

Coulomb charge order, charge glass and bad metallic behavior

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Theory

S. Ciuchi (L'Aquila)

A. Ralko (Néel)

K. Driscoll (Néel)

G. D'Avino (Néel)

J. Merino (UAM)

L. Rademaker (Geneva)

V. Dobrosavljevic (FSU)

Experiments

M. Dressel (Stuttgart)

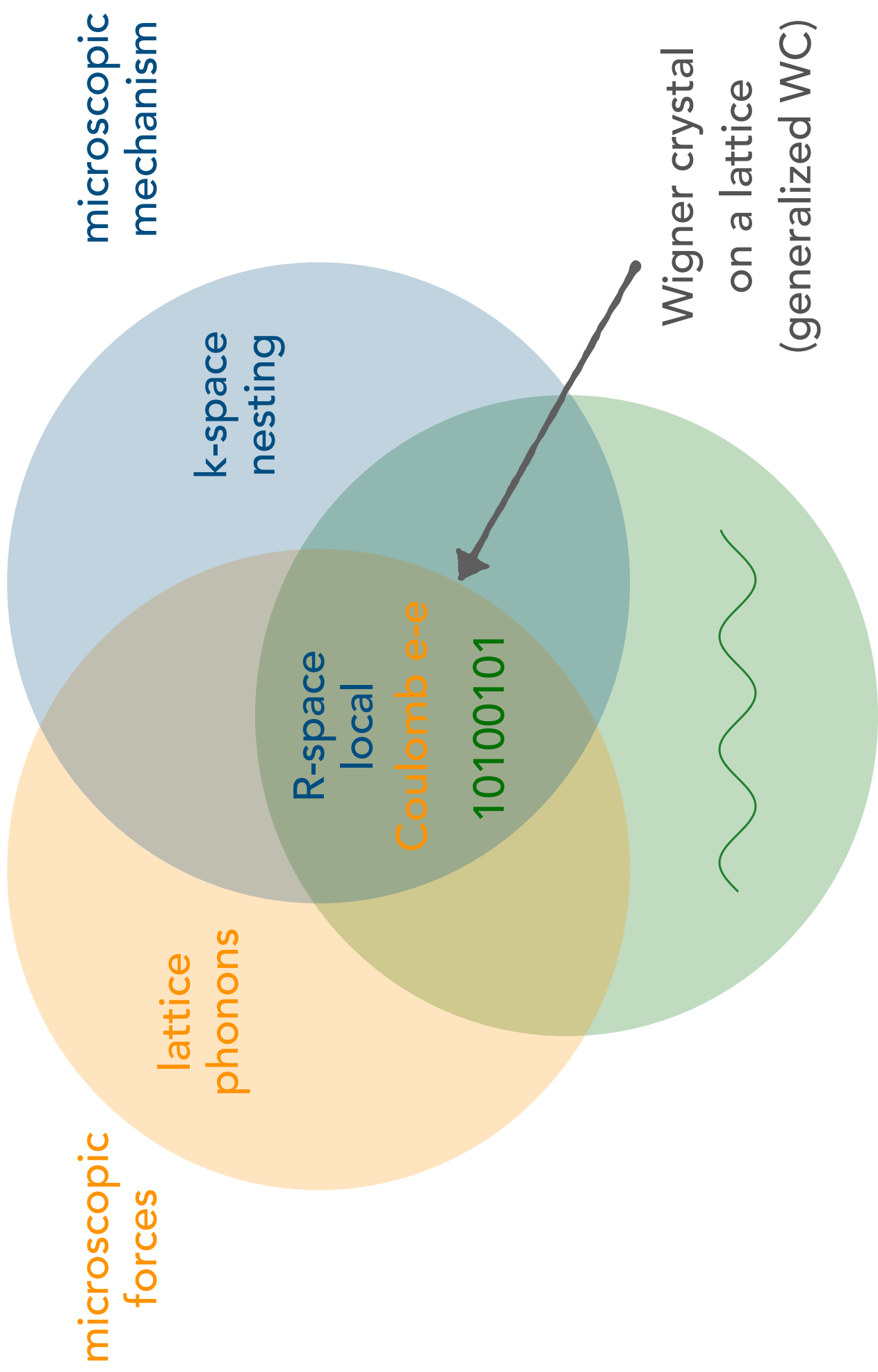
A. Pustogow (Vienna)

H. Siringhaus (Cambridge)

Outline

many electrons with long-range interactions
slow collective charge excitations
(self-generated disorder)

Semantics: CDW vs charge order



Wigner crystallization

DECEMBER 1, 1934

PHYSICAL REVIEW

VOLUME 46

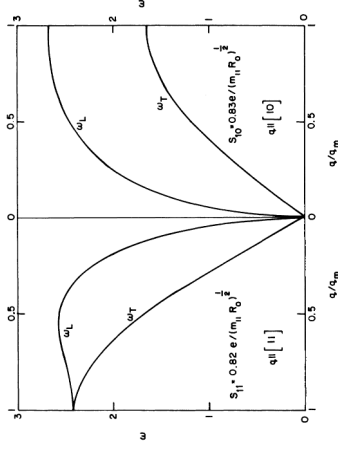
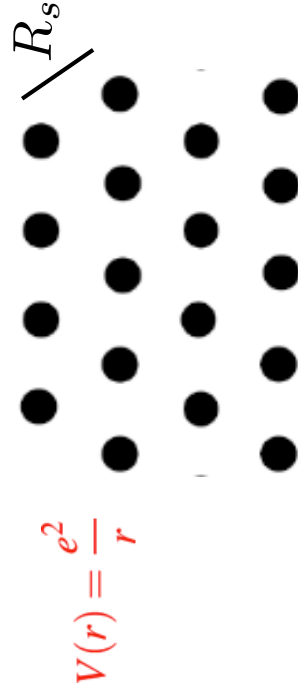


Eugene P. Wigner

On the Interaction of Electrons in Metals

E. WIGNER, *Princeton University*
(Received October 15, 1934)

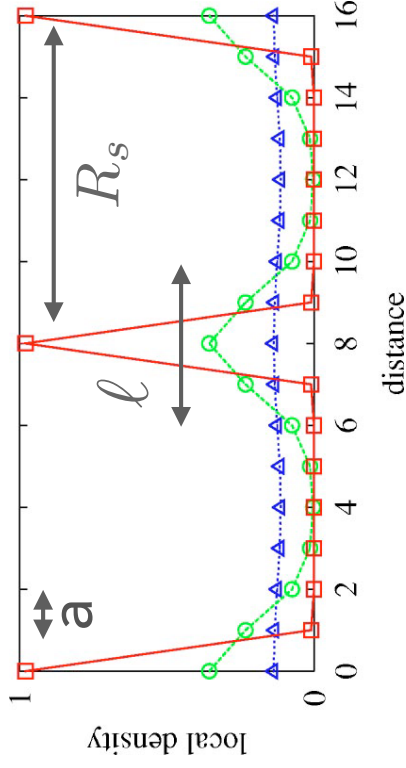
If the electrons had no kinetic energy, they would settle in configurations which correspond to the absolute minima of the potential energy. These are closed-packed lattice configurations, with energies very near to that of the body-centered lattice. Here, every electron is very



- the Wigner crystal has very low energy (shear) collective modes, they are at the origin of its fragility
- WC is notoriously difficult to observe, maybe we can suppress these modes?

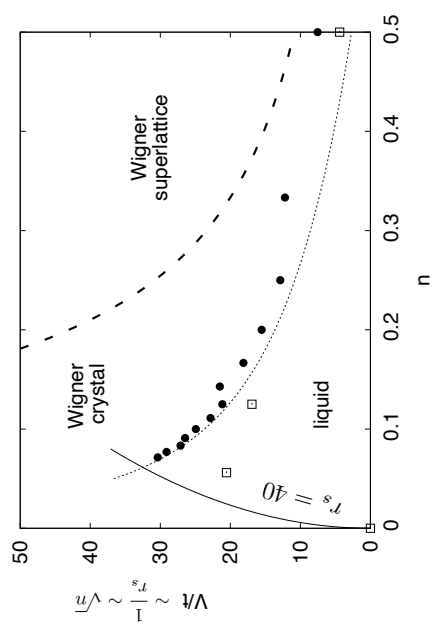
Generalized Wigner crystals

- The WC is notoriously difficult to observe: are there ways to make it more stable?
- route 1): lower the dimension $r_s = 100$ (3D), 40 (2D)
- route 2): get help from the periodic lattice inside materials (pin the modes)



[B. Valenzuela, SF, D. Baeriswyl, PRB 2003]

- continuum limit attained if $R_s \gg l \gg a$



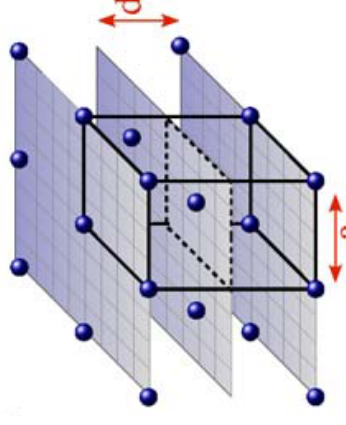
[D. Baeriswyl, SF, ECRYS 2005]

- commensurability with the underlying lattice

-> $r_s \sim 2$ in 2D [Noda & Imada, PRL02]

- layered: 2D motion but keep 3D interactions -> $r_s < 10$
+ naturally obtain charge order with square symmetry (cuprates?)

[G. Rastelli et al EPJB04, G. Rastelli & SF PRB06]



- not surprisingly, commensurate WCs are seen in low dimensional (layered) materials

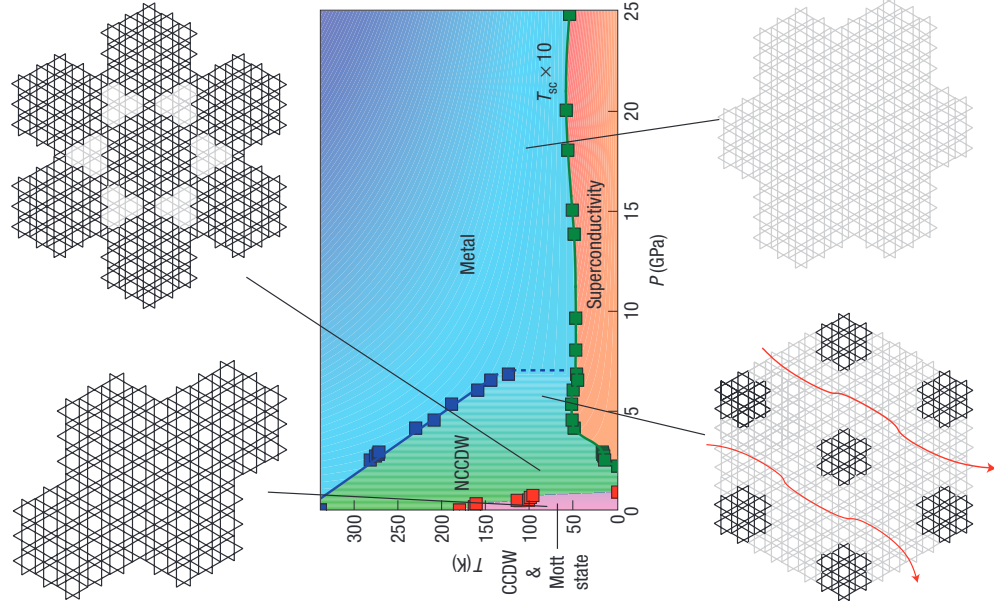
From Mott state to superconductivity in 1T-TaS₂

B. SIPOS^{1*}, A. F. KUSMARTSEVA^{1*}, A. AKRAP¹, H. BERGER¹, L. FORRÓ¹ AND E. TUTIŠ²

¹Ecole Polytechnique Fédérale de Lausanne, IPMC, CH-1015 Lausanne, Switzerland

²Institute of Physics, Bijenička c. 46, Zagreb, Croatia

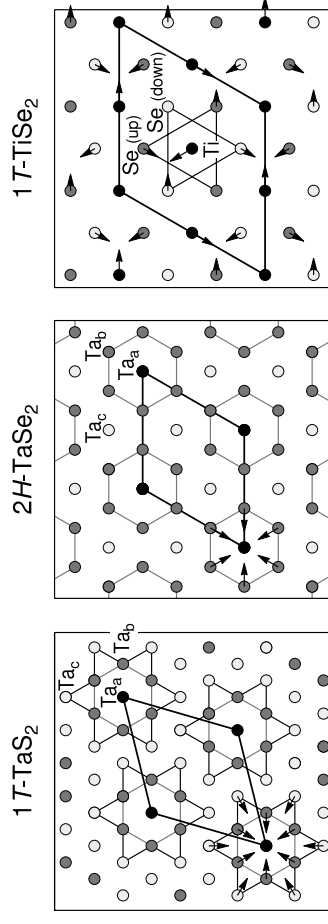
*e-mail: bsipos@gmail.com; anna.kusmartseva@epfl.ch



TOPICAL REVIEW

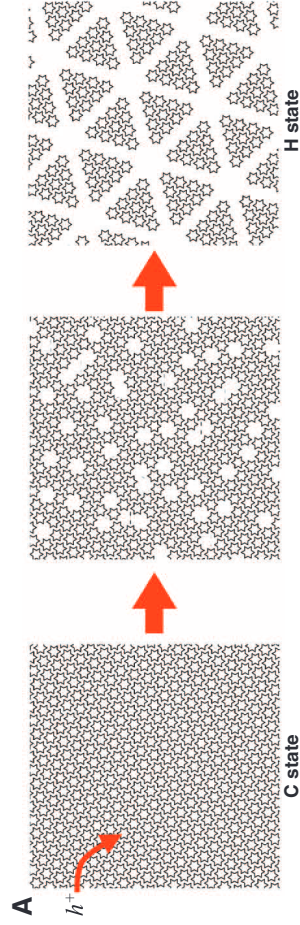
On the origin of charge-density waves in select layered transition-metal dichalcogenides

K Rossmagel



Ultrafast Switching to a Stable Hidden Quantum State in an Electronic Crystal

L. Stojchevska,^{1,2} I. Vaskivskiy,¹ T. Mertelj,¹ P. Kusar,¹ D. Svetin,¹ S. Brazovskii,^{3,4} D. Mihailovic^{1,2,5*}



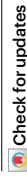
Bilayer Wigner crystals in a transition metal dichalcogenide heterostructure

<https://doi.org/10.1038/s41586-021-03560-w>

Received: 4 October 2020

Accepted: 15 April 2021

Published online: 30 June 2021



You Zhou^{1,2,9}, Jiho Sung^{1,2}, Elise Brutschea¹, Ilva Esterlis², Yao Wang^{2,3}, Giovanni Scuri², Ryan J. Gelly², Hosoek Heo^{1,2}, Takashi Taniguchi^{1*}, Kenji Watanabe⁵, Gergely Zarand⁶, Mikhail D. Lukin², Philip Kim^{2,7}, Eugene Demler^{2,8,2,8,2,8} & Hongkun Park^{1,2,8,2,8}

One of the first theoretically predicted manifestations of strong interactions in many-electron systems was the Wigner crystal^{1–3}, in which electrons crystallize into a

Imaging two-dimensional generalized Wigner crystals

<https://doi.org/10.1038/s41586-021-03874-9>

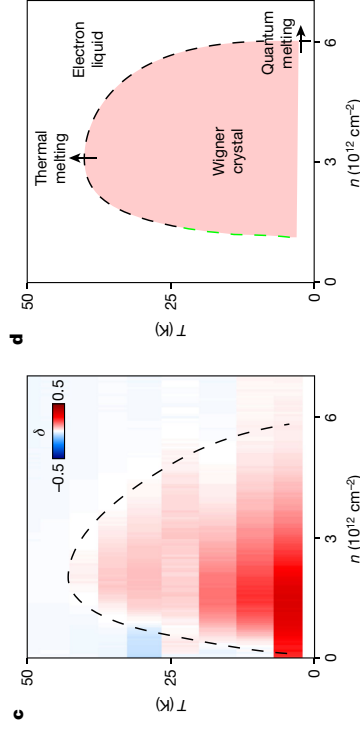
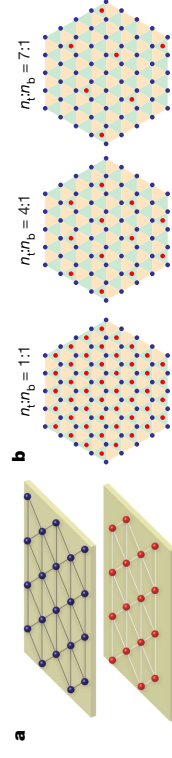
Received: 3 April 2021

Accepted: 4 August 2021

Published online: 29 September 2021

Hongyuan Li^{1,2,9}, Shaowei Li^{1,3,4,5,6,7,8}, Emma C. Regan^{1,2,2}, Danding Wang^{1,2}, Wenyu Zhao¹, Salman Kahn^{1,3}, Kentaro Yumigeta⁶, Mark Blei⁶, Takashi Taniguchi^{1*}, Kenji Watanabe⁵, Sefaattin Tongay⁶, Alex Zettl^{1,3,4}, Michael F. Crommie^{1,3,4,5,7} & Feng Wang^{1,3,4,5,7,8}

The Wigner crystal¹ has fascinated condensed matter physicists for nearly 90 years^{2–14}.



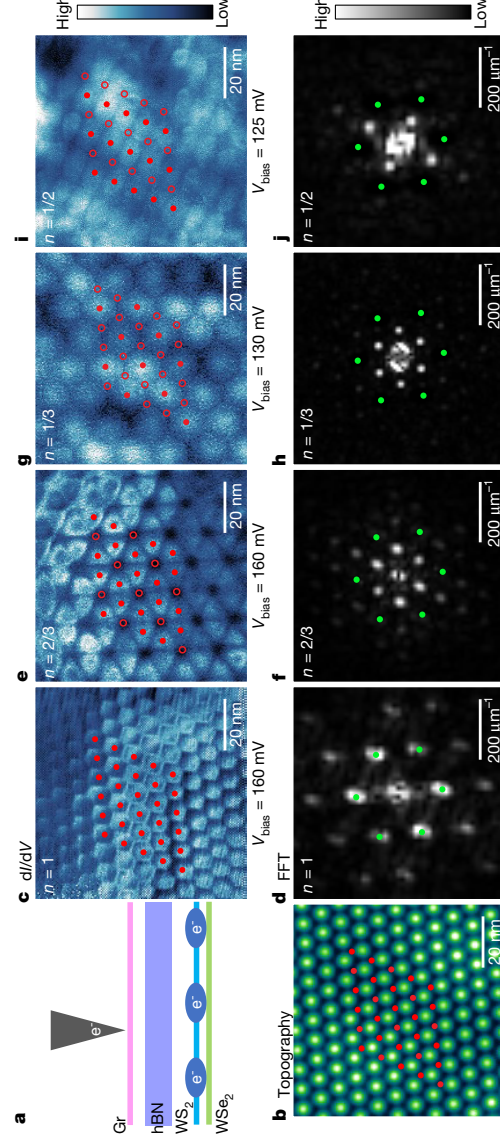
nature
materials

LETTERS
<https://doi.org/10.1038/s41586-021-00959-8>



Stripe phases in WSe_2/WS_2 moiré superlattices

Chenhao Jin^{1,7,8,2}, Zui Tao^{1,2,7}, Tingxin Li^{2,7}, Yang Xu^{1,2}, Yanhao Tang², Jiacheng Zhu², Song Liu³, Kenji Watanabe^{5*}, Takashi Taniguchi^{1*}, James C. Hone³, Liang Fu^{4,5,7}, Jie Shan^{3,2,6,8,2,8} and Kin Fai Mak^{3,2,6,8,2,8}



Charge-cluster glass in an organic conductor

F. Kagawa^{1,2*}, T. Sato¹, K. Miyagawa¹, K. Kanoda¹, Y. Tokura^{1,3}, K. Kobayashi^{1,4}, R. Kumai^{2,4} and Y. Murakami⁴

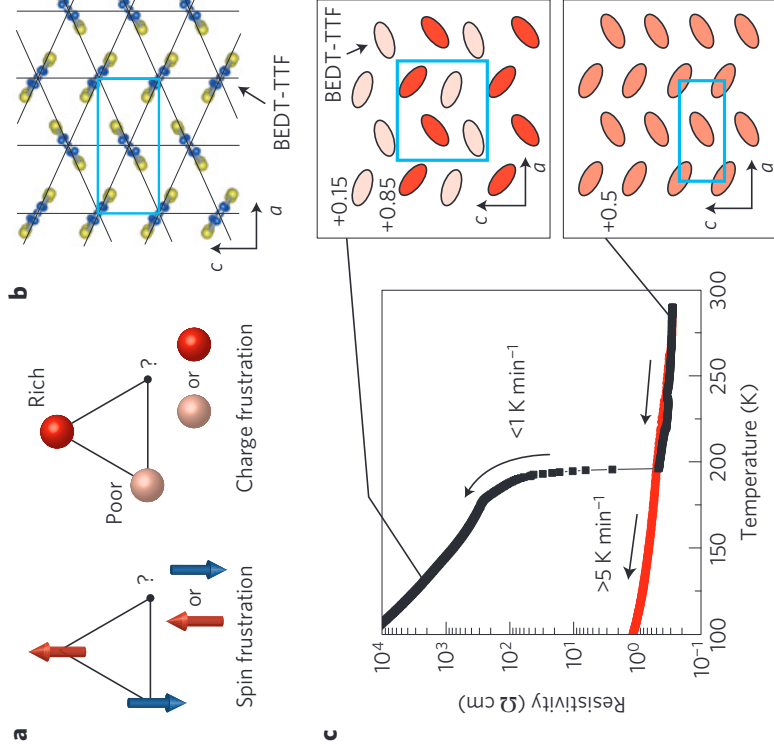
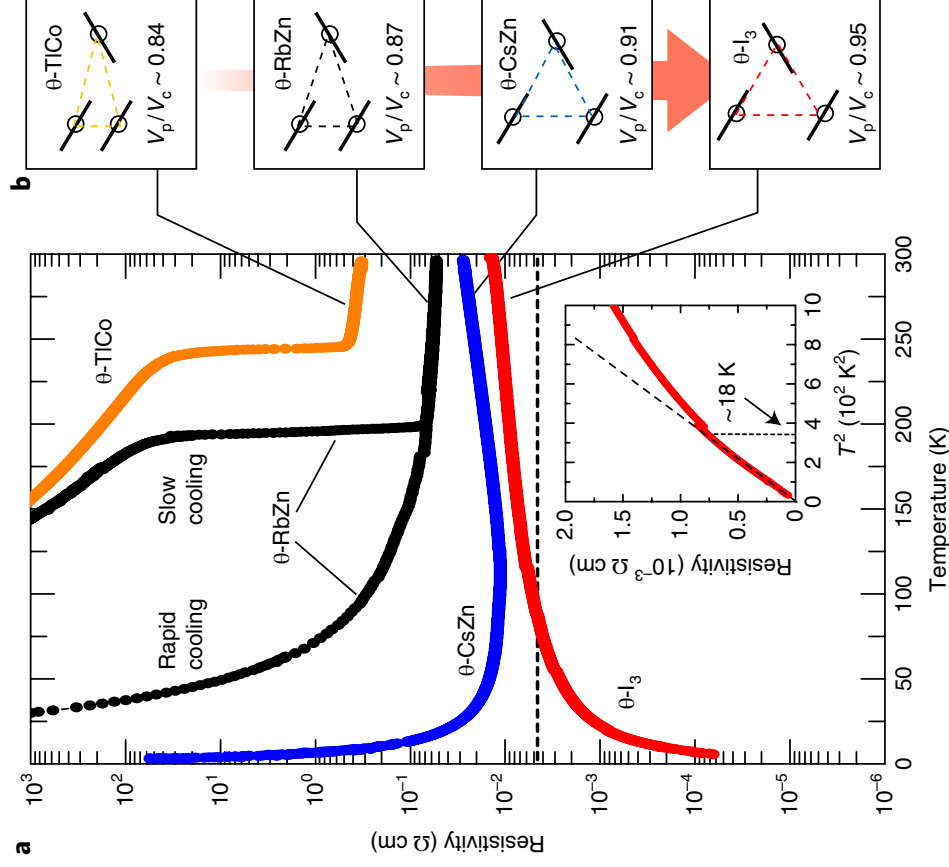


Figure 1 | Charge frustration and crystal structure of θ -(BEDT-TTF)₂RbZn(SCN)₄. **a**, An illustration of the analogy between spin

Strange metal from a frustration-driven charge order instability

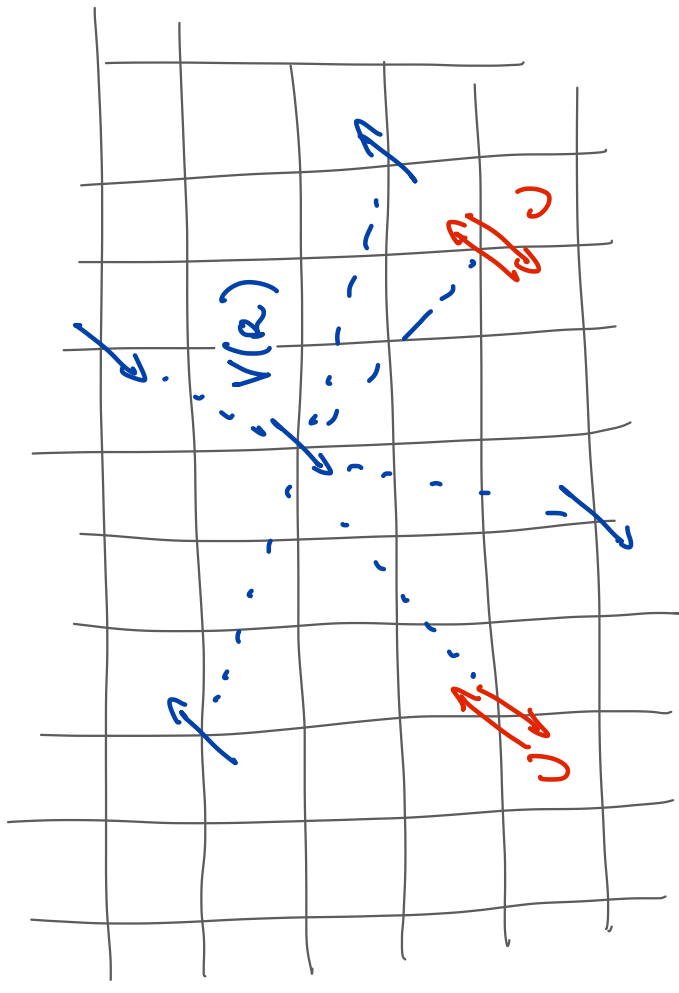
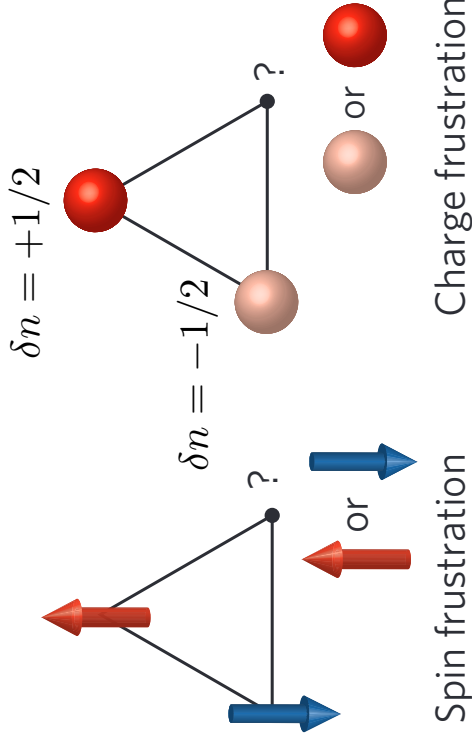
T. Sato^{1,4*}, K. Kitai¹, K. Miyagawa¹, M. Tamura², A. Ueda³, H. Mori³ and K. Kanoda^{1*}



e-e interactions beyond Hubbard: 2 types of frustration

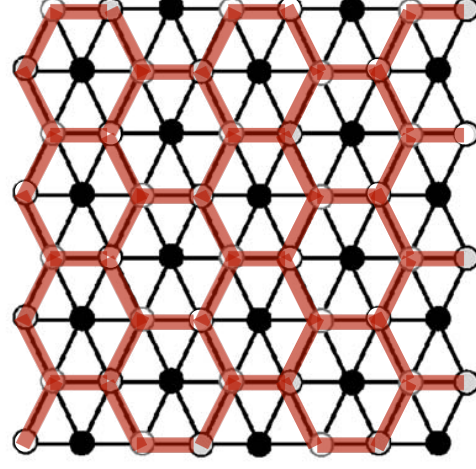
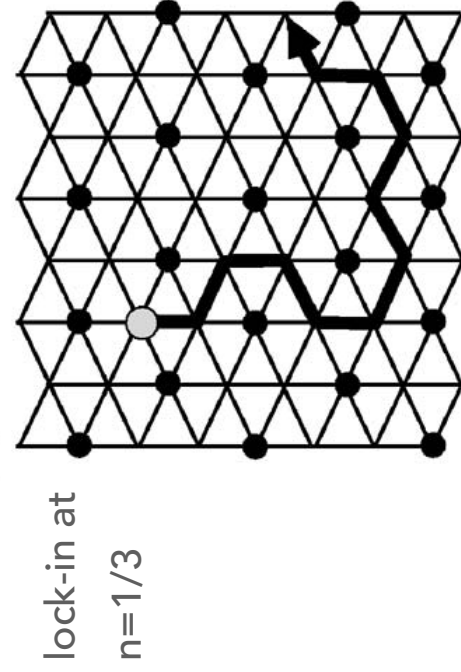
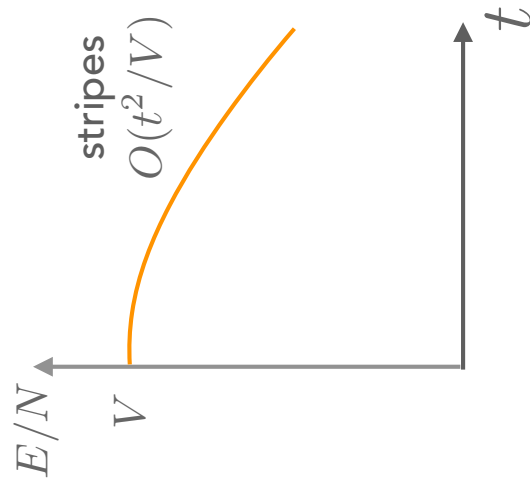
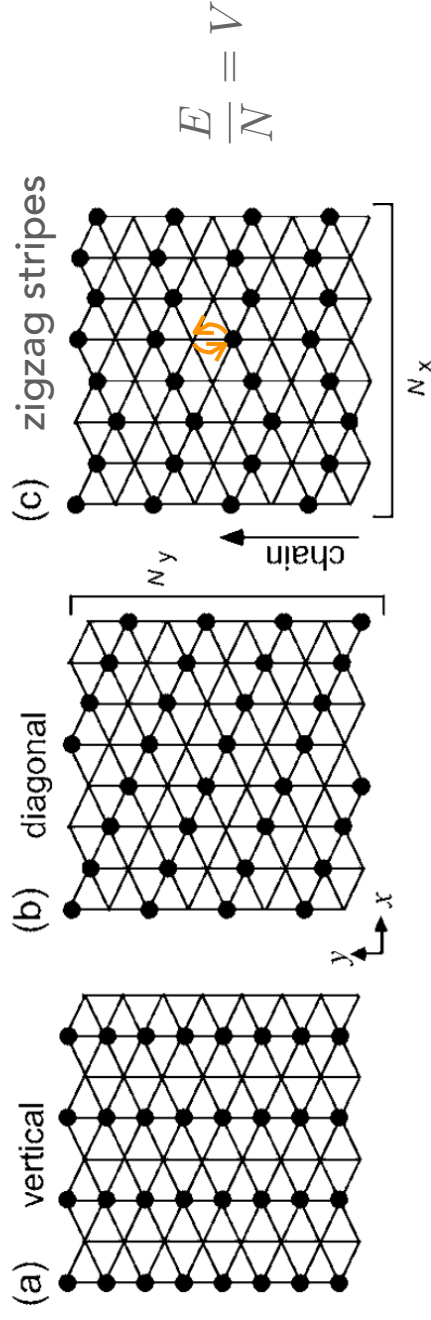
$$H = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{1}{2} \sum_{i \neq j} V(R_{ij}) n_i n_j \quad V(R) = \frac{V}{R^\alpha}$$

- geometric: specific to triangular lattice at $n=1/2$ (appears already for nearest neighbor repulsion)
- long range, all electrons interact together at all distances



Degeneracy and lifting by quantum fluctuations

- n.n. interactions on the triangular lattice at $n=1/2$: infinitely many states with the same electrostatic energy

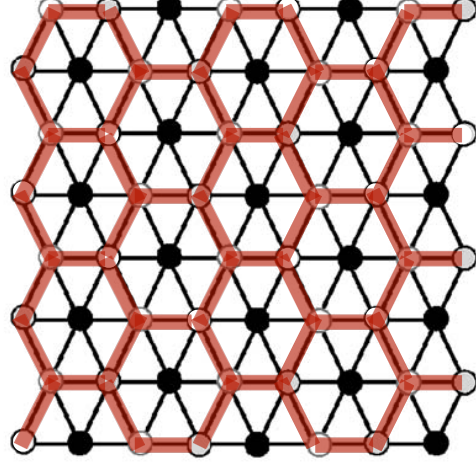
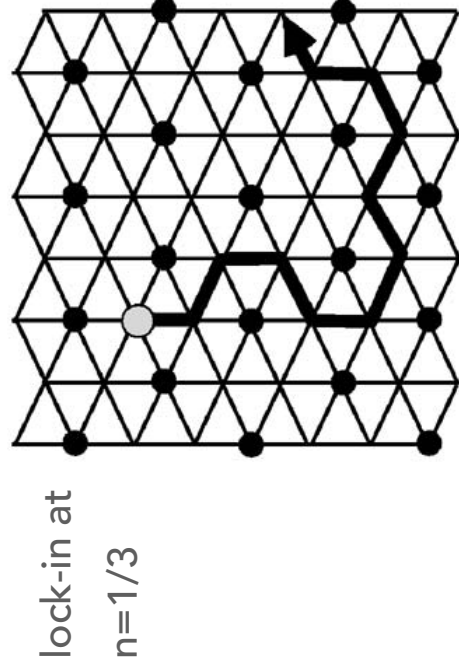
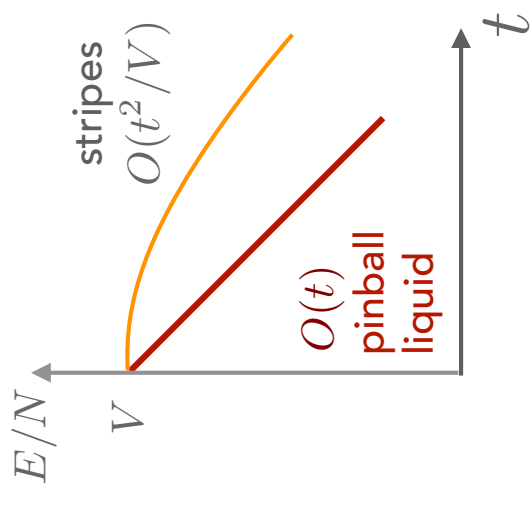
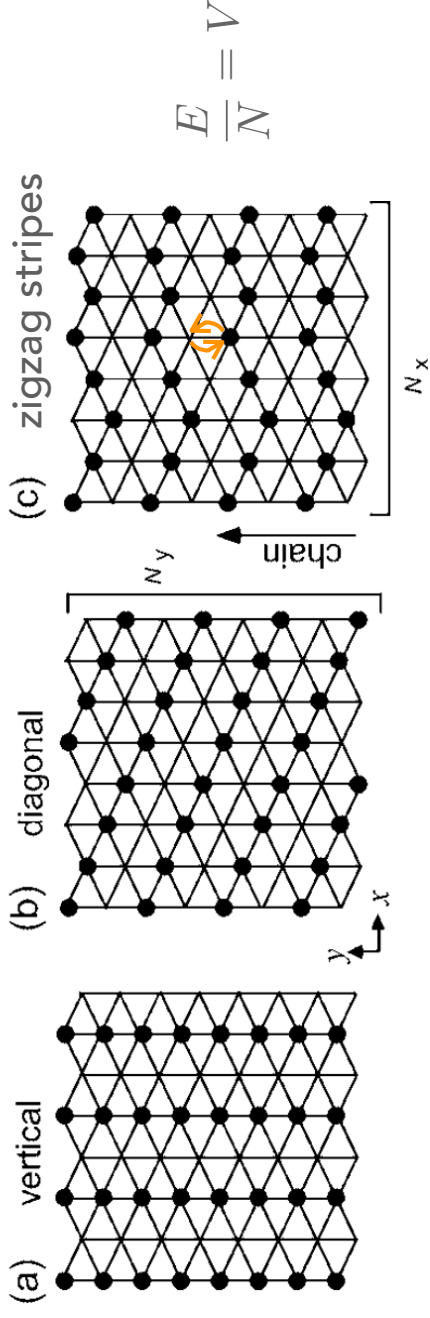


add the remaining $n=1/2-1/3$ on interstitials:
same $E/N = V$

quantum fluctuations:
gain Kinetic energy
 $O(t)$ (not t^2 !)

Degeneracy and lifting by quantum fluctuations

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add the remaining $n=1/2-1/3$ on interstitials:
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quantum fluctuations:
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 $O(t)$ (not t^2 !)

pinball liquid
(unique ground state)

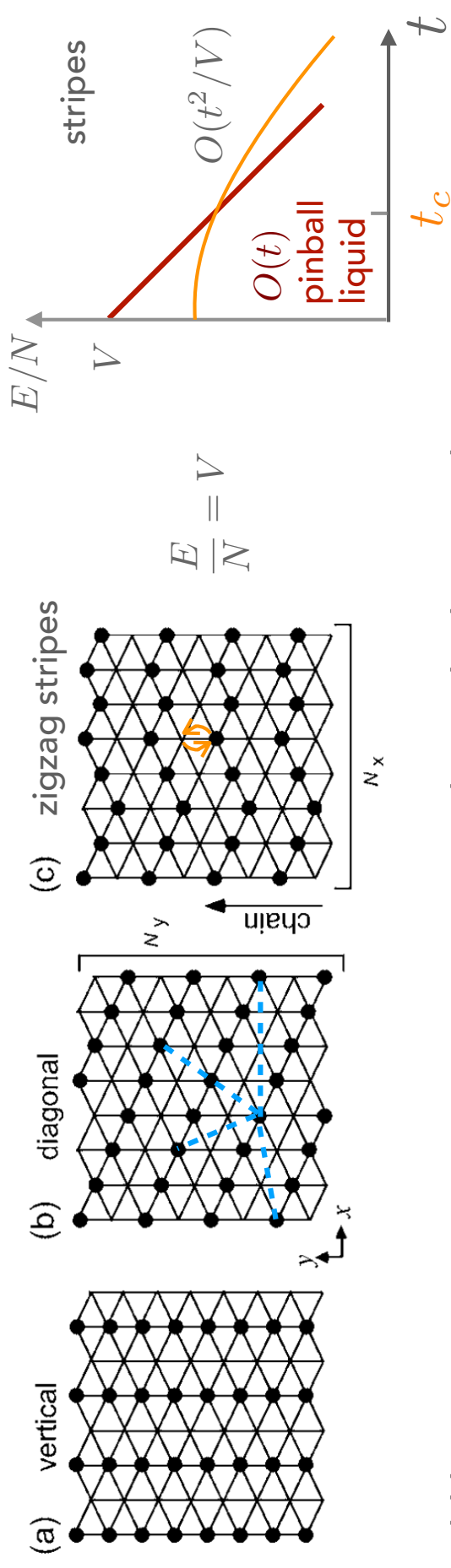
[C. Hotta, N. Furukawa, PRB 06]

observed experimentally in AgNiO_2

[R. Coldea et al PRL 2007, PRL 2011; A. Ralko, J. Merino, SF, PRB 15]

Degeneracy, lifting by long range interactions

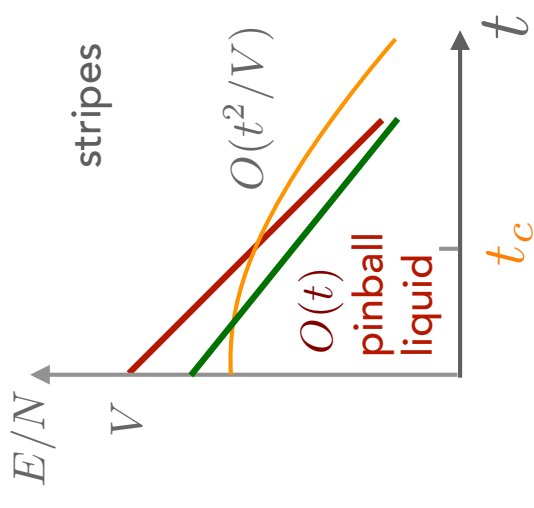
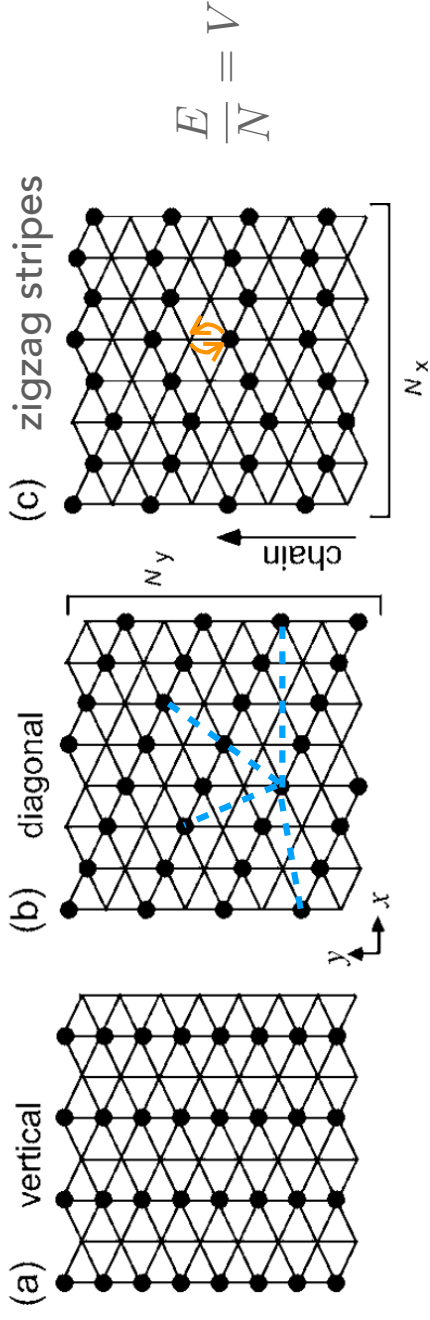
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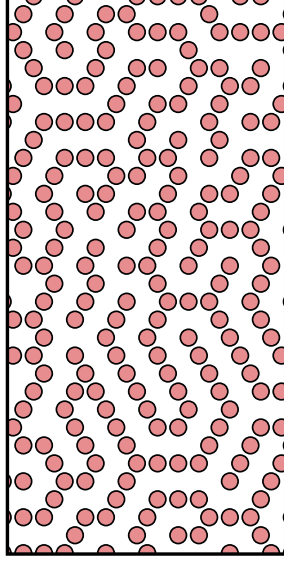
- add long range interactions: stripes now have the lowest electrostatic energy, all other states become metastable
- stripe to pinball transition occurs at a critical t/V

Degeneracy, lifting by long range interactions

- n.n. interactions on the triangular lattice at $n=1/2$: infinitely many states with the same electrostatic energy



- add long range interactions: stripes now have the lowest electrostatic energy, all other states become metastable
- stripe to pinball transition occurs at a critical t/V
- can we find a state that combines the electrostatic gain $\sim V$ of the stripes and the quantum gain $\sim t$ of the pinball, and is therefore more stable than both?



(spoiler: "hidden state" or charge glass)

Long Range Coulomb on triangular lattice

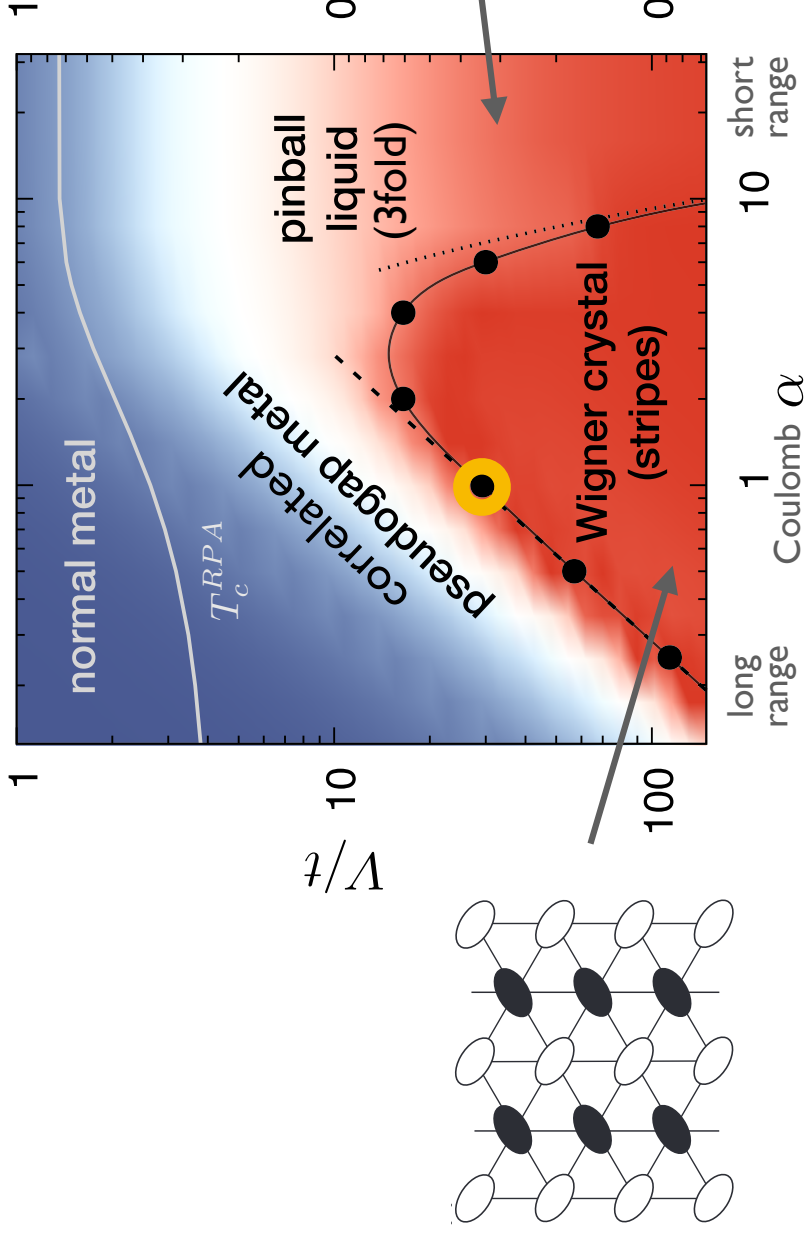
$$H = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^{\dagger} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{1}{2} \sum_{i \neq j} V(R_{ij}) n_i n_j \quad V(R) = \frac{V}{R^\alpha}$$

- **quantum ED $T=0$** : spinful/spinless, $n=0.5$, use Ewald and TBC averaging [SF, J. Merino, PRB 2009; K. Driscoll, A. Ralko and SF, PRB(L) 2021]
- **classical MC $T>0$** : spinless, $n=0.5$ [Mahmoudian et al., PRL 2015]
- **mixed quantum/classical $T>0$** : classical MC + quantum calculation of conductivity, all n [SF, K. Driscoll, S. Ciuchi, A. Ralko, arXiv 2208.06260 this week]
- **fully quantum $T>0$** : FTLM solution in progress, KPM solution in progress

focus on the metal near the Wigner crystal melting where LR interactions matter

Long Range Coulomb on triangular lattice

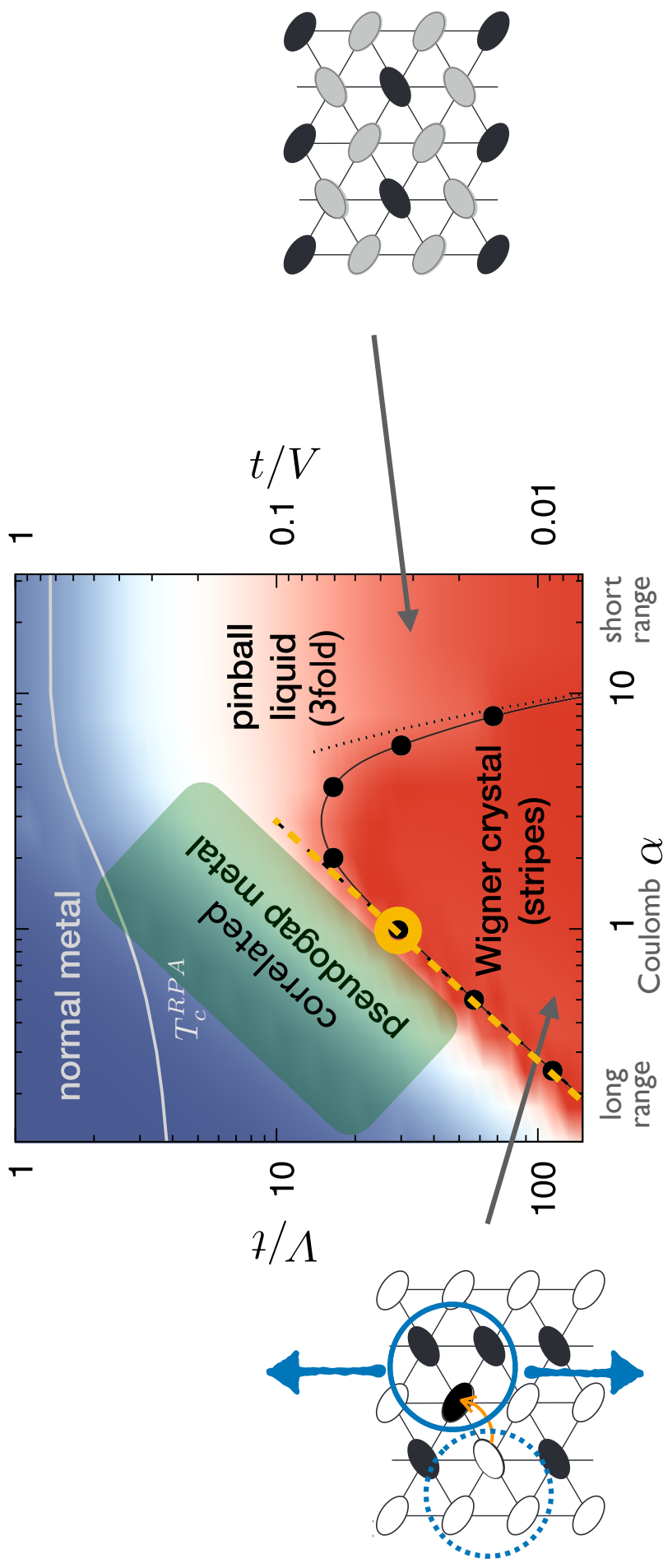
$$H = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{1}{2} \sum_{i \neq j} V(R_{ij}) n_i n_j \quad V(R) = \frac{V}{R^\alpha}$$



- $r_s = 7.2$ due to commensurability with underlying lattice

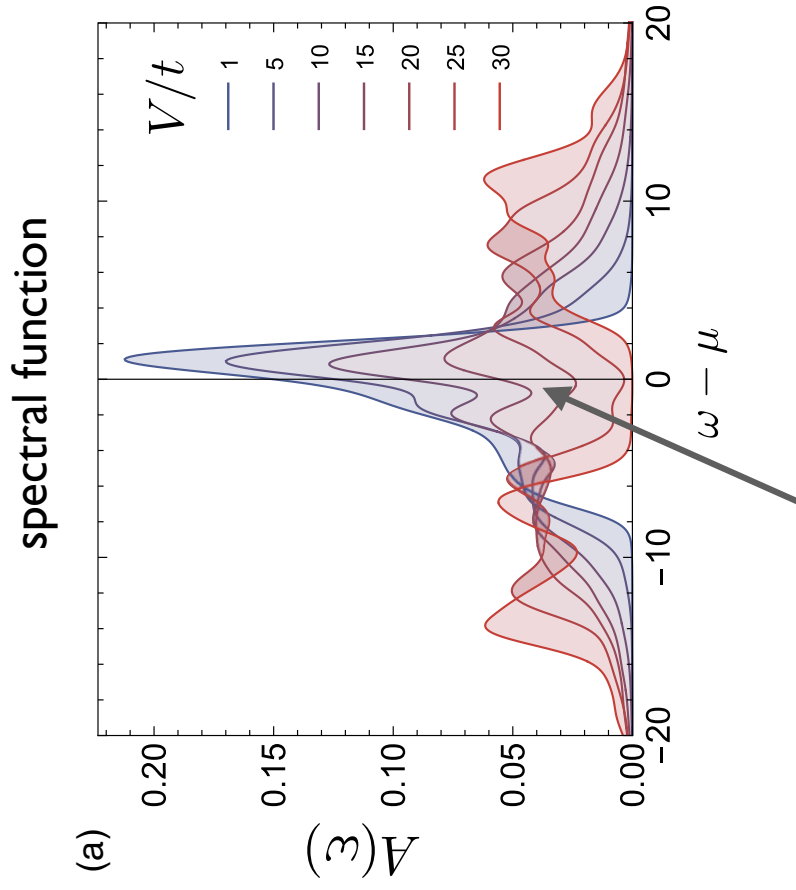
Long Range Coulomb on triangular lattice

$$H = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{1}{2} \sum_{i \neq j} V(R_{ij}) n_i n_j \quad V(R) = \frac{V}{R^\alpha}$$

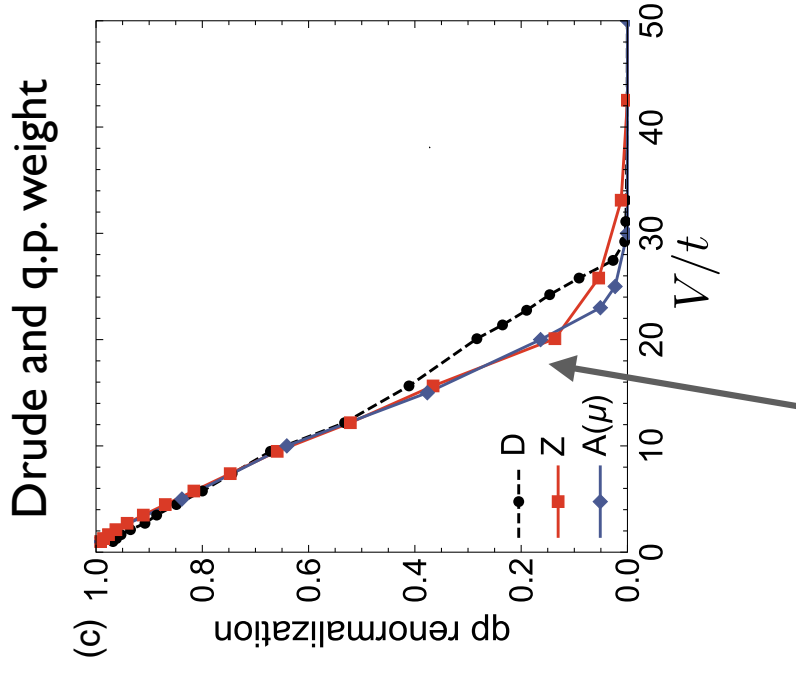


- $r_s = 7.2$ due to commensurability with underlying lattice
- increasing V_c/t with range of interactions indicates **key role of charge defects** for melting of **long range order** (lattice equivalent of soft collective excitations)
- **short range order** persists in the metal up to the mean field transition ($V/t \sim 1$)

Correlated pseudogap metal



pseudogap emerges in the interacting liquid

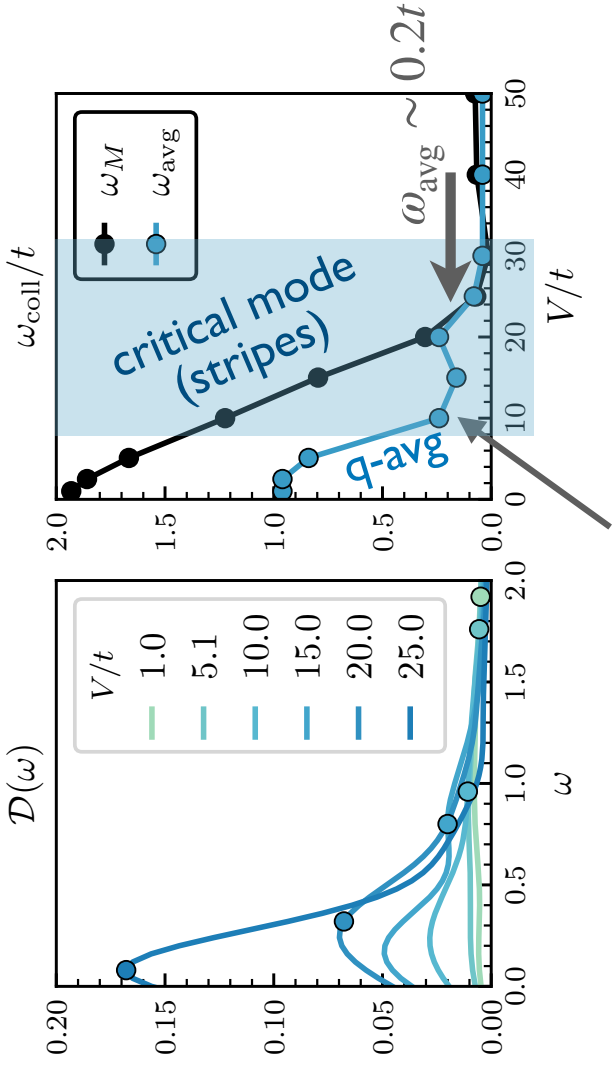


pseudogap regime is strongly correlated,
(no local U, no magnetic moments!)

Self-generated disorder

$$\phi_i = \sum_j V_{ij} n_j$$

distribution of instantaneous site potentials

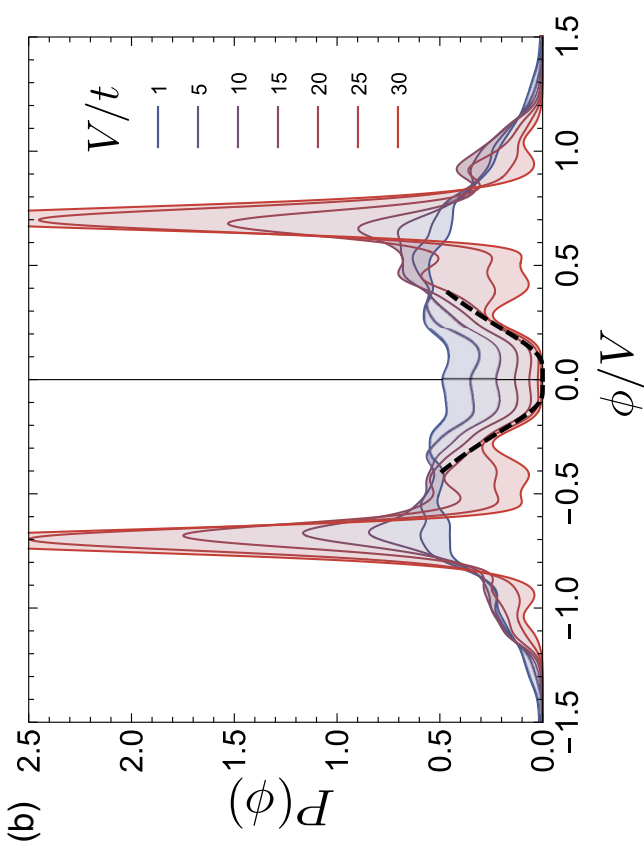


the overall timescale of collective motions is slow within the entire pseudogap phase

separation of timescales

$$\omega_0 \simeq 0.2t \ll \text{bandwidth}$$

the collective motions are much slower than the individual motions:
 the electrons move in a self-generated disordered background
 cf. Andreev and Kosevic (1979) and Efros (1992)



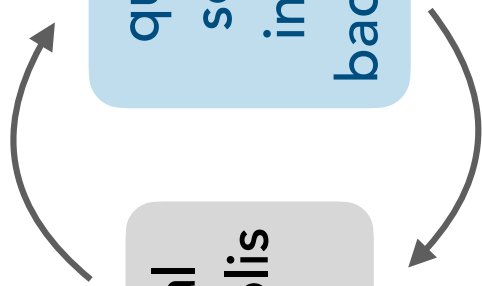
Mixed quantum/classical treatment



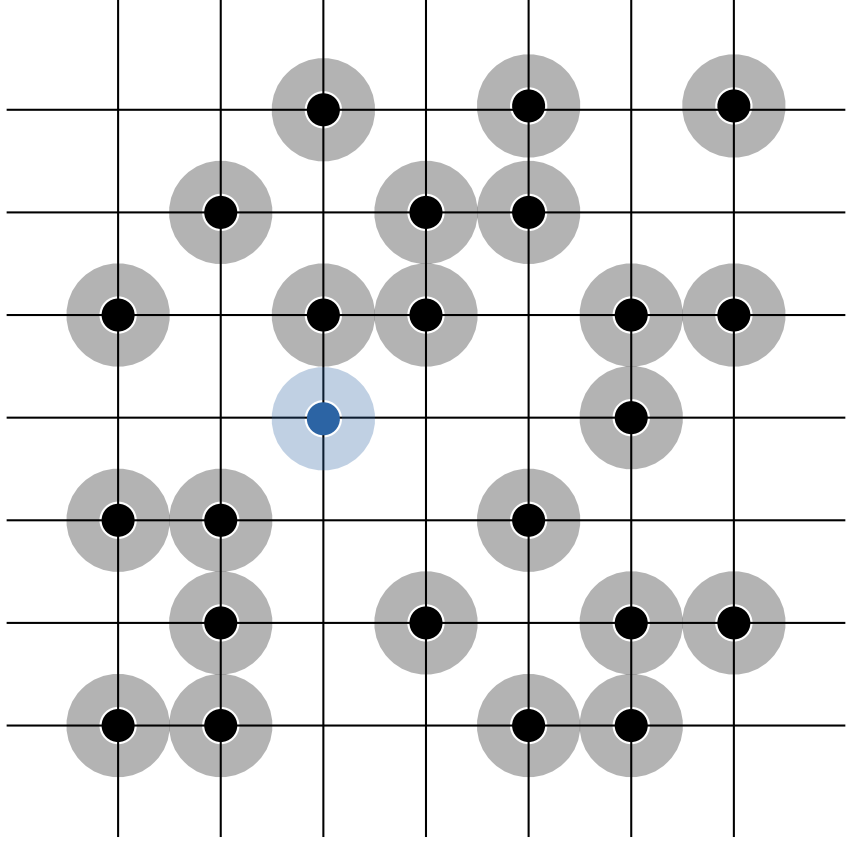
individual motions, fast:
quantum electrons in
the collective landscape

classical
Metropolis
MC

quantum
solution
in frozen
background



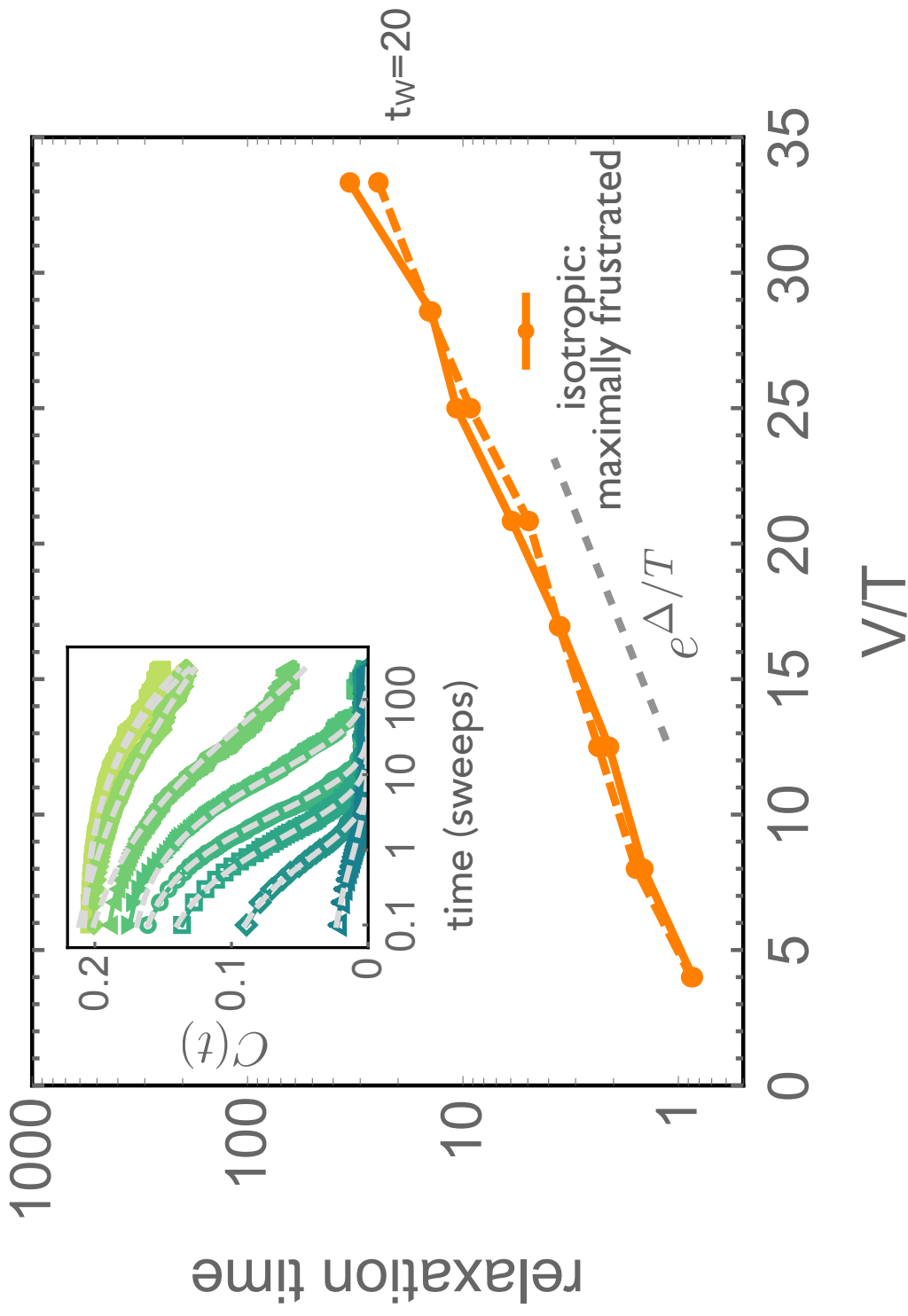
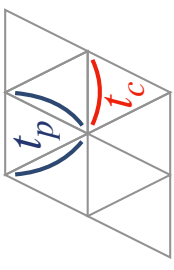
collective motions, slow: classical MC



- metastable states
- crystallization dynamics and aging
- impact on electron transport
- $P(n)$, $C(q)$, $\sigma(w)$, $\rho(T)$, $A(k,w)$, $Cv/T...$

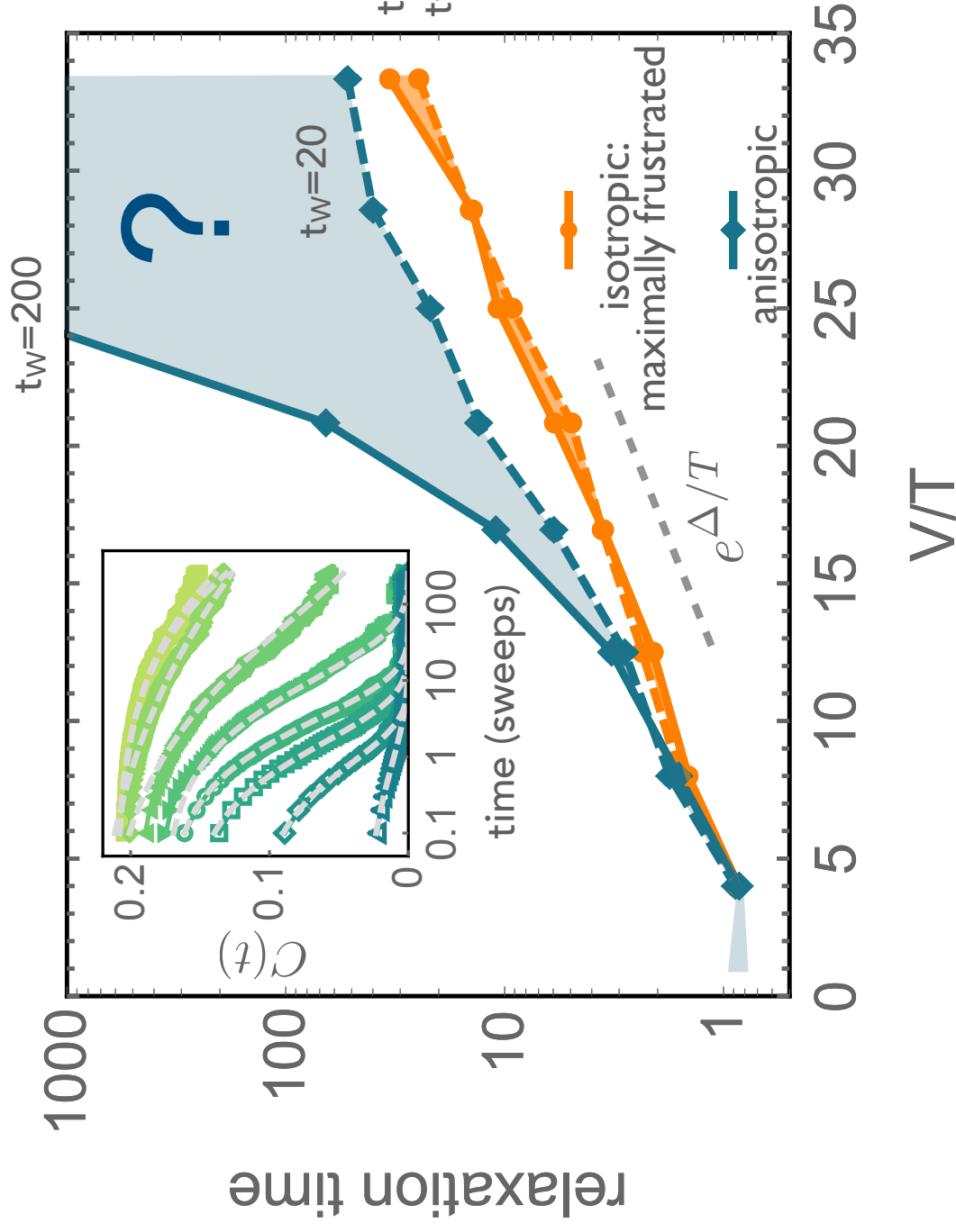
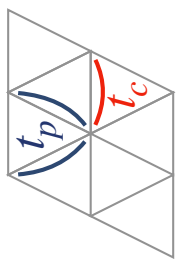
Exponential slowing down vs frustration

$$C(t, t_W) = \sum_i \langle n_i(t + t_W) n_i(t_W) \rangle$$



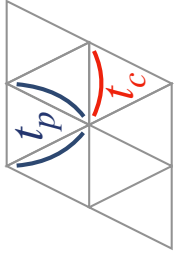
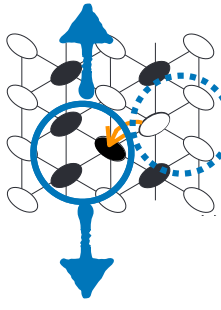
Exponential slowing down vs frustration

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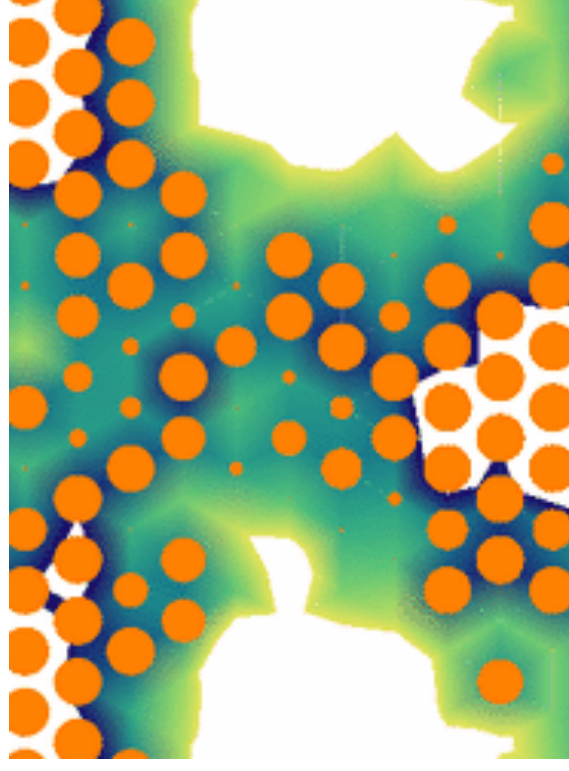
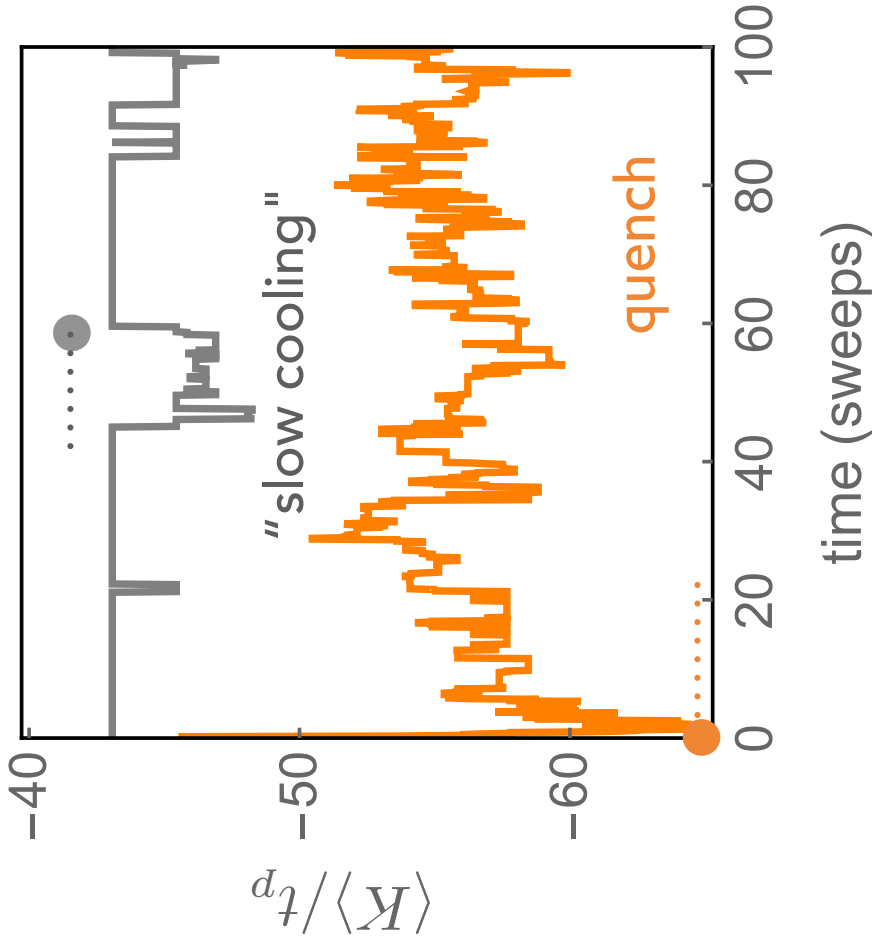
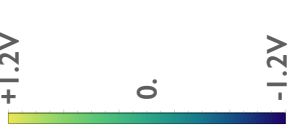
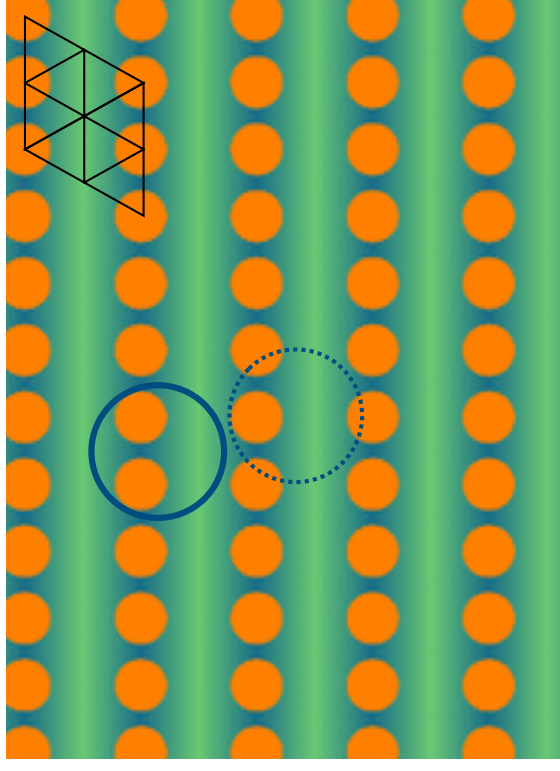
relaxation
time grows
longer than
exponential

Phase dynamics: maximally frustrated

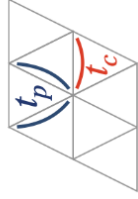


$$t_p = t_c = 0.090V$$

$$T = 0.025V$$

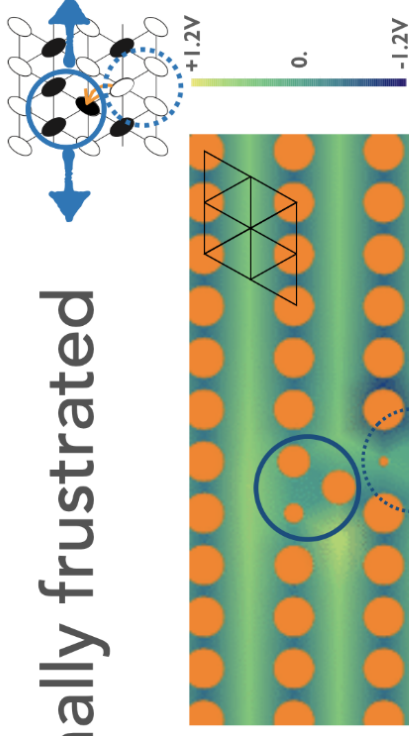
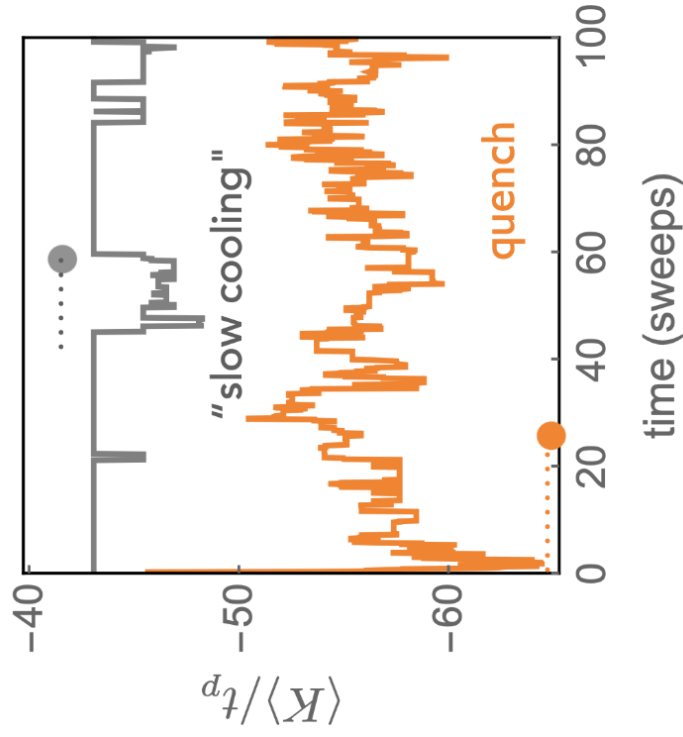


Phase dynamics: maximally frustrated



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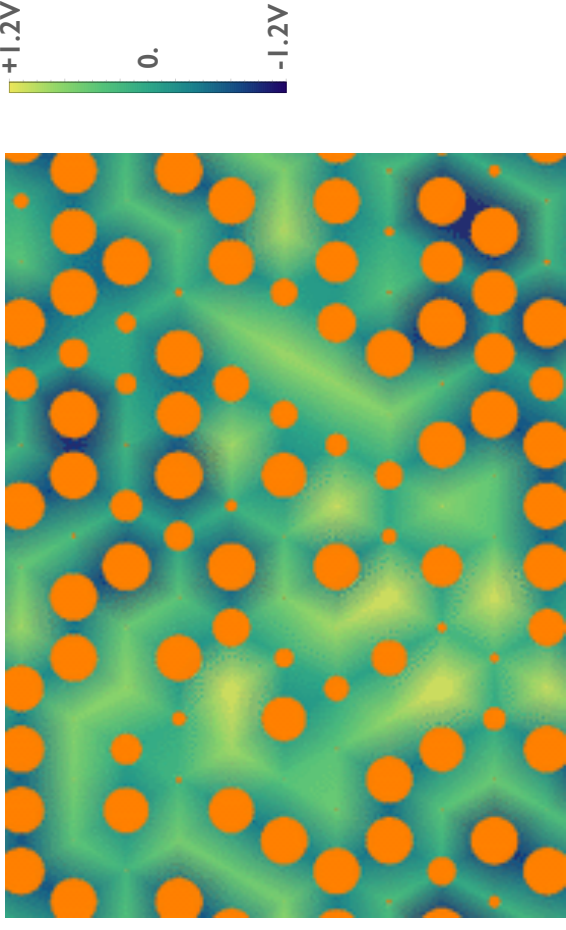
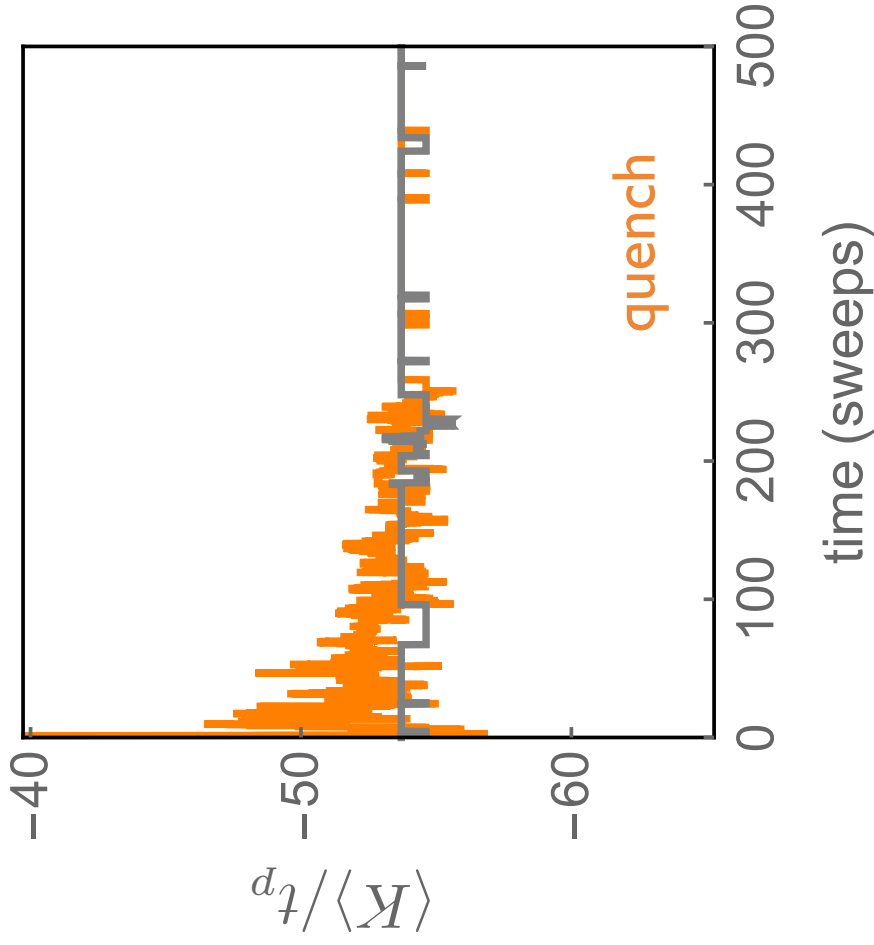
[SF, K. Driscoll, S. Ciuchi, A. Ralko, arXiv 2208.06260 this week]

Phase dynamics: strained (anisotropic)

$$t_c/t_p = 0.1$$

$$t_p = 0.12V$$

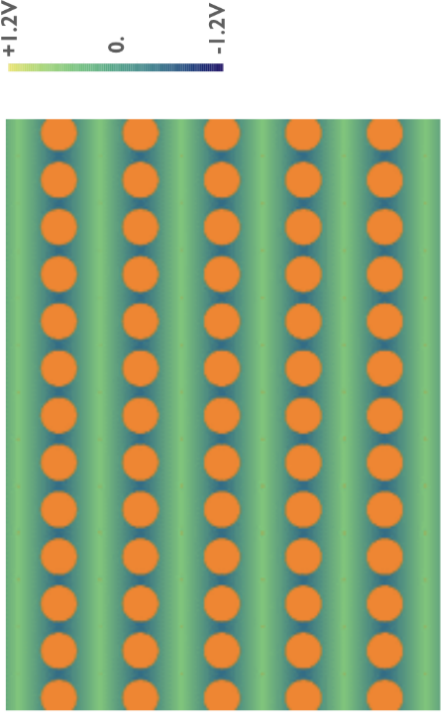
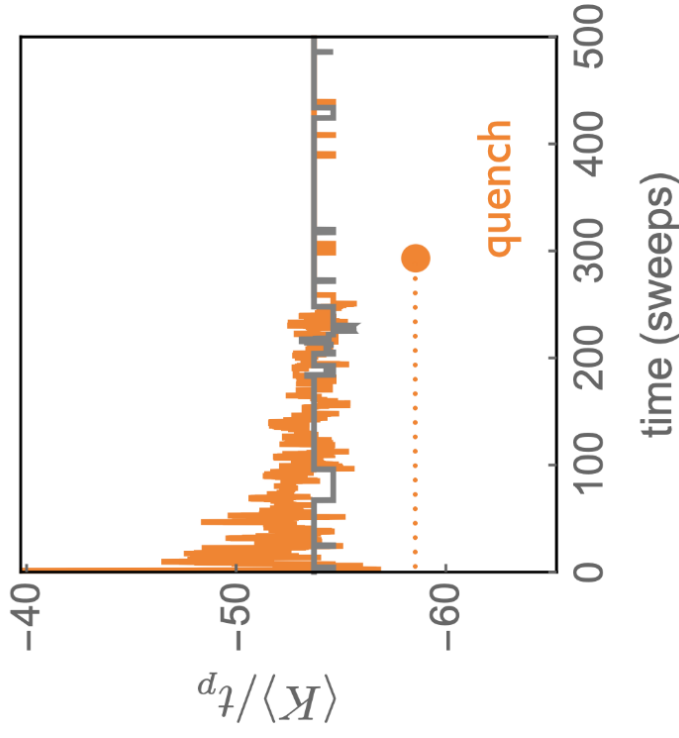
$$T = 0.03V$$



- outcome depends on history
- outcome depends on anisotropy of quantum electrons

Phase dynamics: strained (anisotropic)

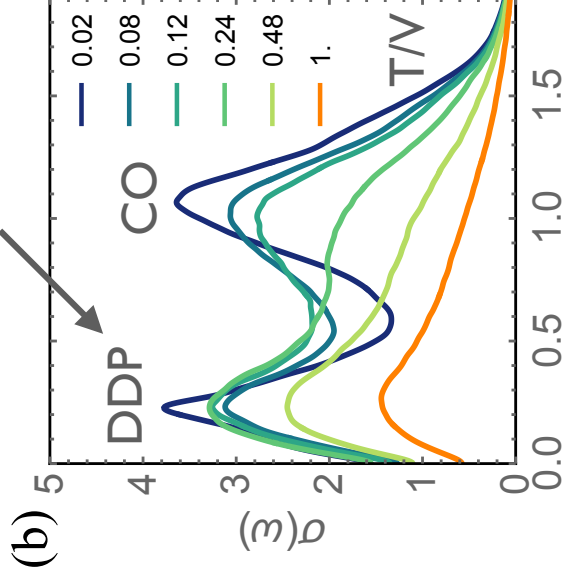
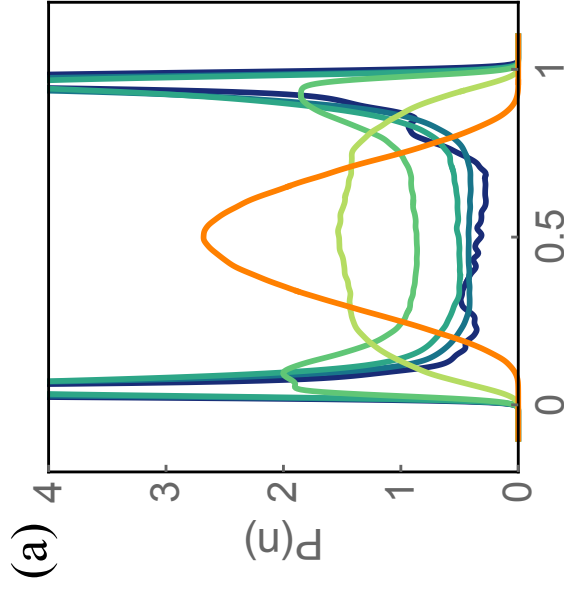
$$\begin{aligned}t_c/t_p &= 0.1 \\t_p &= 0.12V \\T &= 0.03V\end{aligned}$$



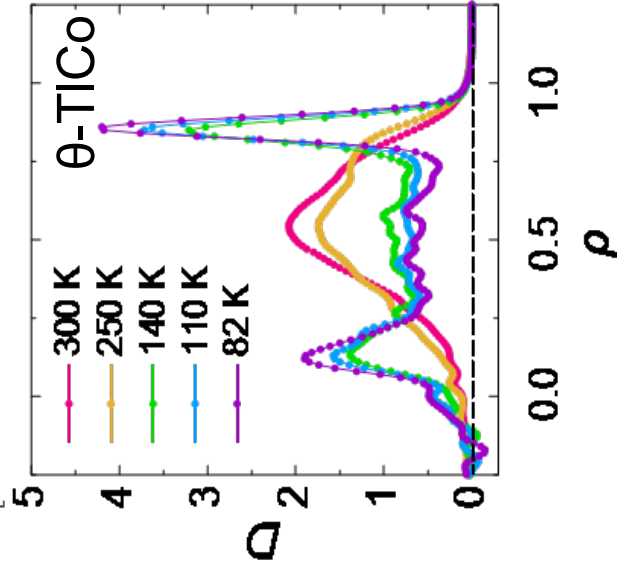
- outcome depends on history
- outcome depends on anisotropy of quantum electrons

Characterization of charge glass

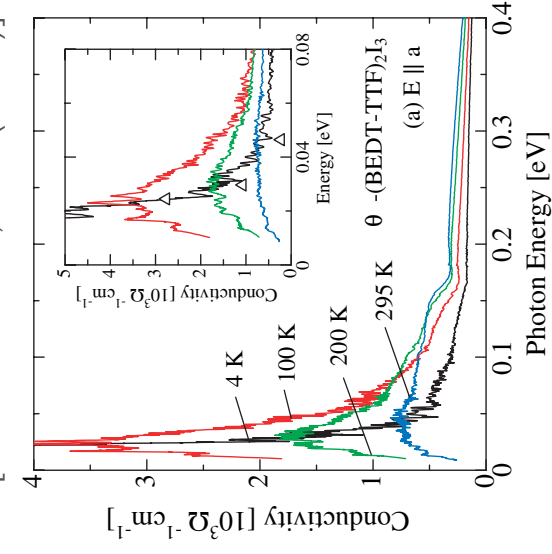
(localization in self generated disorder)



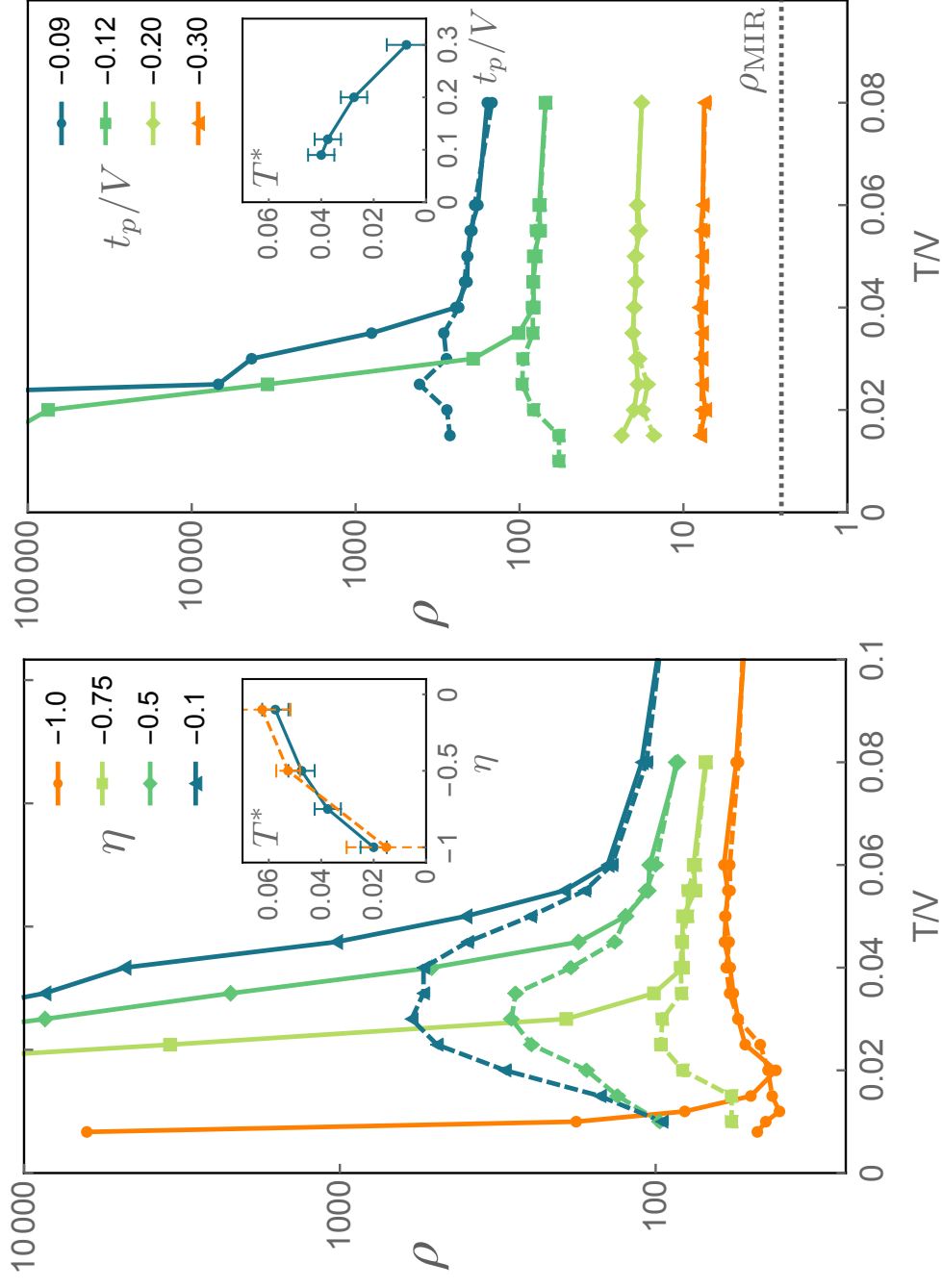
[Murase et al. arXiv:2205.10795]



[Takenaka et al, PRL (2005)]



Self-generated disorder (MC, classical)



Concluding remarks

- Long range interacting electrons on lattices display charge frustration, at the origin of a variety of original physical phenomena.
- One of the most interesting is the emergence of **self-generated disorder due to charge frustration** (very slow collective charge dynamics), which is at the origin of the observed glass phase.
- This is also one plausible mechanism for **bad metal behavior**: it increases the resistivity above the semiclassical value, and Coulomb interactions are ubiquitous after all. It can also lead to **strange T-linear behavior**.