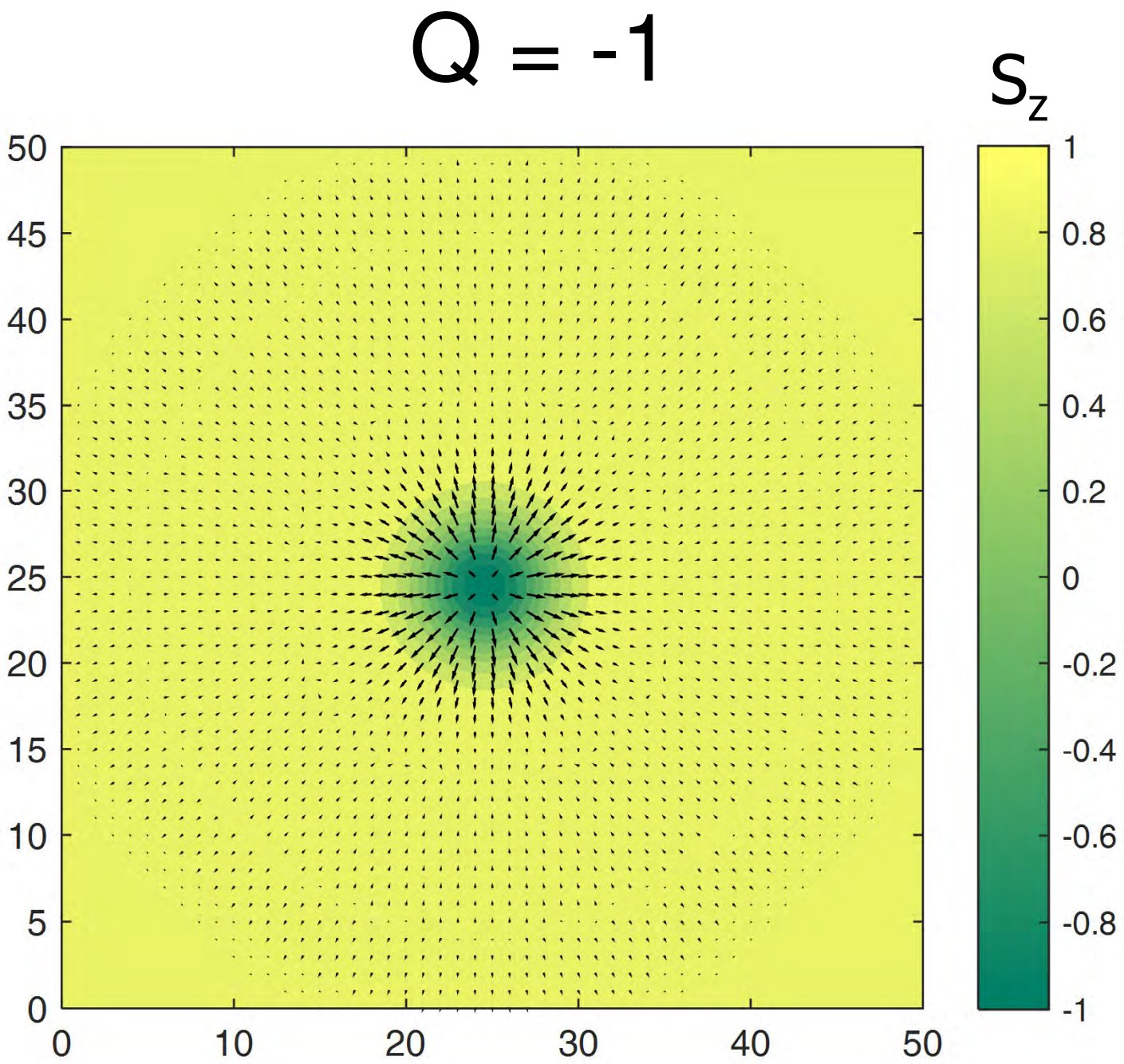
Y. Kharkov, O.P. Sushkov & MM arXiv:1703.09173

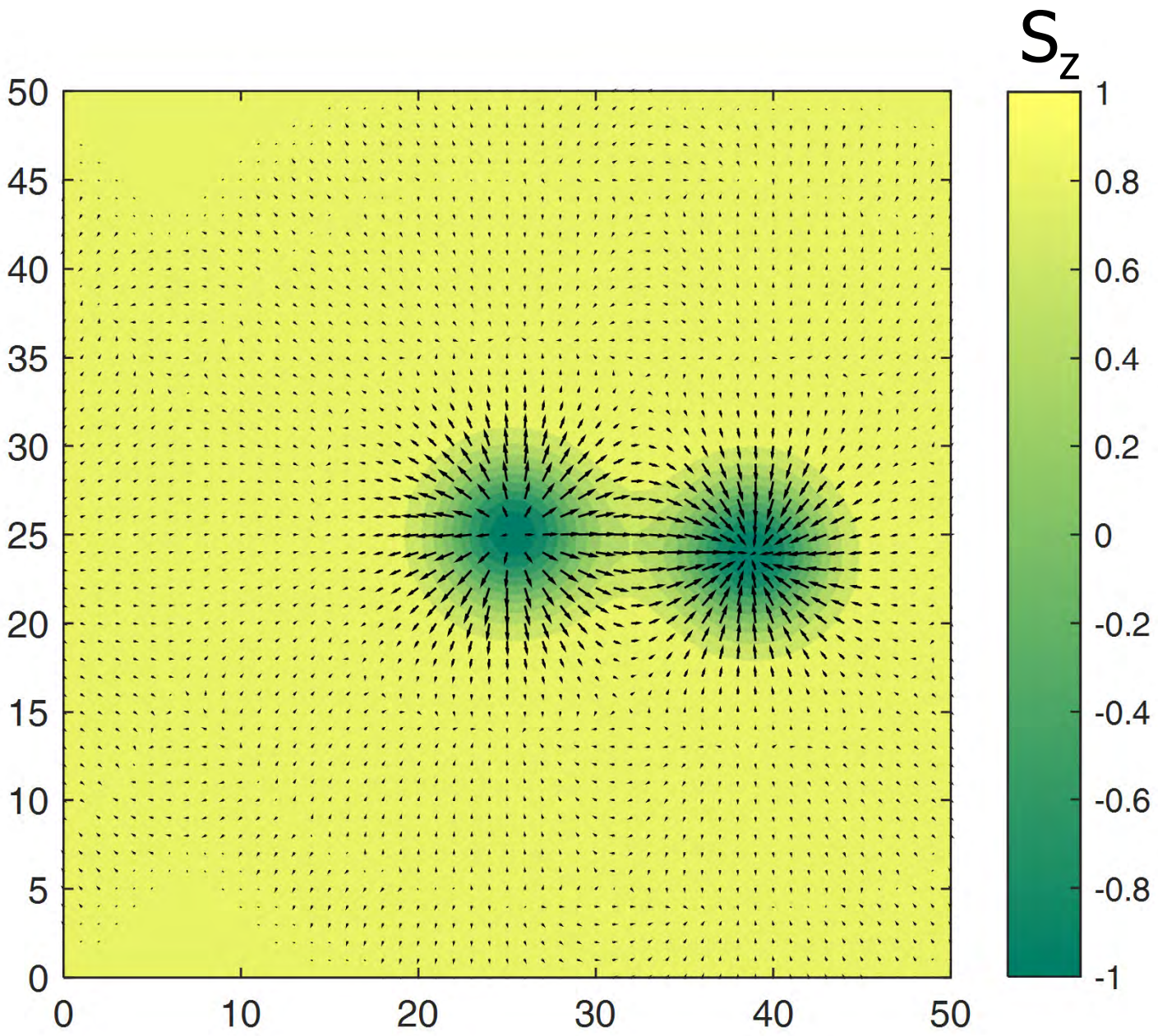
Skyrmions with large Q

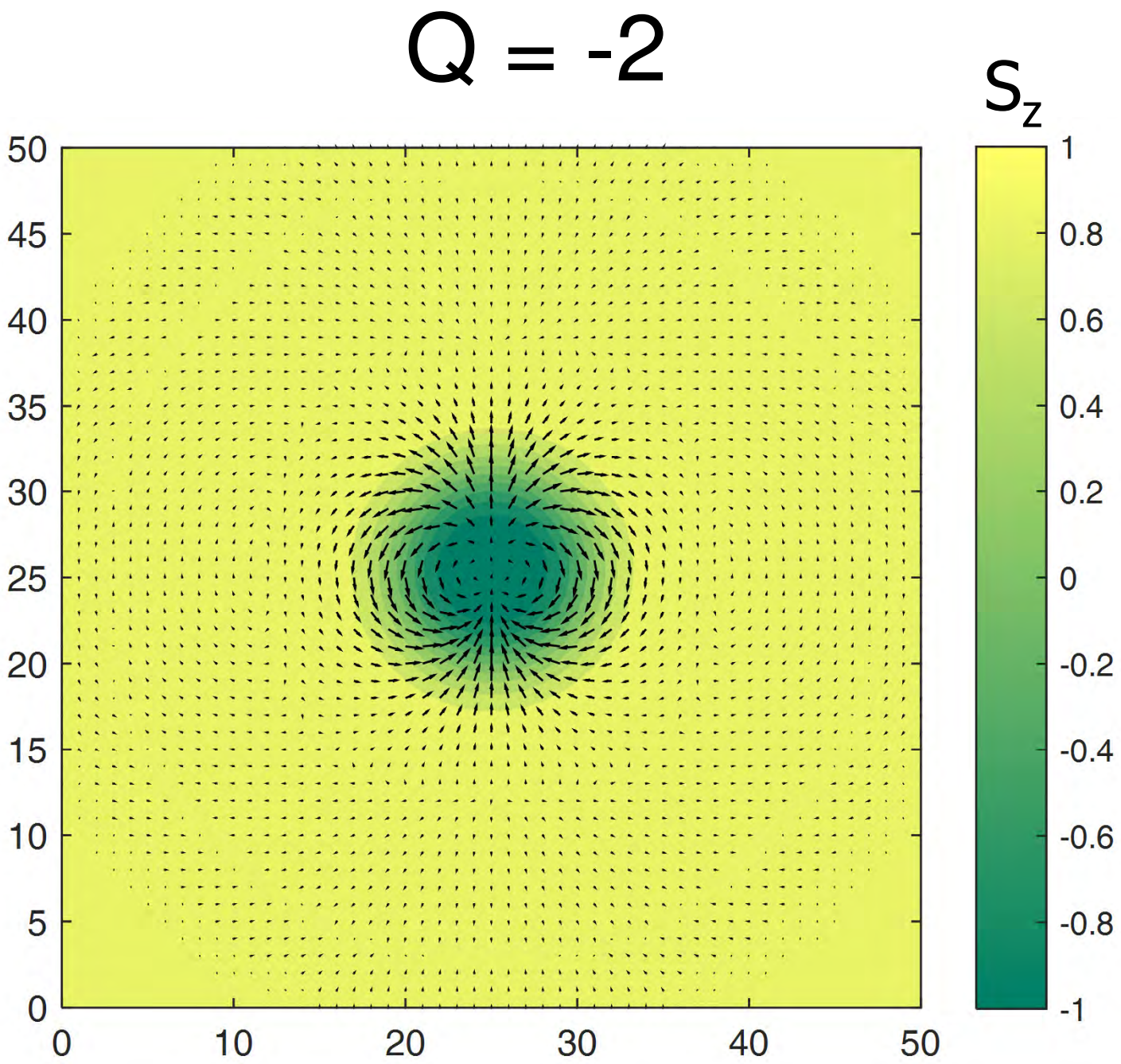
$$\rho > 0, K > 0$$

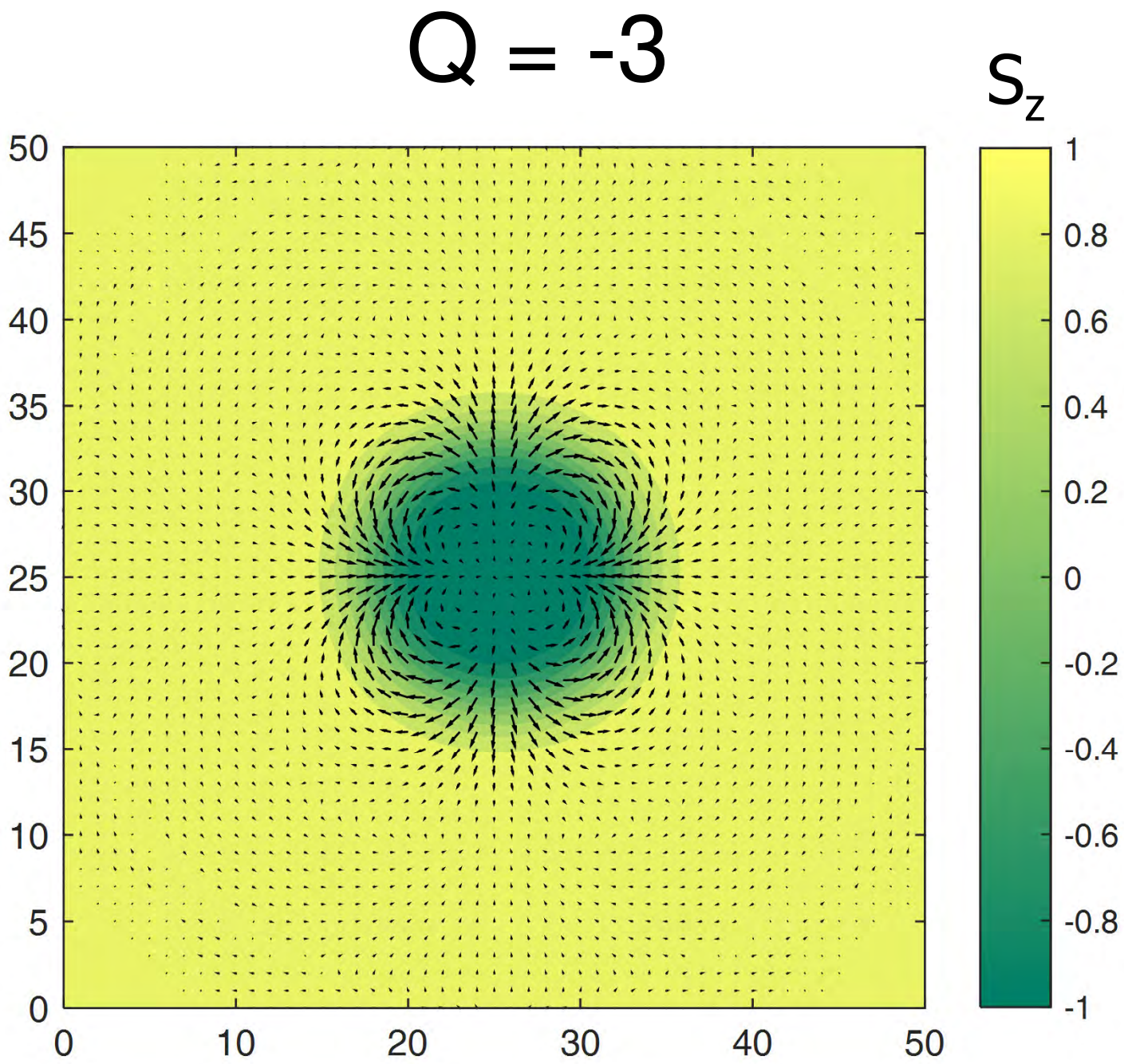


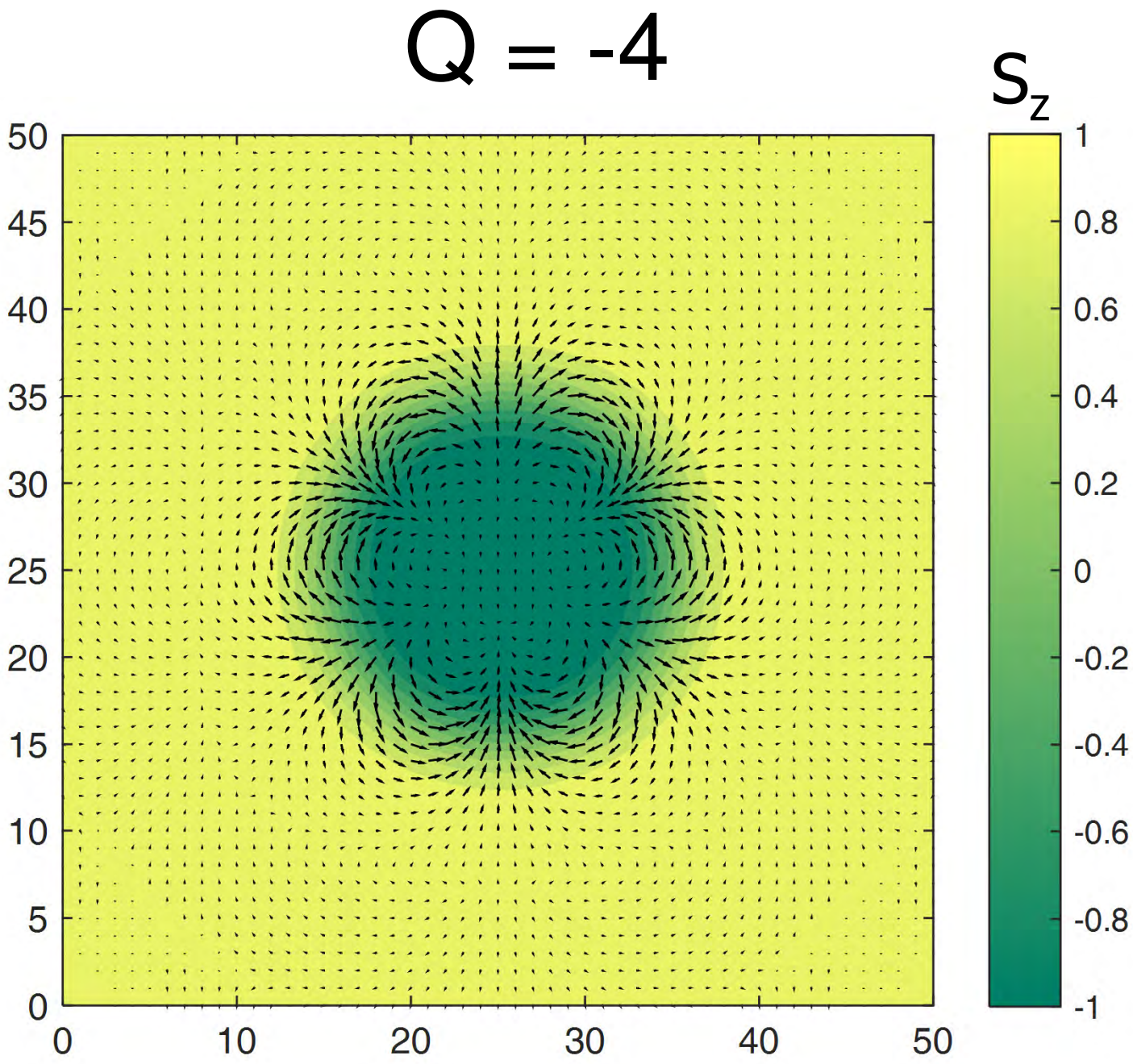
Y. Kharkov, O.P. Sushkov & MM arXive:1703.09173

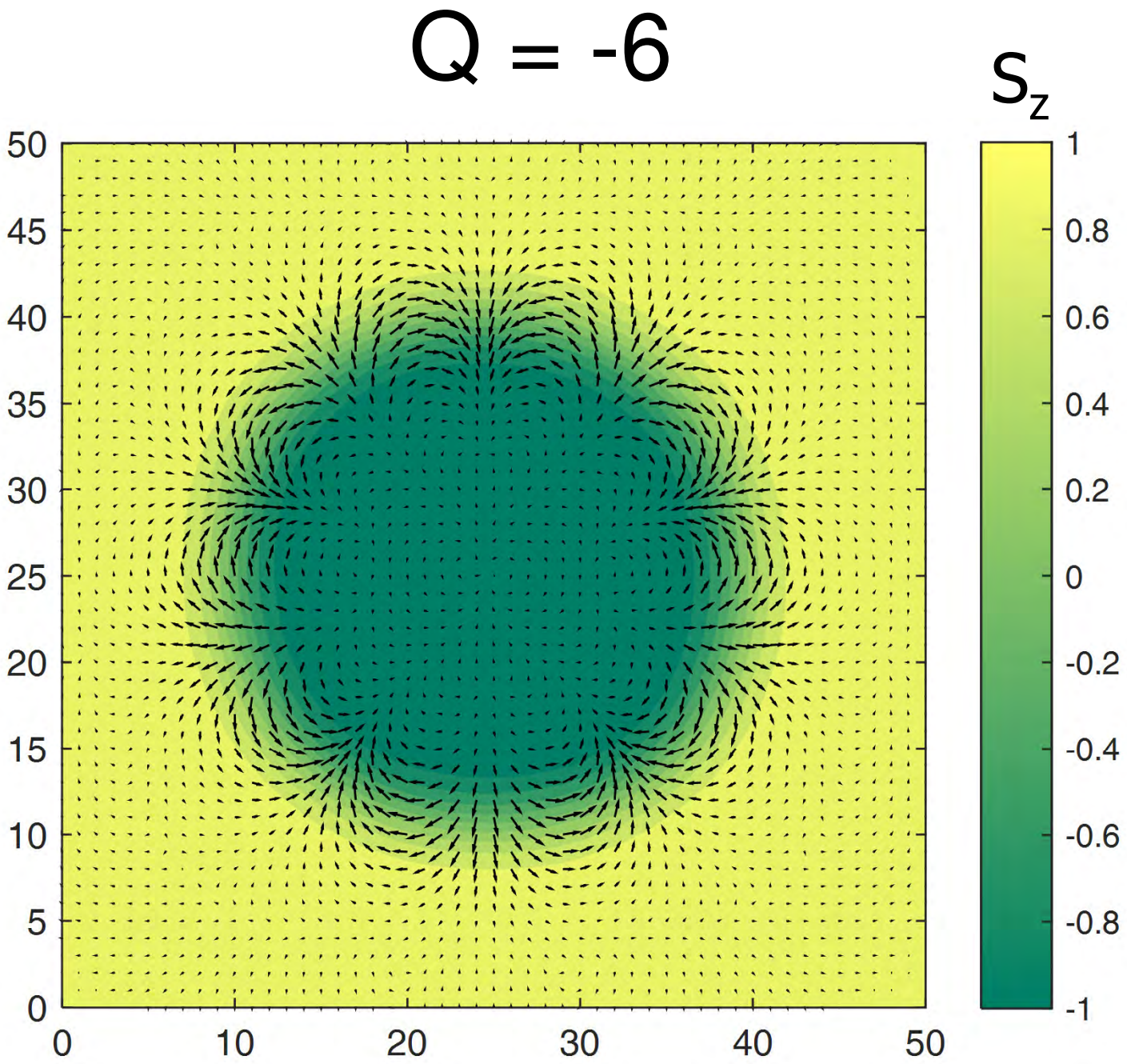

Q = -1
S_z
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


S_z
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

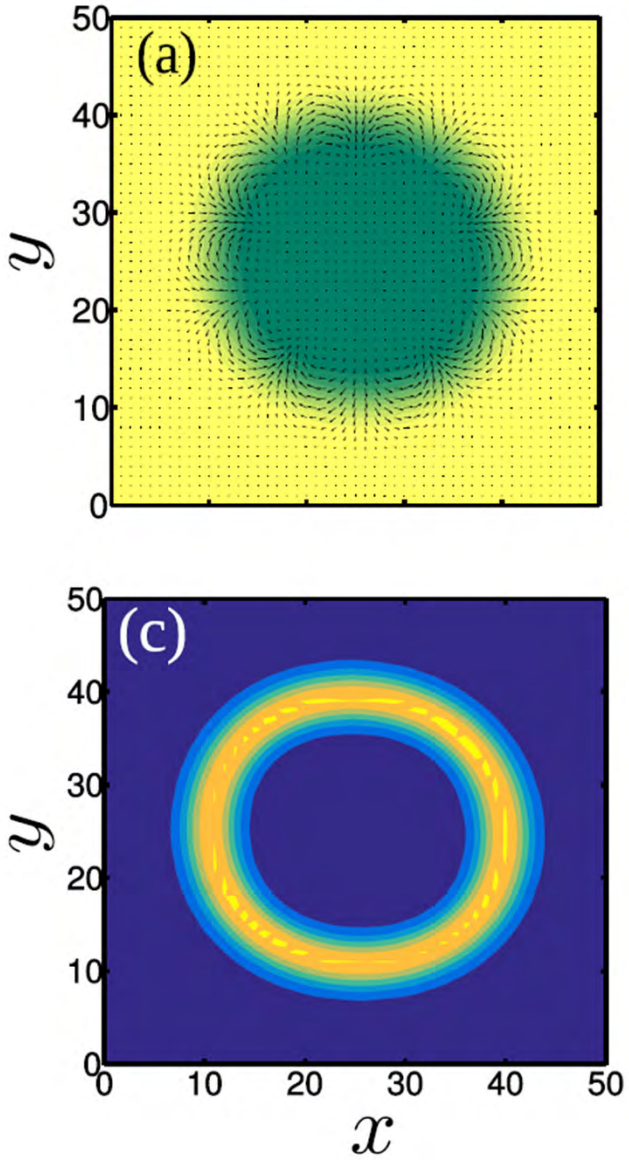

Q = -2
S_z
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

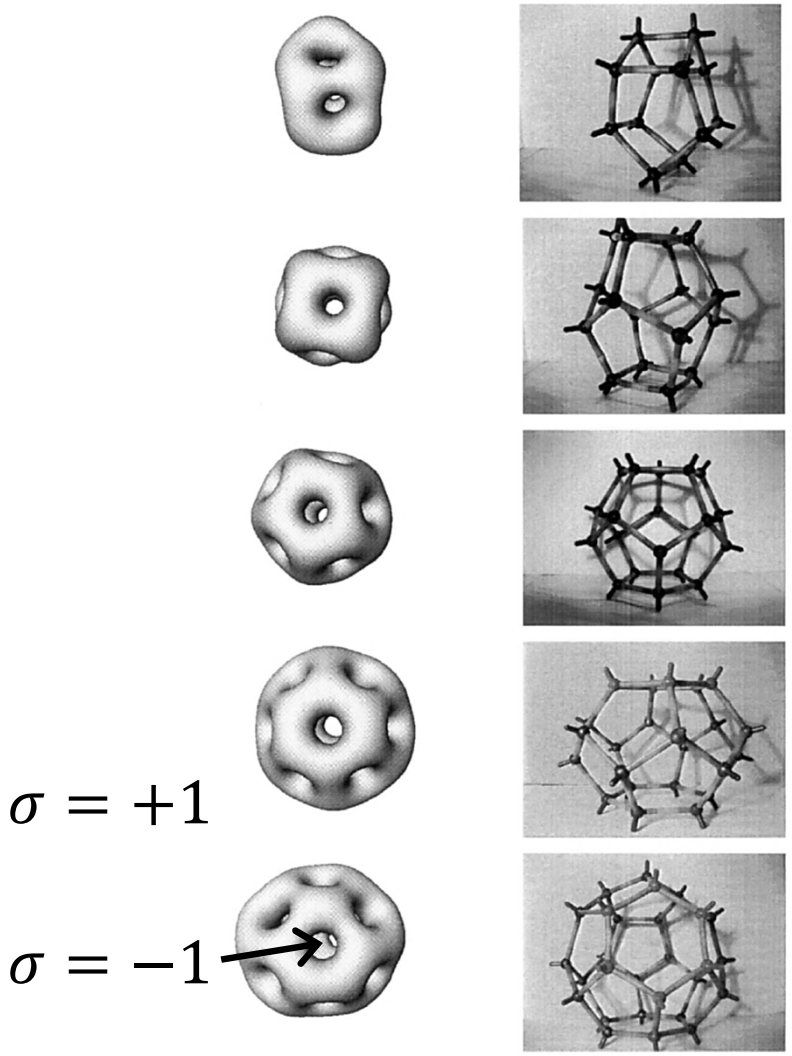

Q = -3
S_z
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


Q = -4
S_z
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


Q = -6
S_z
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

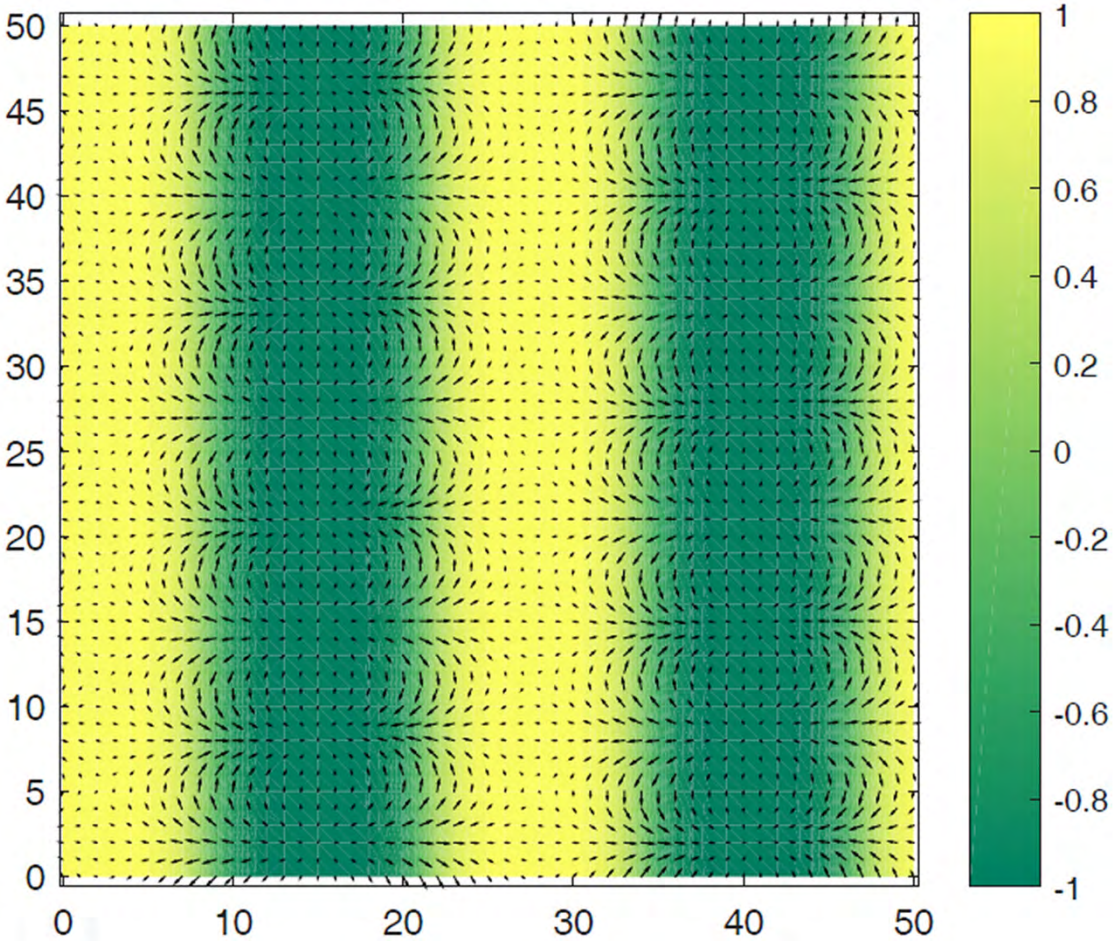
Holographic multi-skyrmions

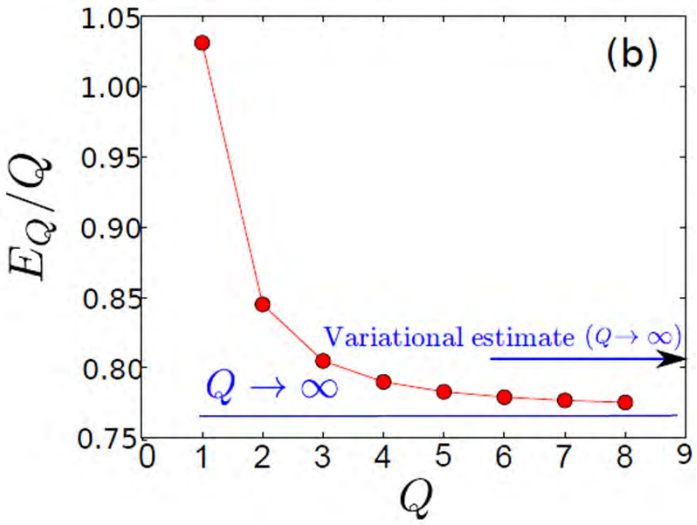




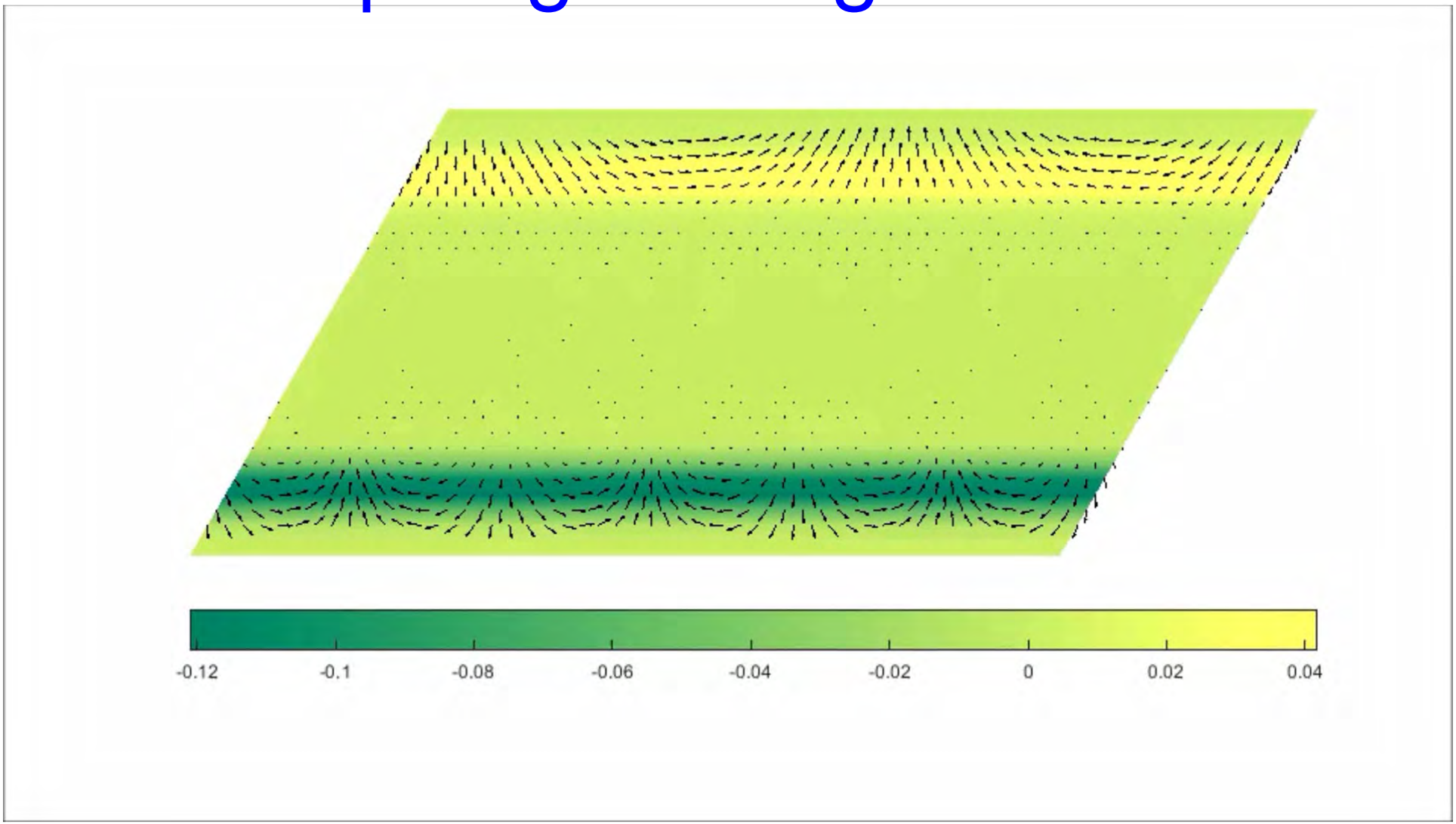
*R. A. Battye & P.M. Sutcliffe, PRL **79**, 363 (1997)*

Topological stripe





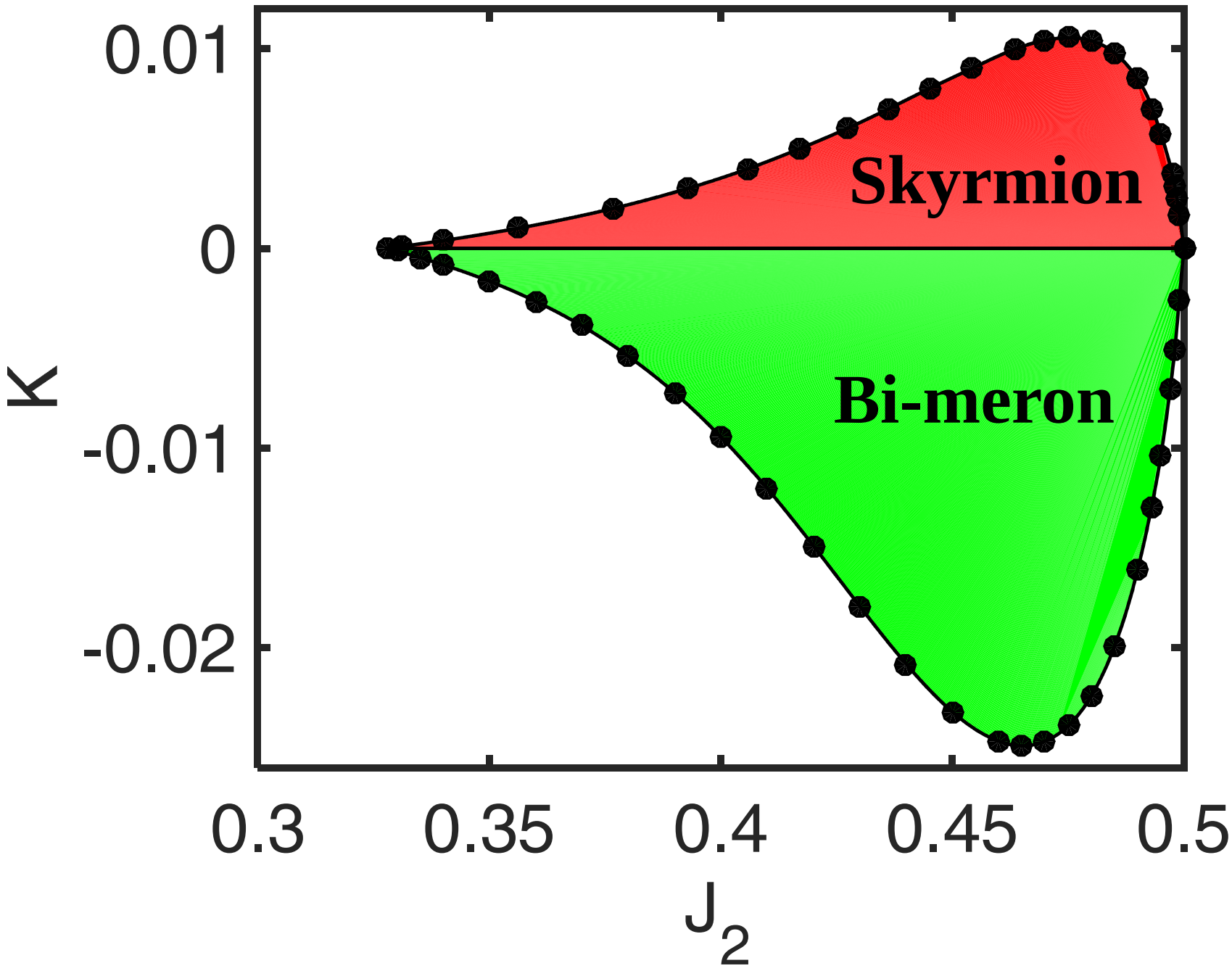
Topological edge states



A. Leonov & MM Nature Commun. ***8****, 14394 (2017)*

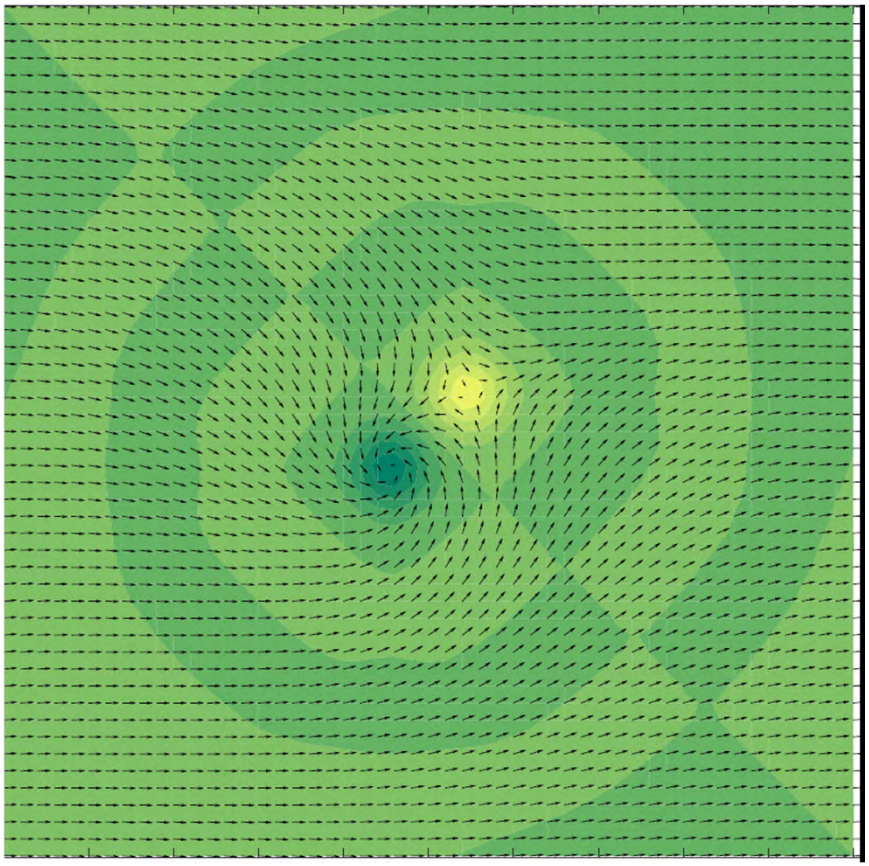
Half-skyrmions

$\rho > 0, K < 0$

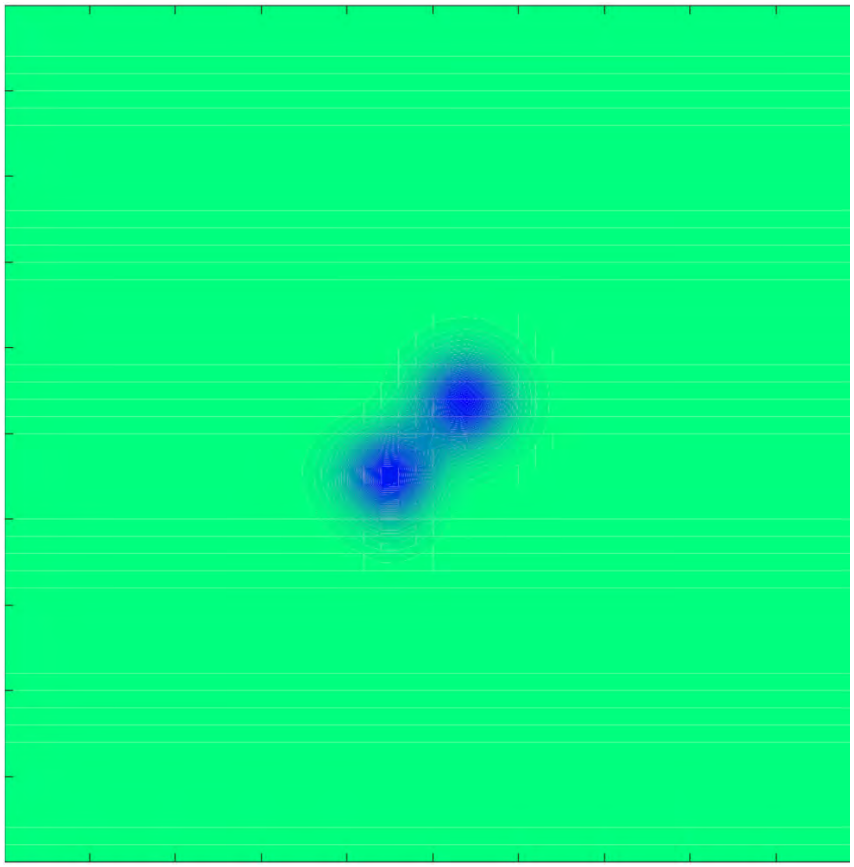

0.01
0
-0.01
-0.02
K
Skyrmion
Bi-meron
0.3
0.35
0.4
0.45
0.5
J2

Bi-merons

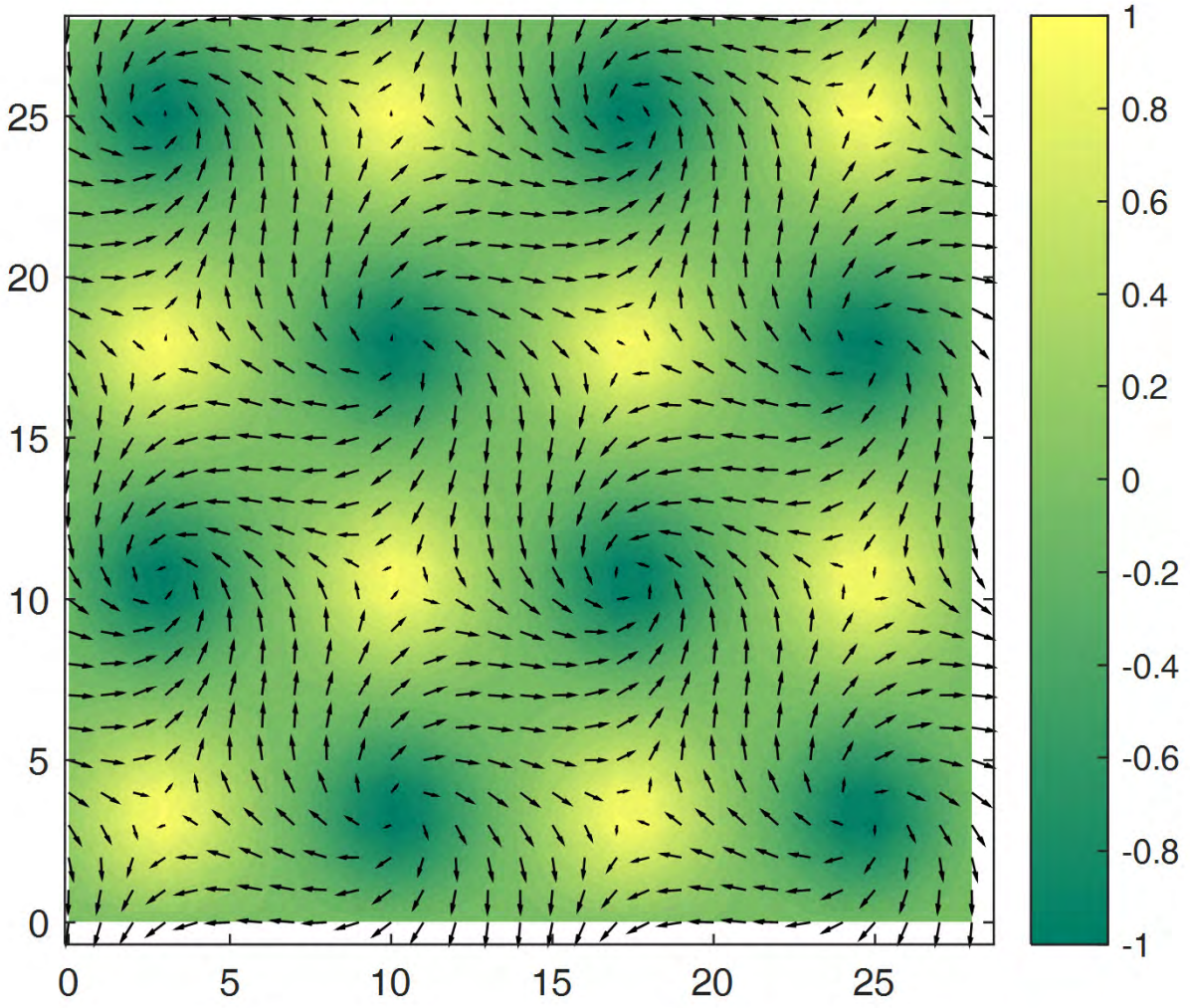
Spin configuration



Topological density



Meron-antimeron crystal



Superfluid ^{3}He

$$\boldsymbol{S}=1 \quad \boldsymbol{L}=1$$

$$\left\langle \boldsymbol{c}_{-\mathbf{k}\beta}\boldsymbol{c}_{\mathbf{k}\alpha}\right\rangle = \boldsymbol{d}_{\mu}(\mathbf{k})\left(\sigma_{\mu}\boldsymbol{i}\sigma_{2}\right)_{\alpha\beta}$$

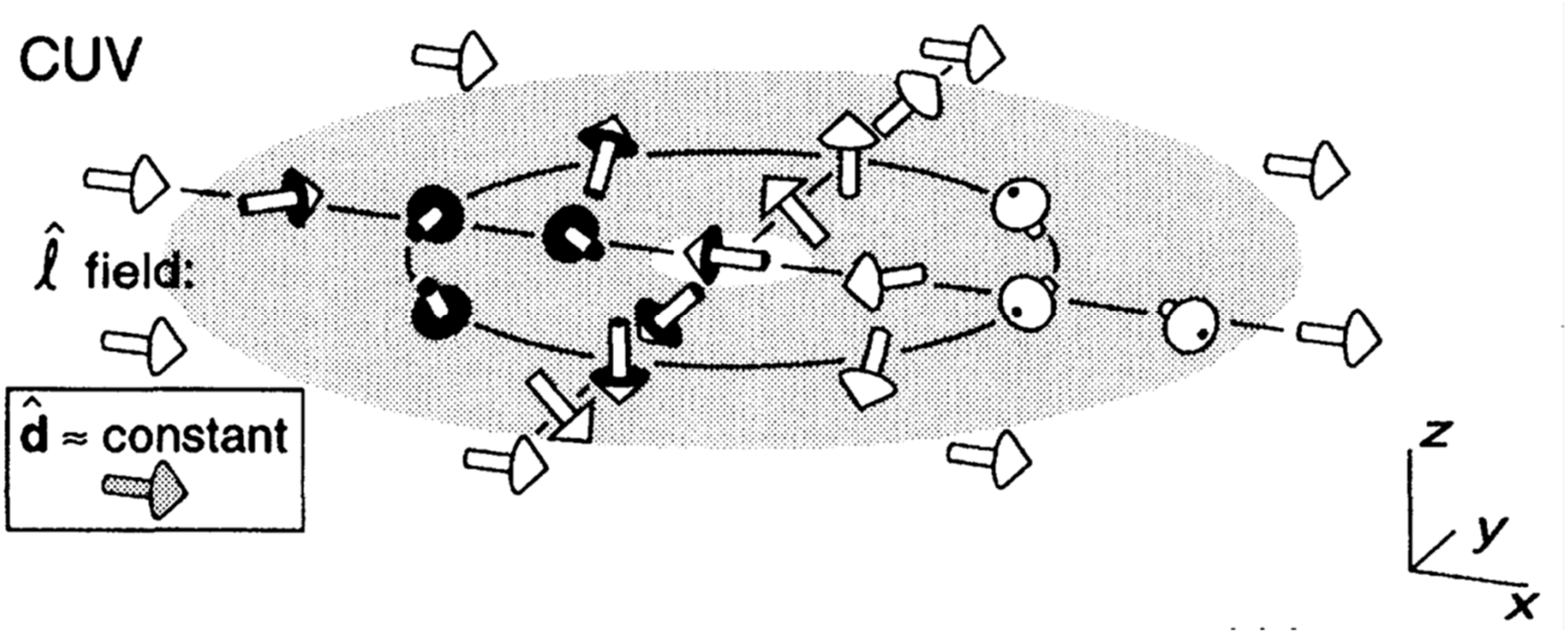
$$\boldsymbol{d}_{\mu}(\mathbf{k}) = \boldsymbol{d}_{\mu\boldsymbol{j}}\hat{\boldsymbol{k}}_{\boldsymbol{j}}$$

^{3}He-A $$\boldsymbol{d}_{\mu\boldsymbol{j}} = \Delta \boldsymbol{e}^{\boldsymbol{i}\phi}\hat{\boldsymbol{d}}_{\mu}\left(\boldsymbol{e}_{1\boldsymbol{j}} + \boldsymbol{i}\boldsymbol{e}_{2\boldsymbol{j}}\right)$$

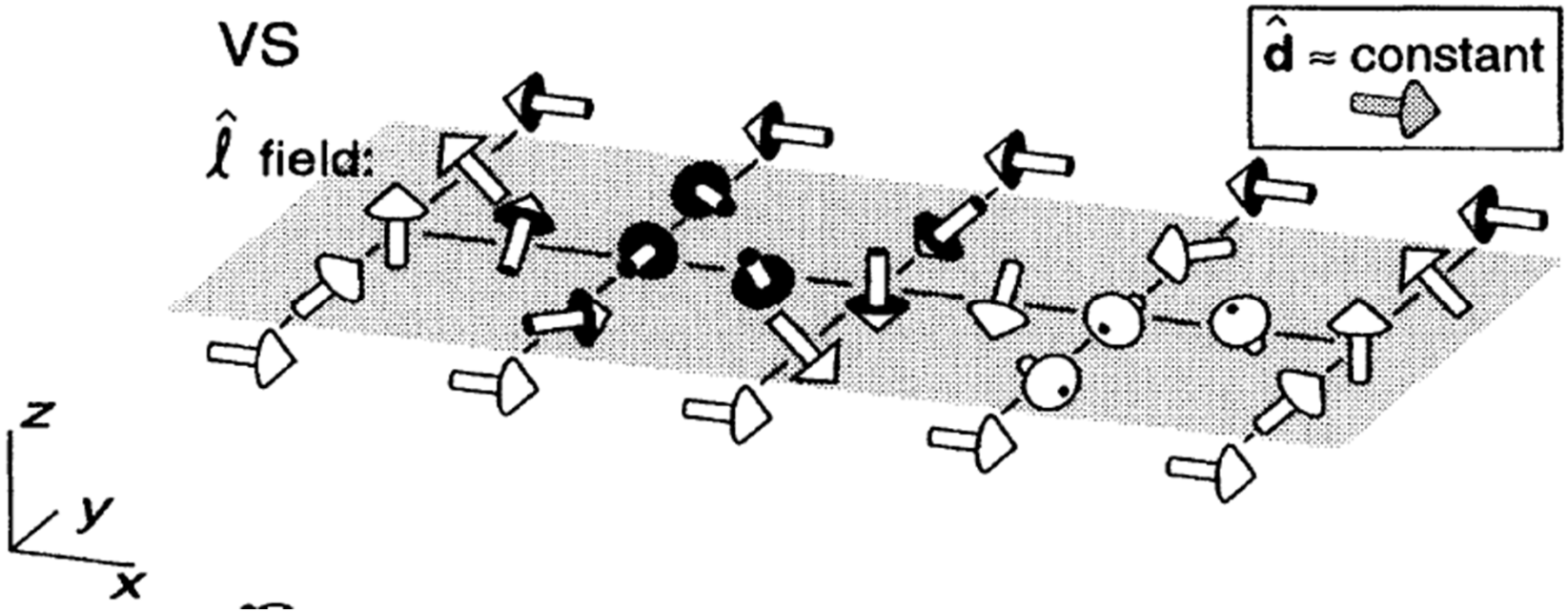
^{3}He-B $$\boldsymbol{d}_{\mu\boldsymbol{j}} = \Delta \boldsymbol{e}^{\boldsymbol{i}\phi}\delta_{\mu\boldsymbol{j}}$$

Bi-meron

*U. Parts et al. PRL **75**, 3320 (1975)*

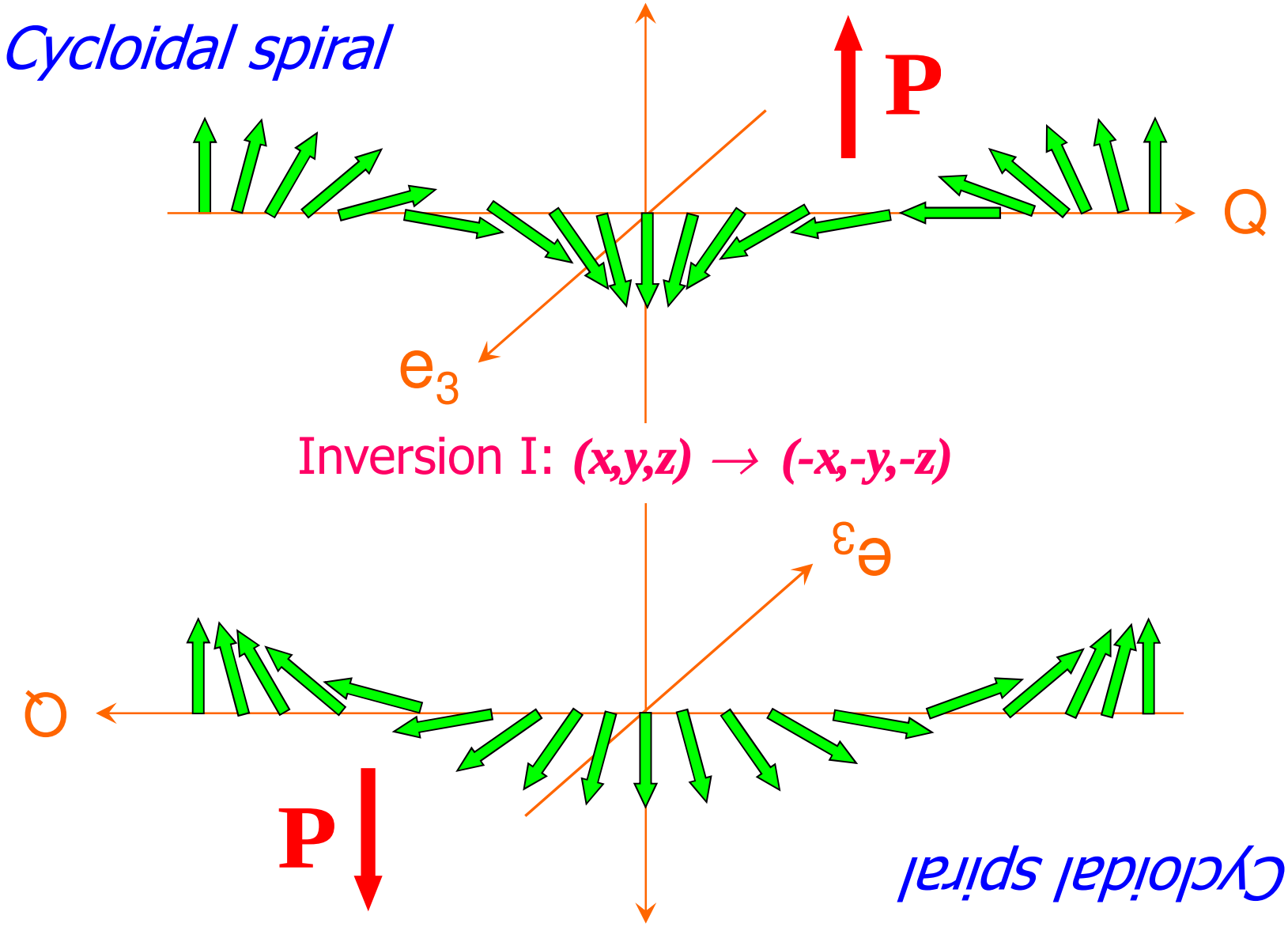


Topological stripe



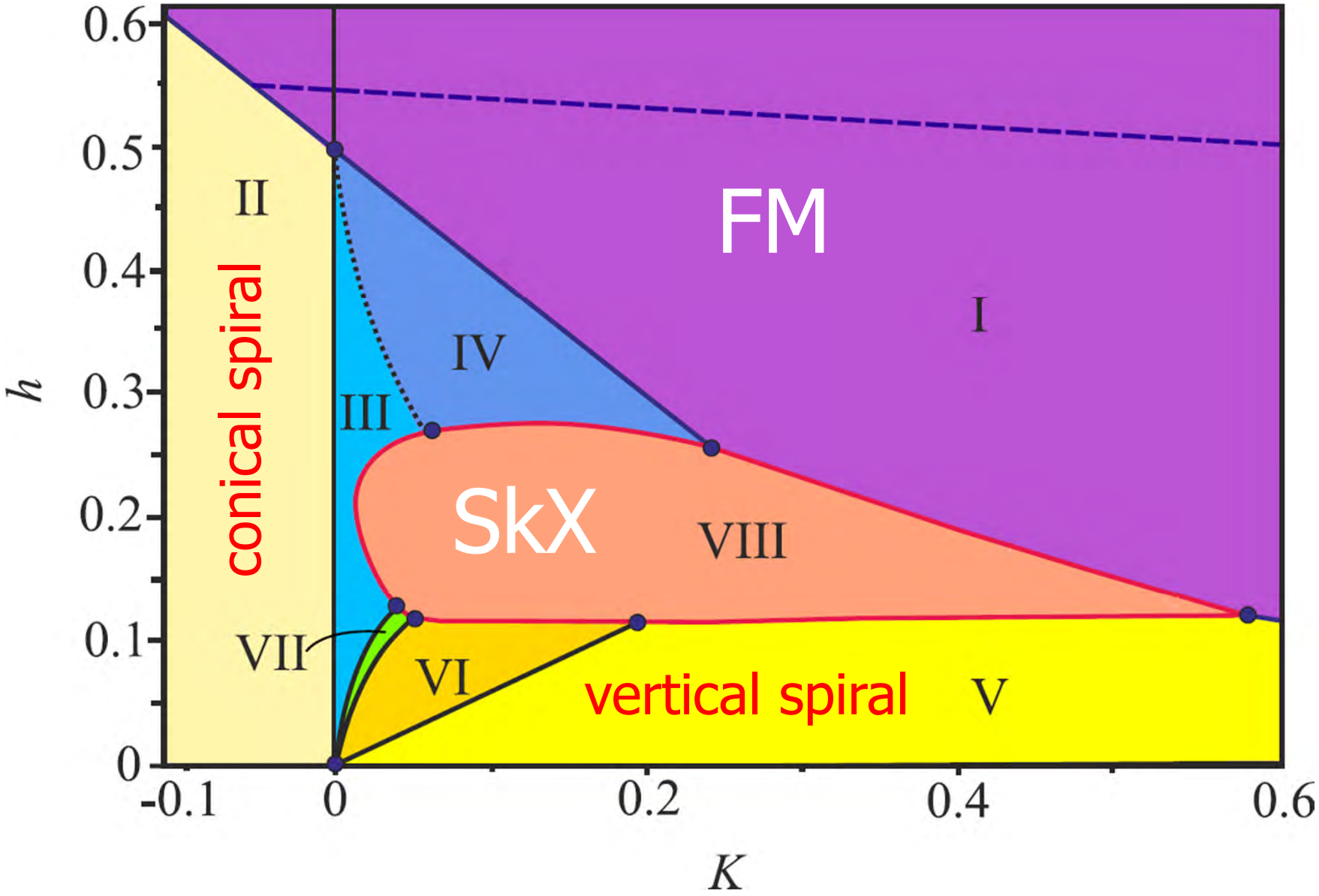
Magnetoelectric properties of skyrmions in Mott insulators

Breaking of inversion symmetry by spin ordering

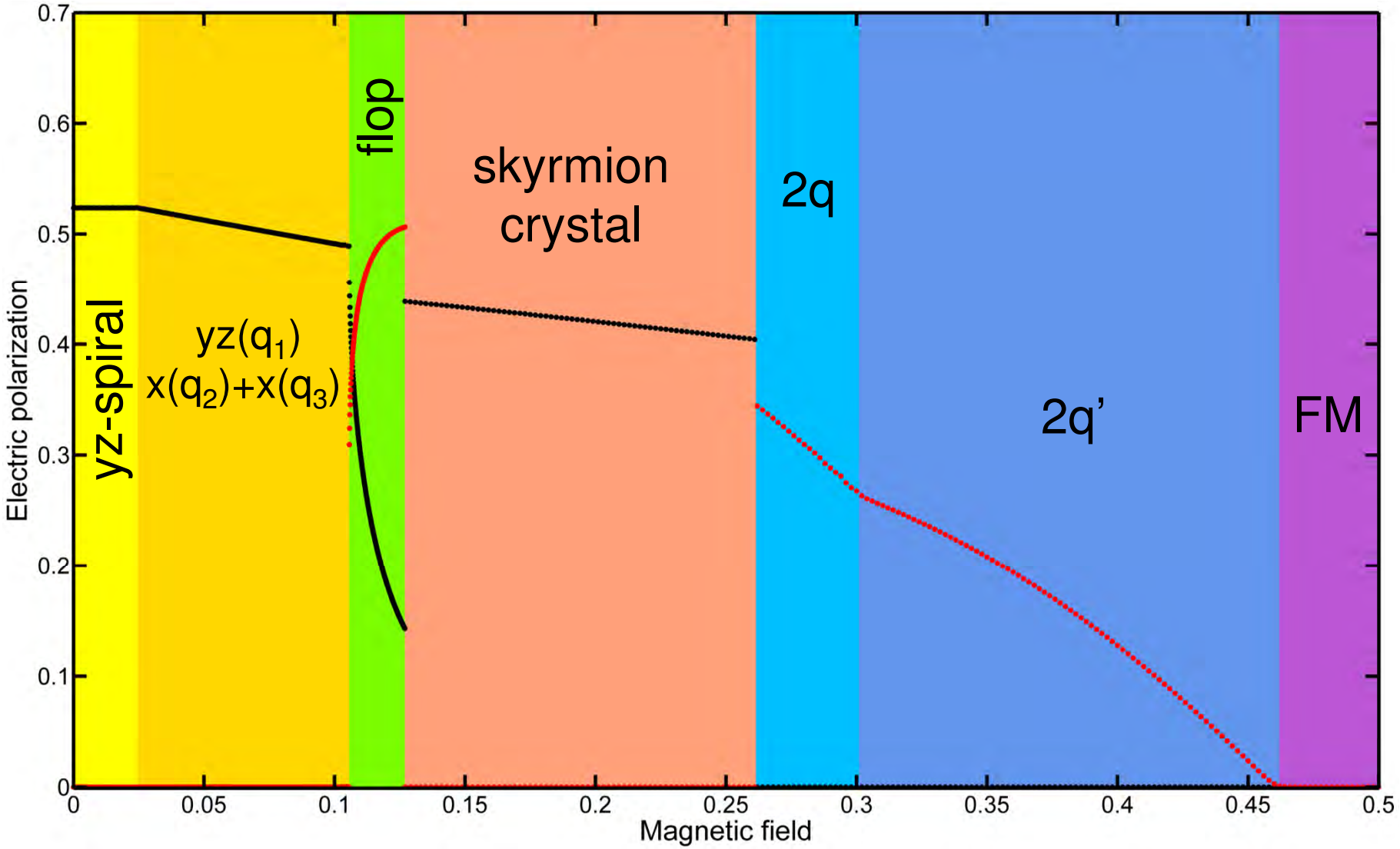


Phase Diagram ($\rho < 0$)

A. Leonov and MM, Nature Commun. 6, 8275 (2015)



Polarization at K = 0.04



Skyrmion electric dipole moment

Magneto-electric coupling *MM PRL **96**, 067601 (2006)*

$$f_{me} = -g\,\mathbf{E}\cdot\left[\mathbf{m}(\nabla\cdot\mathbf{m}) - (\mathbf{m}\cdot\nabla)\mathbf{m}\right]$$

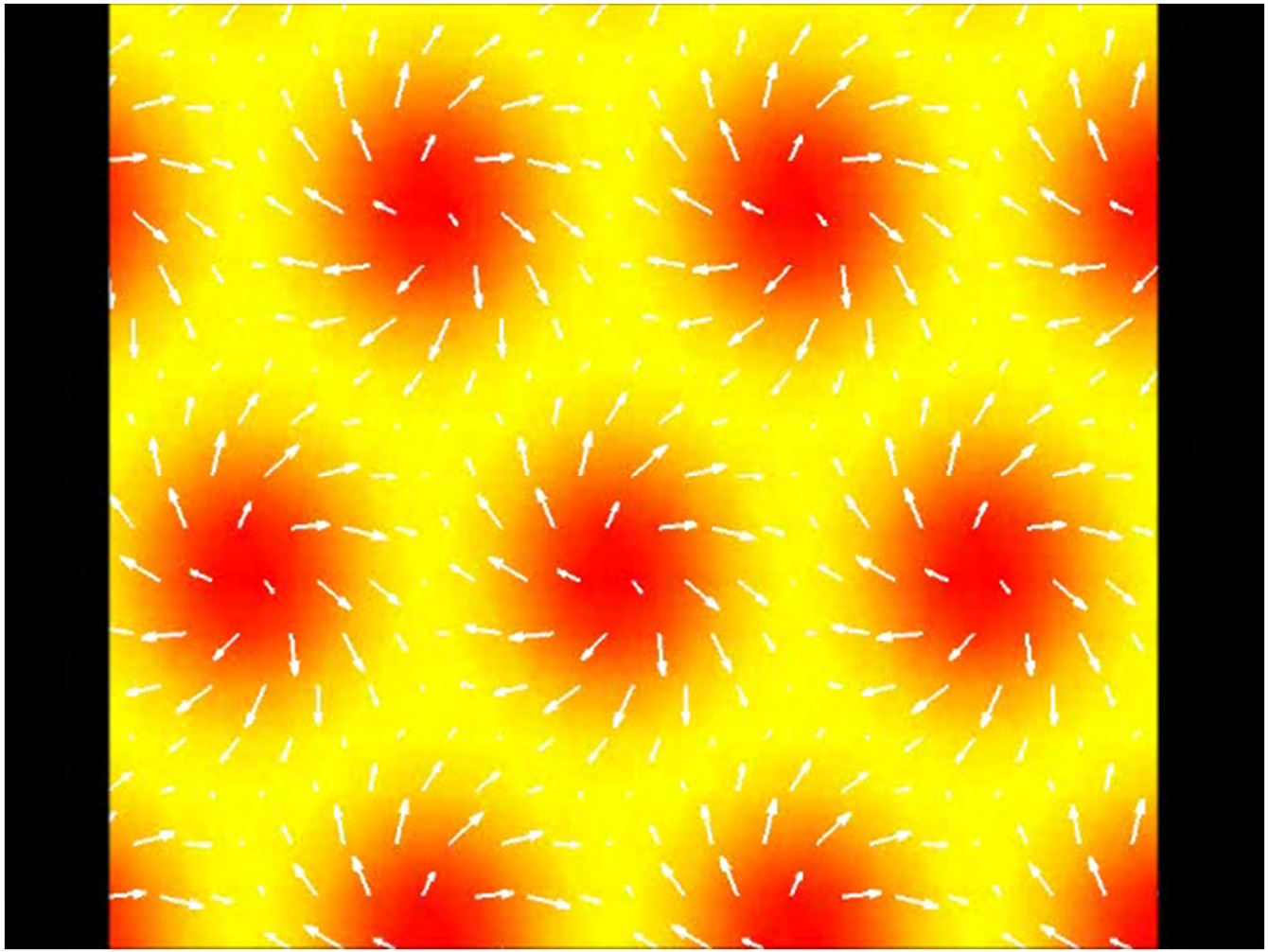
Polarization induced by skyrmion

$$P_z = -\frac{\partial f}{\partial E_z} = g\left[\frac{d\Theta}{dr} + \mathrm{v}\frac{\sin 2\Theta}{2r}\right]\cos\left((\mathrm{v}-1)\varphi + \chi\right)$$

Dipole moment for vorticity v = +1

$$D_z(\chi) = D(0)\cos\chi$$

Magnetically-induced electric oscillations



H_x

Conclusions

- Non-trivial topology of magnetic defects gives rise to interesting physics
- Skyrmions, merons, monopoles, chiral bobbers, hopfions, …
- Electric field control of skyrmions in magnetic insulators
- Topological memory