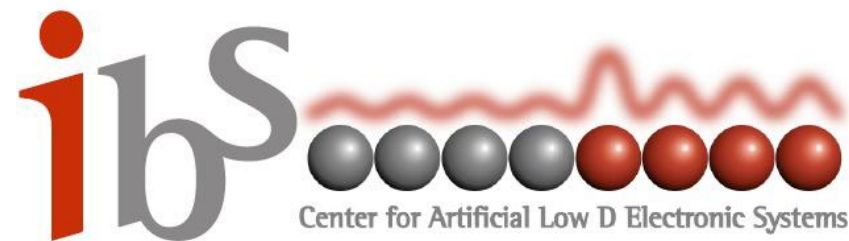


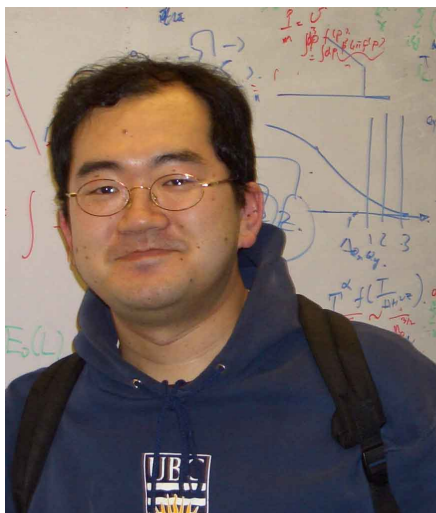
Superconductivity & Non-Fermi Liquids in domain wall networks of 2D CDW systems

Gil Young Cho

¹ POSTECH ² IBS CALDES



Collaborators

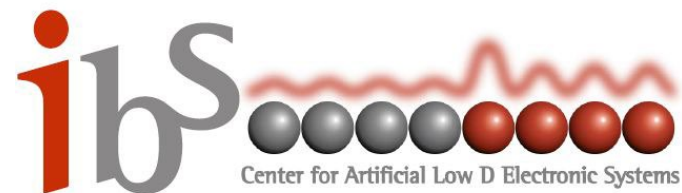


Masaki Oshikawa
(ISSP/Univ. Tokyo)



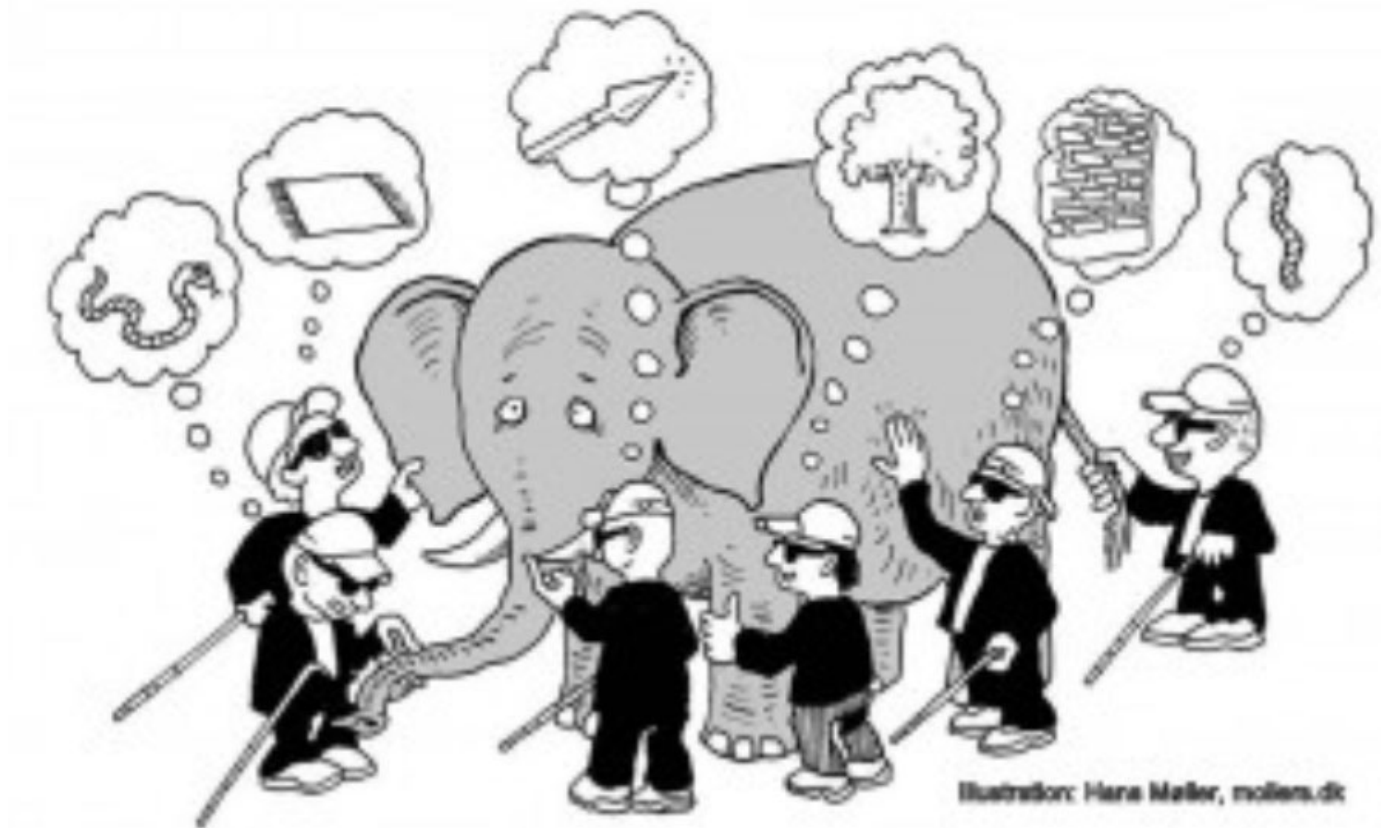
Han Woong Yeom
(POSTECH/IBS CALDES)

Funding



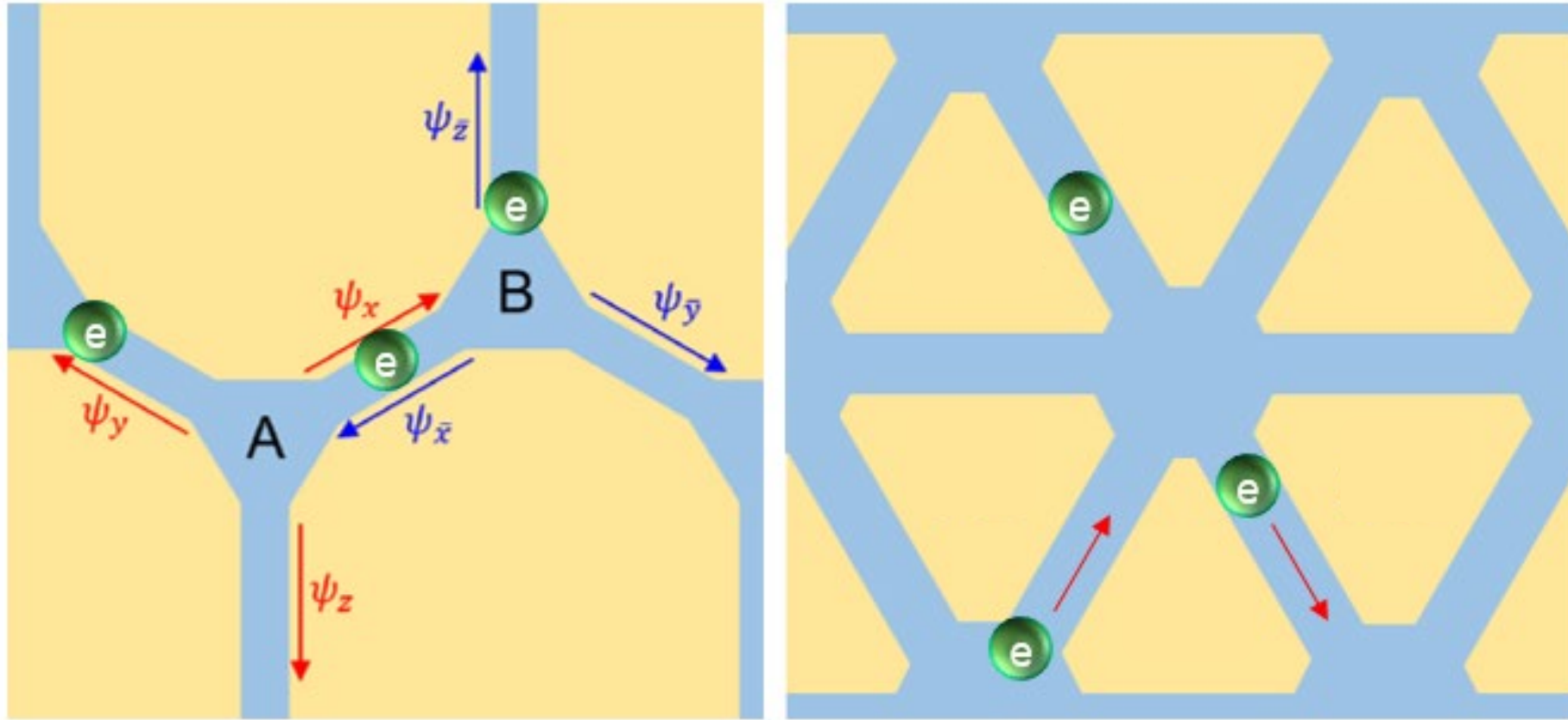
- [1] Park, **GYC**, Lee, Yeom, Nature Communications (2019)
- [2] Lee, Geng, Park, Oshikawa, Lee, Yeom, **GYC**, Physical Review Letters (2020)
- [3] Lee, Oshikawa, **GYC**, Physical Review Letters (2021)
- [4] Kang, Lee, **GYC**, Physical Review Letters (2021)
- [5] Park, Kim, **GYC**, Lee, Physical Review Letters (2019)
- [6] Park, Lee, (...), **GYC**, Lee, Nature (2022) - if time allows...

Warning: I am *not really a specialist of a particular material...*



Based on my (*possibly wrong*) understandings on materials/experiments.
(although I consulted with many experimentalists)

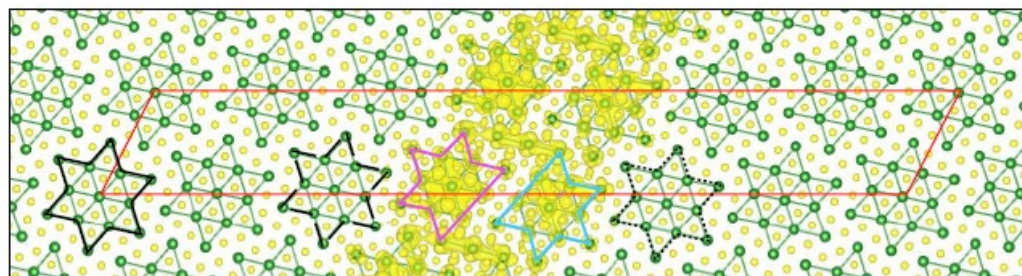
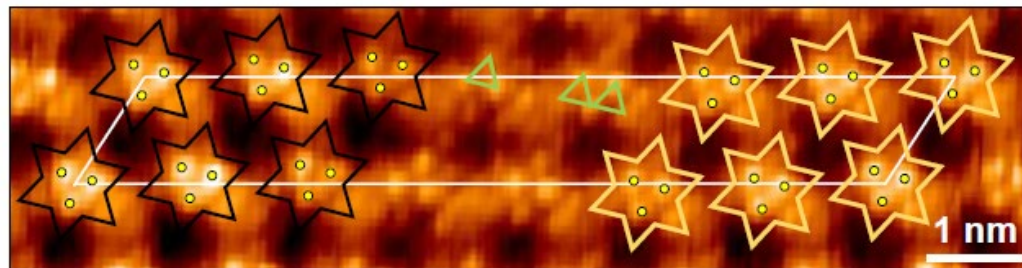
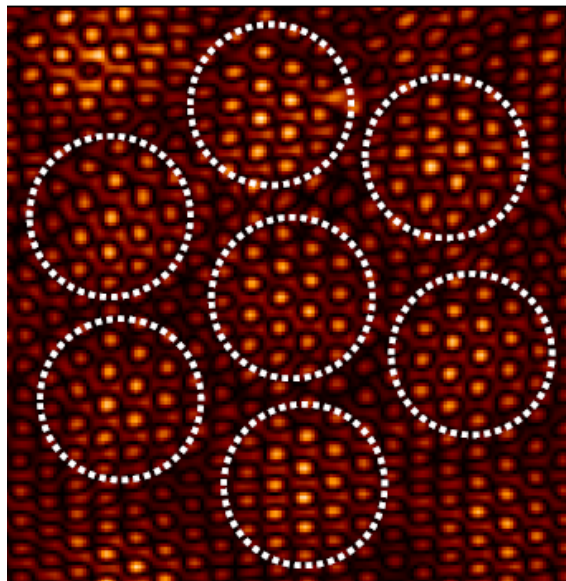
Goal: Exploring Electronic Properties of Quasi-1d Systems



Flat bands, superconductivity, topological states & Non-Fermi liquids

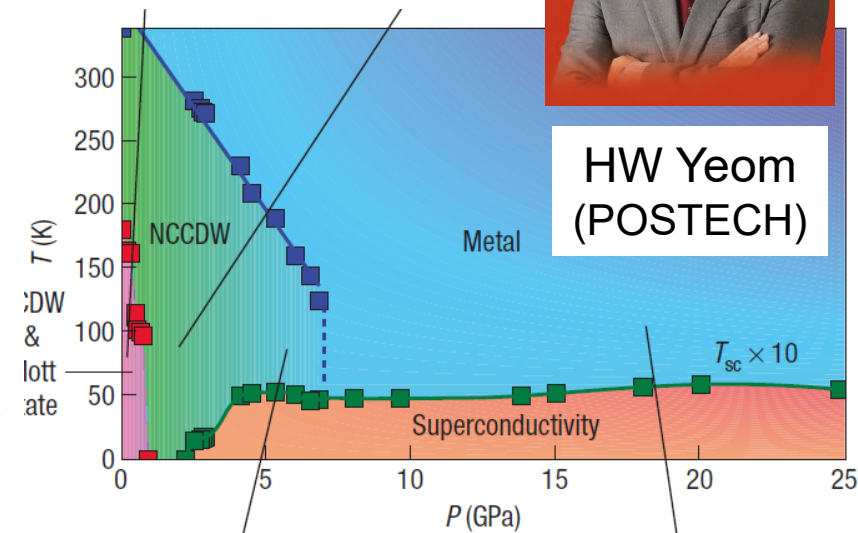
The pattern that captured my eyes:

Experiments: STM on nearly-commensurate charge-density wave in 1T-TaS₂



[Park, GYC, Lee, Yeom, Nat. Comm. (2019)]

1. Lateral size of domain wall network $\sim O(80)$ Å.
2. *Only* domain walls are metallic.



Phase diagram
[Sipos et. al. Nat. Mat. (2008)]



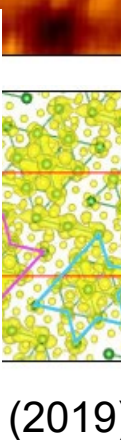
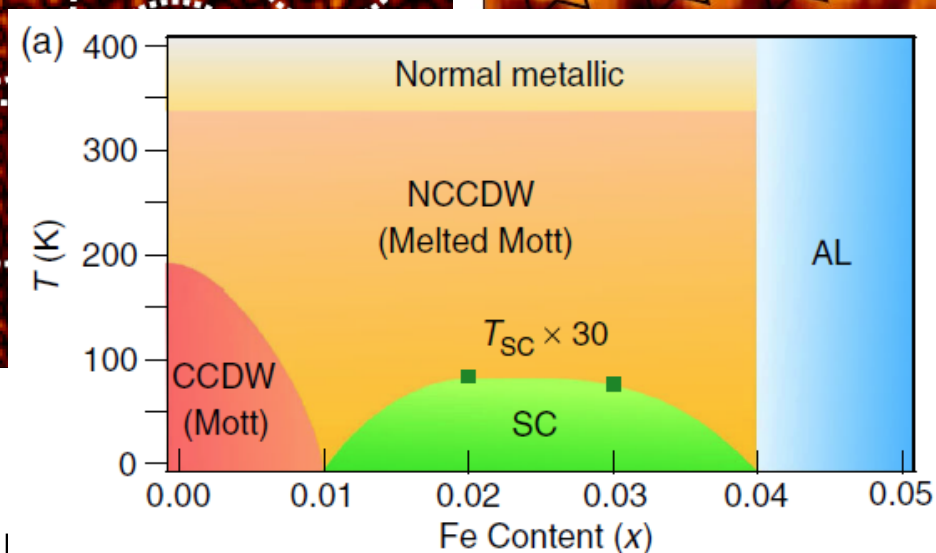
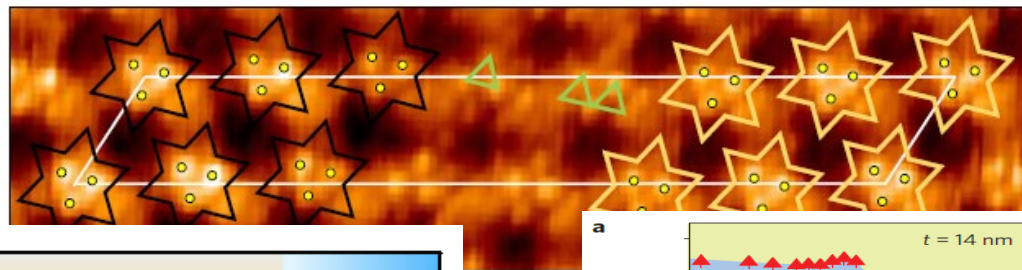
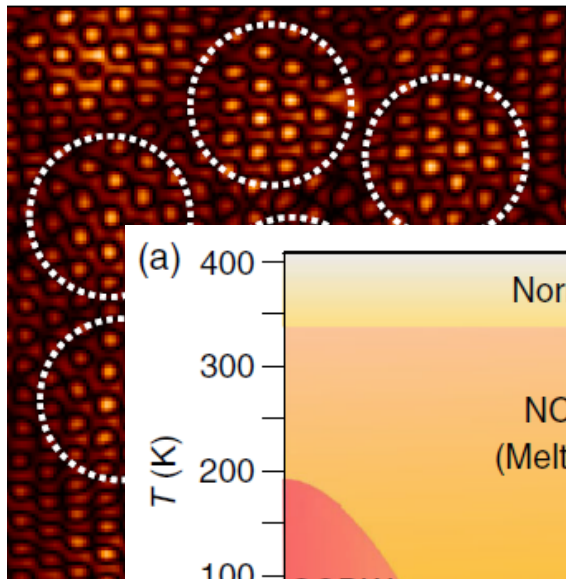
HW Yeom
(POSTECH)

The pattern that captured my eyes:

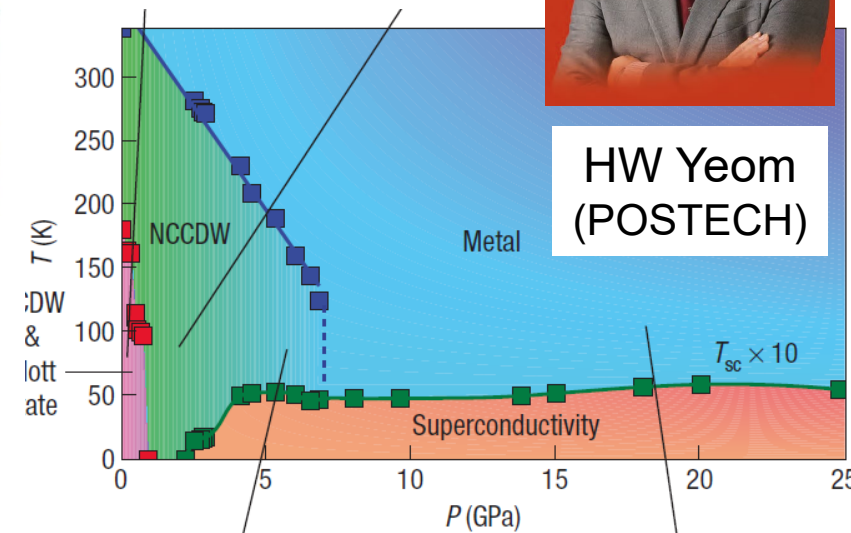
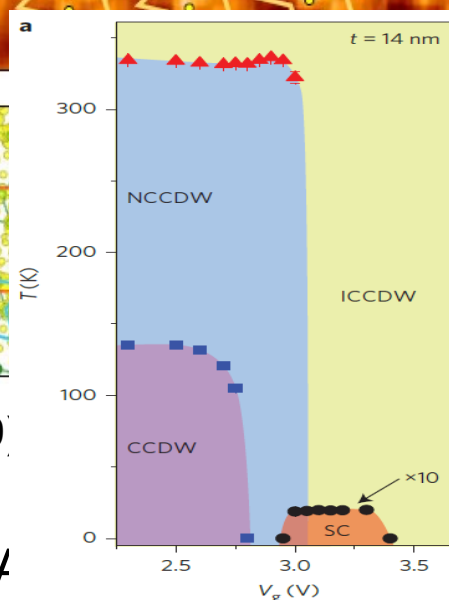


HW Yeom
(POSTECH)

Experiments: STM on nearly-commensurate charge-density wave in 1T-TaS₂



(2019)



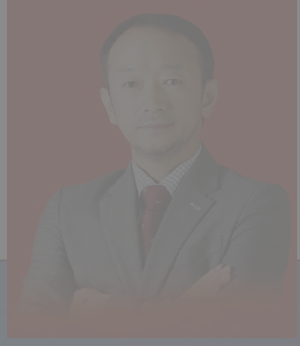
Phase diagram
[Sipos et. al. Nat. Mat. (2008)]

1. Lateral size of domain wall network $\sim \lambda(80) / \lambda$
2. Only domain walls are metallic.

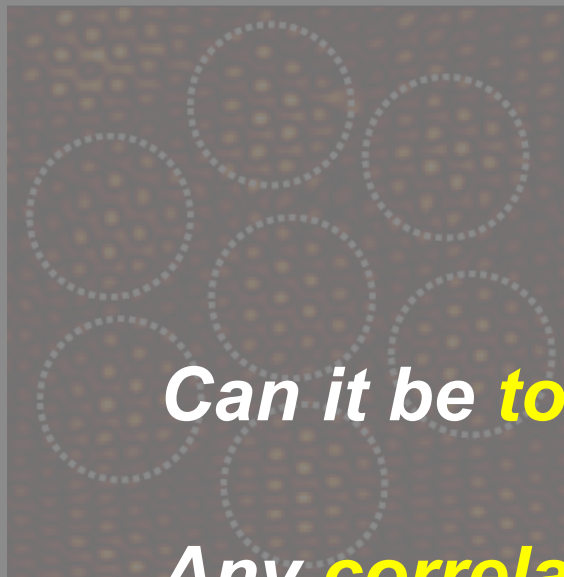
What's the electronic structure? Why superconducting?

The pattern that captured my eyes:

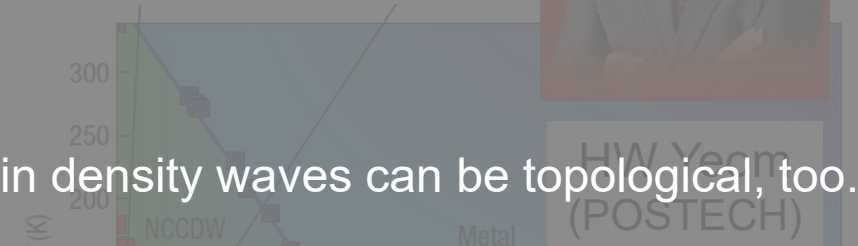
Experiments: STM on nearly-commensurate charge-density wave in 1T-TaS₂



HW Yeom (POSTECH)



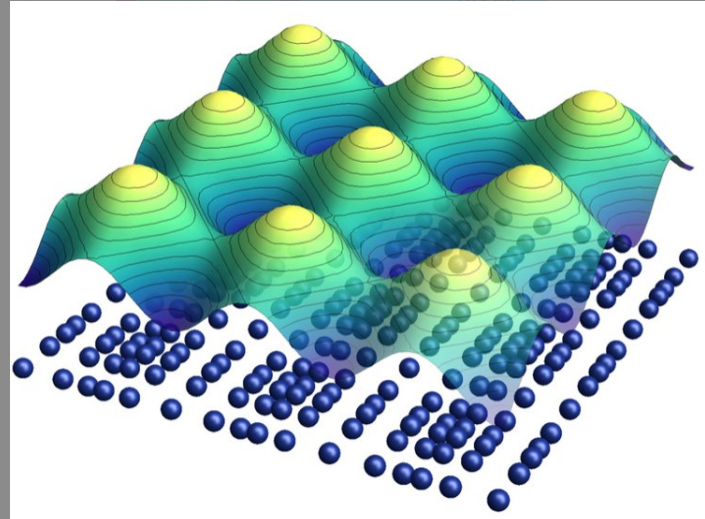
Cf: certain density waves can be topological, too.



Can it be **topological**?

Any **correlation-driven** phenomena?

[Ref. JW Park, GYC, J Lee, HW Yeom, Nat. Comm. (2019)]

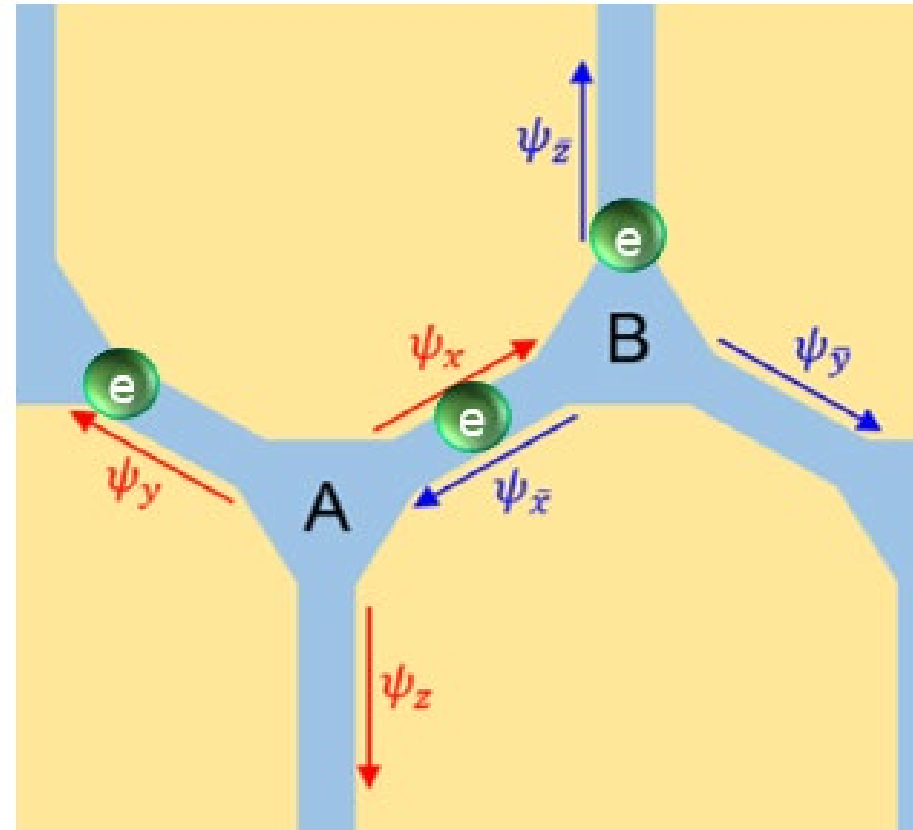
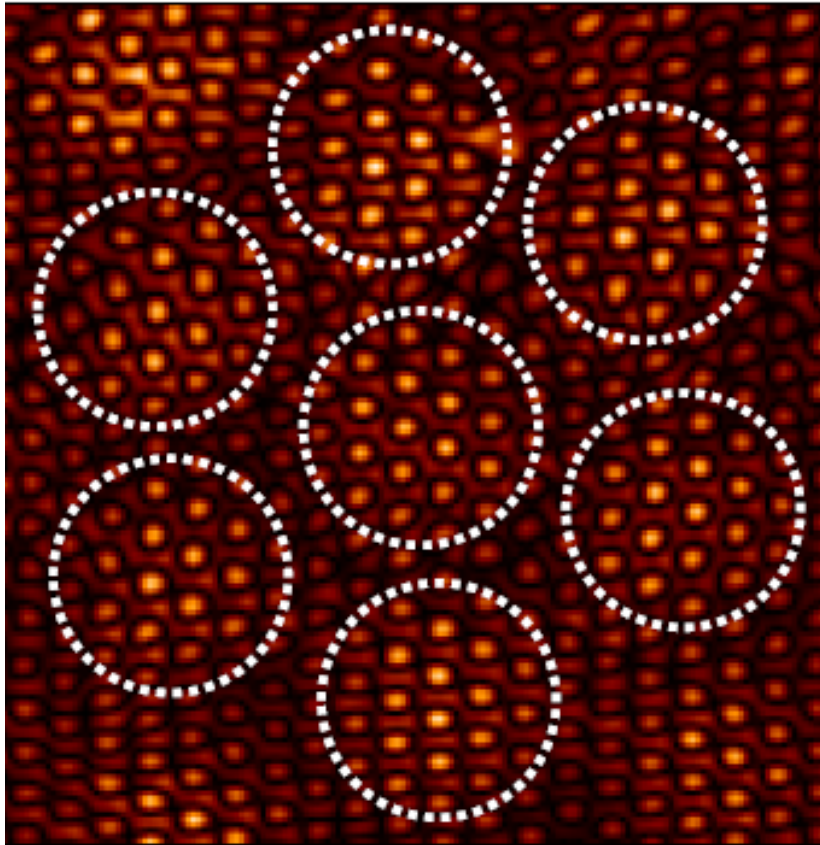


1. Lateral size of domain wall network ~ O(80) Å.
2. *Only* domain walls are metallic.

GYC, Soto-Garrido, Fradkin, Phys. Rev. Lett. (2015)

What's the electronic structure? Why superconducting?

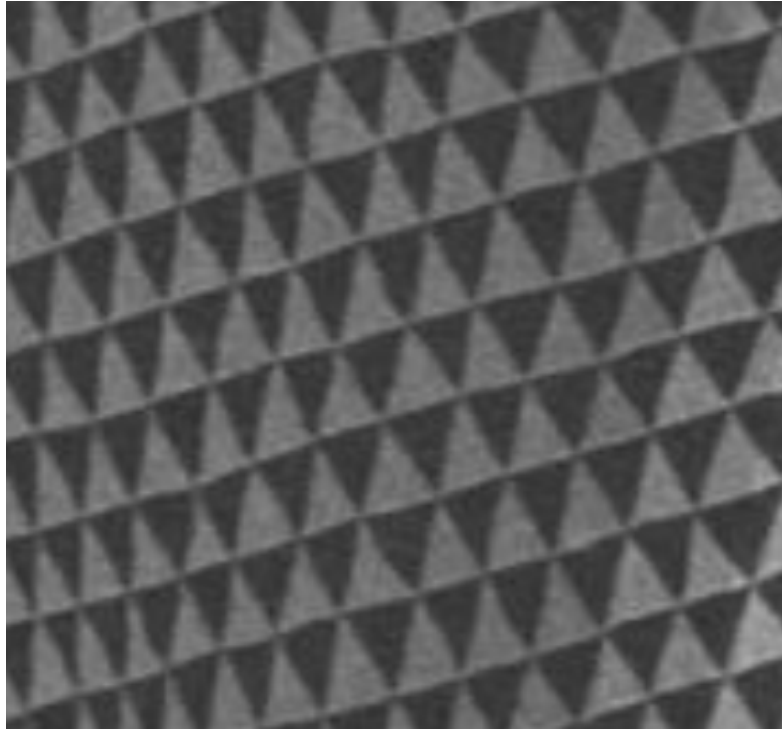
To the 1st order approximation:



Problem: **1D** mobility of electrons, combined with **2D** superstructures.

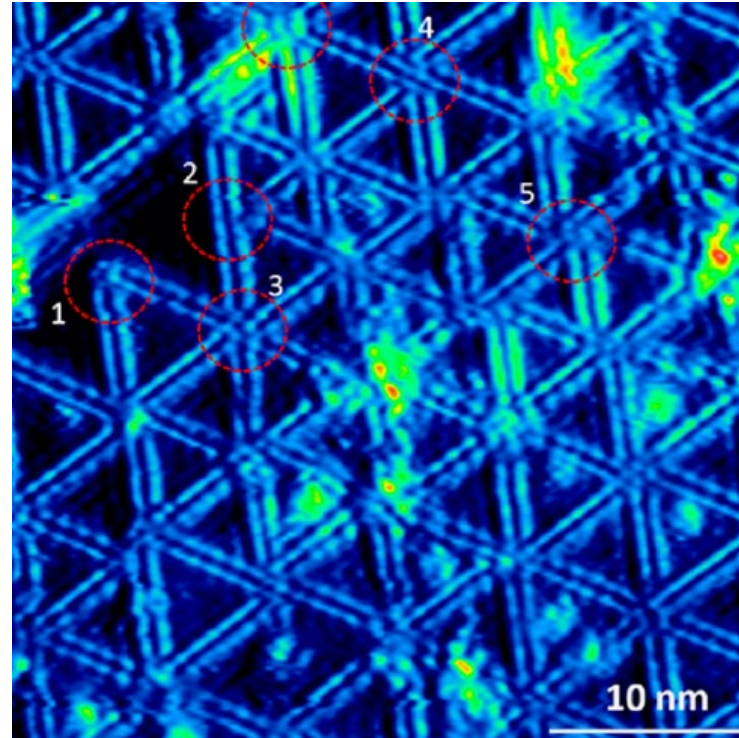
[Lee, Geng, Park, Oshikawa, Lee, Yeom, **GYC***, Physical Review Letters (2020)]

This structure is *more common* than one naively thinks:



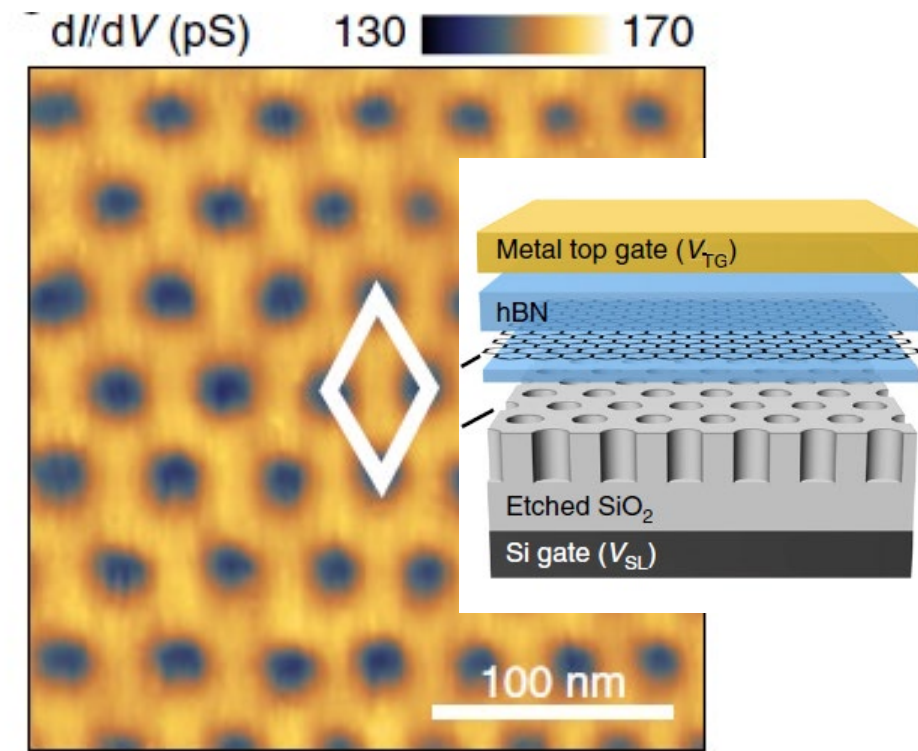
Twisted Bilayer Graphene

[Yoo et.al. Nat. Matt. (2019)]



Moiré MoSe₂

[Ma et.al. ACS Nano (2019)]

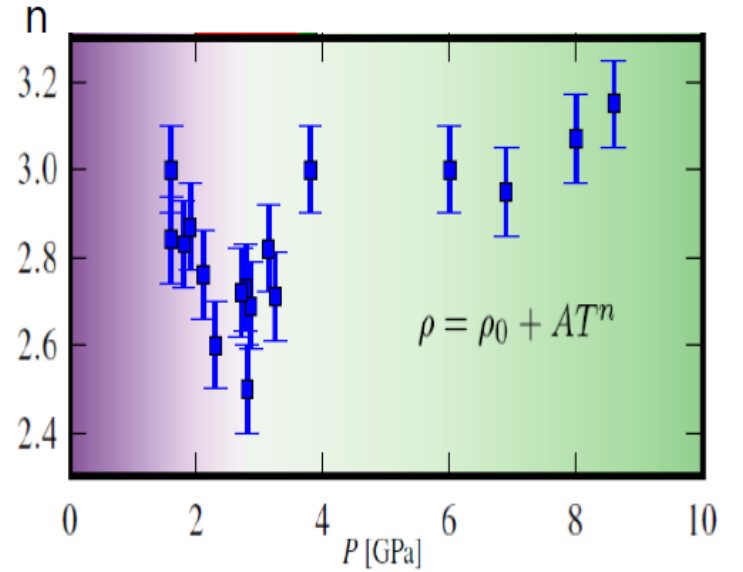
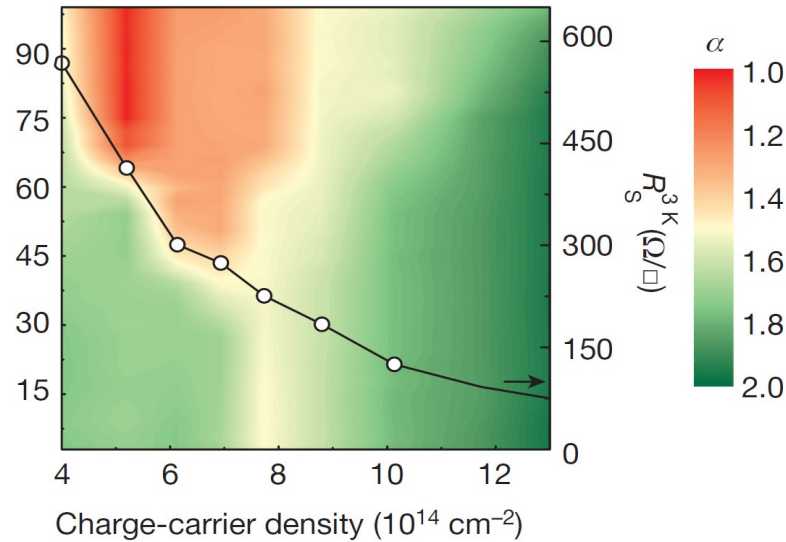
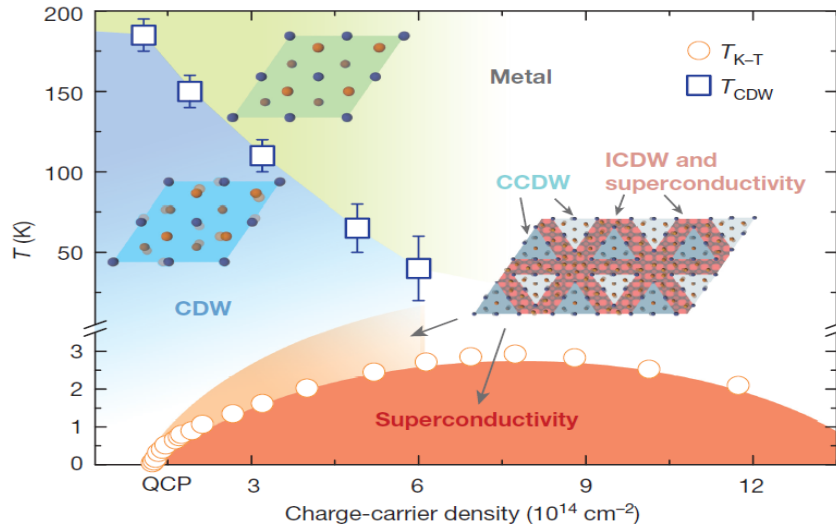


Patterned Network on Graphene

[Forsythe et.al. Nano Lett (2018)]

More interesting examples?

Experiments: 1T-TiSe₂



[Ref. Li et.al., Nat. (2016)]

[Ref. Kusmartseva et. al. PRL (2009)]

1. Non-Fermi Transport $R \sim R_0 + AT^n$, varying exponent $n \approx 1 \sim 3$
2. Emerging superconductivity, and domain wall networks (Little-Park effect)

Correlation effect seems important in these materials!

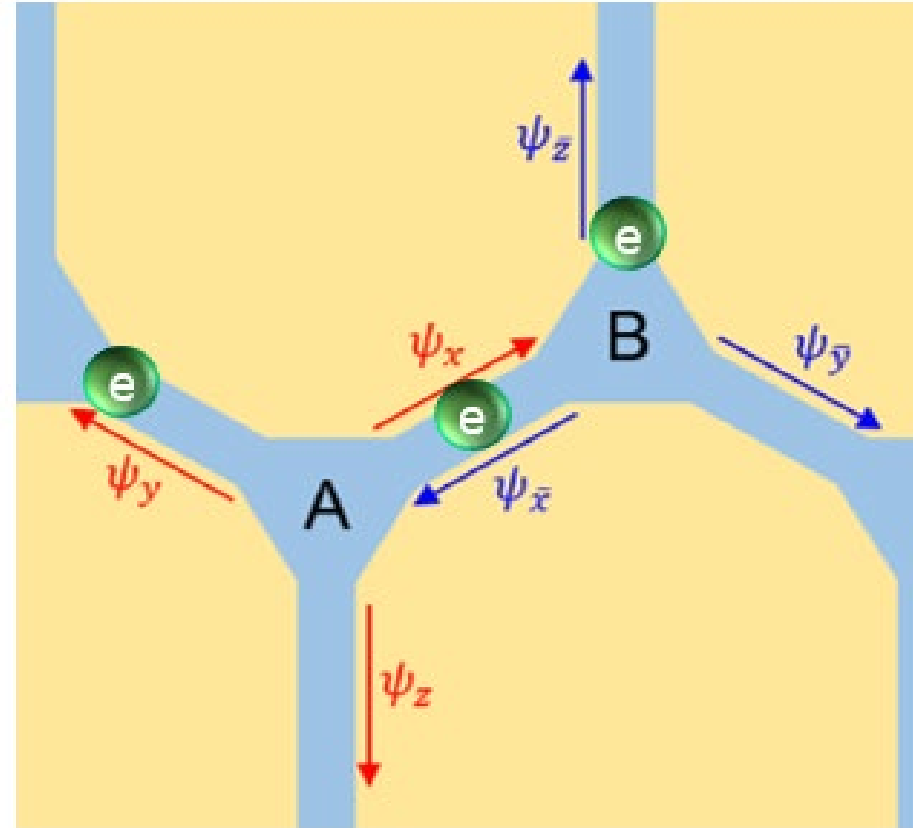
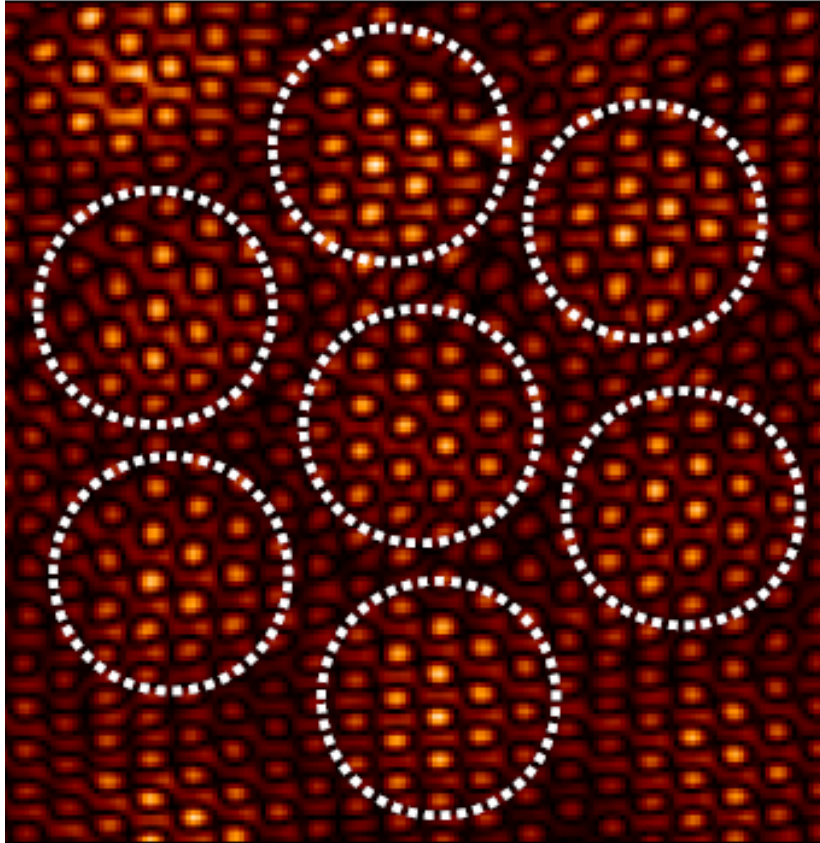
Can we *explain* these?

[1] Park, **GYC**, Lee, Yeom*, Nature Communications (2019)

[2] Lee, Geng, Park, Oshikawa, Lee, Yeom, **GYC***, Physical Review Letters (2020)

[3] Lee, Oshikawa, **GYC***, Physical Review Letters (2021)

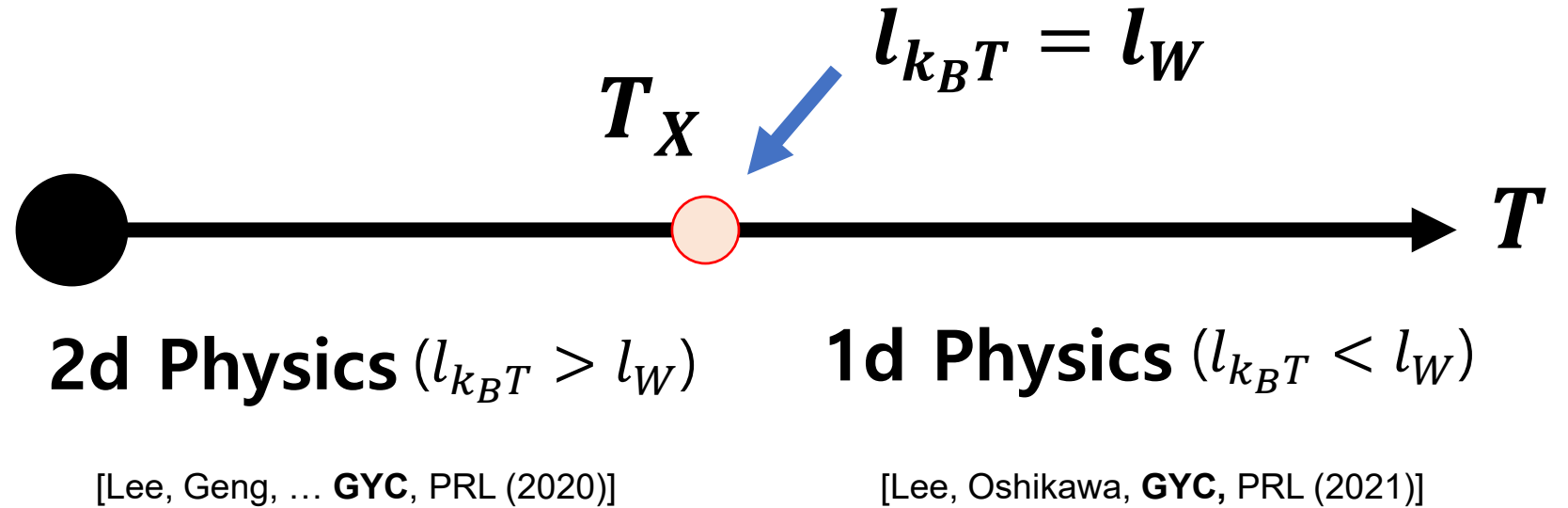
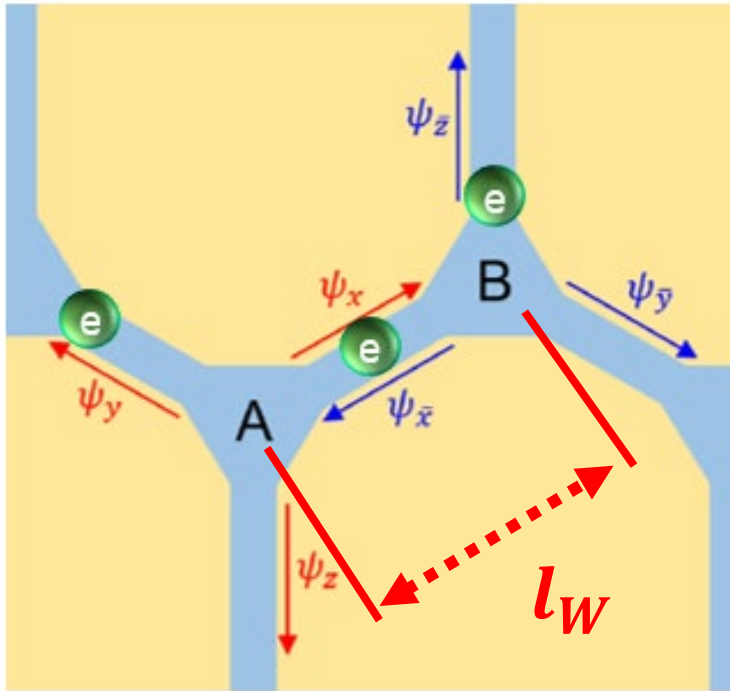
To the 1st order approximation:



Problem: *1D* mobility combined with *2D* superstructures.

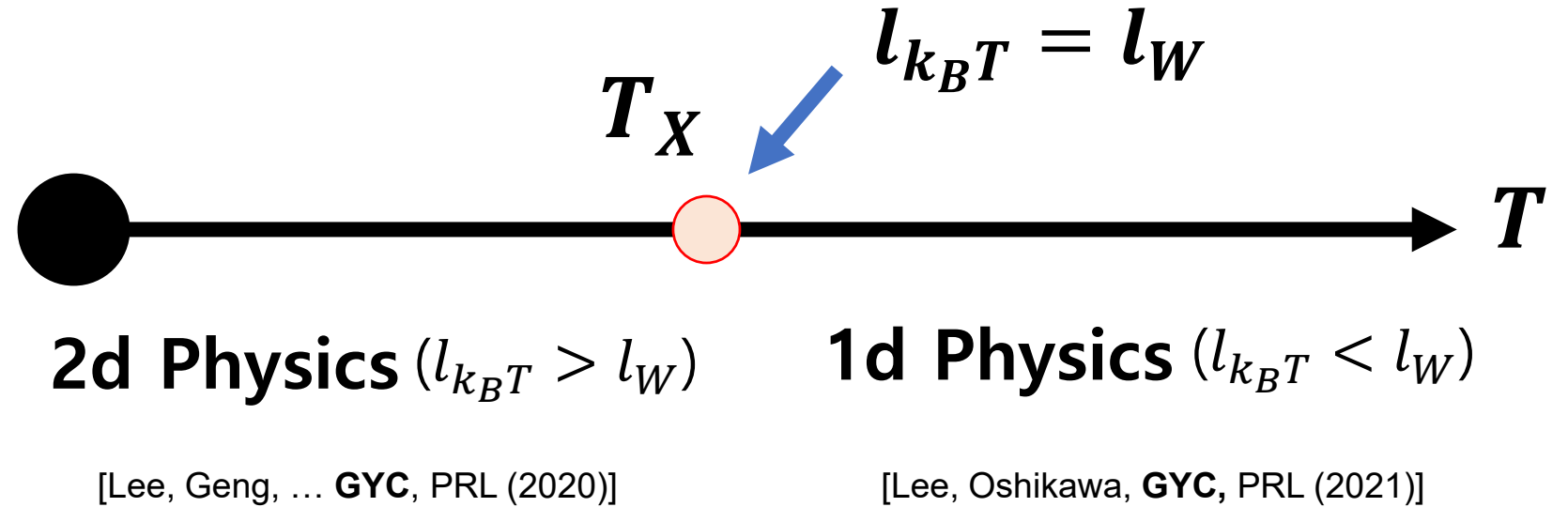
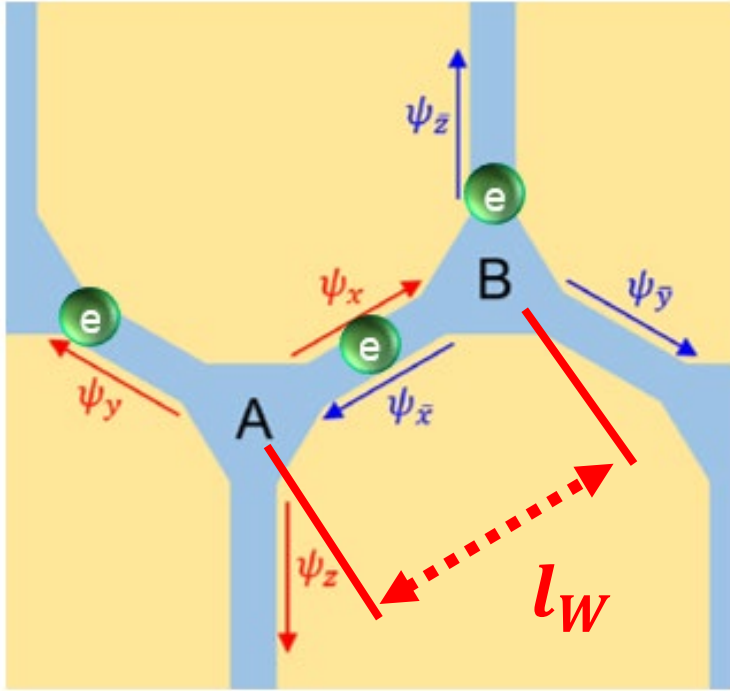
[Lee, Geng, Park, Oshikawa, Lee, Yeom, **GYC***, Physical Review Letters (2020)]

Key observation: Dimensional Crossover T_X



Wire length l_W vs. Thermal coherence $l_{k_B T} = \frac{\hbar v_F}{k_B T}$

Key observation: Dimensional Crossover T_X



- **TiSe₂: O(10)K** (assuming e.g. that the electrons are flowing along 1D domain walls)
- **Twisted bilayer graphene: O(50)K** (for 140 nm)
- **TaS₂: O(100)K**

Key observation: Dimensional Crossover T_X

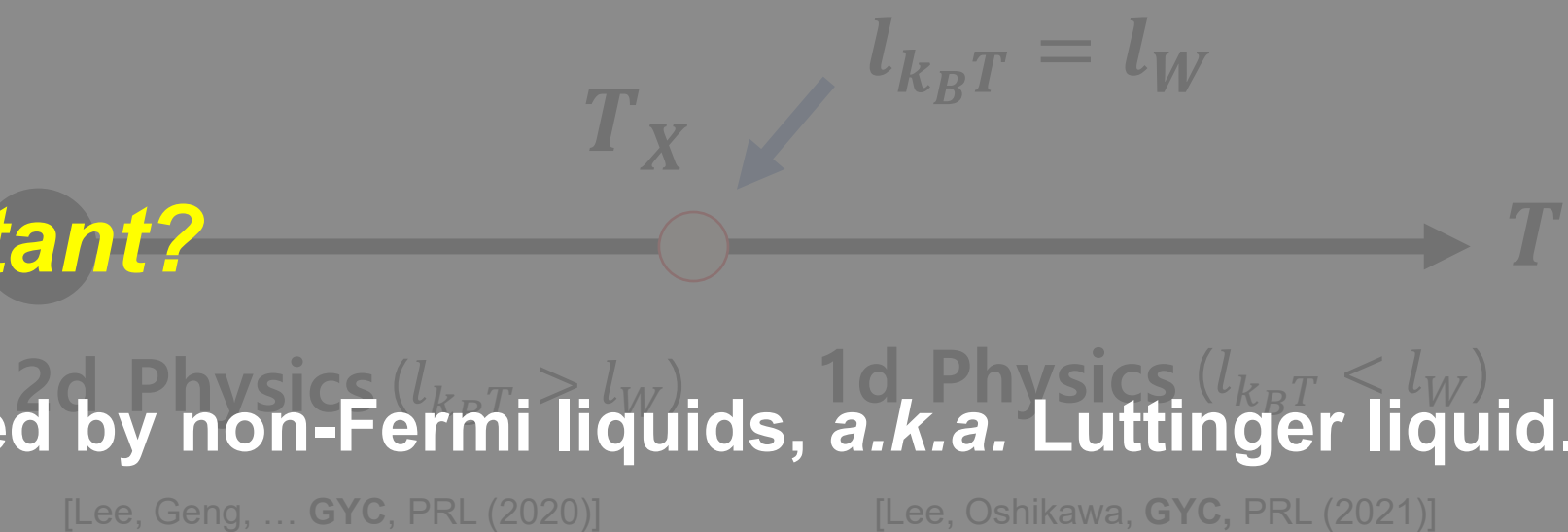
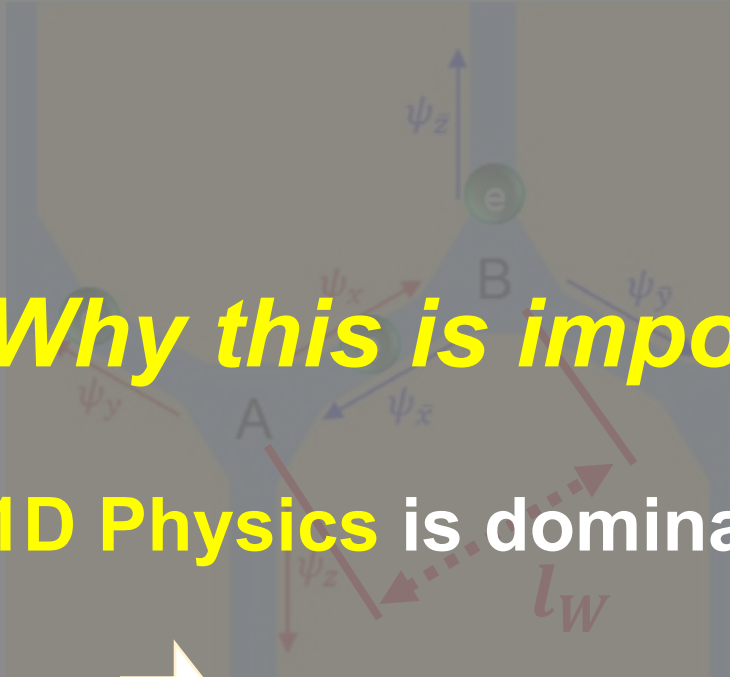
Why this is important?

1D Physics is dominated by non-Fermi liquids, *a.k.a.* Luttinger liquid.

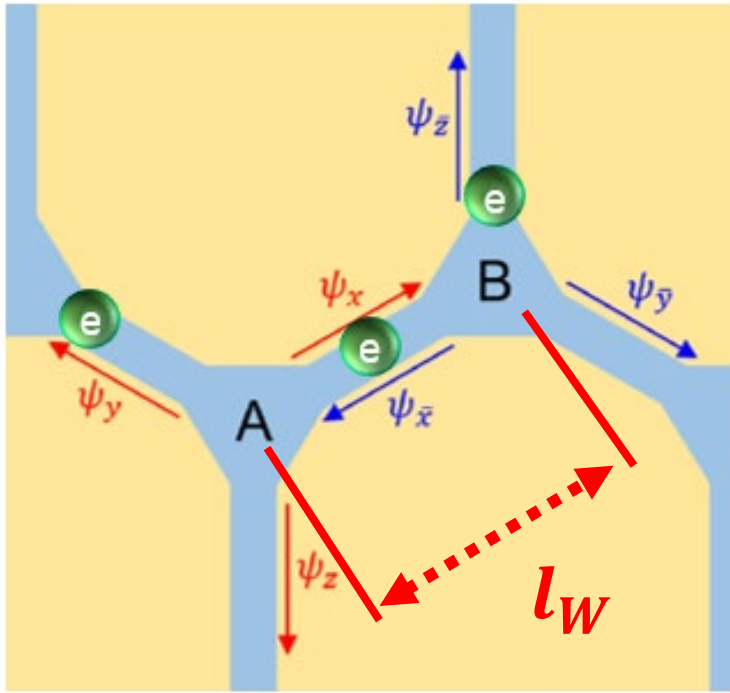
➔ **Low-T and high-T should be differently accessed in theory.**

- **TiSe₂: O(10)K** (assuming e.g. that the electrons are flowing along 1D domain walls)
- **Twisted bilayer graphene: O(50)K** (for 140 nm)
- **TaS₂: O(100)K**

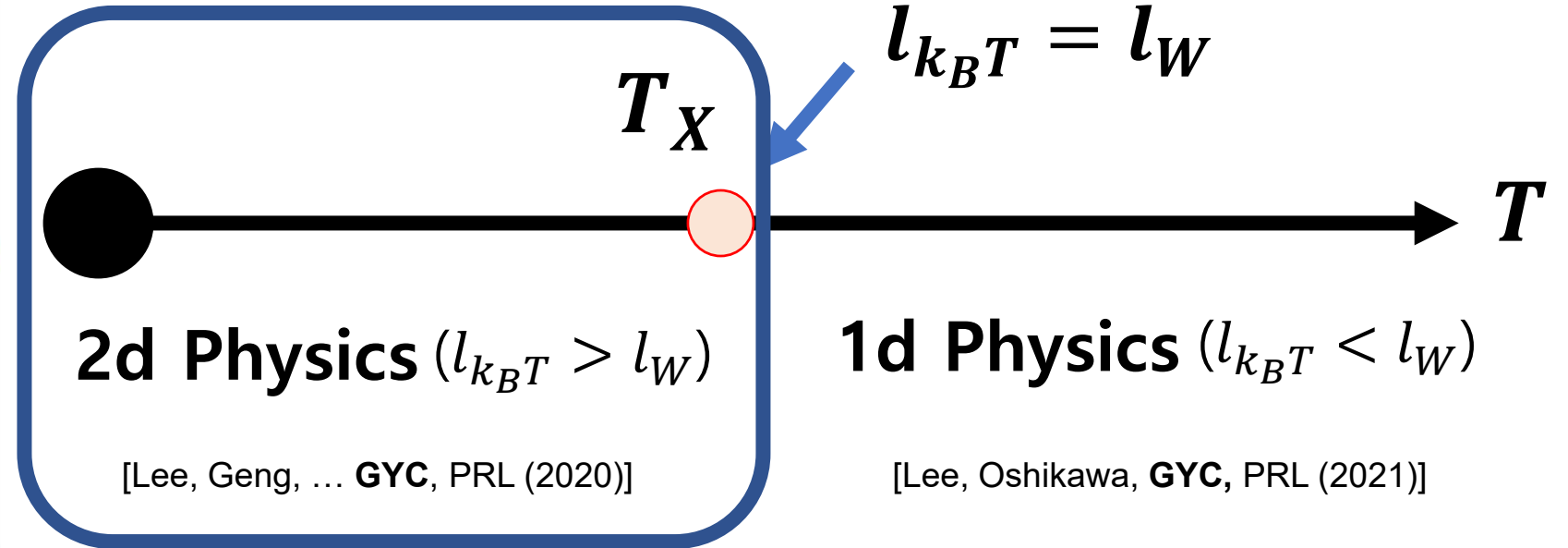
[Lee, Oshikawa, **GYC***, Phys. Rev. Lett. 126, 186601 (2021)]



Key observation: Dimensional Crossover T_X



Relatively easier problem.

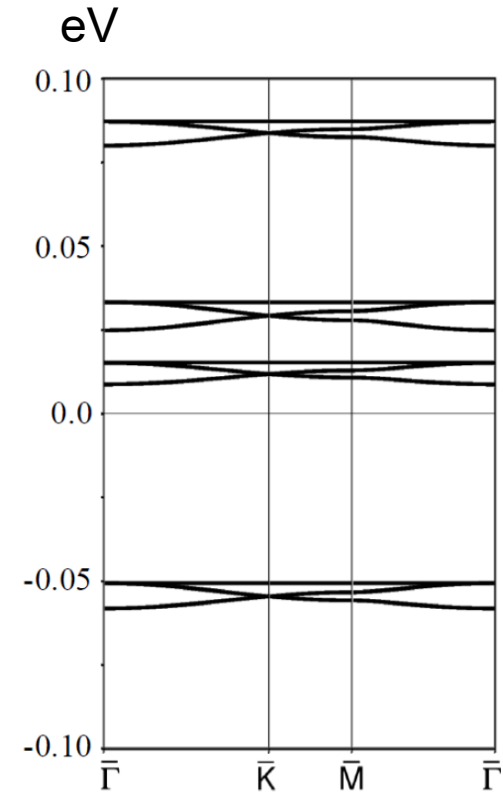
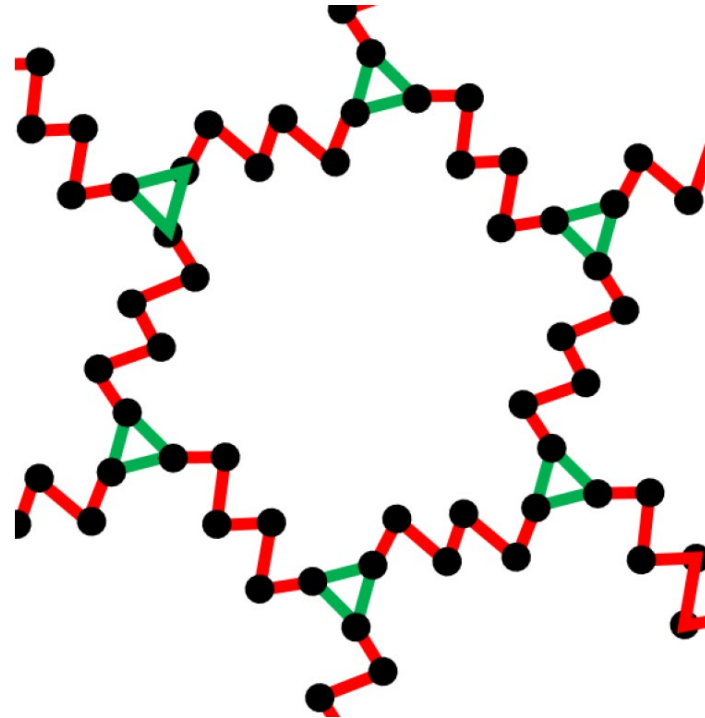
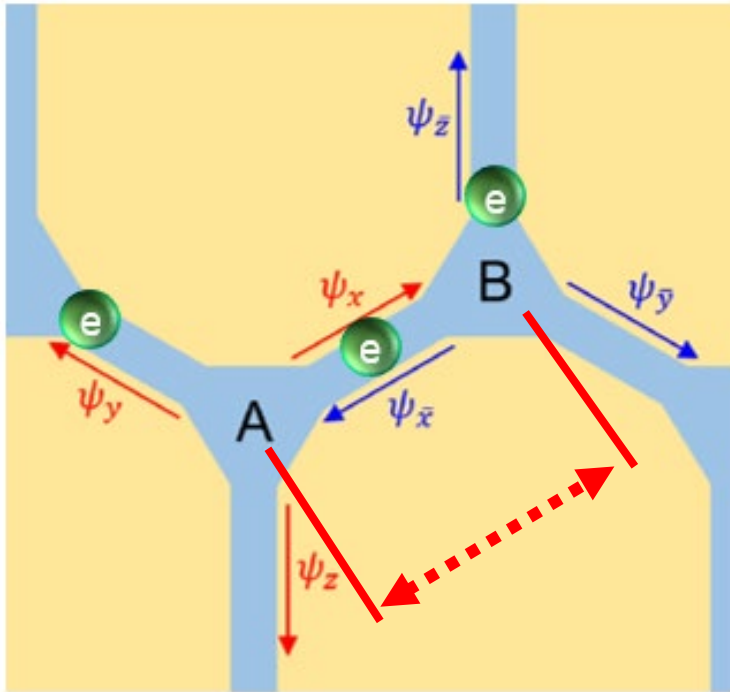


- **TiSe₂: O(10)K** (assuming e.g. that the electrons are flowing along 1D domain walls)
- **Twisted bilayer graphene: O(25)K** (for 300 nm)
- **TaS₂: O(100)K**

2D limit: $T < T_X$

2D limit: Cascade of *stable* flat bands & Superconductivity

[Lee, Geng, Park, Oshikawa, Lee, Yeom, GYC*, Phys. Rev. Lett. 124, 137002 (2020)]



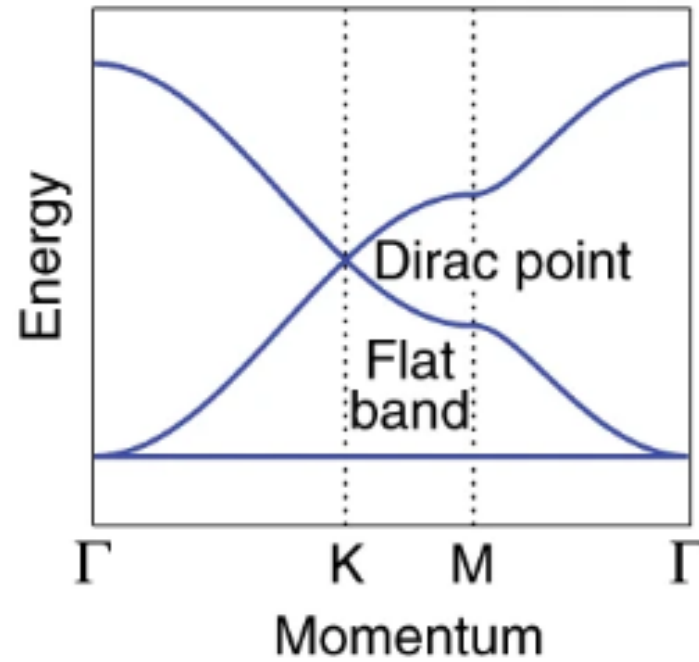
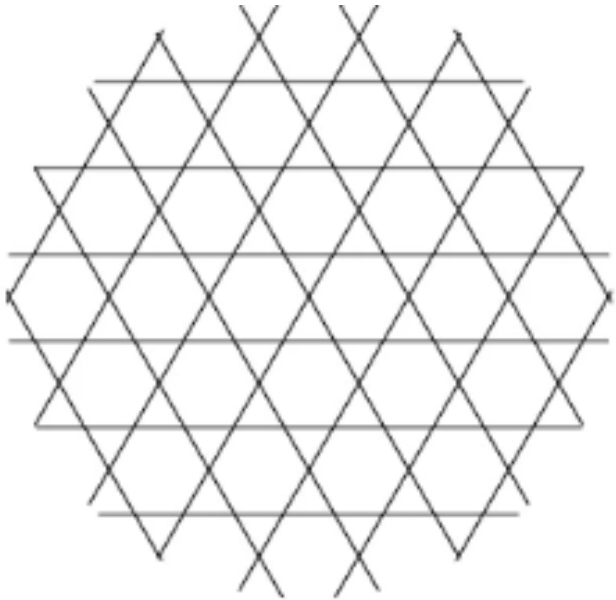
(with realistic parameters, geometry)

1. Not only one, but *many, many flat bands*.
2. Flatness is *stable*, i.e. protected by locality, time-reversal & crystal symmetry.

Stability = relevance to the real, experimental systems!

Of course, flat band is *not* new.

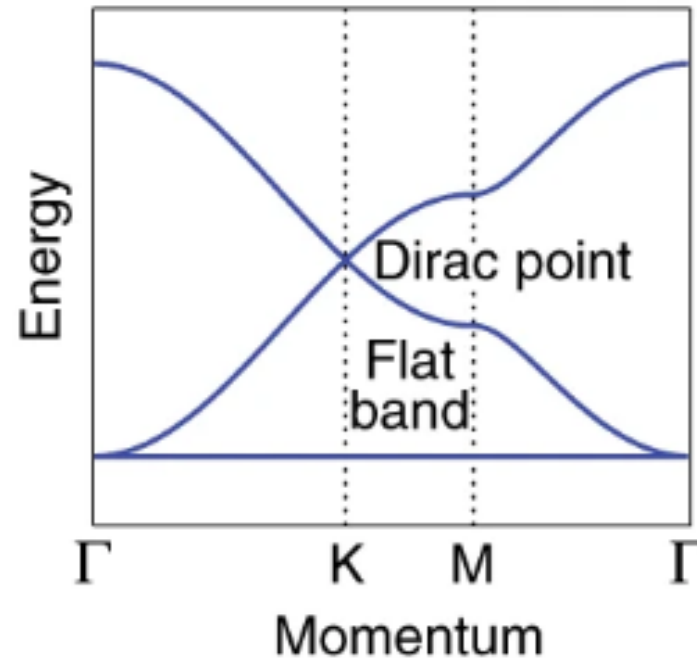
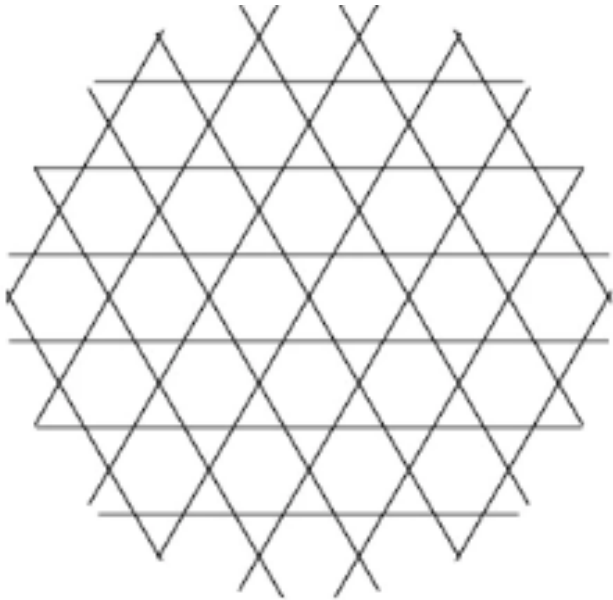
1. Kagome Lattice



- Single flat band
- NN-Hopping *Only*

Of course, flat band is *not* new.

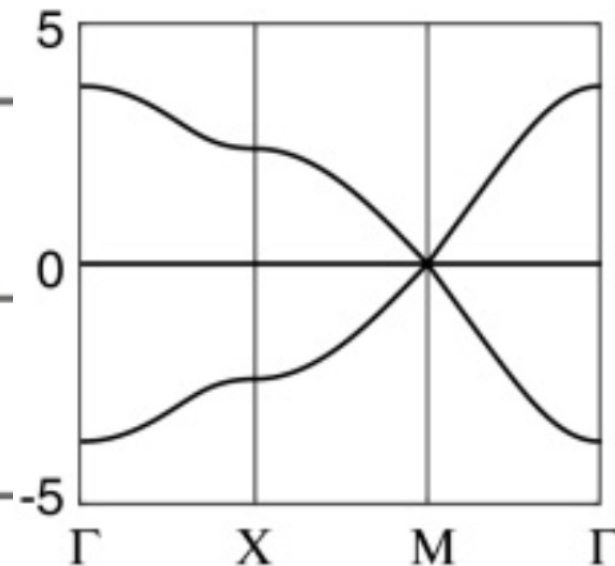
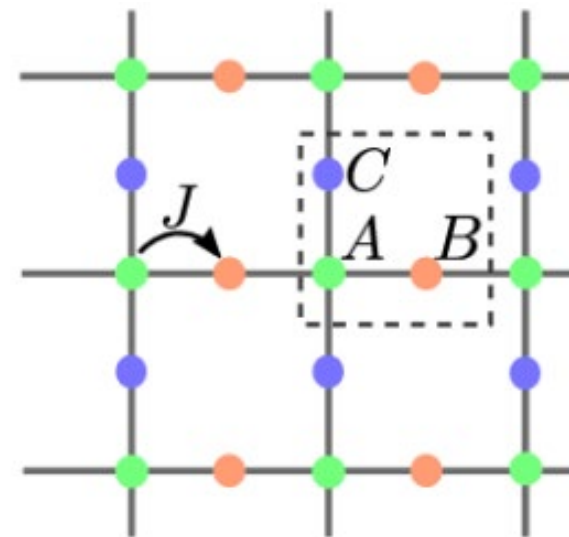
1. Kagome Lattice



- Single flat band
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- Single flat band
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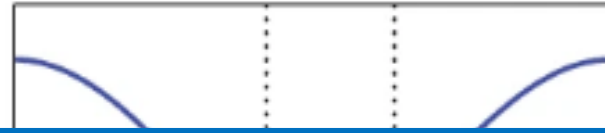
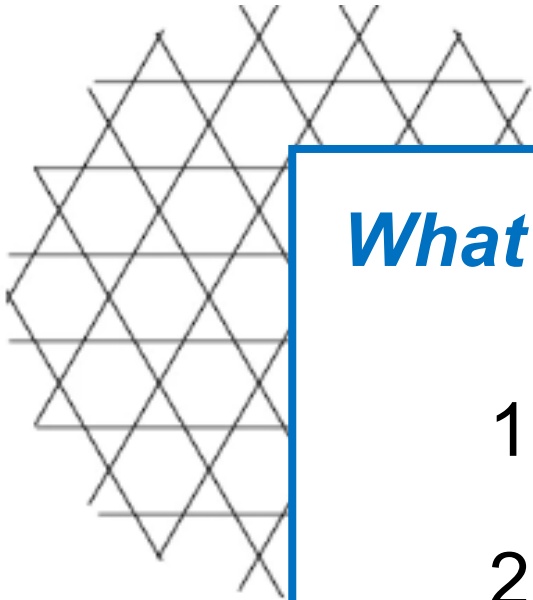
(sublattice symmetric hopping)



2. Lieb Lattice

Of course, flat band is *not* new.

1. Kagome Lattice



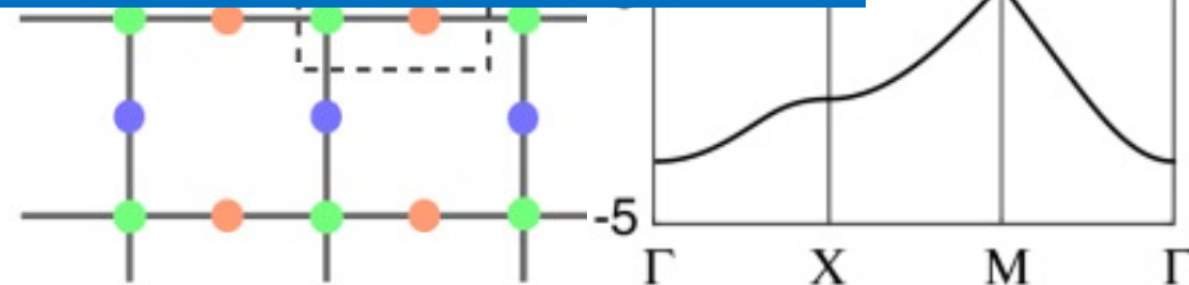
What's new in our model is:

1. Multiple, *not just one*, Flat bands
2. Unusual Stability & Protecting symmetries

Easier to observe **the physics of flat bands without much tuning**

- NN-Hopping

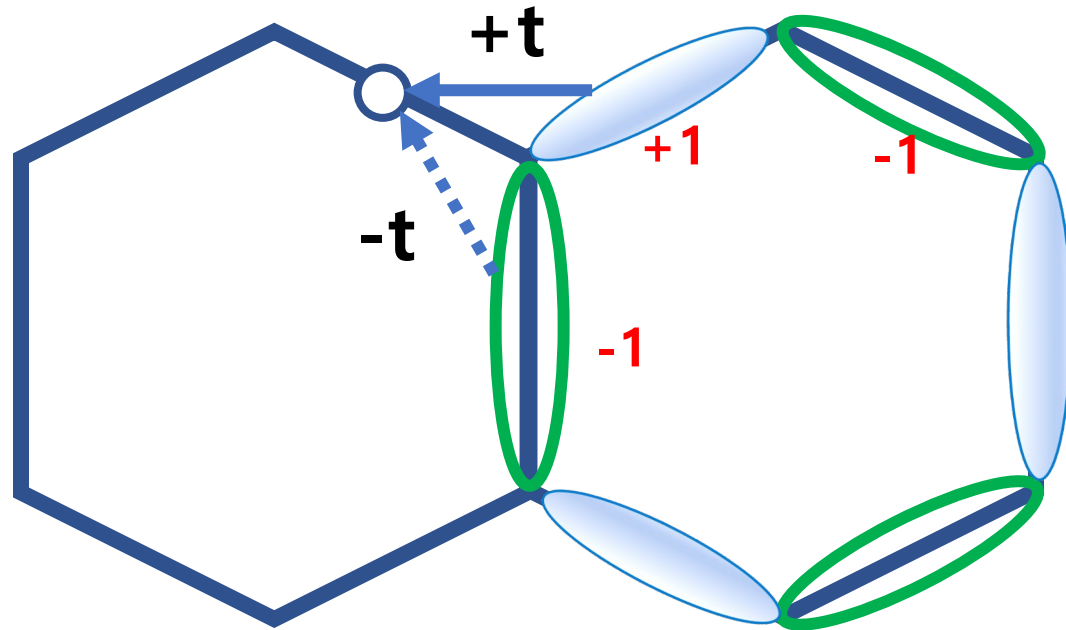
(sublattice symmetric hopping)



eb Lattice

Emergence of flat bands & their stability

Numerics: flat band states have $\Psi(\mathbf{x}) = 0$ at the nodes. [Cf. Bergman, Wu, Balents 2008]



Standing waves inside each wire.

Hence, flat bands are protected by:

- (1) **[Locality]** Hopping is shorter than wire length [$\sim 80\text{\AA}$ in 1T-TaS_2]
- (2) **[Symmetry]** $D_6 \times T$ symmetry
- (3) Multiple standing waves = Repeated, Multiple flat bands

Emergence of flat bands & their stability

Numerics: flat band states have $\Psi(\mathbf{x}) = 0$ at the nodes

Including **interactions**

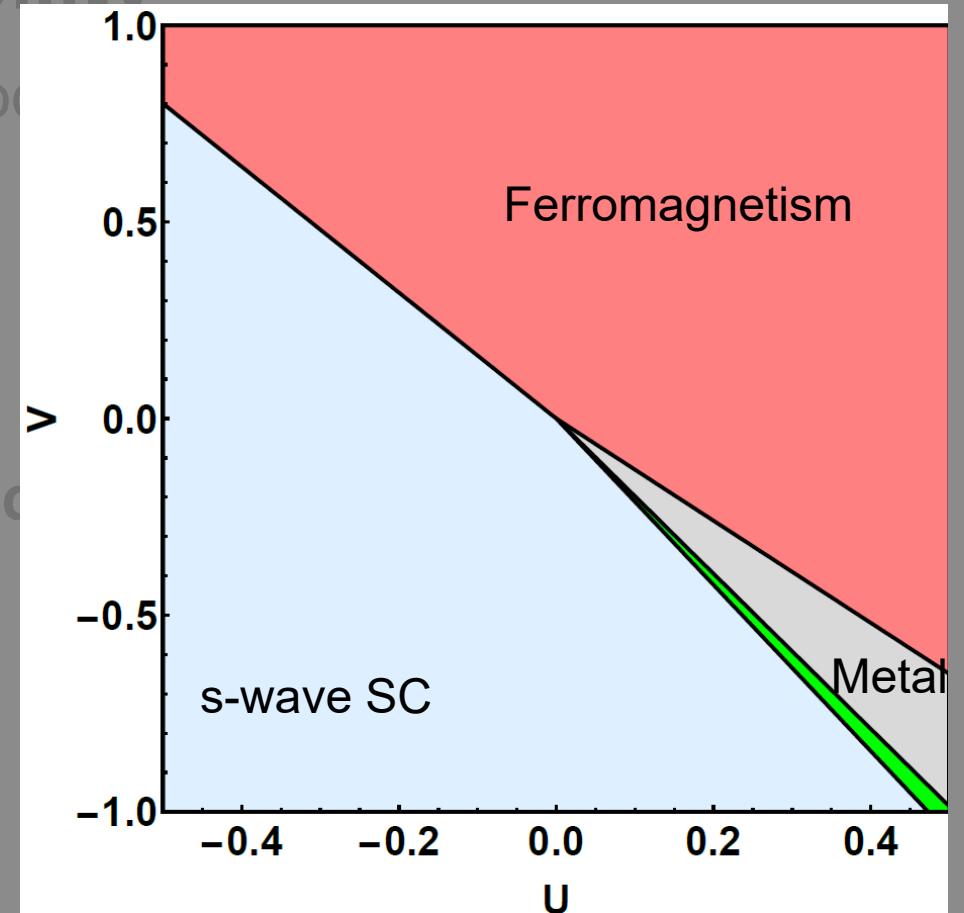
$$H_{int} = U \sum_{\mathbf{r}} n_{\mathbf{r}}^2 + V \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} n_{\mathbf{r}} n_{\mathbf{r}'}$$

(**cf.** phonon-electron interaction.)

Broad range of **s-wave SC** & **Ferromagnetism**

Hence, flat bands are protected by:

- (1) [Locality] Hopping is shorter than wire length [$\sim 80\text{\AA}$ in 1T-TaS_2]
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Emergence of flat bands & their stability

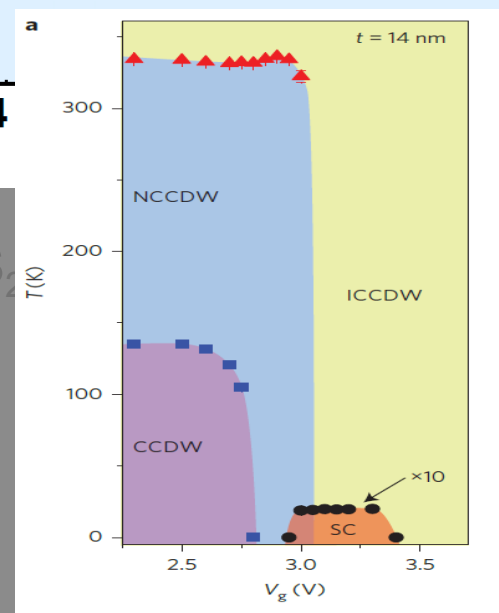
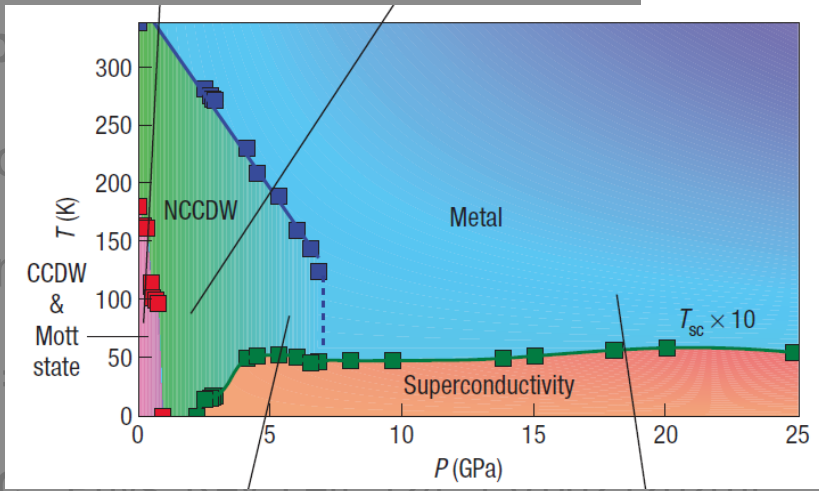
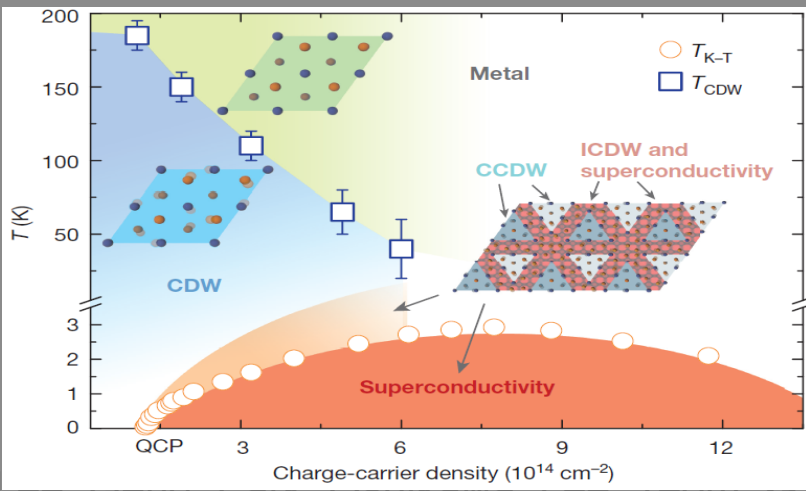
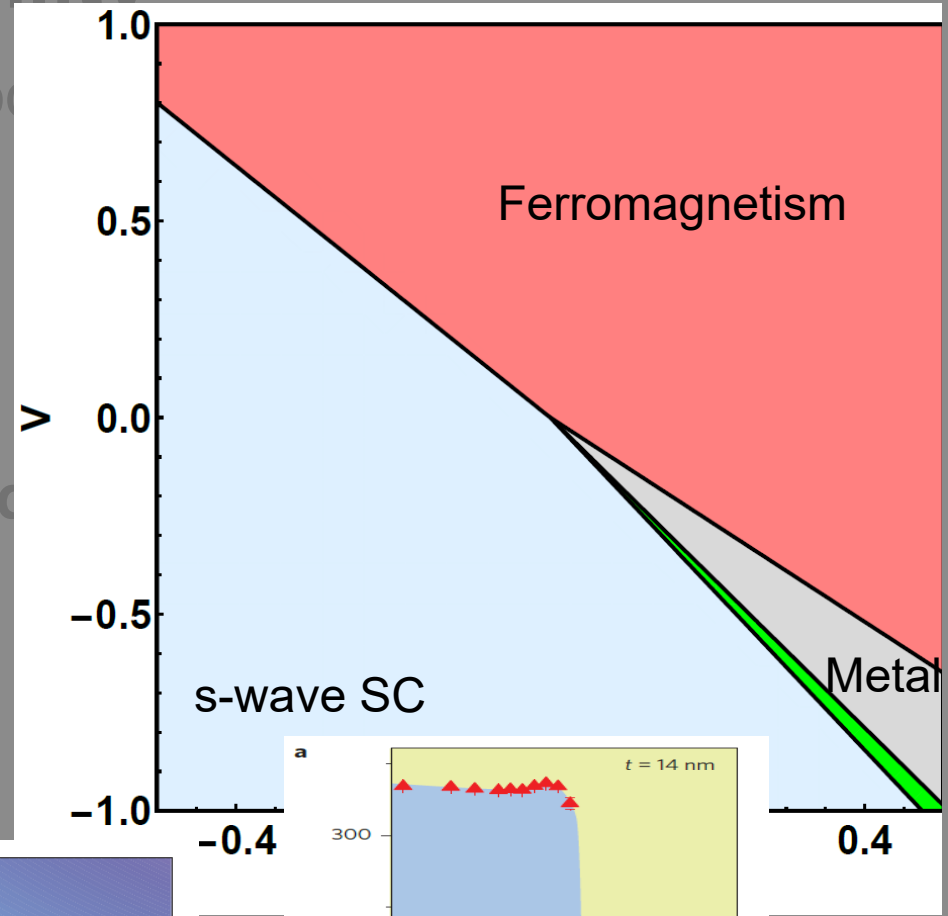
Numerics: flat band states have $\Psi(x) = 0$ at the node

Including **interactions**

$$H_{int} = U \sum_{\mathbf{r}} n_{\mathbf{r}}^2 + V \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} n_{\mathbf{r}} n_{\mathbf{r}'}$$

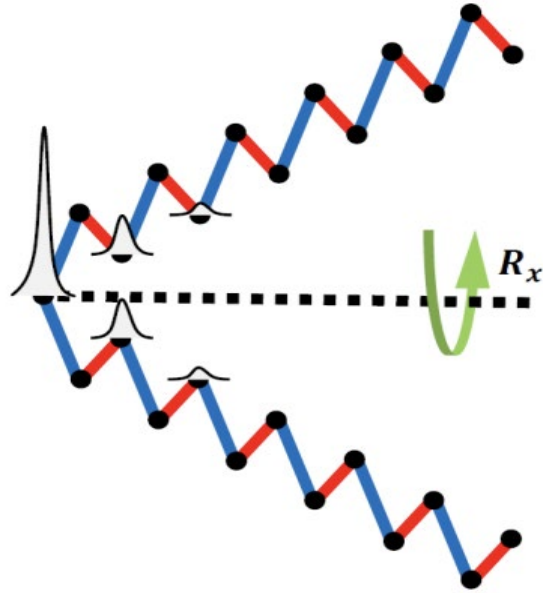
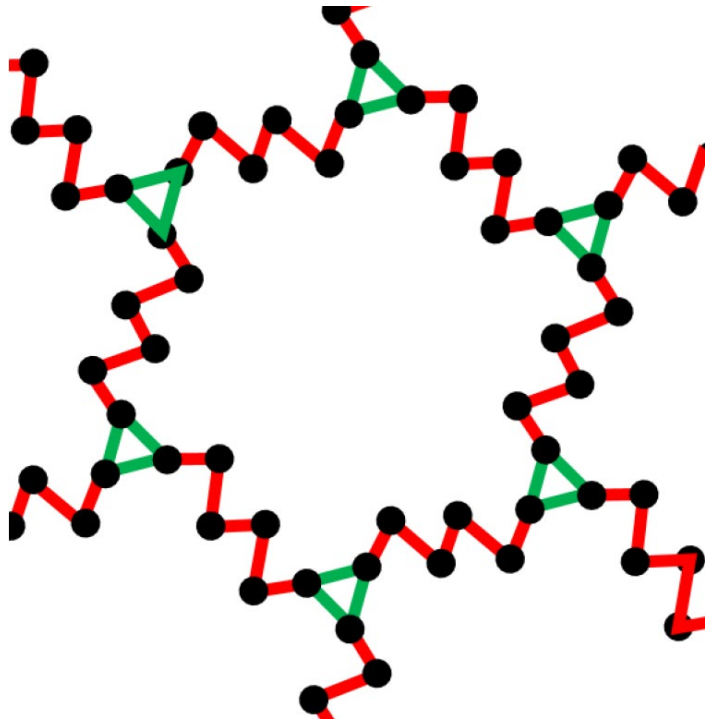
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Broad range of **s-wave SC** & **Ferromagnetism**

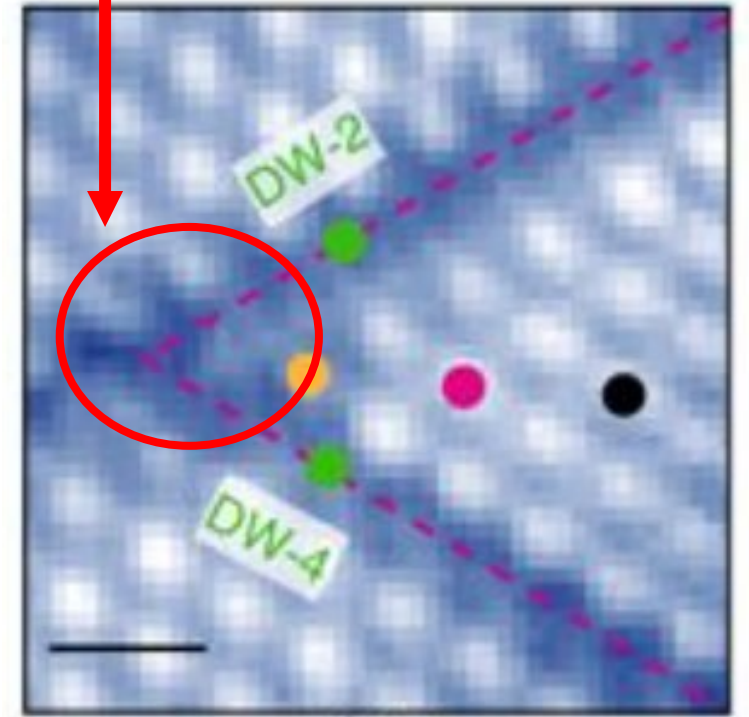


2D limit: “Higher-order” topology

[Lee, Geng, Park, Oshikawa, Lee, Yeom, **GYC***, Phys. Rev. Lett. 124, 137002 (2020)]



LDOS peak ?



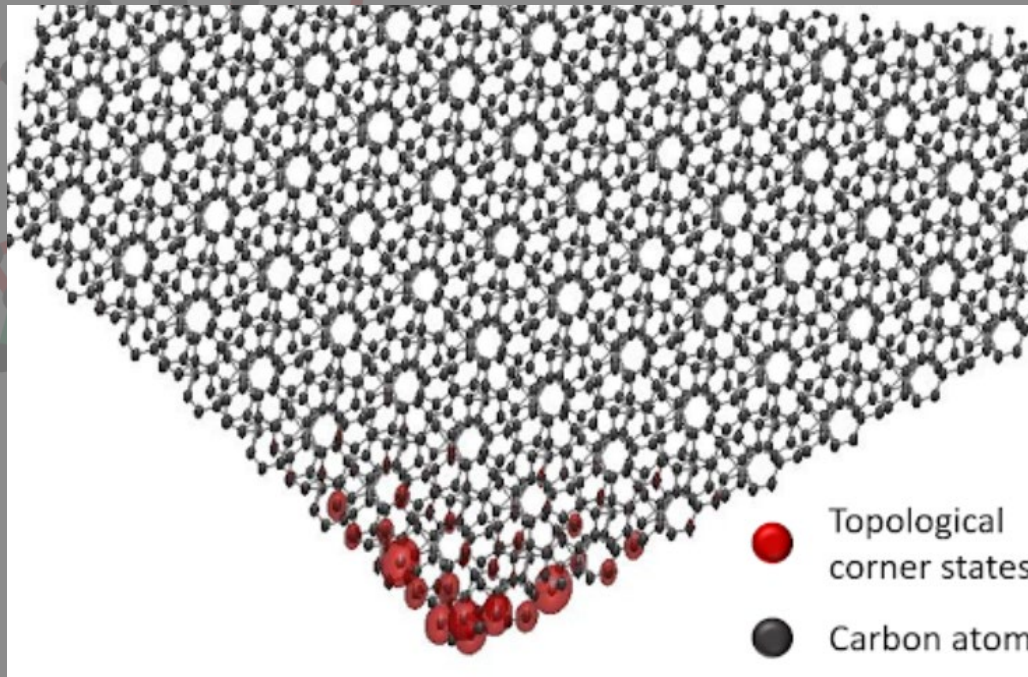
[Ref: Cho et al, Nat. Comm.(2017)]

2D limit: “Higher-order” topology

[Lee, Geng, Park, Oshikawa, Lee, Yeom, GYC*, Phys. Rev. Lett. 124, 137002 (2020)]

LDOS peak ?

This looks very similar to the corner state of **the higher-order TI**



PHYSICAL REVIEW LETTERS **123**, 216803 (2019)

Editors' Suggestion

Higher-Order Topological Insulator in Twisted Bilayer Graphene

Moon Jip Park,^{1,*} Youngkuk Kim^{2,†} Gil Young Cho,^{3,‡} and SungBin Lee^{1,§}

Hidden Higher-order Topology

[Ref: Yeom's group, Nat. Comm.(2017)]

...which one can actually make a more precise connection.

Domain wall networks provide a natural platform for “higher-order” topology.

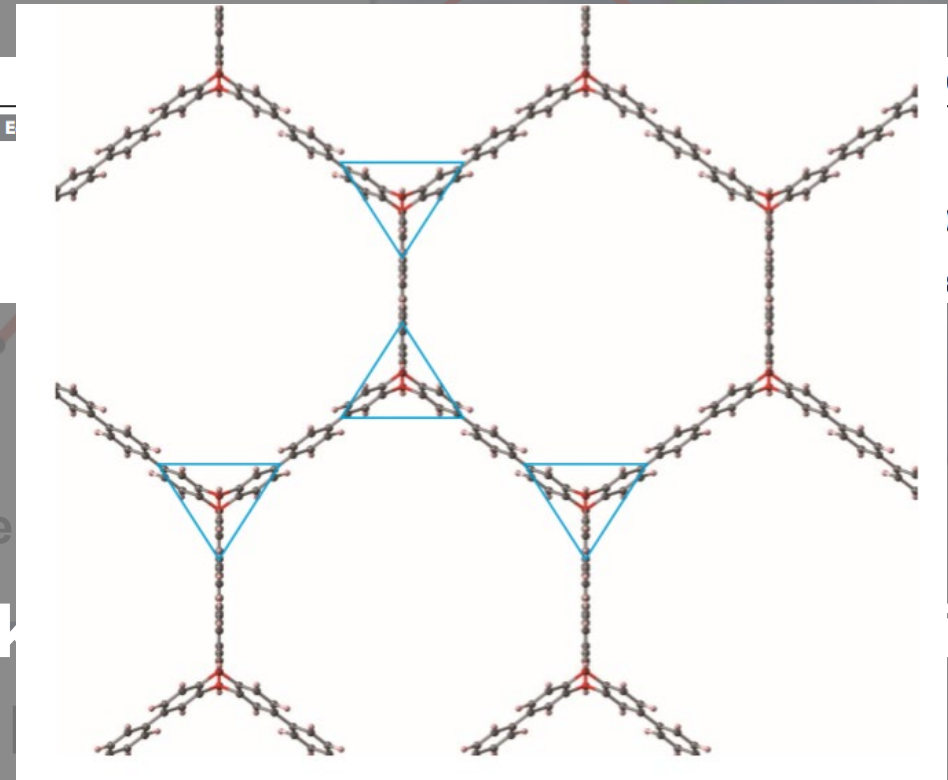
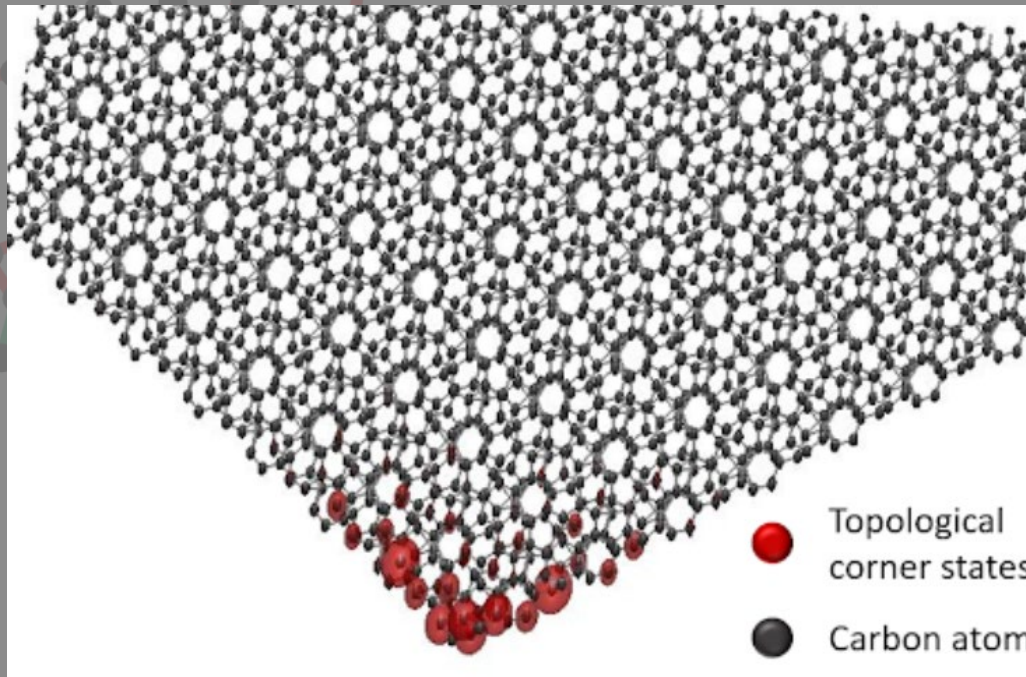
(when gapped)

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[Lee, Geng, Park, Oshikawa, Lee, Yeom, GYC*, Phys. Rev. Lett. 124, 137002 (2020)]

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...which one can actually make

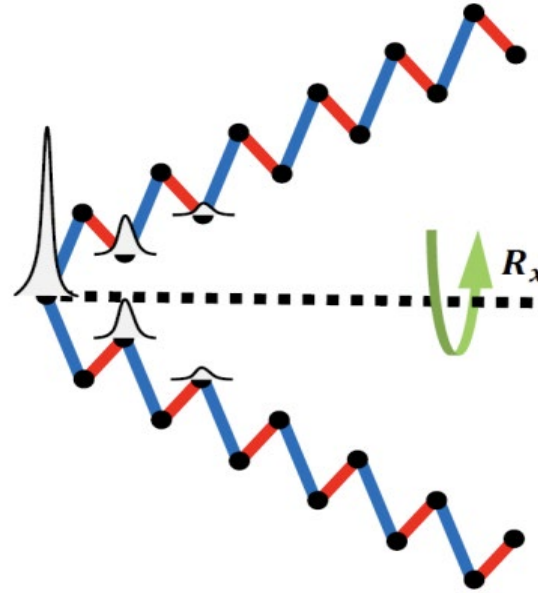
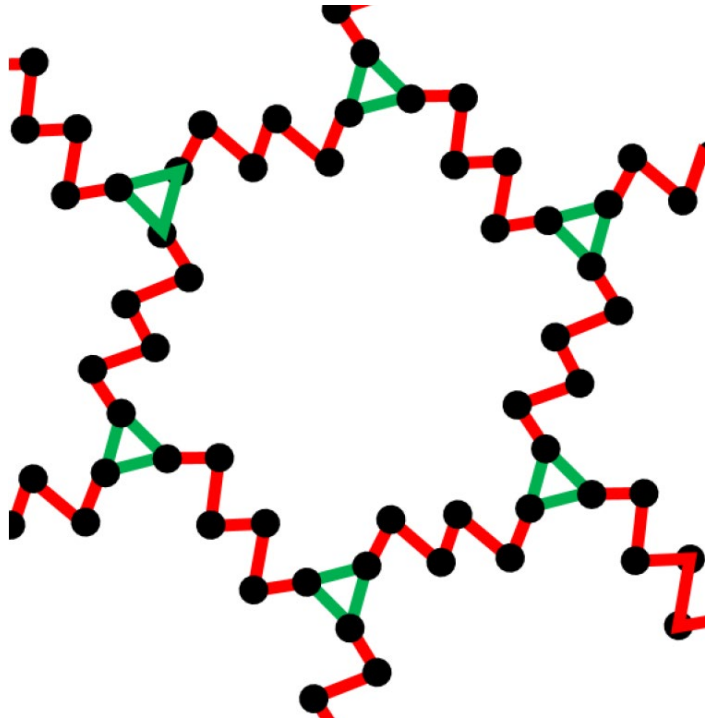
Domain wall networks provide a natural

(when gapped)

Triptycene (Hatsugai's group, Phys. Rev. Mat. 2019)

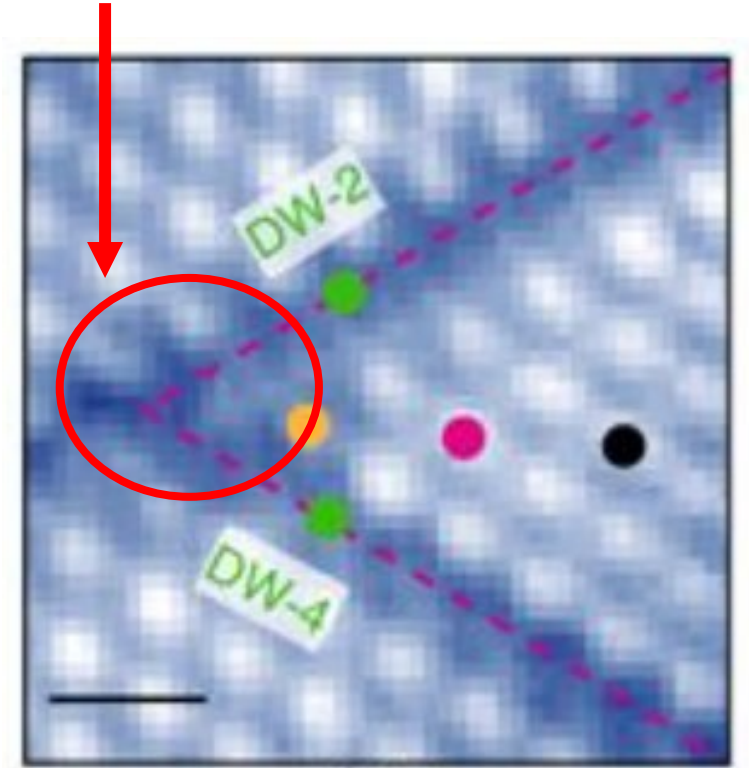
2D limit: “Higher-order” topology

[Lee, Geng, Park, Oshikawa, Lee, Yeom, **GYC***, Phys. Rev. Lett. 124, 137002 (2020)]



Hidden Higher-order Topology

LDOS peak ?



[Ref: Yeom's group, Nat. Comm.(2017)]

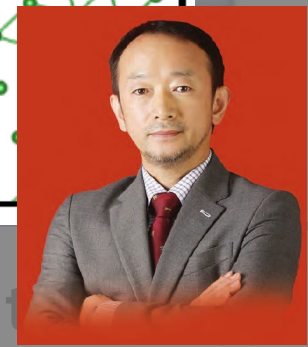
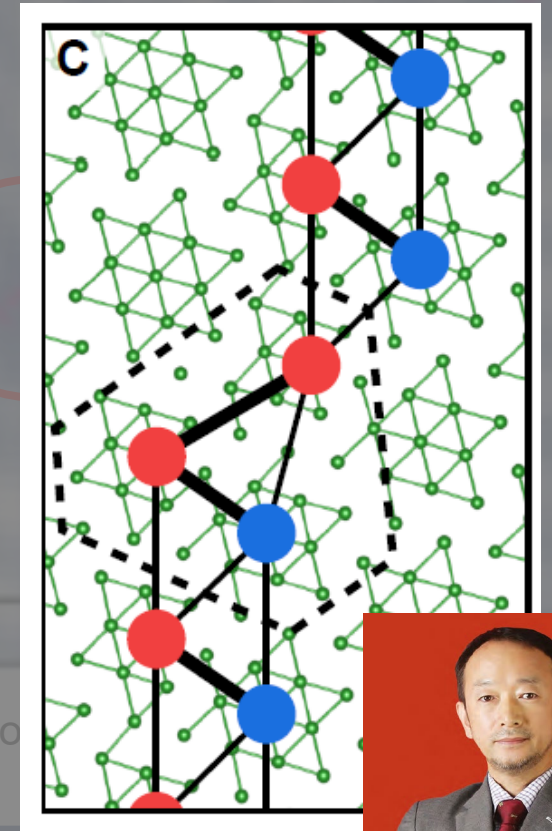
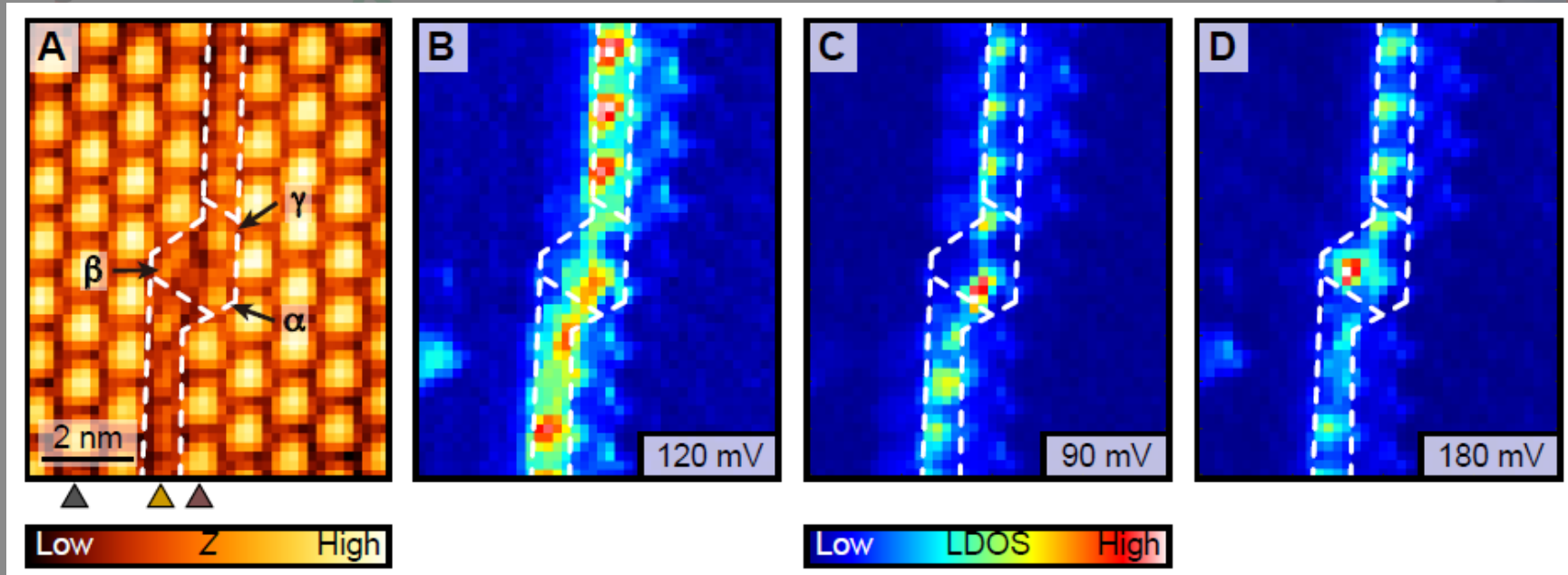
Domain wall networks provide a natural platform for “higher-order” topology.

(when gapped)

2D limit: Higher-order topology

[Lee, Geng, Park, Oshikawa, Lee, Yeom, **GYC***, Phys. Rev. Lett. 124, 137002 (2020)]

LDOS peak ?

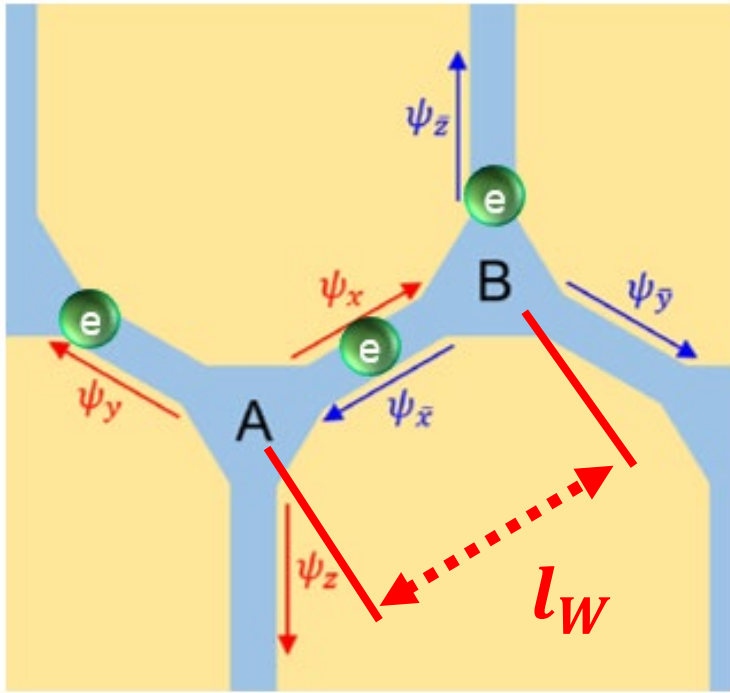


HW Yeom
(POSTECH)

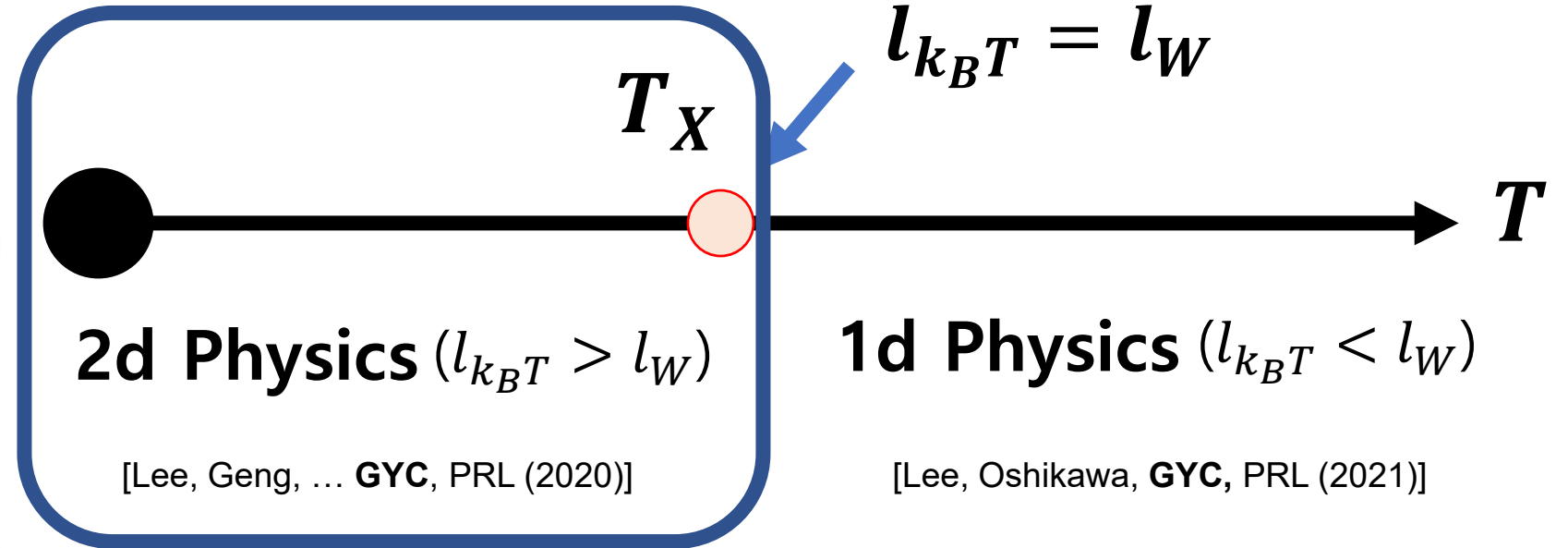
Lee, Park, **GYC**, Yeom, submitted to *PRL*

Domain wall networks provide a natural platform for higher-order topology (when gapped)

Key observation: Dimensional Crossover T_X



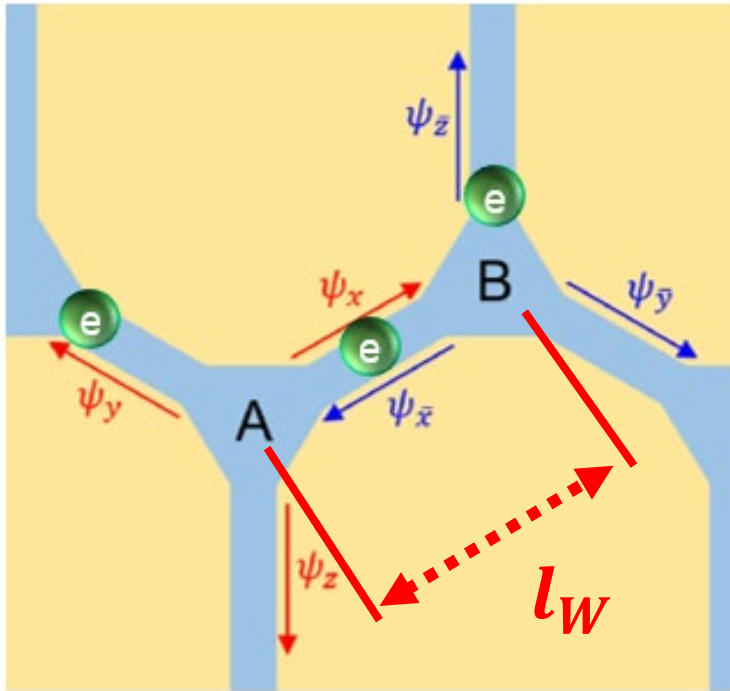
Relatively easier problem.



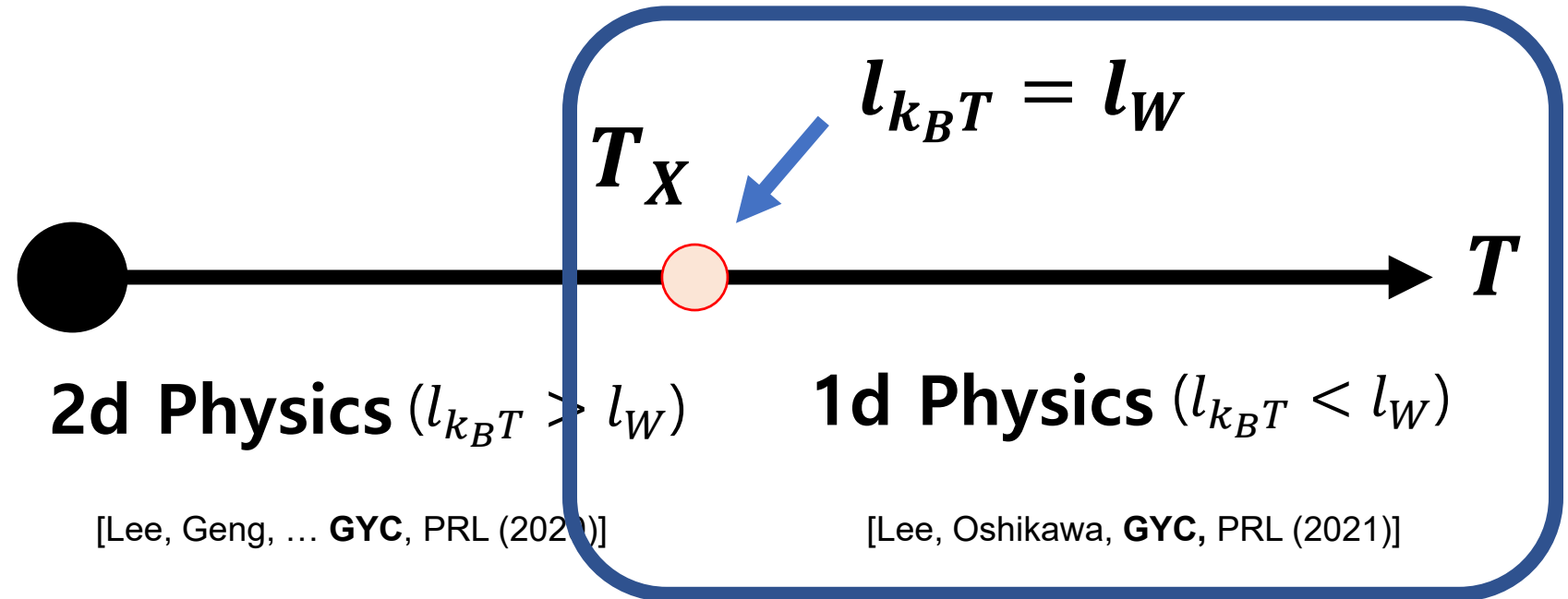
- Cascades of stable flat bands
- Superconducting instabilities & experimental implications
- Higher-order topology & STM experiments

1D limit: $T > T_X$

Key observation: Dimensional Crossover T_X



What do we expect?



[Lee, Geng, ... **GYC**, PRL (2020)]

[Lee, Oshikawa, **GYC**, PRL (2021)]

- **TiSe₂: O(10)K** (assuming e.g. that the electrons are flowing along 1D domain walls)
- **Twisted bilayer graphene: O(25)K** (for 300 nm)
- **TaS₂: O(100)K**

[Lee, Oshikawa, **GYC***, Phys. Rev. Lett. 126, 186601 (2021)]

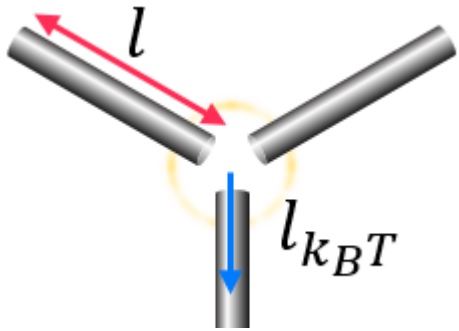
Quasi-1D: Plethora of Non-Fermi Liquids above T_x

[Lee, Oshikawa, **GYC***, Phys. Rev. Lett. 126, 186601 (2021)]

Dimensionally

1D & 0D systems

For $T > T_x$:



Luttinger Liquids

& Junctions

Quasi-1D: Plethora of Non-Fermi Liquids above T_x

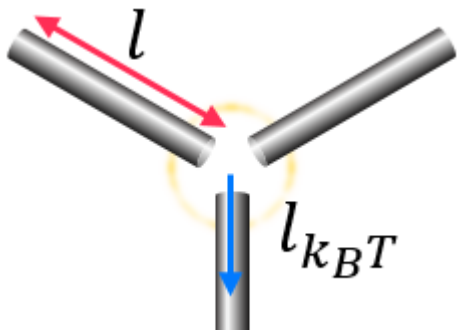
[Lee, Oshikawa, **GYC***, Phys. Rev. Lett. 126, 186601 (2021)]

Dimensionally
1D & 0D systems

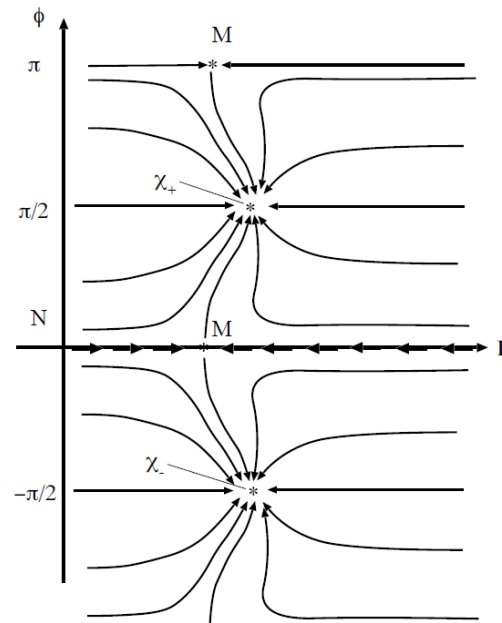
“Fixed Points”
of an Y-junction

2D NFLs

For $T > T_x$:



Luttinger Liquids
& Junctions



[Oshikawa, Chamon, Affleck (2005)]

(1) Electric conductivity

$$g(T) \sim g_0 + c T^{\alpha(K)}$$

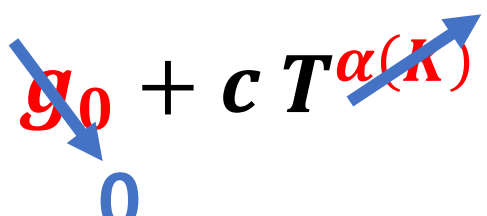
...where $\{g_0, \alpha(K)\}$ are *universal*.

- Classification is possible.
(e.g. triangular ~ 220 types)

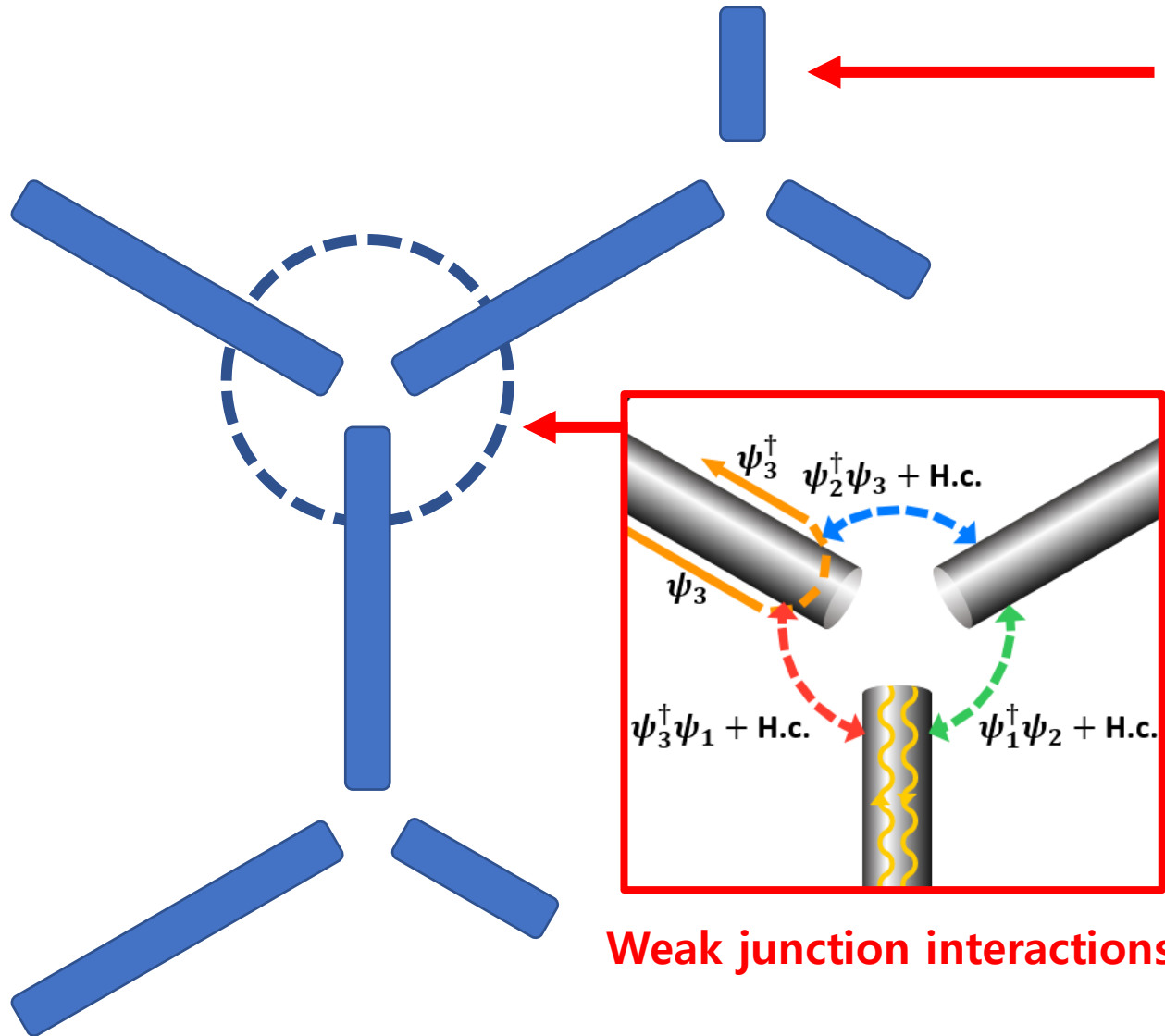
(2) Exotic states

- Emergent nematic phase
- Spin-charge separation

Toy Model: strange insulator phase

$$g(T) \approx g_0 + c T^{\alpha(K)} 2K - 2$$
A diagram illustrating the equation $g(T) \approx g_0 + c T^{\alpha(K)} 2K - 2$. A blue arrow points from the g_0 term down to a blue 0 below it. Another blue arrow points from the $\alpha(K)$ term up and to the right towards the $2K - 2$ term.

Toy model: repulsive, spinless electrons



Luttinger liquid: $H = \frac{v}{2} \int dx \left[\frac{1}{K} (\partial_x \theta)^2 + K (\partial_x \phi)^2 \right]$

(Luttinger parameter $K > 1$, repulsive)

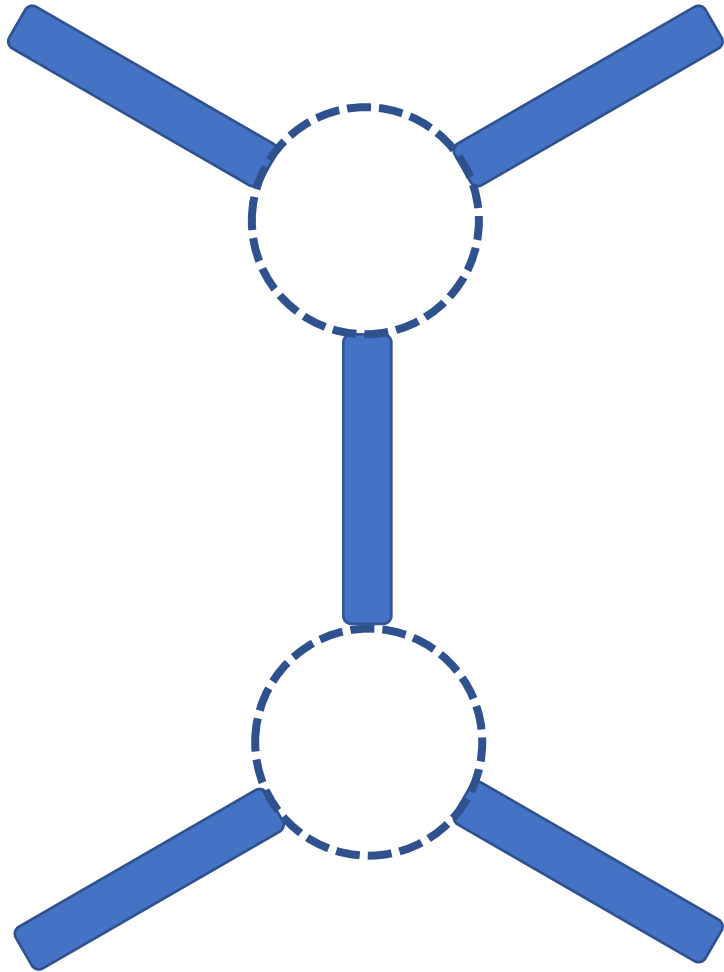
Q. What would happen

if the temperature gets lowered?

A. System evolves along the RG flow.

Weak junction interactions

Toy model: repulsive, spinless electrons



Decoupled fixed point emerges.

(Cf. it is also known as Neumann BC)

What do we expect out of this fixed point?

(1) Electrically insulating for $T > T_X$.

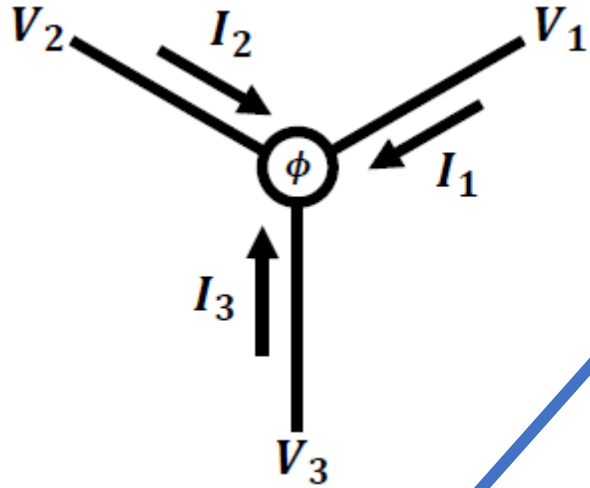
- Even the electron hopping is irrelevant (!)

(2) Thermodynamically metal

- Luttinger liquid excitations are *intact*.

Electric conductivity in temperature T ?

Relating “microscopic” junction to “macroscopic” network



$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} G_S & (G_A - G_S)/2 & -(G_A + G_S)/2 \\ -(G_A + G_S)/2 & G_S & (G_A - G_S)/2 \\ (G_A - G_S)/2 & -(G_A + G_S)/2 & G_S \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$$I_a = \sum_b G_{ab} V_b$$

Conductance of the junction

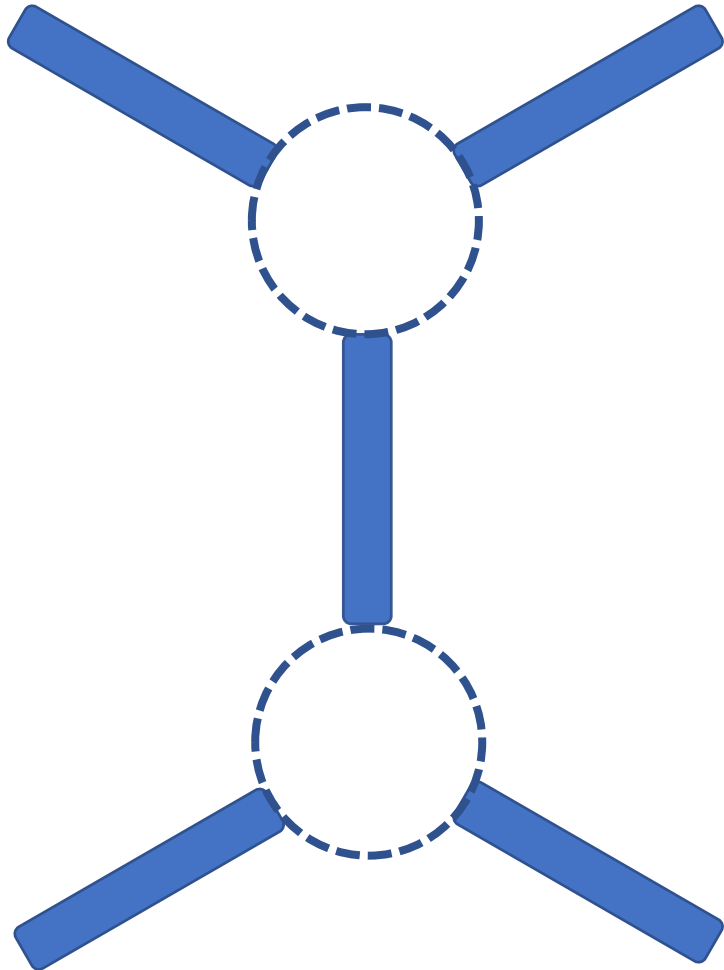
Key observations:

- (1) G_S and G_A are **determined by the fixed point of the 0D junction**
- (2) G_S and G_A determine **the 2D conductivity:**

$$g_{xx}(T) = \sqrt{3} \frac{G_S(T)}{4} \quad \& \quad g_{xy}(T) = \frac{G_A(T)}{4}$$

(g_{ab} = “macroscopic” data, $G_{S/A}$ = “microscopic” data)

Toy model: repulsive, spinless electrons



Decoupled fixed point emerges.

(Cf. it is also known as Neumann BC)

Perturbative expansions to the leading order are...

$$\mathbf{g}_{xx}(\mathbf{T}) \approx \mathbf{0} + \frac{\sqrt{3}\pi e^2 t^2}{2h} \tau_c^{2K} \frac{\pi^{2K-1} \Gamma\left(\frac{1}{2}\right) \Gamma(K)}{\Gamma\left(\frac{1}{2} - K\right)} \mathbf{T}^{2K-2} \quad (\text{for } K > 1)$$

$$\mathbf{g}_{xy}(\mathbf{T}) \approx \mathbf{0} + \mathcal{O}(\mathbf{T}^{2K-2})$$

Determined by the *leading irrelevant operator* at the fixed point

(universal data of the fixed point)

For this “decoupled” fixed point, $\Delta = K \rightarrow \delta g \sim T^{2\Delta-2}$

Toy model: repulsive, spinless electrons

Locally critical umklapp scattering and holography

(AdS/CFT duality)

Sean A. Hartnoll^b and Diego M. Hofman[‡]

Abstract

Efficient momentum relaxation through umklapp scattering, leading to a power law in temperature d.c. resistivity, requires a significant low energy spectral weight at finite momentum. One way to achieve this is via a Fermi surface structure, leading to the well-known relaxation rate $\Gamma \sim T^2$. We observe that local criticality, in which energies scale but momenta do not, provides a distinct route to efficient umklapp scattering. We show that umklapp scattering by an ionic lattice in a locally critical theory leads to $\Gamma \sim T^{2\Delta_{k_L}}$. Here $\Delta_{k_L} \geq 0$ is the dimension of the (irrelevant or marginal) charge density operator $J^t(\omega, k_L)$ in the locally critical theory, at the lattice momentum k_L . We illustrate this result with an explicit computation in locally critical theories described holographically via Einstein-Maxwell theory in Anti-de Sitter spacetime. We furthermore show that scattering by random impurities in these locally critical theories gives a universal $\Gamma \sim (\log \frac{1}{T})^{-1}$.

$$R(T) \sim \frac{1}{T^{2\Delta}}$$

**We have achieved a microscopic model
of strange insulator behaviors!**

Experiment in graphene ?

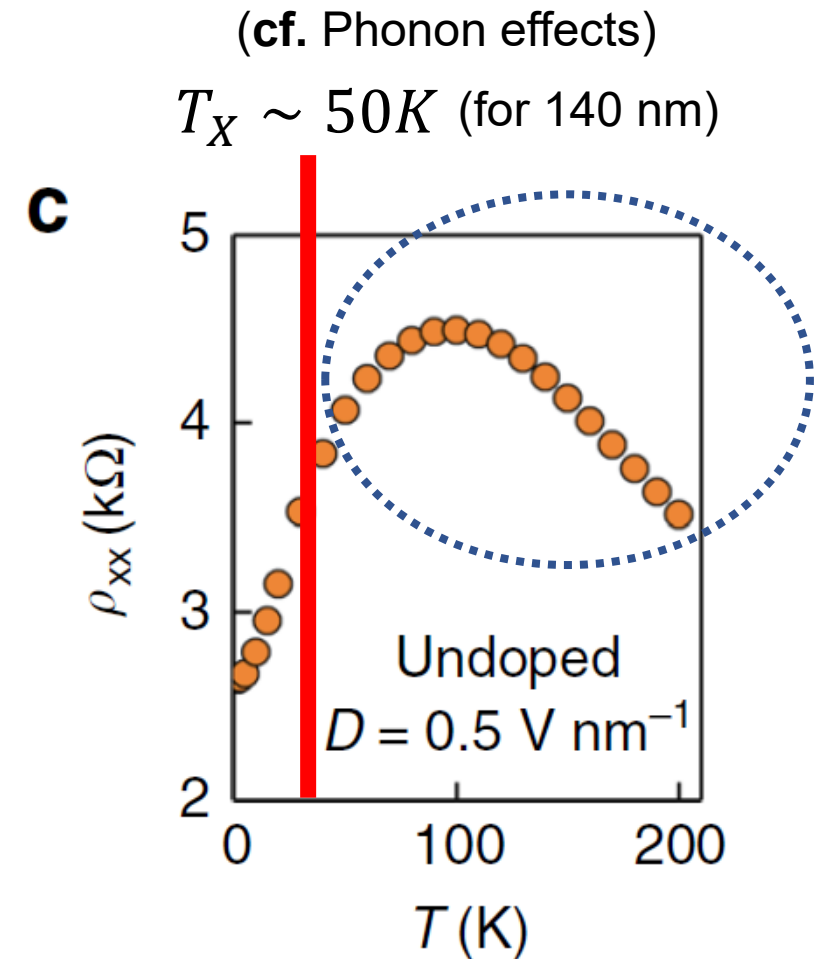
ARTICLE

<https://doi.org/10.1038/s41467-019-11971-7>

OPEN

Giant oscillations in a triangular network of one-dimensional states in marginally twisted graphene

S.G. Xu^{1,2,5}, A.I. Berdyugin^{1,5}, P. Kumaravadivel^{1,2}, F. Guinea¹, R. Krishna Kumar^{1,2}, D.A. Bandurin¹, S.V. Morozov³, W. Kuang¹, B. Tsim^{1,2}, S. Liu⁴, J.H. Edgar⁴, I.V. Grigorieva¹, V.I. Fal'ko^{1,2}, M. Kim¹ & A.K. Geim^{1,2}



Hopefully to see this in the future experiments

Remark:

In all cases, we find:

$$g(T) \approx \mathbf{g_0} + c T^{\alpha(K)}$$

Universal conductance of the junction

(determined by the junction BCs)

Power-law correction in temperature T
(fixed by the leading irrelevant operator)

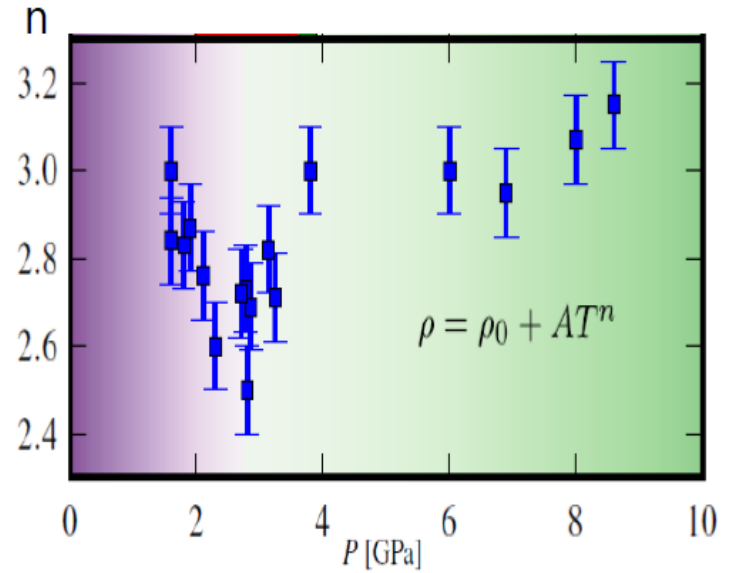
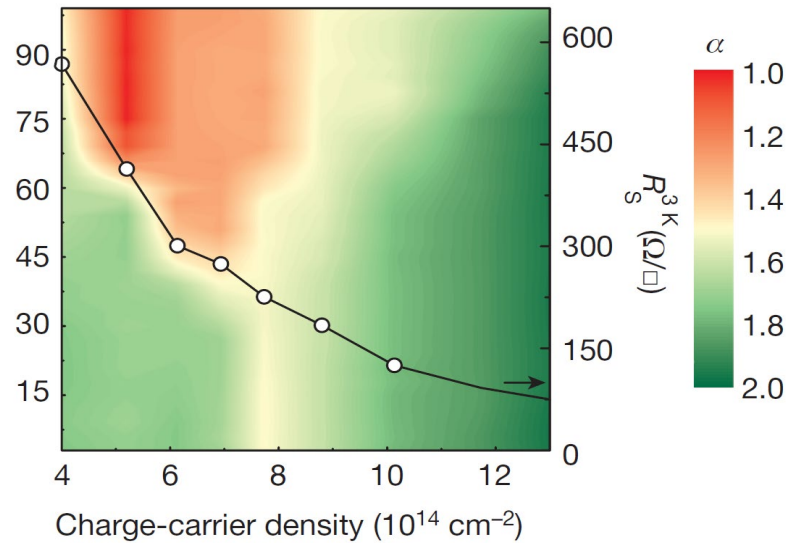
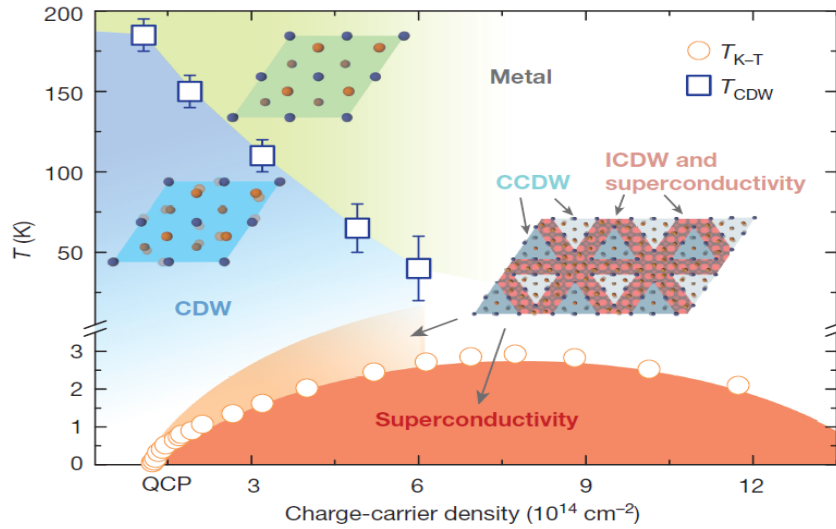
“K” is the *exactly marginal parameter*.

This will *continuously evolve* when experimental parameters, e.g., gating or pressure, are changed.

This is markedly different from a regular 2D Fermi liquid!

[Lee, Oshikawa, **GYC***, Phys. Rev. Lett. 126, 186601 (2021)]

Strongly Reminiscent of Experiments...



[Ref. Li et.al., Nat. (2016)]

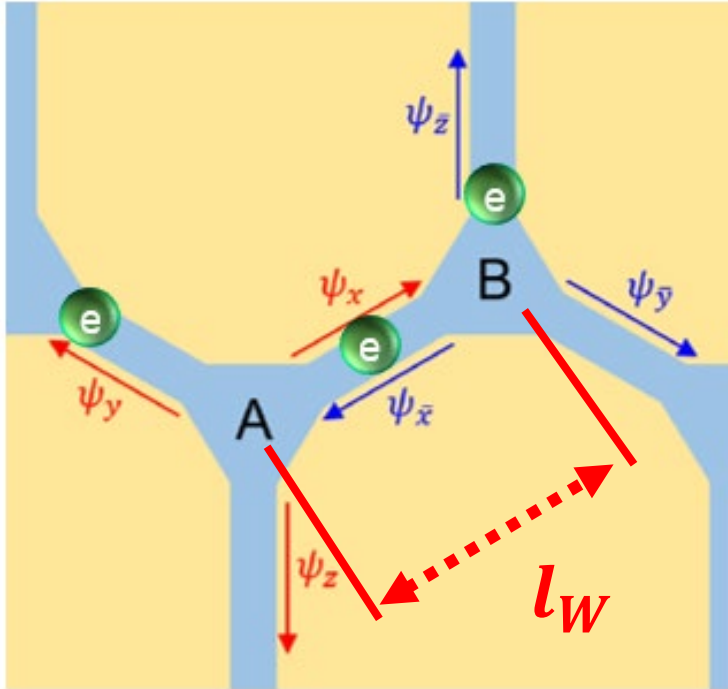
[Ref. Kusmartseva et. al. PRL (2009)]

1. Non-Fermi Transport $R \sim R_0 + AT^n$, varying exponent $n \approx 1 \sim 3$
2. Emerging superconductivity, and domain wall networks (Little-Park effect)

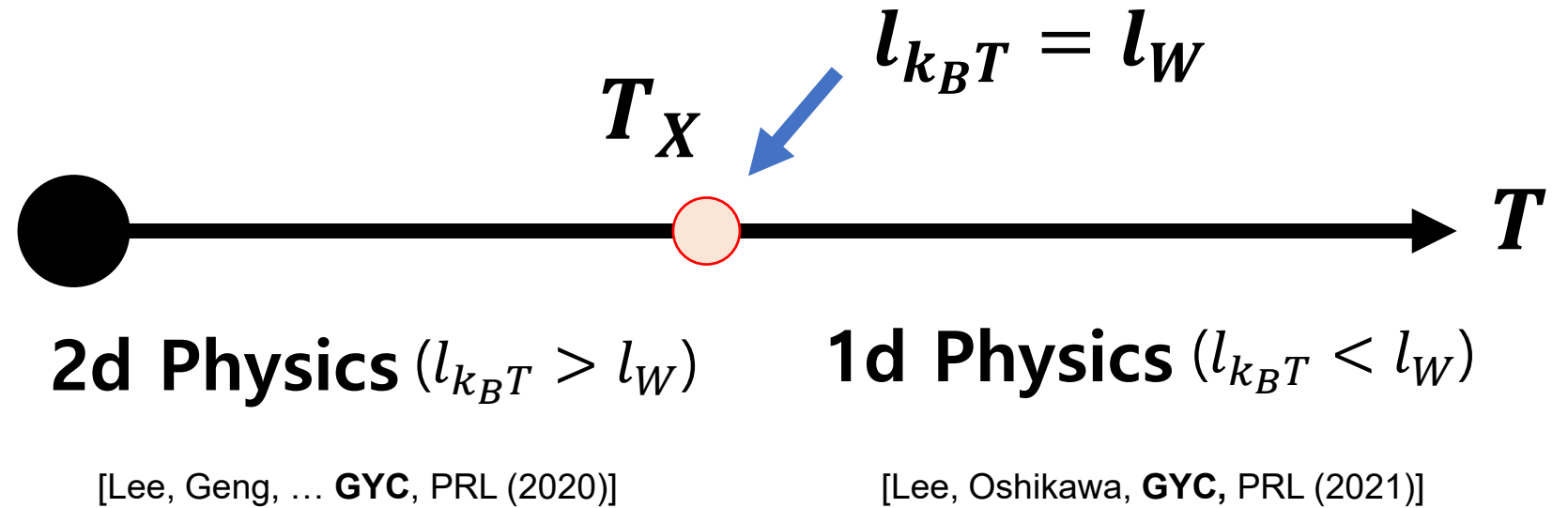
Correlation effect seems important in these materials!

Conclusions

Key observation: Dimensional Crossover T_X



Relatively easier problem.



- Cascades of stable flat bands
- Superconducting instabilities & experimental implications
- Higher-order topology & STM experiments
- Non-Fermi liquids (high- T regime)

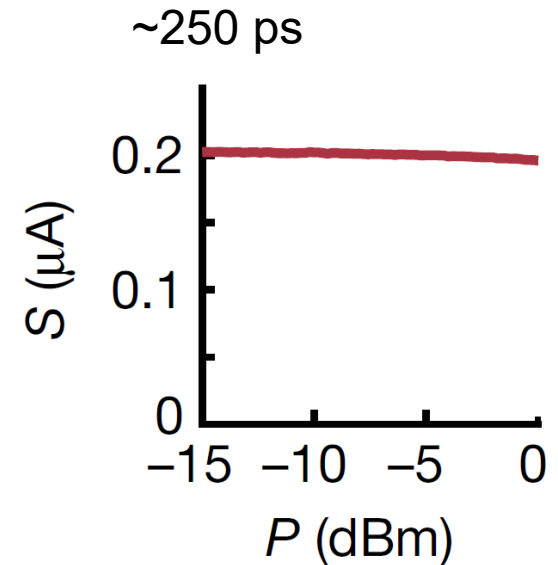
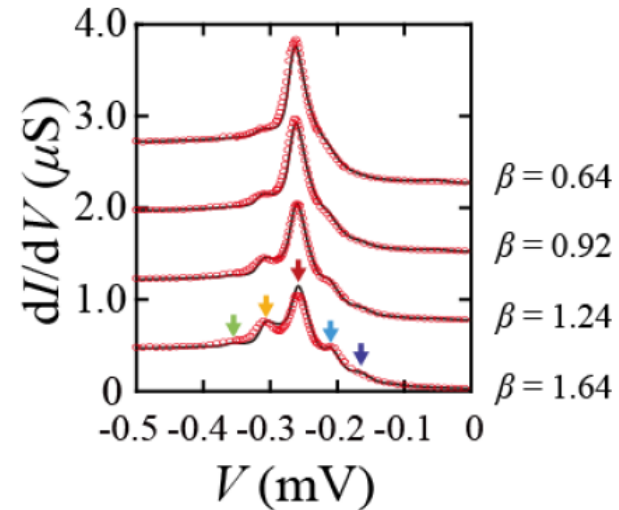
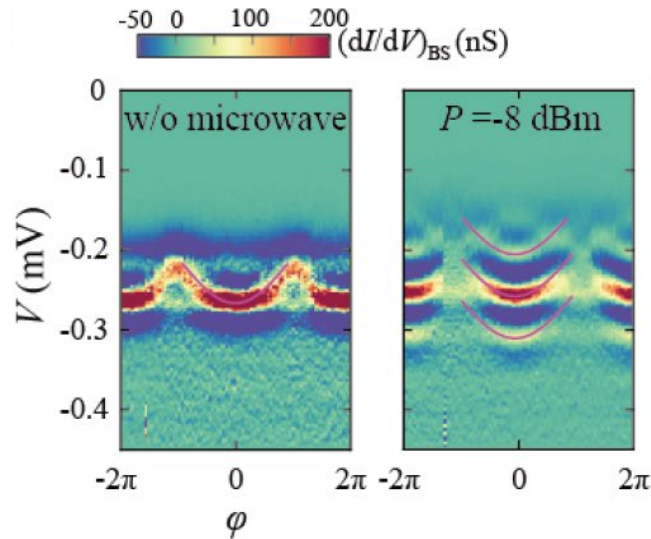
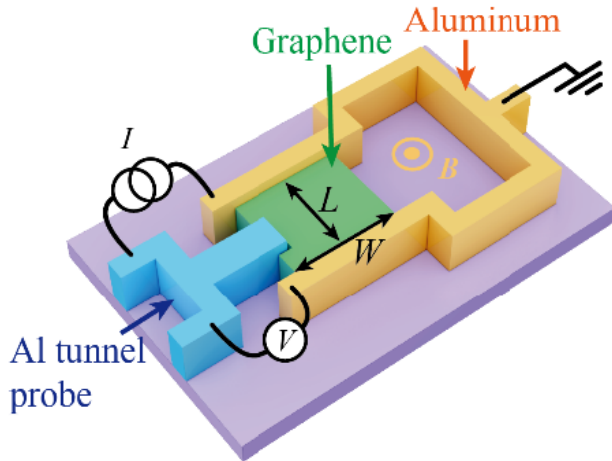
If there's *1 min...*

Steady non-equilibrium quantum states:

Article

Steady Floquet–Andreev states in graphene Josephson junctions

~ $O(10^{17})$ longer life time than previous pulse-generated Floquet states
> 25 hr



Upshot: graphene + continuous irradiation of microwaves = steady Floquet states

$$\beta = ev_F |\vec{E}| / \hbar\omega$$

Diagnostics of steady states: spectral sum rules

[Park, Lee, (...), GYC, Lee, Nature (2022)]

**THANK
YOU!**

