Anomalous Diffusion of Magnetic Monopoles in Spin Ice

Emergence of a Dynamical Fractal in a Clean Magnet

mpipks

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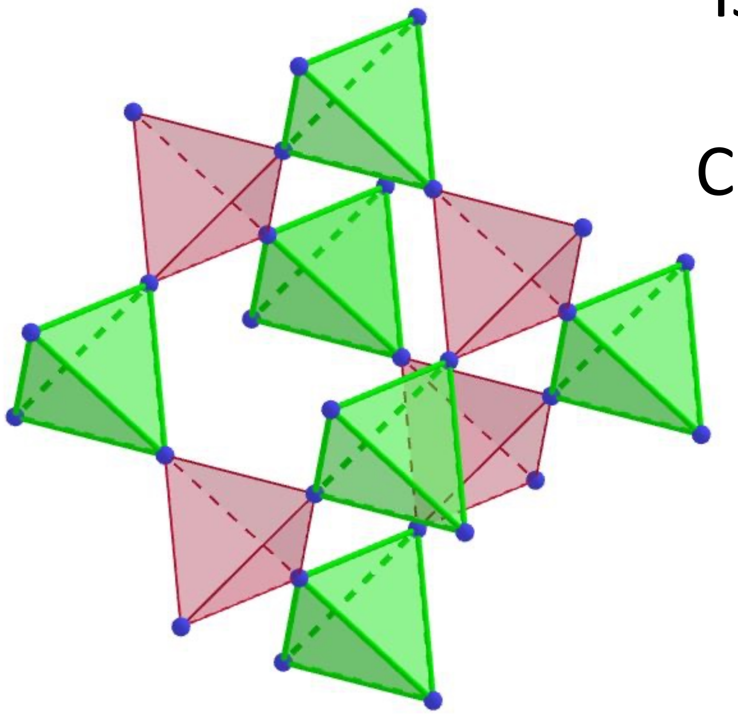
Spin Ice Basics

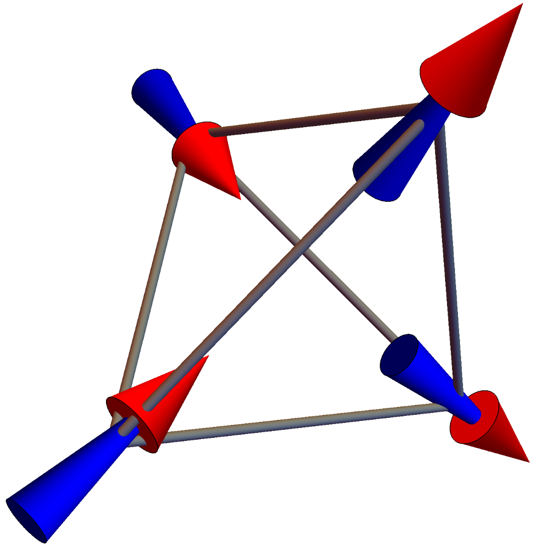


Ising spins on the pyrochlore lattice

Constrained to point along easy axes (in or out)

Ferromagnetic nearest-neighbour interactions

→ Frustrated magnet with highly degenerate 2-in-2-out groundstates

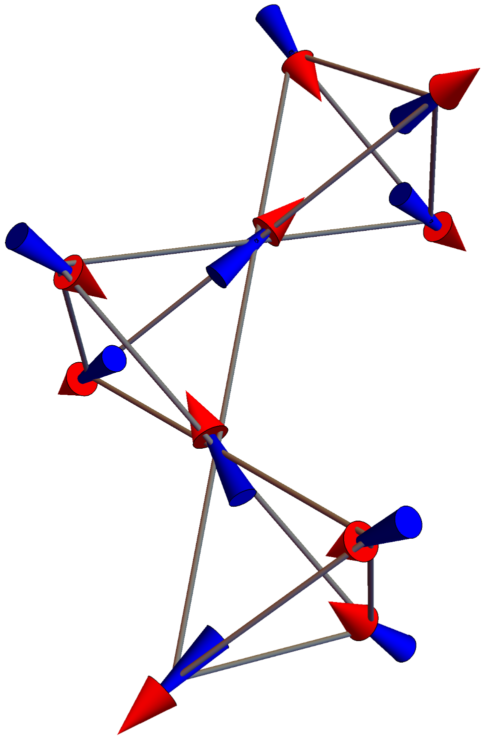


Ice Rules
2 spins in and 2 spins out per tetrahedron

Bramwell & Harris, *J. Phys.: Cond. Mat.* 32 (2020).

Dysprosium Titanate ($Dy_2Ti_2O_7$)

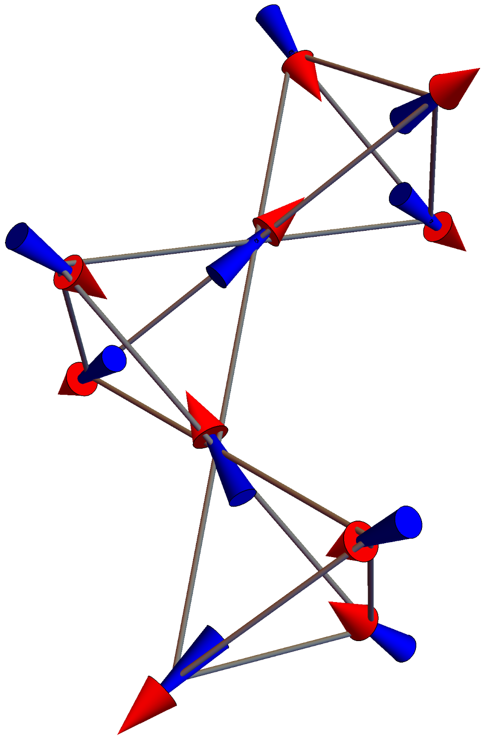
$J = 15/2$ spins with Ising-like single ion states



Long-ranged dipolar spin-spin interactions

Follows the ice rules due to screening

Dysprosium Titanate ($Dy_2Ti_2O_7$)



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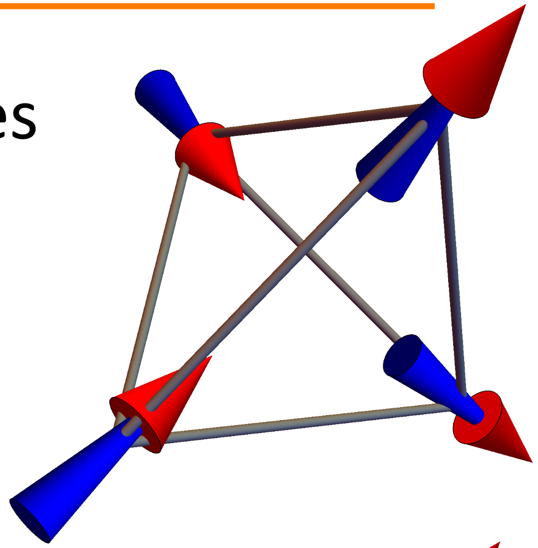
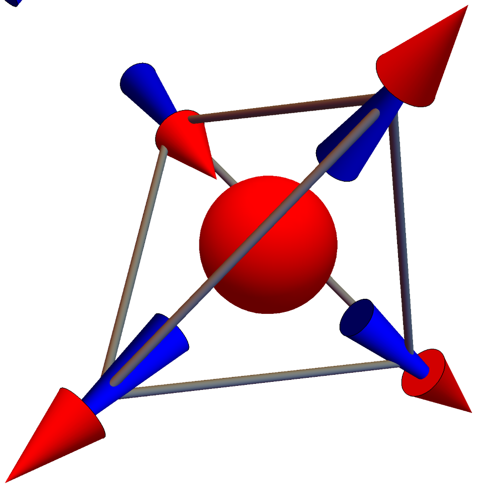
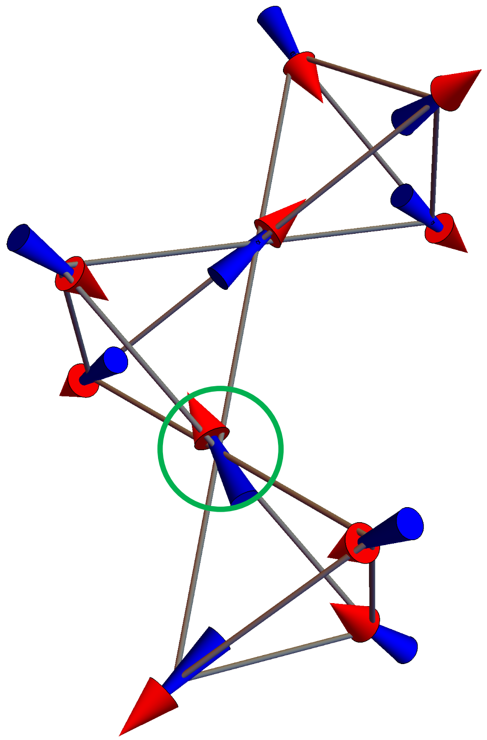


Emergent **magnetic monopole** excitations

Monopoles live on the diamond lattice

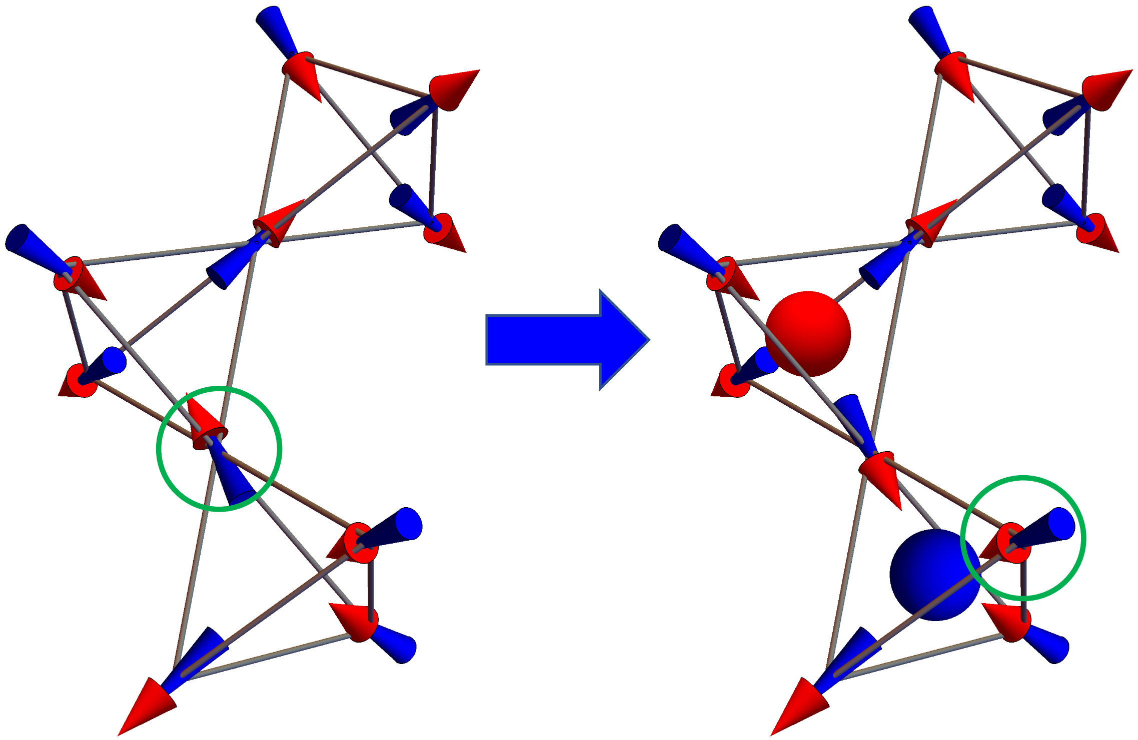


Emergent Magnetic Monopoles



Castelnovo, Mossner & Sondhi, *Nature* 451, 7174 (2008)

Emergent Magnetic Monopoles



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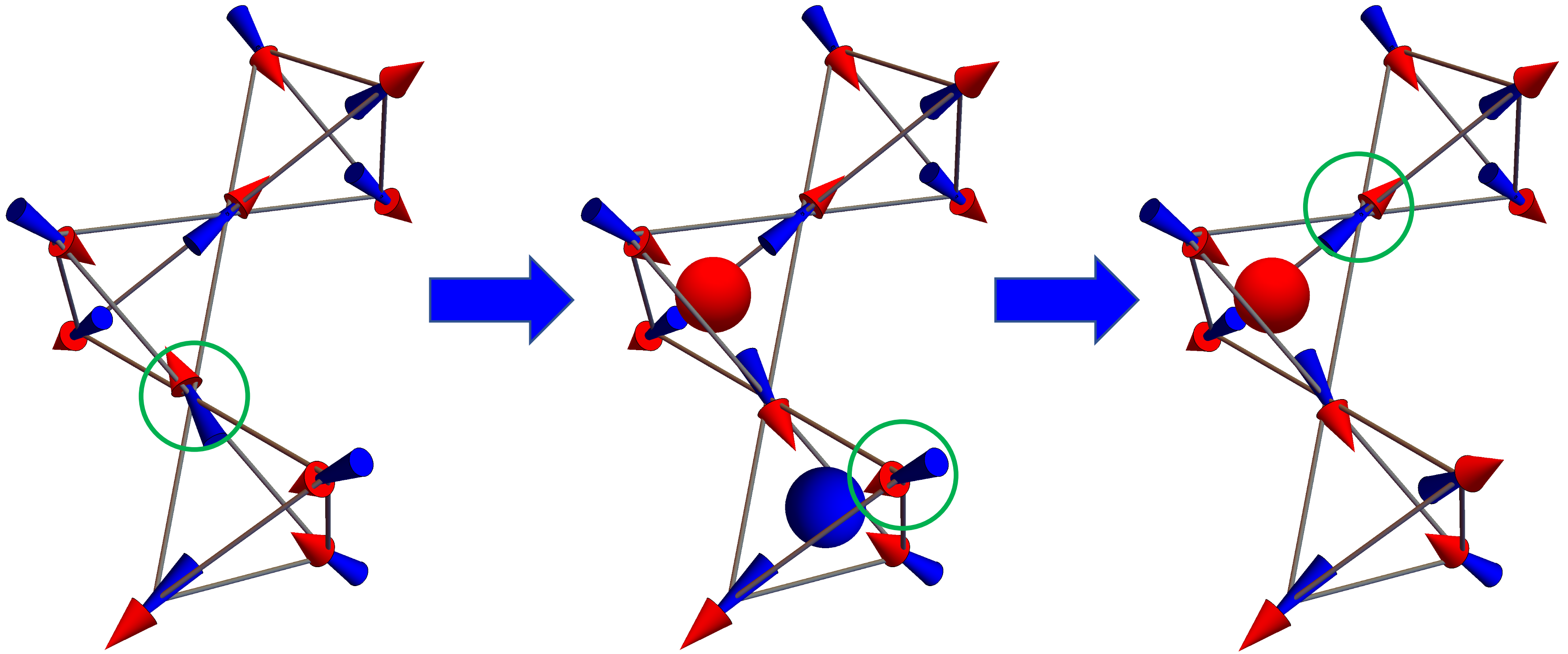

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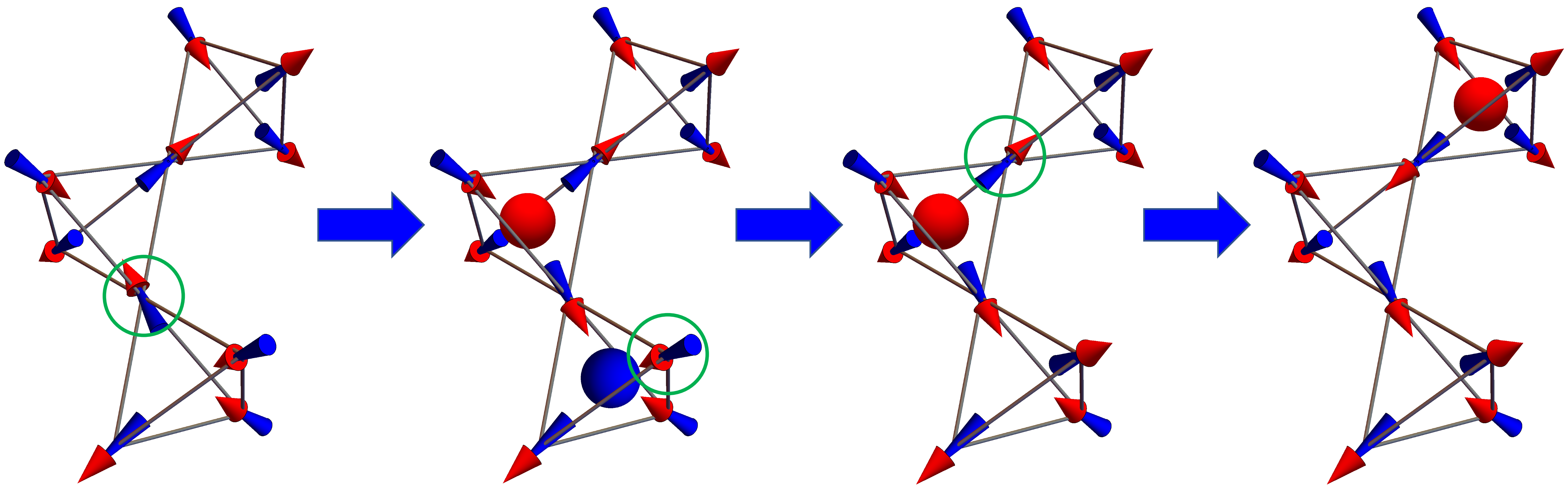
Emergent Magnetic Monopoles



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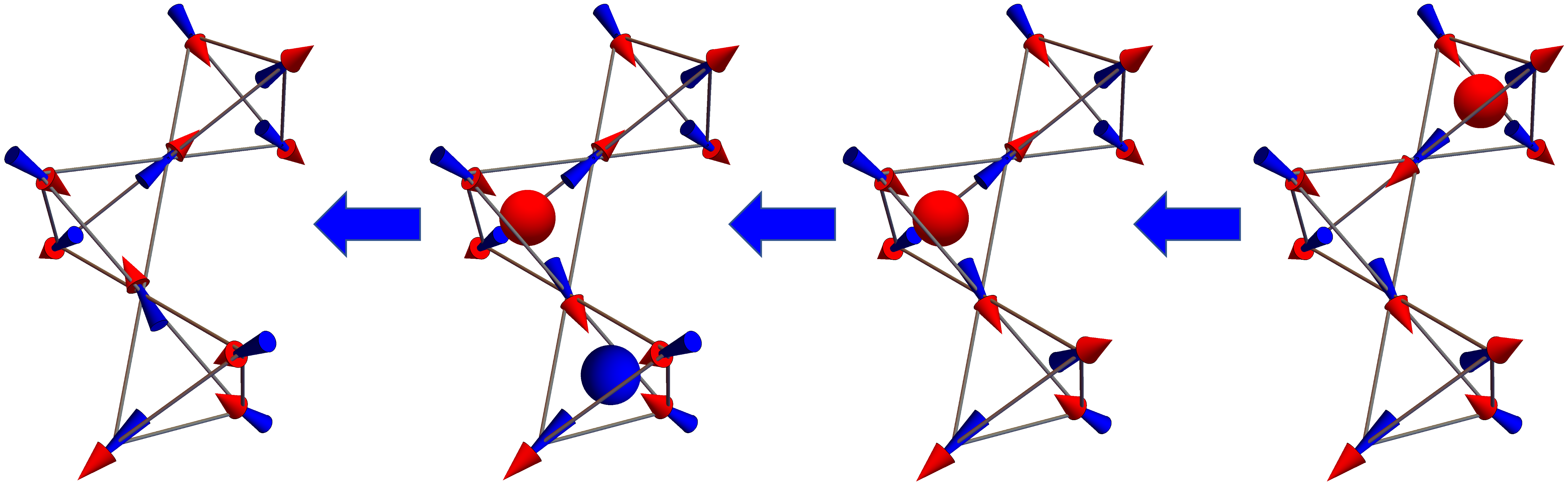

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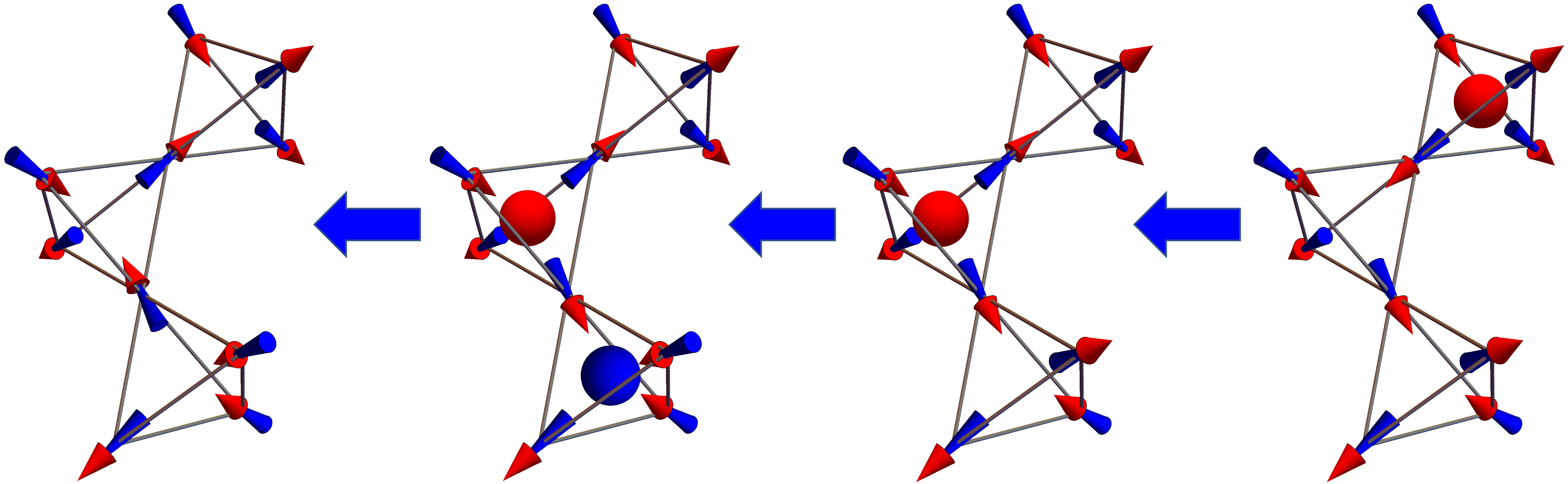
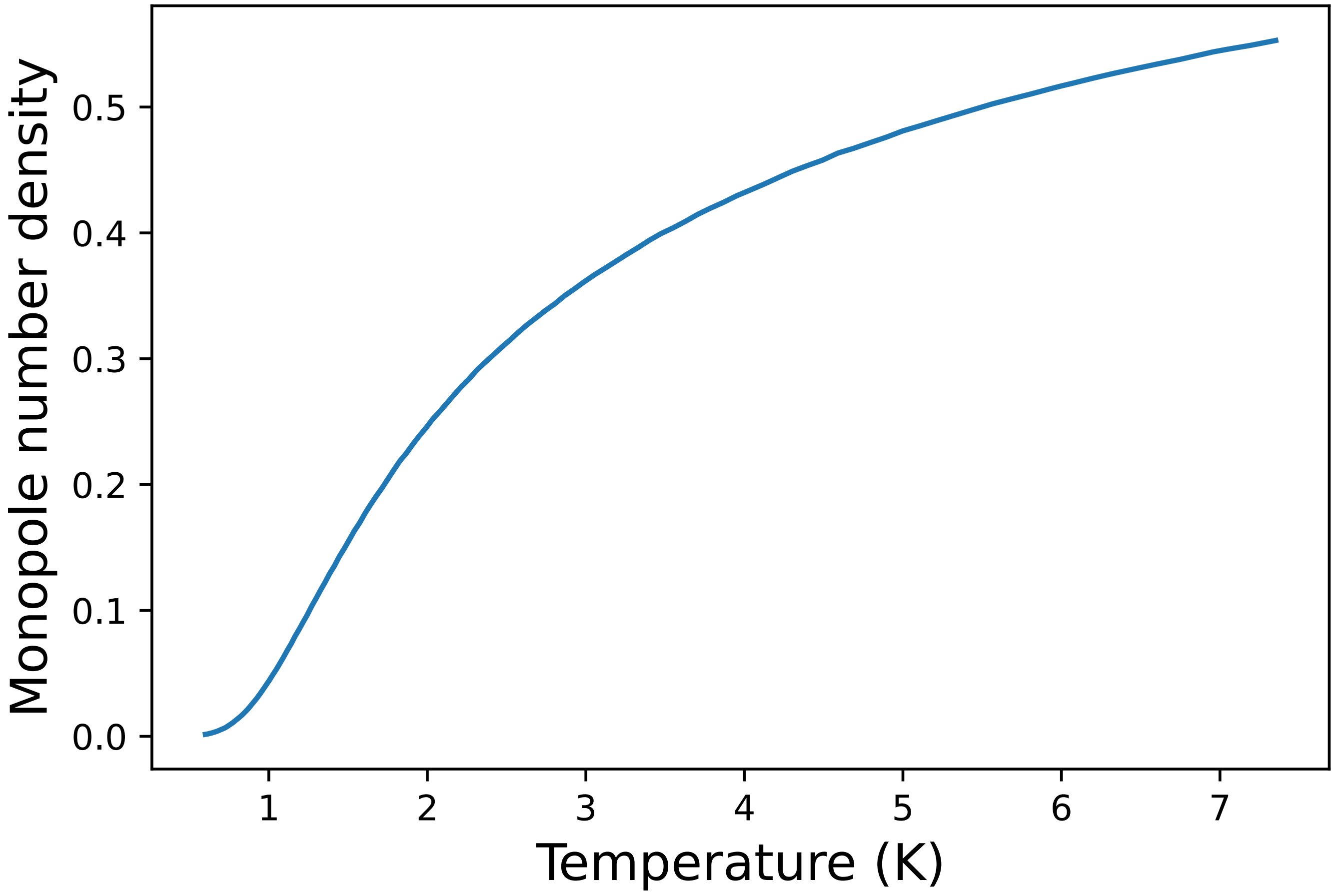


Monopole creation cost ~ 4 K

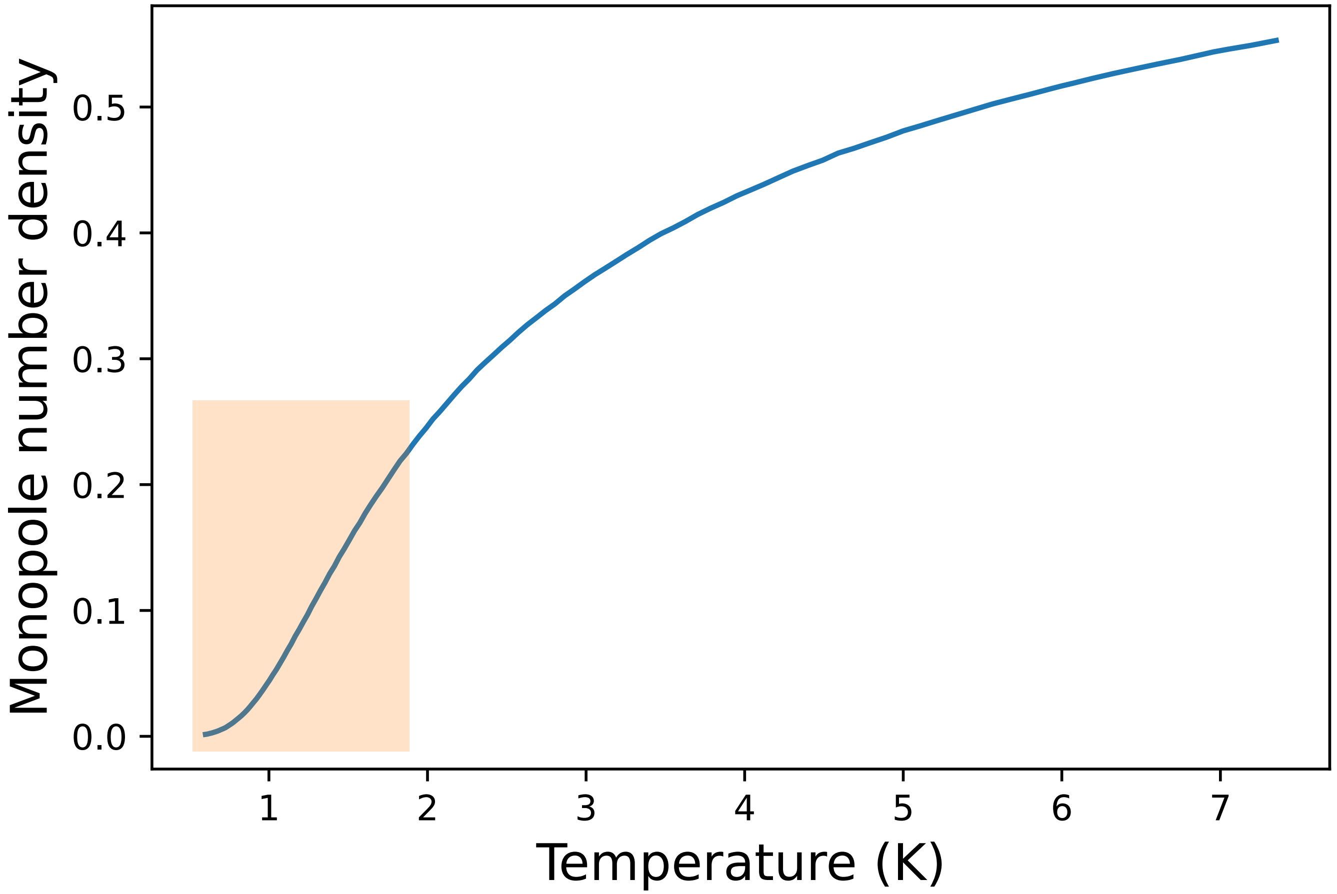
Monopole movement cost $\sim \pm\, 0.05$ K

Castelnovo, Mossner & Sondhi, *Nature* 451, 7174 (2008)

Emergent Magnetic Monopoles



Emergent Magnetic Monopoles



”Standard Model” (SM) of Spin Ice Dynamics

Spin flip dynamics ($T < 10$ K)

Quantum tunnelling between Ising states, enabled by transverse fields.

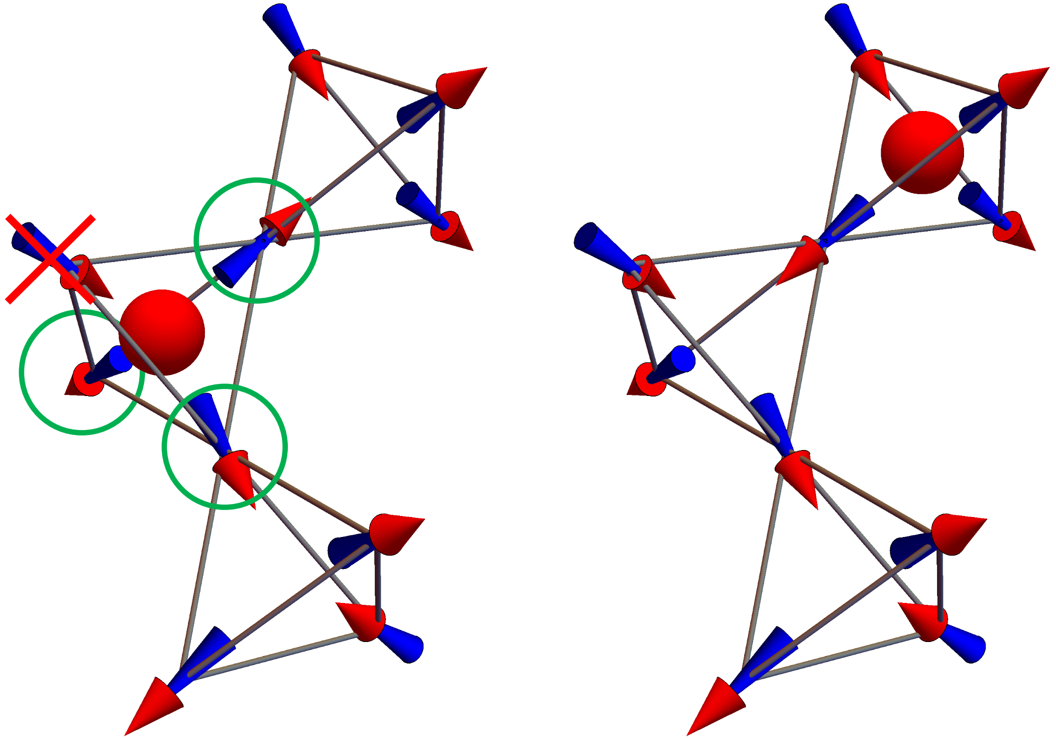
Flip attempts at constant rate $1/\tau_0$.

Monte Carlo time ∝ real time.

Monopole creation is rare at low T.

Monopole motion dominates dynamics.

Monopole motion

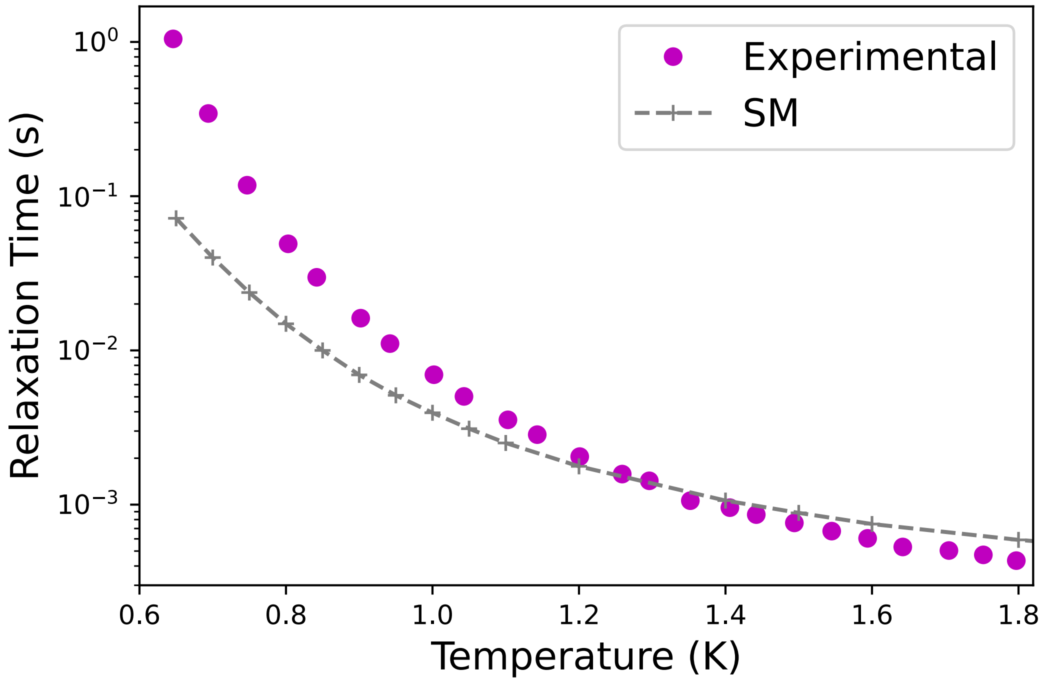


Random choice between 3 directions.

Ryzhkin, *J. Theor. and Exp. Phys.*, 128, 559 (2005).
Jaubert & Holdsworth, *Nature Physics* 5, 258 (2009).

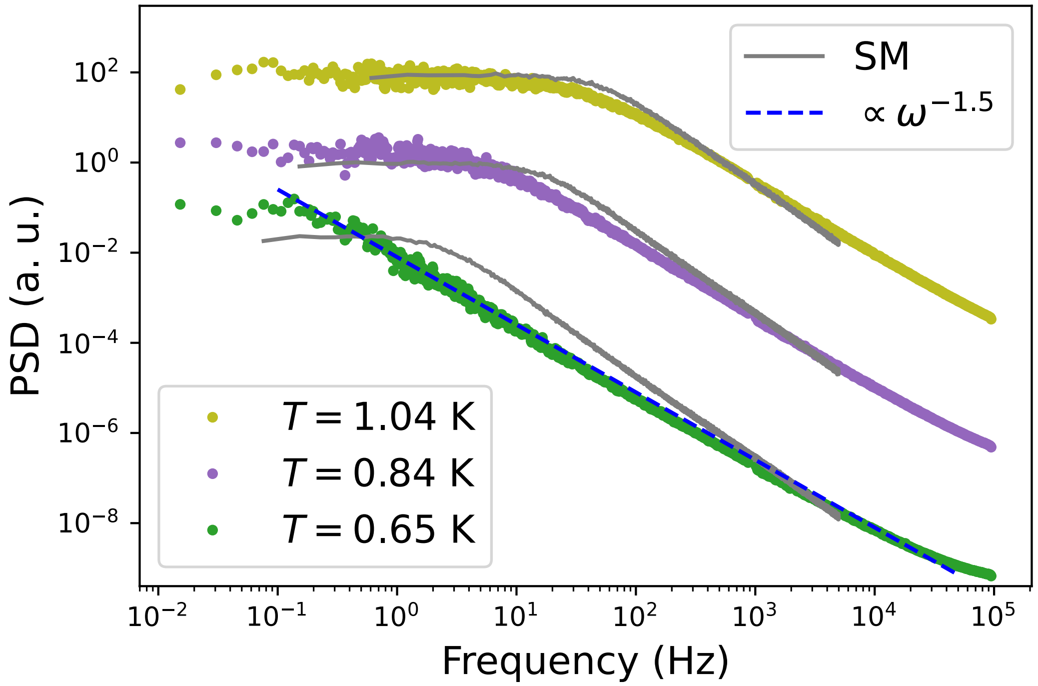
The Puzzles

Rapidly Diverging Relaxation Time



Previous explanations invoked extrinsic contributions (e.g. disorder, boundary effects).

Anomalous Magnetic Noise

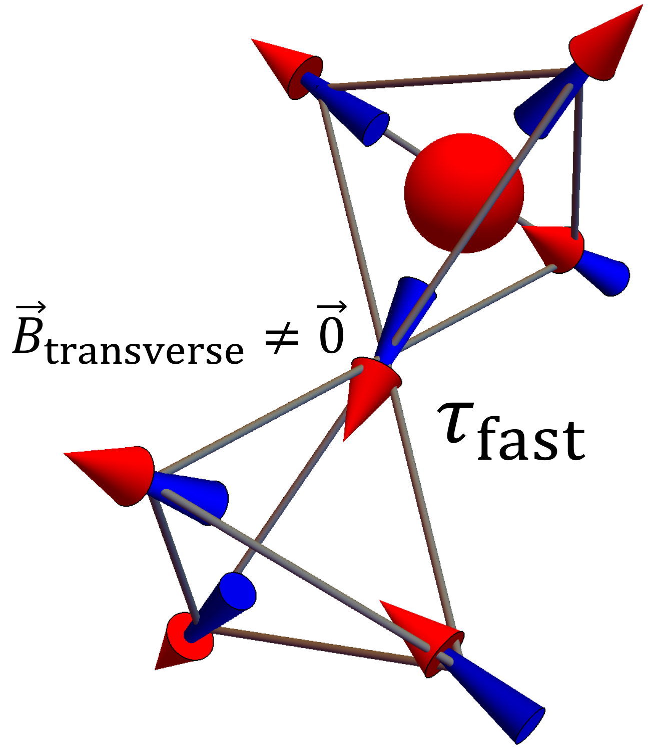


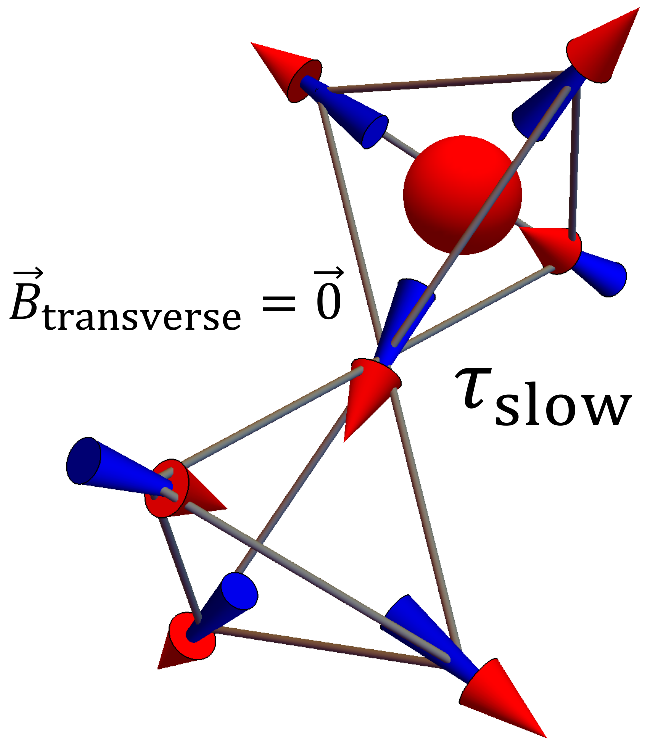
SM → Lorentzian $\propto [1 + (\omega\tau)^2]^{-1}$

Experiments → anomalous $\sim\omega^{-1.5}$

Experimental results from: A. M. Samarakoon, *et al.*, *Proceedings of the National Academy of Sciences* 119, e2117453119 (2022).

Beyond the "Standard Model" (bSM)





$\sim\frac{1}{3}$ of spins neighbouring monopole have $\vec{B}_{\text{transverse}} = \vec{0}$.

These attempt flips at lower rate $1/\tau_{\text{slow}}$.

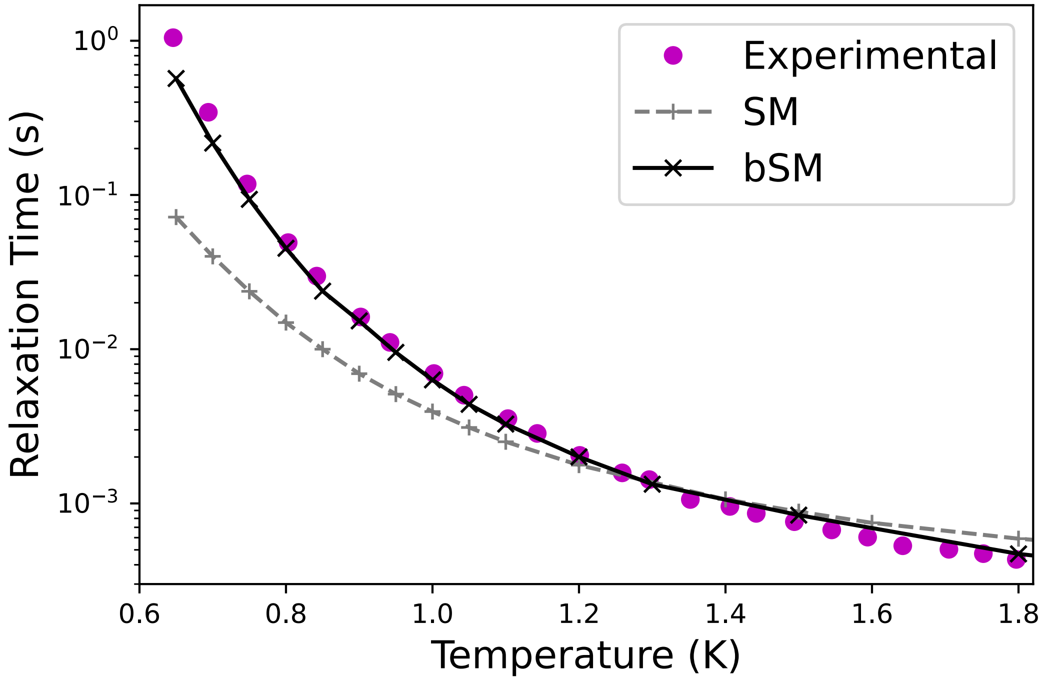
$Dy_2Ti_2O_7$: $\frac{\tau_{\text{slow}}}{\tau_{\text{fast}}} \approx 10^4$

(We approximate $\tau_{\text{slow}} = \infty$)

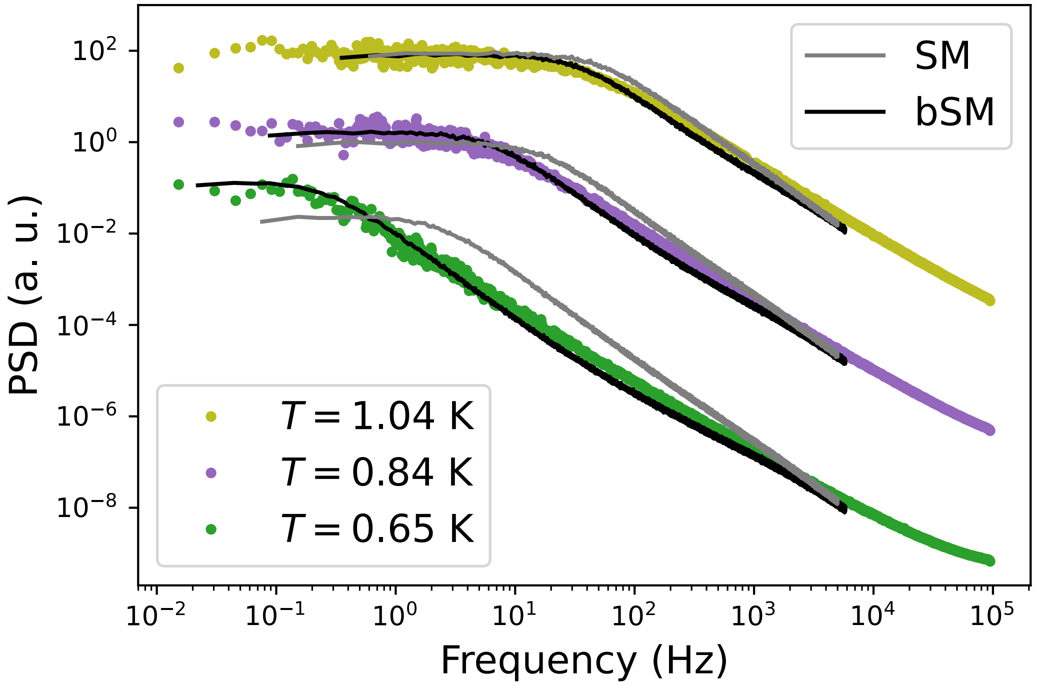
B. Tomasello, C. Castelnovo, R. Moessner, J. Quintanilla, *PRL* 123, 067204 (2019).

The Puzzles Revisited

Rapidly Diverging Relaxation Time



Anomalous Magnetic Noise



Rapidly diverging relaxation time and anomalous magnetic noise explained through purely intrinsic effects!

Fitting parameter SM: $\tau_0 = 200\ \mu s$

bSM: $\tau_{fast} = 85\ \mu s$

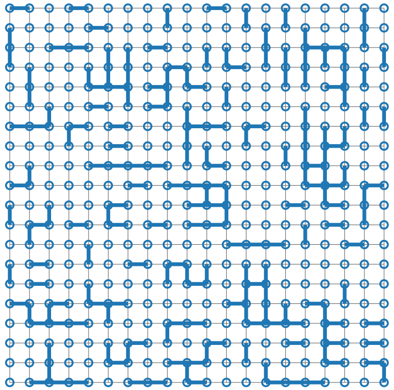
Aside: Percolation Theory

Pick your lattice of choice

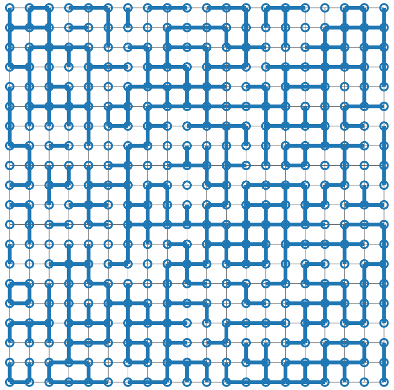
Randomly fill p $(0 \leq p \leq 1)$ of the bonds

A single percolating cluster appears at the critical point $p = p_c$

The percolating cluster is self-similar
→ **Fractal**



$p < p_c$



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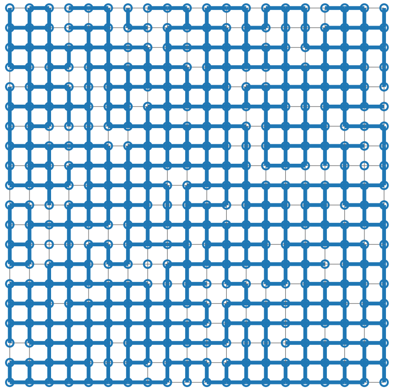


$p > p_c$

Links to Percolation Theory

Ice rules and slow spins leave on average 2 directions for a monopole to move in.

→(Dynamical) bond percolation problem on the diamond lattice.

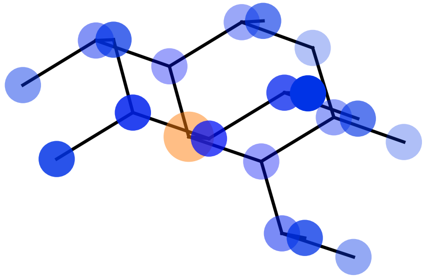
→Monopoles are random walkers on percolation clusters.

Close to critical filling fraction $p_c \approx 0.39$ → Fractal structure on length scales up to correlation length.

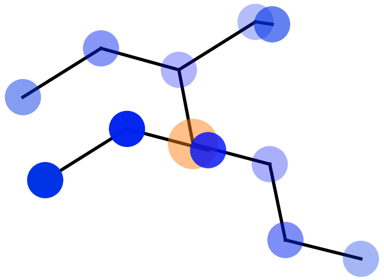
→ Monopoles move on an **emergent dynamical fractal**!

The Emergent Dynamical Fractal

SM



bSM

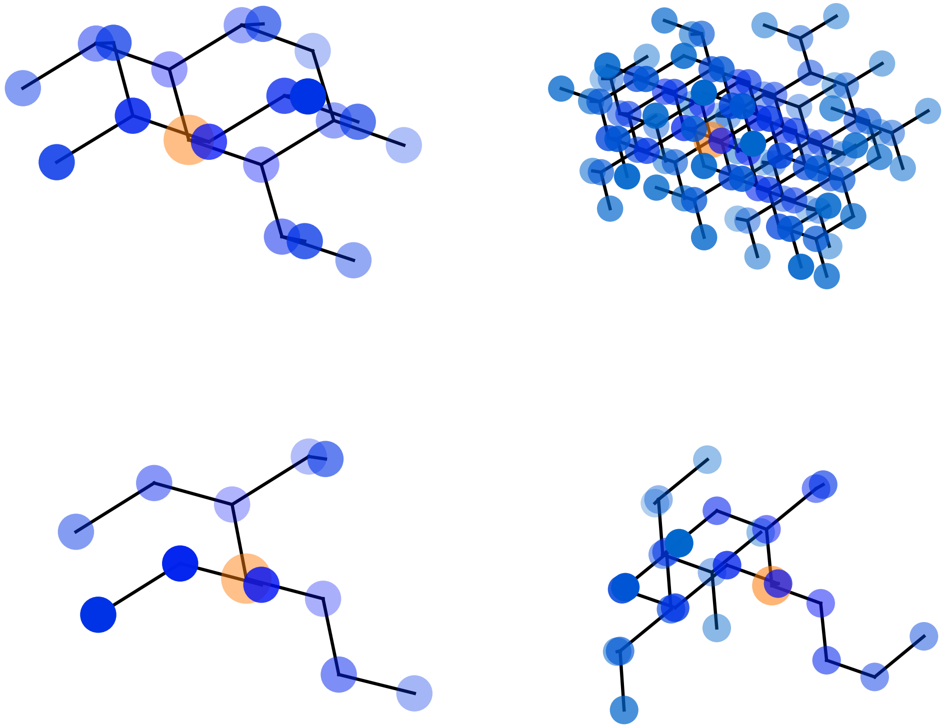


$n = 3$

The Emergent Dynamical Fractal

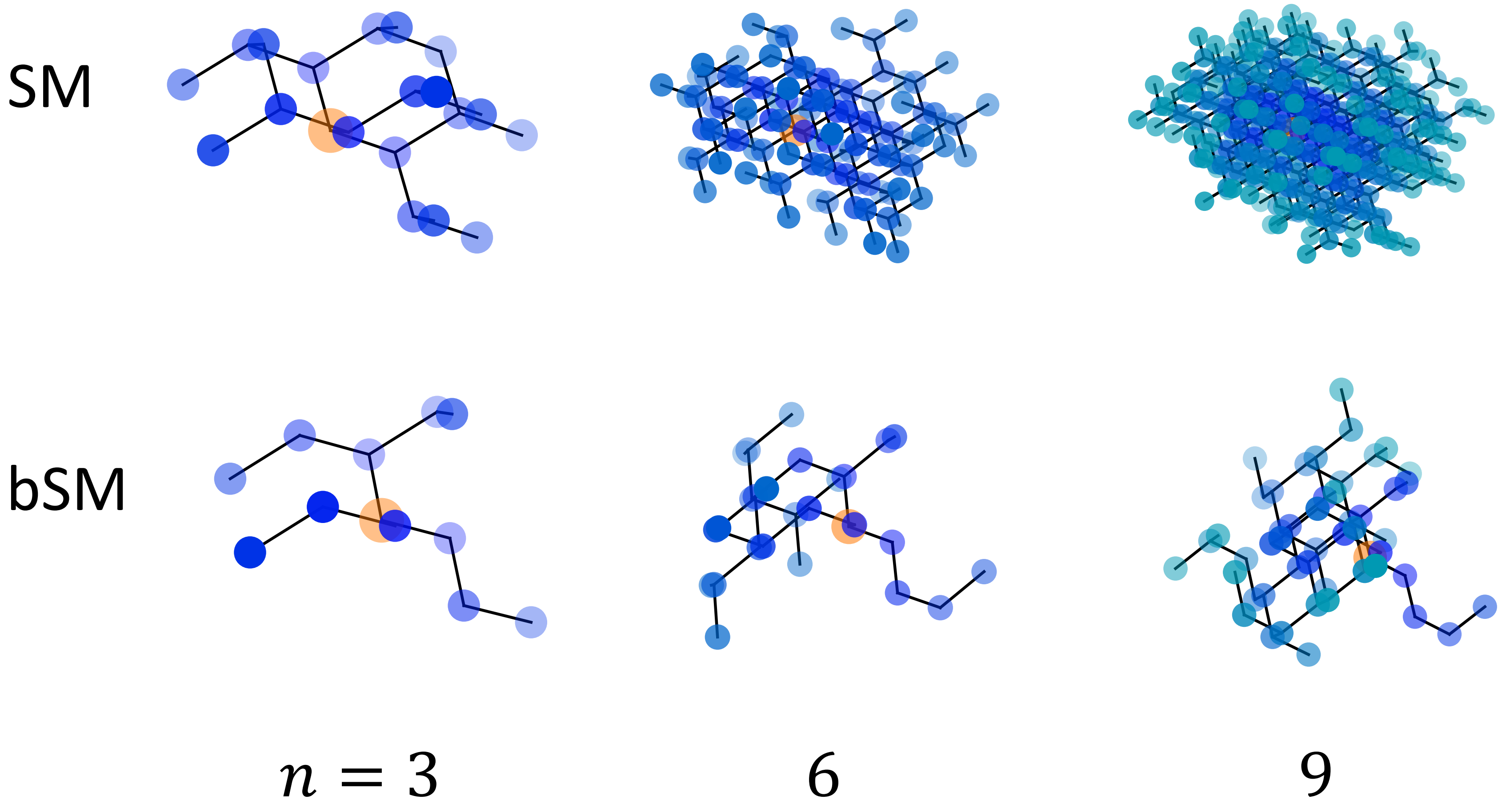
SM

bSM

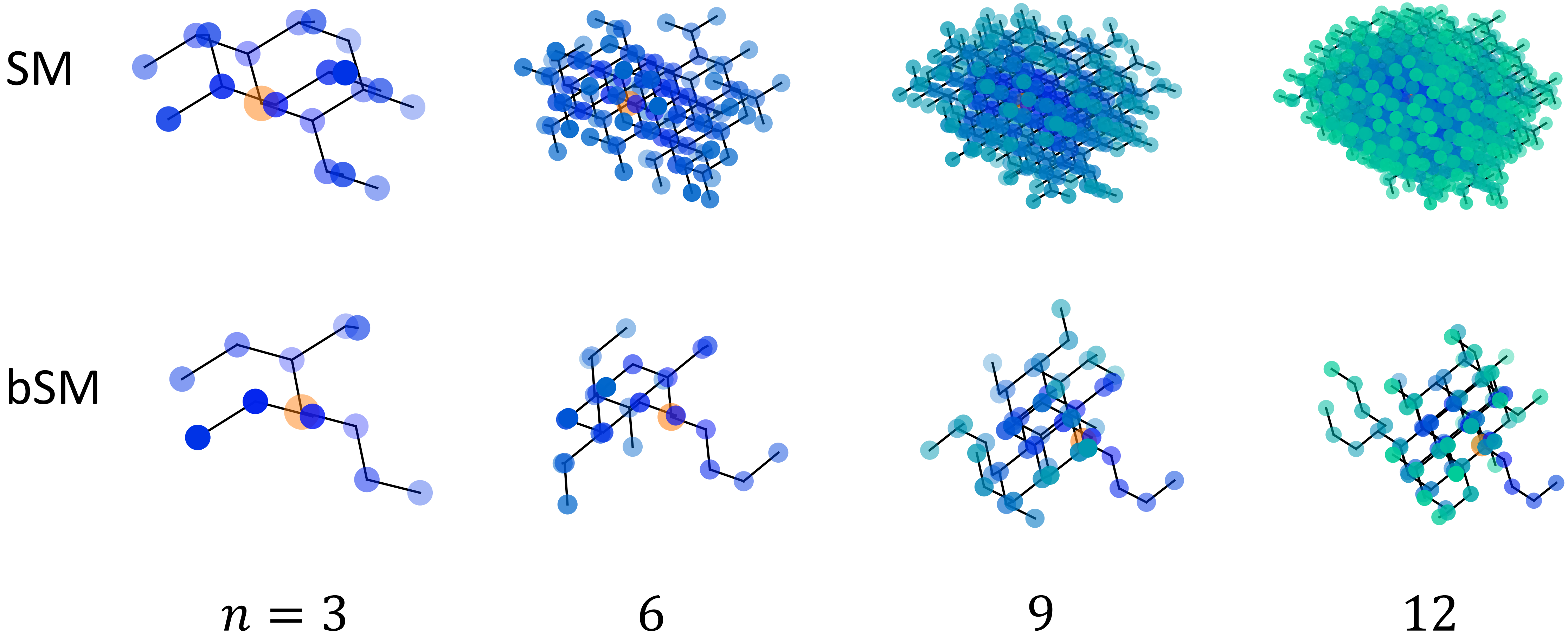


$n = 3$ 6

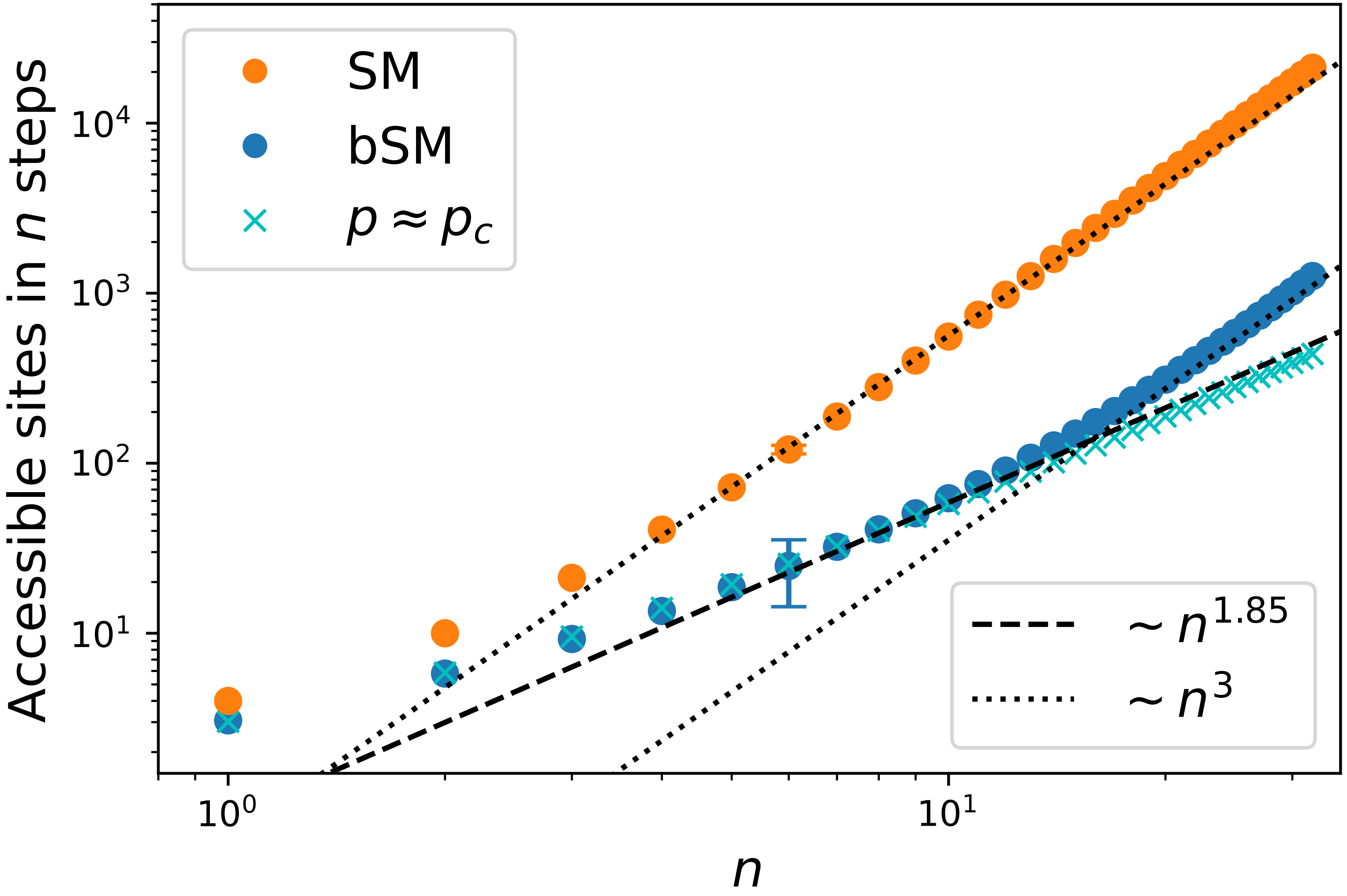
The Emergent Dynamical Fractal



The Emergent Dynamical Fractal



Cluster Growth



Percolation theory:

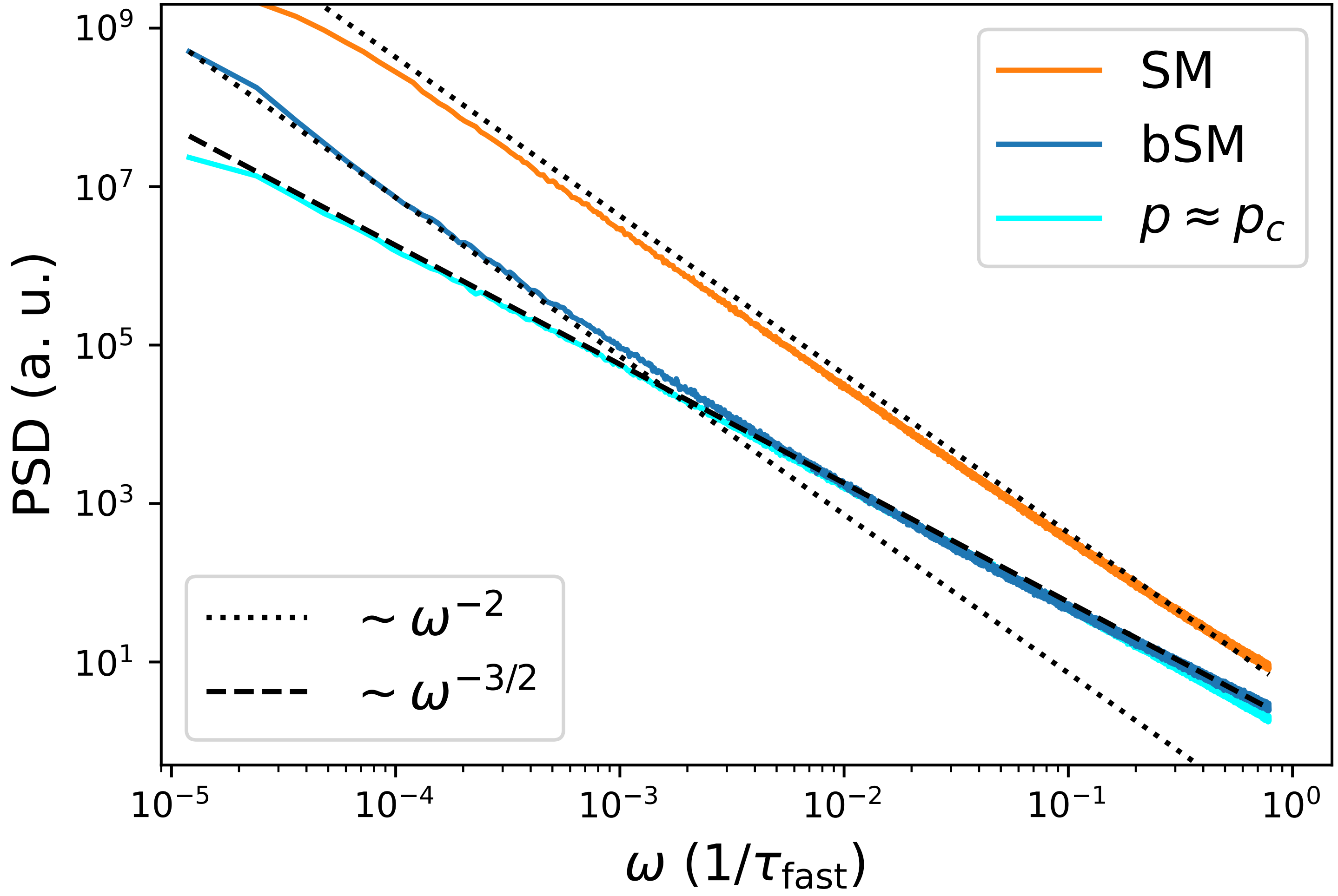
fractal exponent

$$S \sim \begin{cases} n^{1.85}, & n < n_\xi \\ n^3, & n > n_\xi \end{cases}$$

Fractal up to $n_\xi \approx 14$!

bSM monopoles can reach $\sim 130/2000$ sites in 14 steps.

Monopole Noise



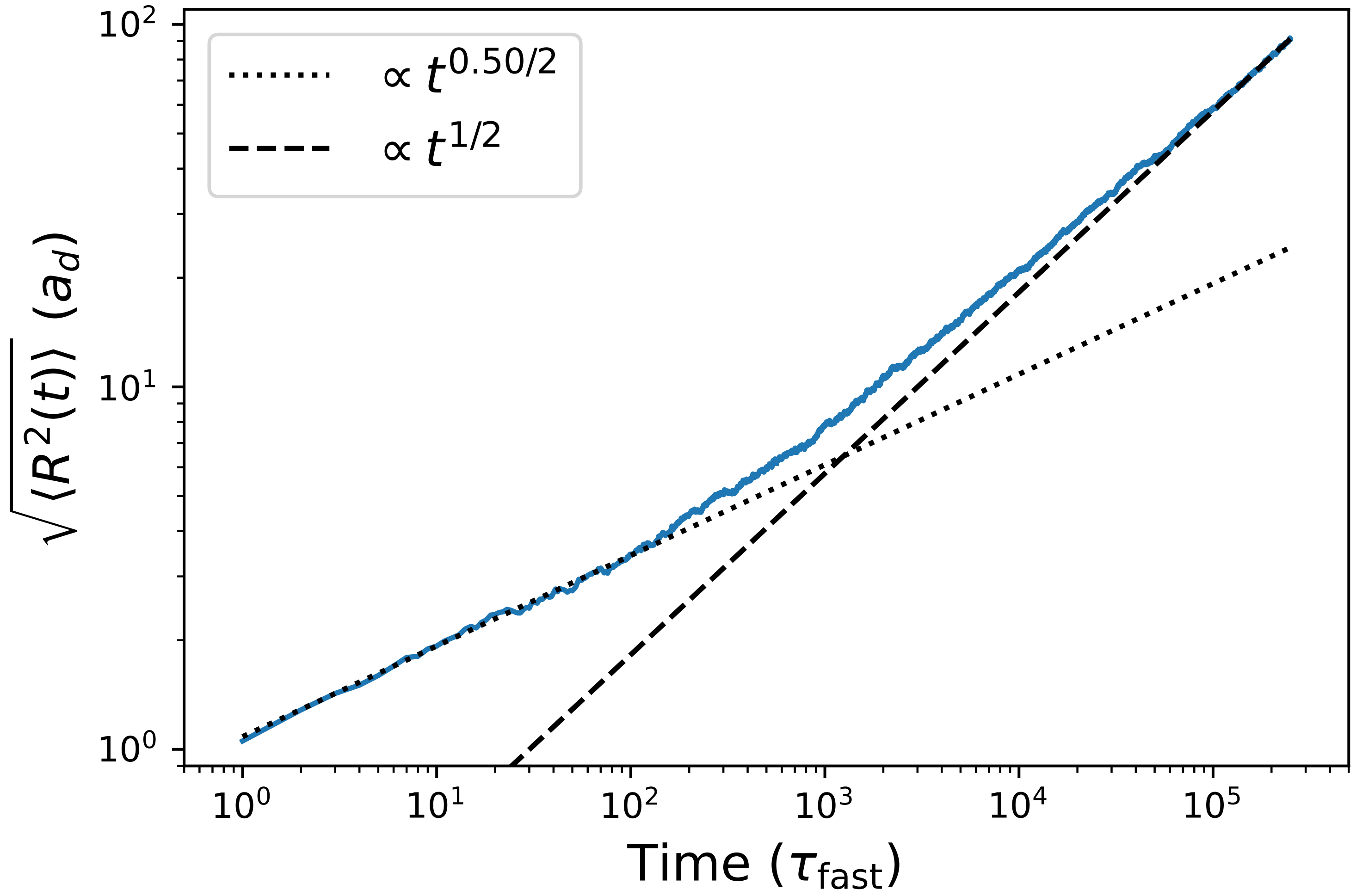
Percolation theory:

$$\mathrm{PSD} \sim \begin{cases} \omega^{-2}, & \omega < \omega_\xi \\ \omega^{-1.50}, & \omega > \omega_\xi \end{cases}$$

fractal exponent

Explains anomalous exponent seen in experiments!

Subdiffusive Monopoles



Percolation theory:

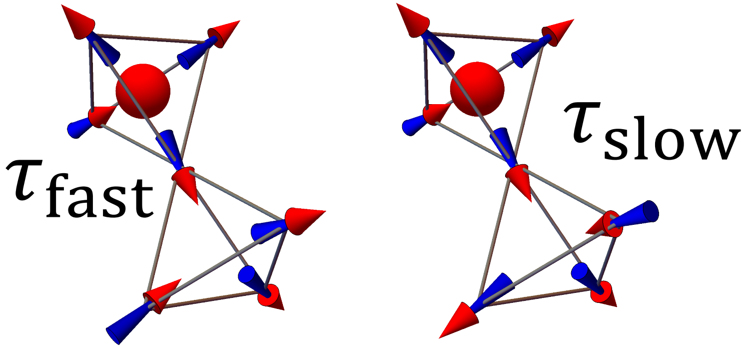
fractal exponent

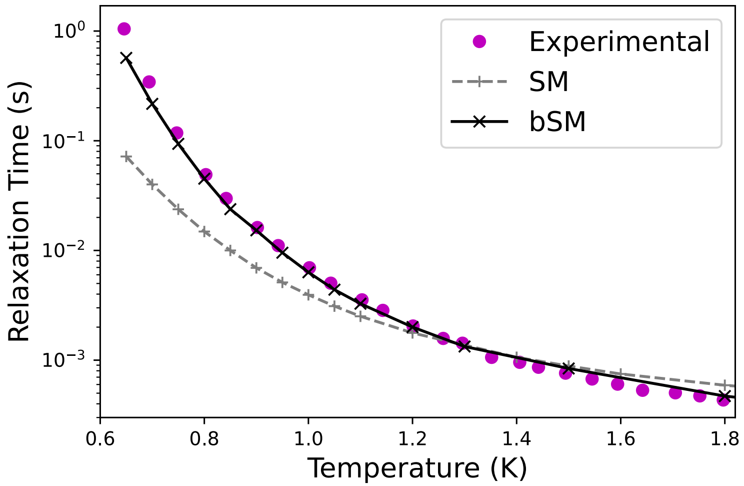
$$\langle R^2(t)\rangle \sim \begin{cases} t^{0.50}, & t < t_\xi \\ t, & t > t_\xi \end{cases}$$

Subdiffusive monopole motion on timescales up to $t_\xi \approx 10^3\ \tau_{\text{fast}}$.

Summary

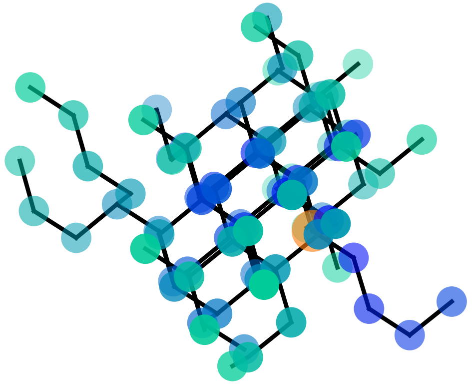
Bimodal distribution of internal transverse fields proves crucial to spin ice dynamics.



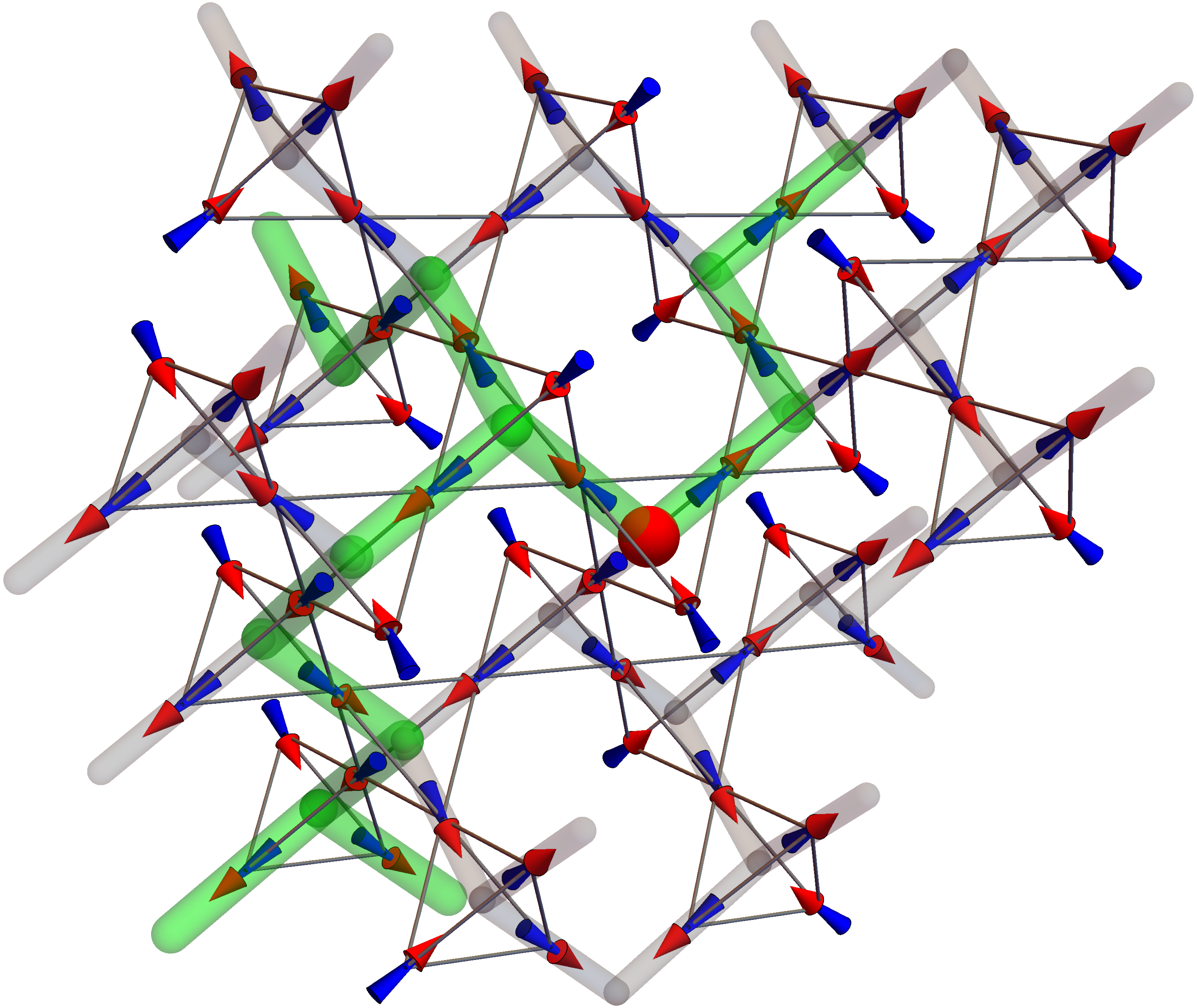


Explains dynamical properties of spin ice as a consequence of *intrinsic* effects.

Emergent fractal structure in a uniform, disorder-free bulk magnetic crystal.



Extra slides



Hamiltonians

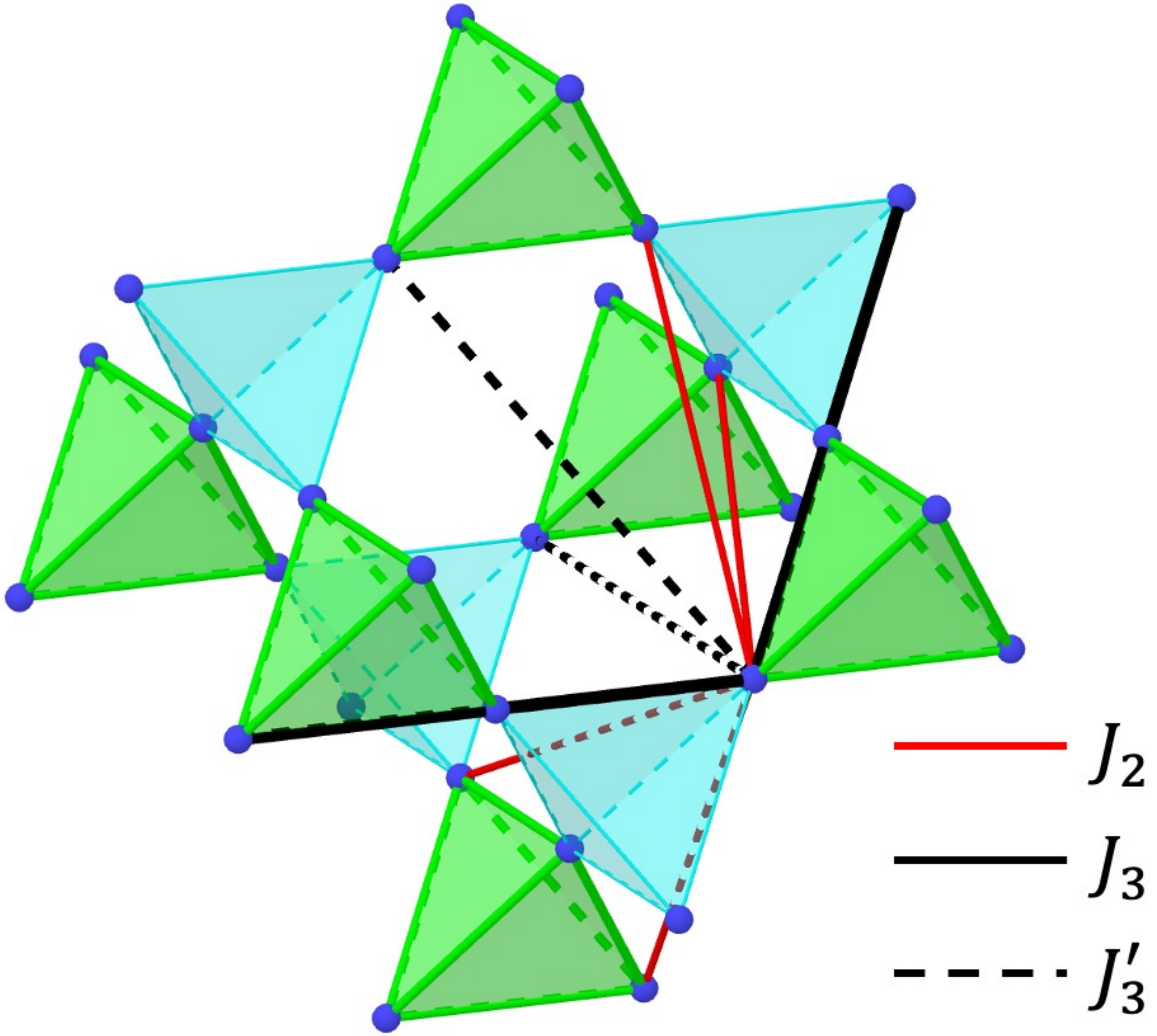
$$\mathcal{H}_{\mathrm{NN}} = -J_{\mathrm{eff}} \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$\mathcal{H} = Da^3 \sum_{i<j} \left[\frac{\boldsymbol{S}_i \cdot \boldsymbol{S}_j}{r_{ij}^3} - \frac{3(\boldsymbol{S}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{S}_j \cdot \boldsymbol{r}_{ij})}{r_{ij}^5} \right] + J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$+J_2 \sum_{\langle i,j \rangle_2} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_3 \sum_{\langle i,j \rangle_3} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_3' \sum_{\langle i,j \rangle_{3'}} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$Da^3 = 1.3224$ K $\quad J_1 = 3.41$ K $\quad J_{\mathrm{eff}} = J_{\mathrm{eff}}(T) \sim 5.7$ K

$J_2 = 0.0$ K $\quad J_3 = -0.00466$ K $\quad J_3' = 0.0439$ K



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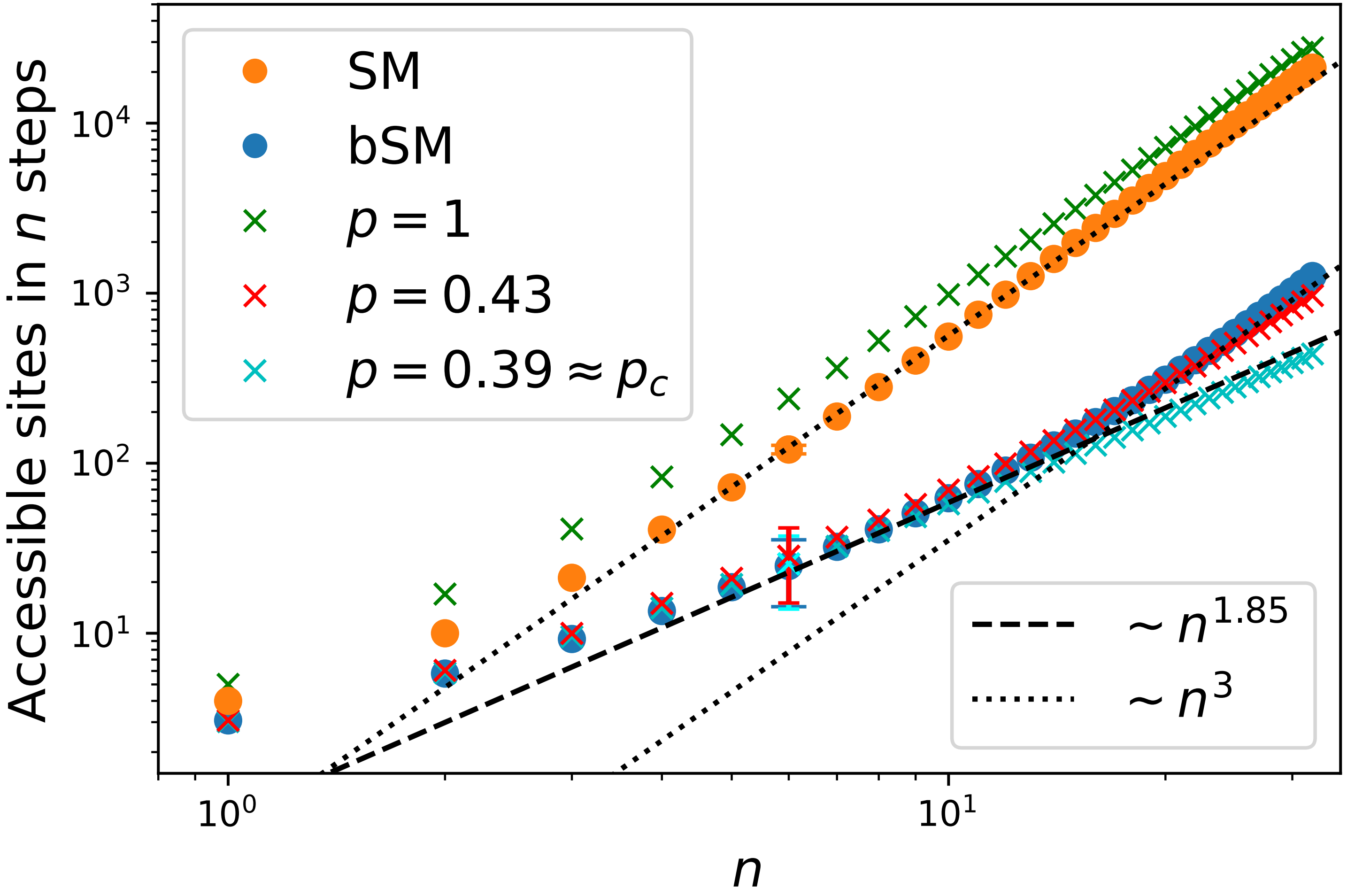

Percolation theory:

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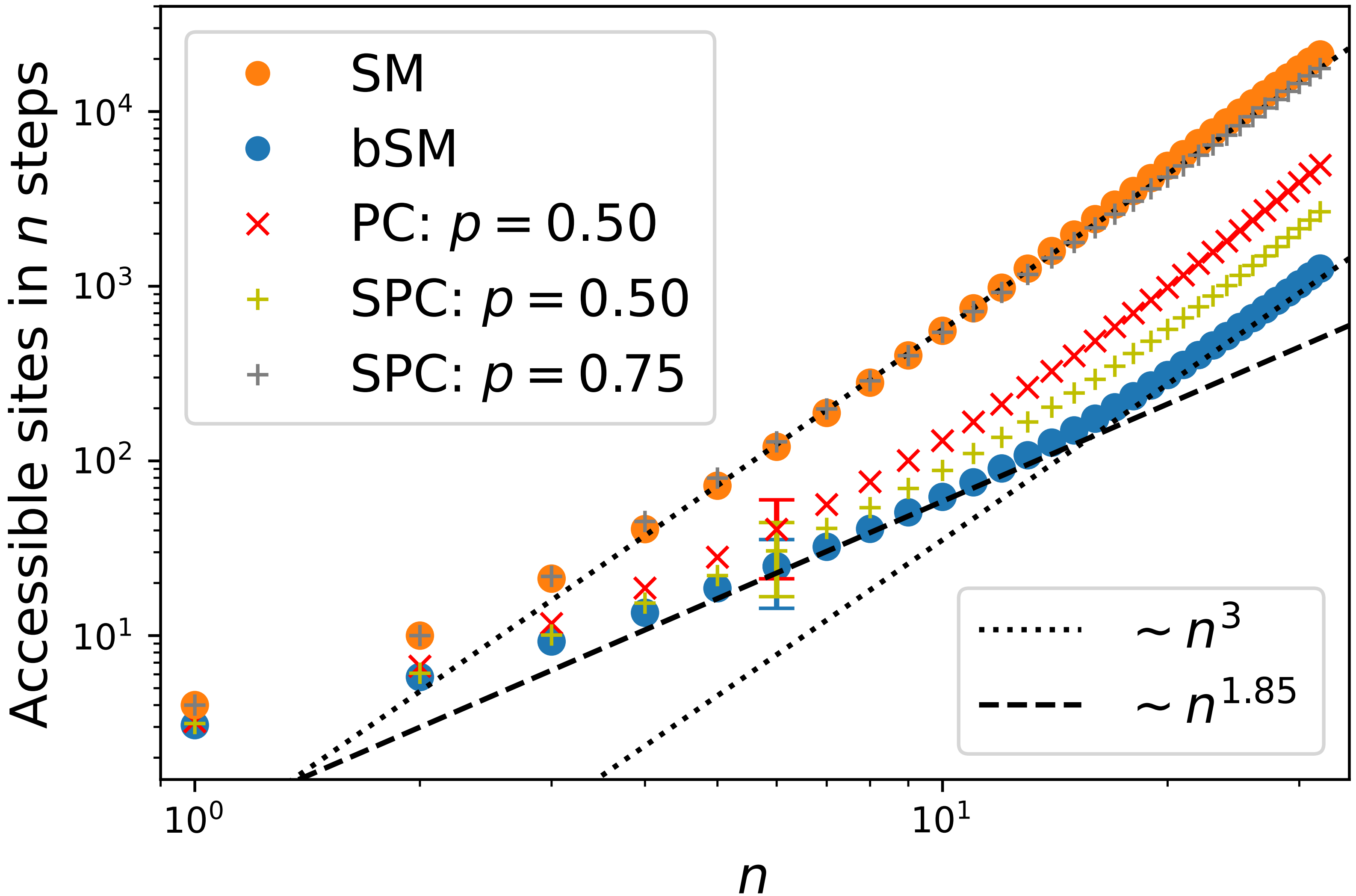
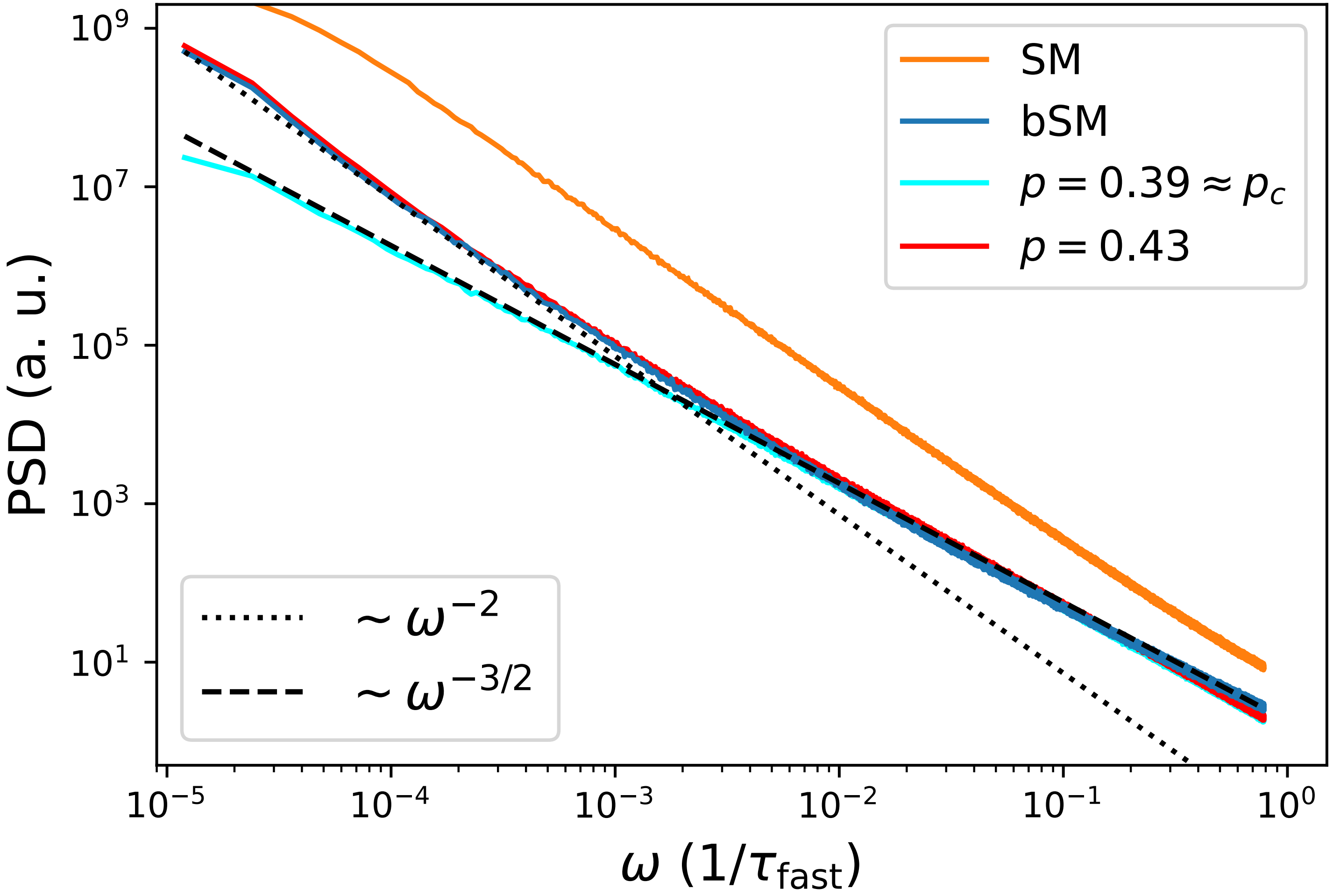
Fractal up to $n_\xi \approx 14$!

Structured percolation



Monopole Noise



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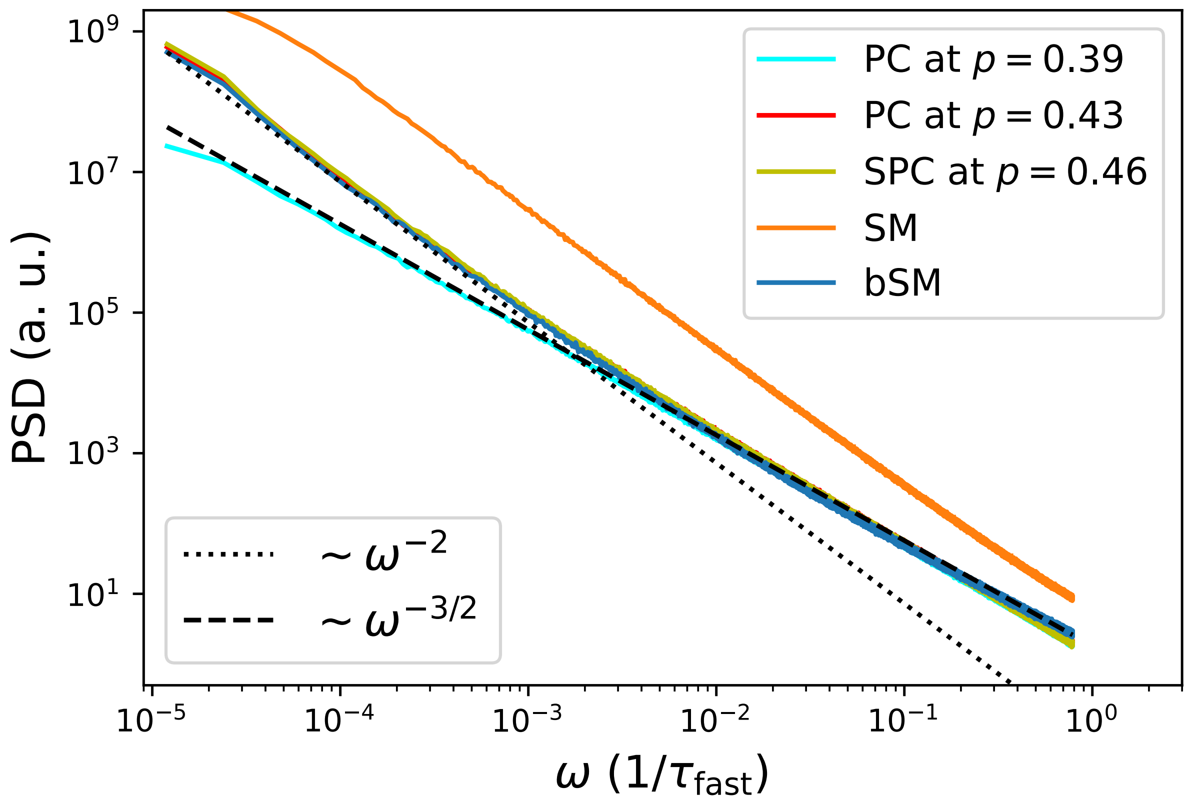
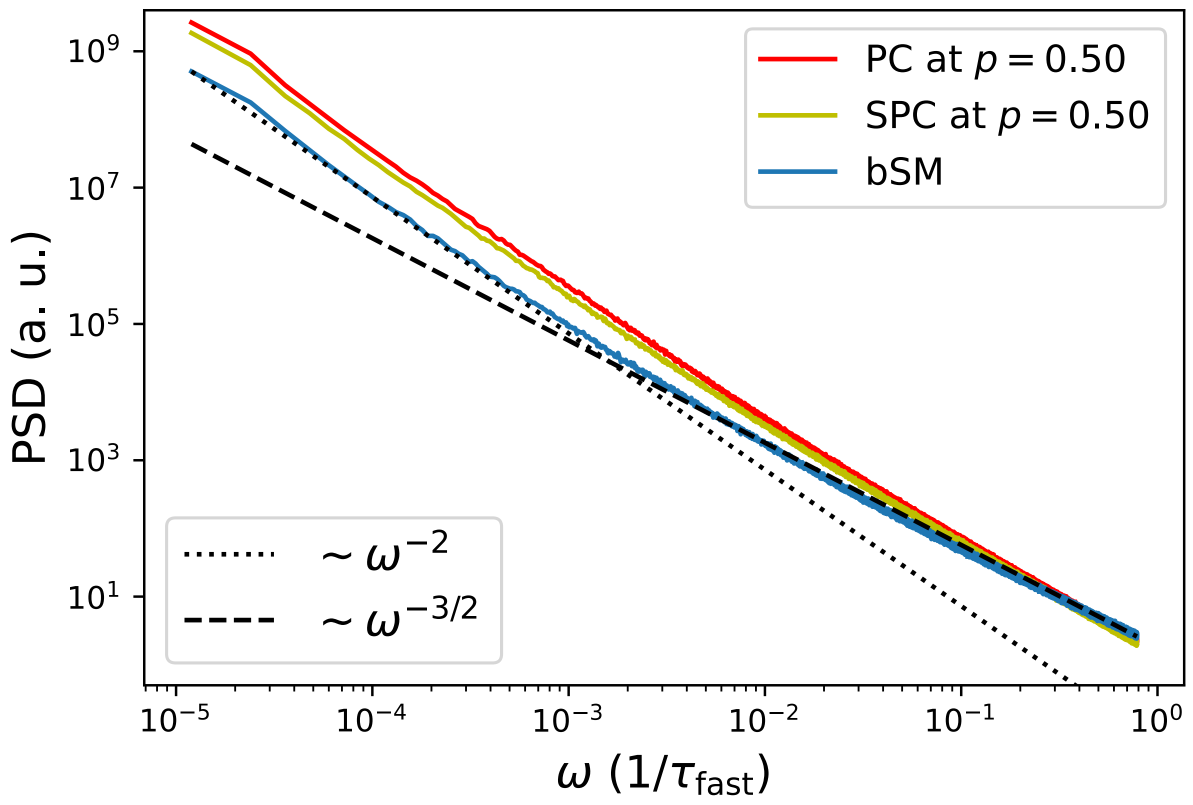
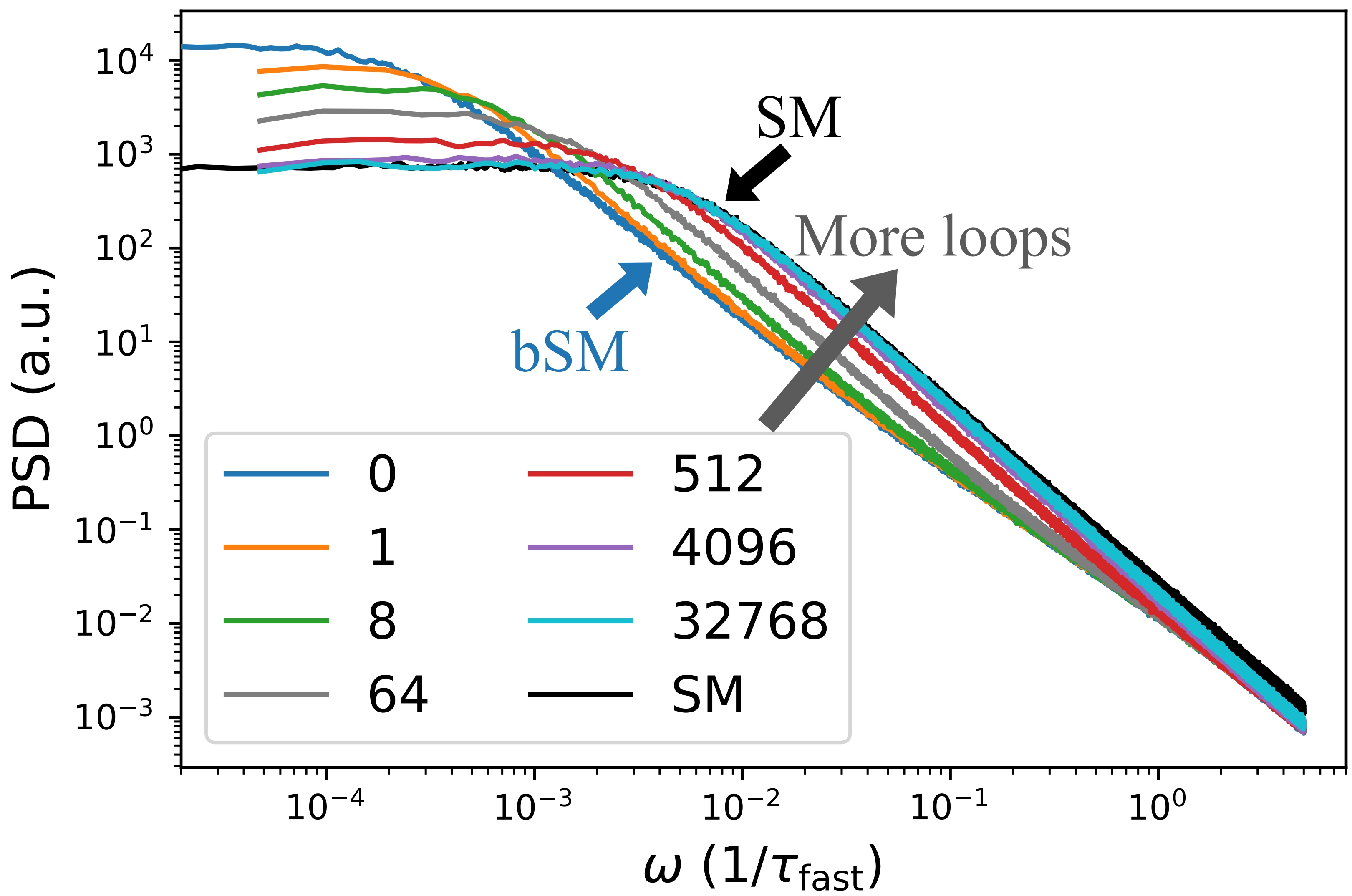
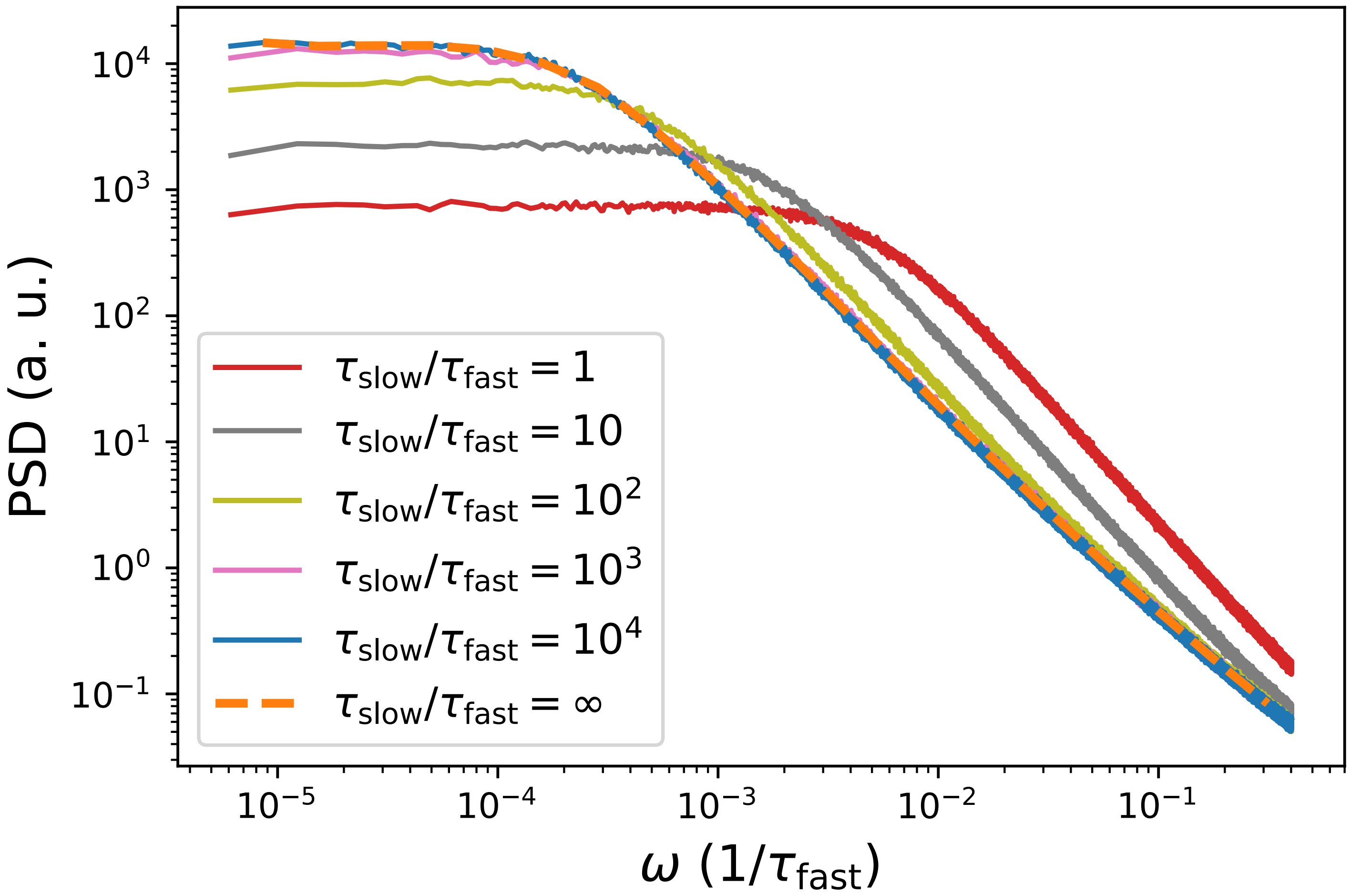




Loop updates



Varying slow timescale



Relaxation Time