

Electronic and structural order in oxides

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Kimber et al, arXiv:2202.05565 Guzman-Verri, Liang, PBL, arXiv:2205.14171 Guzman-Verri, Brierley, PBL, Nature 576, 429-432 (2019)

Functional materials often exploit broken symmetries induced by electronic correlations

Ferroelectric memory





LiMO₂ battery cathode



Electrocatalysis, fuel cells





Transparent conducting oxide



Memristor array



Electrocalorics



Magnetoresistive materials (Courtesy Soh, Aeppli)



These are all very complicated systems

Each has their own Hamiltonian, different fluctuations, different order parameters

Anything systematic in common?

Structural motifs

Electronically driven phase transitions

Tuning via chemical composition

Structural motifs

Perovskite RMO₃



M = 'active' transition metal ion R= Rare earth (3+); alkaline earth (2+); alkali (1+) Layered RMO₄



With these simple ingredients, can mix and match superconductors, ferroelectrics, ferroelastics, ferromagnets, antiferromagnets, charge density waves, ion-transporting compounds, photoabsorbers, catalysts, cathodes, ...

Routes to control - chemical doping - e.g. cuprate superconductors



Keimer et al, Nature 518 (2015)

Control via ionic size

A= Rare Earth (3+) or Alkaline Earth (2+); B = transition metal; O = oxygen



A schematic metal-insulator diagram for the filling-control (FC) and bandwidth-control (BC) 3*d* transition-metal oxides with perovskite structure. From Fujimori, 1992.



Phase transitions due to electron correlation

- There are many classes of materials where Coulomb correlations cause a phase transition from a high temperature disordered metallic phase to a low temperature ordered insulating phase $H = \sum t_{i:i} \hat{c}_{i}^{\dagger} \hat{c}_{i} + \sum U \hat{n}_{i:i} \hat{n}_{i:i}$
- Paradigm is the Hubbard model

$$H = \sum_{ij} t_{ij} \hat{c}_i^{\dagger} \hat{c}_j + \sum_i U \hat{n}_{i\uparrow} \hat{n}_{j\downarrow}$$

- But many possible broken symmetries, spin, orbit, lattice, charge etc.
- Central dogma is that the important parameters are all electronic
 - Ratio of interaction to bandwidth "U/W"
 - Number of carriers in the band
 - Coupling to lattice degrees of freedom is mostly irrelevant, or simply renormalizes parameters
- If the energy scales are all electronic, why isn't T_c typically eV
 - There has to be some source of entropy that lowers T_c from electronic energy scales
 - All "Mott" transitions couple to elastic degrees of freedom.

Response function for free fermi gas

Attractive interactions -> "charge density wave" (CDW)

$$\delta\rho(\mathbf{q},\omega) = \chi(\mathbf{q}\omega)V(\mathbf{q},\omega) \qquad \chi_o(\mathbf{q},\omega) = \frac{\delta\rho(\mathbf{q},\omega)}{V(\mathbf{q},\omega)} = 2\sum_k \frac{n(\mathbf{k}) - n(\mathbf{k} + \mathbf{q})}{\epsilon_{\mathbf{k}+\mathbf{q}} - \epsilon_{\mathbf{k}} - \hbar\omega}$$

$$\chi_{charge}(q,\omega) = \frac{\chi_0(q,\omega)}{1 + U_{charge} \chi_0(q,\omega)} \qquad \chi_o'/g(\mu)$$

$$\chi_{spin}(q,\omega) = \frac{\chi_0(q,\omega)}{1 - U_{spin} \chi_0(q,\omega)} \qquad \chi_o'/g(\mu)$$
Hubbard model: $U_{charge} = U_{spin} = U$
Repulsive interactions -> "spin density wave" (SDW)

Importance of low dimensionality – "nesting" of fermi surfaces Implies CDW's are always lattice-mediated – "Peierls" distortion

This is a good start but overly simplified

q/2k

Weak to strong coupling in the Hubbard model with one electron per atom

$$\chi_{\sigma}(\mathbf{q}, \omega) = \frac{\mu_B^2 \chi_o(\mathbf{q}, \omega)}{1 - \frac{1}{2} U \chi_o(\mathbf{q}, \omega)}$$

- Weak coupling: SDW at q=2k_f
- Intermediate coupling: SDW at q=2k_f ; CDW at $4k_{\rm f}$
- Strong coupling
 - Charge localised on site
 - Spins weakly coupled by superexchange



"Striped" phases of La_{0.33}Ca_{0.67}MnO₃



TEM image shows periodic ordered lattice

Interpreted as periodic array of 3+/4+ ions



S Mori, CH Chen and S-W Cheong, Nature **392** (1998) 473

Hemberger et al Phys. Rev. B 66, 094410 (2002)

UHF spin density in La_{0.5}Ca_{0.5}MnO₃



V Ferrari et al, Phys. Rev. Lett. 91, 227202 (2003)

Little true charge disproportionation – reapportionment between orbitals and screening Strong coupling to lattice – here evident because Mn(III) is Jahn-Teller active

Why are the stripes straight? Are they always?

Electronic phase transitions couple to structure at the atomic scale

These local degrees of freedom interact via long range elastic forces

These long-range correlations profoundly change the nature of the phase transitions Examples

- 1. Dynamics of domain walls in ferroelectrics
- 2. Ferroelectric phase transition in GeTe
- 3. Tuning of phase transitions by atomic size in transition metal oxides
- 4. "Quantum paraelectricity" in SrTiO₃
- 5. Better batteries and refrigerators through control of strong correlation

SrTiO3 – nearly ferroelectric, very low carrier density superconductor



105K: cubic to tetragonal 37K: quantum paraelectric



Müller, PRB (1979)

Softening transverse optic phonon, Curie-law dielectric constant

$$\omega_{TO}^2 \propto 1/\epsilon \propto T - T_0$$

Saturates below 35 K – quantum paraelectric? Or something else?

Ferroelectricity couples to elastic strain

Primary order parameter – polarization P – couples to elastic strain ϵ In a (nearly) cubic crystal

Electro-striction – ϵP^2

Produces a non-linear long-range coupling between polarization fluctuationsMacroscopic crystalline deformations at phase transitionExpected to drive ferroelectric phase transition first-order

Flexoelectricity – ε dP/dx

Coupling is harmonic – mixed optic (TO)/acoustic(TA) modes

Coupling vanishes as q -> 0

Softening TO mode hybridizes with TA – incommensurate fluctuations?

GG Verri, C Liang, PBL, unpublished

Landau theory for ferroelectric polarization *P* with electrostrictive coupling with a transition temperature T₀

$$F(P,\epsilon) = \int d^3r \left[a(T-T_0)P^2 + bP^4 + K\epsilon^2 + c\epsilon P^2 \right]$$

minimise F to eliminate strain variable ϵ gives $\epsilon \propto P^2$ and thus

$$F(P) = \int d^3r \left[a(T - T_0)P^2 + bP^4 - (K/c^2)P^2 \langle P^2 \rangle \right]$$

looks innocuous except the last term is infinitely long range

elastic strain is the metric of space and cannot be arbitrarily deformed

strain compatibility condition
$$\nabla \times (\nabla \times \epsilon)^T = 0$$

Warm-up problem : a domain wall in a pseudo-cubic ferroelectric



2D elastic theory from Ginzburg-Landau

$$\begin{split} S &= \int d\mathbf{r} \left[\overbrace{-(\partial_t \mathbf{P})^2 + \gamma(\nabla \mathbf{P})^2 + r|\mathbf{P}|^2 + u|\mathbf{P}|^4}^{\text{Polarization}} + \underbrace{\frac{a_1}{2}\phi_1^2 + \frac{a_2}{2}\phi_2^2 + \frac{a_3}{2}\phi_3^2}_{\text{Strain}} + \underbrace{q_2\phi_2(P_x^2 - P_y^2)}_{\text{Strain}} \right] \\ &+ \sum_{\mathbf{q}} \sum_{\alpha,\beta} \tilde{P}_\alpha \left(f_\alpha q_\alpha^2 \delta_{\alpha\beta} + g_{\alpha\beta} \frac{q_\alpha q_\beta}{q^2} + h_{\alpha\beta} q_\alpha q_\beta \right) \tilde{P}_\beta \\ &\xrightarrow{\text{Dipolar interactions}} \\ \phi_1 &= (\epsilon_{xx} + \epsilon_{yy})/\sqrt{2}, \quad \phi_2 = (\epsilon_{xx} - \epsilon_{yy})/\sqrt{2} \quad \phi_3 = \epsilon_{xy} \\ \nabla^2 \phi_1 - (\partial_x^2 - \partial_y^2)\phi_2 - 2\sqrt{2}\partial_x \partial_y \phi_3 = 0 \quad \Longrightarrow \quad \text{Elastic compatibility condition} \\ S &= \int d\mathbf{r} \left[\gamma(\nabla \mathbf{P})^2 + r|\mathbf{P}|^2 + u|\mathbf{P}|^4 \right] - \frac{m_2}{2} \int d\mathbf{k} |\Gamma(k)|^2 H(\mathbf{k}) + \sum_{\alpha,\beta} \tilde{P}_\alpha \left(g \frac{q_\alpha q_\beta}{q^2} + h_{\alpha\beta} q_\alpha q_\beta \right) \tilde{P}_\beta \\ H(\mathbf{k}) &= 1 - \frac{(D\Omega^4 + \Omega^2(k_x^2 + k_y^2) + \frac{4}{A}k_x^2k_y^2)}{(k_x^2 + k_y^2)^2 + Ck_x^2k_y^2 + D\Omega^4 + \Omega^2k_x^2(1 + D)} \quad \Longrightarrow \quad \begin{array}{c} \text{Infinite range,} \\ \text{retarded, strainmediated coupling} \end{array}$$

Mode dispersion for small amplitude, long wavelength displacements



k^{1/2} dispersion is analogous to 2D plasmon
Gap is analogous to plasmon-polariton
Gap magnitude ~ shift in T_c by clamping of strain

Slow phonons: response function depends on how efficiently gapped modes can decay into bulk phonons

- For finite v_s, strain interaction is retarded
- $v_s \rightarrow 0 \implies$ static strain potential, energy gap $\Delta \sim \sqrt{\eta |P_0|^2} \sim 10 \text{meV}$
- Wall is coupled to phonons with $k_x < k_c \sim 1/L;$ $\omega_c = v_s k_c \sim 10 \text{meV}$
- Excitations can decay if $v_s k_y < \omega < \sqrt{\omega_c^2 + v_s^2 k_y^2}$



Large elastic "gap"

- Coupling to low |k| phonons introduces response at ω ~ v_sk_y
- Fast excitations with $\omega > \omega_{\rm acoustic}$ are damped
- For faster v_s, larger ω_c, gapped mode is absorbed by phonon continuum



Small elastic "gap"

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GeTe: cubic (rocksalt) to rhombohedral (ferroelectric) transition at 670 K



r (Å) 5.0 <100>c <112>c а <110>c а 2.0 3.0 4.0 6.0 7.0 8.0 r-GeTe 300 K Temperature (K) 009 009 009 <111>c 1.5 0.5 G(r) G(r) (a.u) -0.5 c-GeTe 825 K -1.0 b [100]_c [111]_c [100]_c[120]_c [112]_c [110]_c M-N M-M M-N M-M M-N M-M b 8 10 12 6 [100] r (Å) [110] 1.5 b С Peak height $(Å^{-2})$ [111] k_z 800 [112] e (K) 700 600 500 0.5 300 0.0 -0.4 -0.2 0.0 0.2 0.4 Off-centre Ge distortion (Å) 400 500 700 800 600 Temperature (K)

Correlations become strongly anisotropic above Tc – long-range in (100)



Predicted long-range kernel

Kimber et al, arXiv:2202.05565

Ionic size effects in perovskite transition metal oxides ABO₃

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An example: Perovskite manganites

- A "doped" oxide e.g. La_{1-x}Ca_xMnO₃ where the formal valence of Mn varies between Mn³⁺ and Mn⁴⁺
- A "strongly correlated" electron system close to a (Mott) metal-insulator transition





Doping dependence of transition



Urushibara et al 1995

Atomic size dependence of the transition





• Is there another mechanism that can contribute to the tilt dependence?

Cooperative Jahn Teller effect in perovskite manganites

Jahn-Teller Breathing Shear
$$H=\sum_{\bf k}a_T|T_{\bf k}|^2+a_B|B_{\bf k}|^2+a_S|S_{\bf k}|^2 \quad {\rm Elastic\ energy}$$

In manganites, J-T is often a locally broken symmetry that produces a "bad metal" phase competing with ferromagnetic metal

As before, eliminate the other components while maintaining elastic compatibility



Interaction V(k) is moderated by rotation angle ϕ_0 Long-range coupling is quadratic (primary order parameter)

G Guzman-Verri et al, 2019

κ , $\gamma, \delta h$ fixed



Jahn-Teller





Cartoon of single Jahn-Teller polaron





Data: Rodriguez-Martinez & Attfield, PRB (1996)



Guzman-Verri. Brierley and PBL Nature 2019

SrTiO3 – nearly ferroelectric, very low carrier density superconductor





Gervais, PRB (1993)

1000

 $\omega_{TO}^2 \propto 1/\epsilon \propto T - T_0$

Softening transverse optic phonon, Curie-law dielectric constant

Saturates below 35 K – quantum paralectric? Or something else?

Ferroelectricity in STO couples to elastic strain

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Softening TO mode hybridizes with TA – incommensurate fluctuations?

GG Verri, C Liang, PBL, arXiv:2205.14171

Harmonic Ginzburg-Landau theory for isotropic flexoelectric elastic displacement u(q)polarization order parameter P(q)

Stabilising non-linearity $H = \int \frac{d^3q}{(2\pi)^3} \omega_{\text{TO}}^2(\boldsymbol{q}) \left| P(\boldsymbol{q}) \right|^2 + \omega_{\text{TA}}^2(\boldsymbol{q}) \left| u(\boldsymbol{q}) \right|^2 + (1/2)V(\boldsymbol{q}) \left[P(\boldsymbol{q})u^*(\boldsymbol{q}) + P^*(\boldsymbol{q})u(\boldsymbol{q}) \right] + \frac{\gamma}{4} \int d^x P^4(\boldsymbol{x}) d^x P^4$

$$\omega_{\text{TO}}^2(\boldsymbol{q}) = r + cq^2 + hq^4, \qquad r = r_0(T - T_0)$$
$$\omega_{\text{TA}}^2(\boldsymbol{q}) = v_s^2 q^2, \qquad V(\boldsymbol{q}) = fq^2$$

 $\Omega_{\mathrm{TO}}^{2}(\boldsymbol{q}) = \frac{1}{2} \left(\omega_{\mathrm{TA}}^{2}(\boldsymbol{q}) + \omega_{\mathrm{TO}}^{2}(\boldsymbol{q}) + \sqrt{V^{2}(\boldsymbol{q})} + \left[\omega_{\mathrm{TO}}^{2}(\boldsymbol{q}) - \omega_{\mathrm{TA}}^{2}(\boldsymbol{q}) \right]^{2} \right)$

 $\Omega_{\mathrm{TA}}^{2}(\boldsymbol{q}) = \frac{1}{2} \left(\omega_{\mathrm{TA}}^{2}(\boldsymbol{q}) + \omega_{\mathrm{TO}}^{2}(\boldsymbol{q}) - \sqrt{V^{2}(\boldsymbol{q}) + \left[\omega_{\mathrm{TO}}^{2}(\boldsymbol{q}) - \omega_{\mathrm{TA}}^{2}(\boldsymbol{q})\right]^{2}} \right)$

Mean field ferroelectric transition at T₀

Coupled harmonic modes



Parameters approximate for KTaO3

Mean field phase diagram (harmonic)



Conventional Paraelectric = follows Curie-Weiss law, no modulation in the acoustic branch.

Modulated Paraelectric = follows Curie-Weiss law, modulated acoustic branch.

Modulated Solid = Acoustic branch has condensed @ finite q @ Tmod.

Self-consistent treatment of non-linearities - classical phonons

$$\frac{\gamma}{4} \int d^{x}P^{4}(x) \rightarrow \frac{6\gamma}{4} \int d^{3}x \langle P(x)P(x) \rangle P^{2}(x) = \frac{3\gamma}{2} \left(\underbrace{\langle P(x)P(x) \rangle}_{=s} \right) \left(\int d^{3}x P^{2}(x) \right) \qquad 4^{\text{th}} \text{ order term treated at "Gaussian" level}$$

$$\psi(r) = \int \frac{d^{3}q}{(2\pi)^{3}} \langle |P(q)|^{2} \rangle = \int \frac{d^{3}q}{(2\pi)^{3}} \frac{T}{r + (c - f^{2}/v_{s}^{2})q^{2} + hq^{4}} \qquad \text{Diverges near T}_{ic} \text{ in isotropic/cubic system} - \text{self-consistent theory necessary}$$

$$\tilde{\omega}_{TO}^{2}(q) = \omega_{TO}^{2}(q) + 3\gamma\psi(r) = r + cq^{2} + hq^{4} + 3\gamma\psi(r), \qquad \psi(\tau) = \int \frac{d^{3}q}{(2\pi)^{3}} \frac{T}{\tau + (c - f^{2}/v_{s}^{2})q^{2} + hq^{4}} \qquad \tau \equiv \tilde{\omega}_{TO}^{2}(0) = r + 3\gamma\psi(\tau).$$

TO phonon stabilizes at $T >> T_0$

Incommensurate phase stable at $T < T_{ic} \ll T_0$

Brazovski S. A. Sov.Phys.JETP 41 85. (1975).

Phase diagram (classical thermal fluctuations only)



Phase diagram (including quantum zero-point fluctuations)







Estimates for KTaO₃ and SrTiO₃



Quantum Criticality

Coak et al. PNAS June 9, 2020 117 (23) 12707-12712; https://doi.org/10.1073/pnas.1922151117



Why are there so many cross-overs?

Evidence for modulated phase



FIG. 2. TA-TO coupling in the quantum paraelectric phase of SrTiO₃: a) Energy scans at $\mathbf{Q} = (2,-0.05,-0.05)$ for $\mathbf{T} = 250$ K down to 4 K. Fits including a convolution with the experimental resolution are shown in dotted lines (see [29]). b) Intensity of the acoustic (in red) and optical modes (in blue) as a function of temperature. c) energy position of the acoustic mode deduced from a). We note a loss of intensity of the TA mode below T_{AFD} accompanied by a softening of the TA branch

Mesoscopic tunneling in strontium titanate

Benot Fauqué,^{1,*} Philippe Bourges,² Alaska Subedi,³ Kamran Behnia,⁴ Benot Baptiste,⁵ Bertrand Roessli,⁶ Tom Fennell,⁶ Stéphane Raymond,⁷ and Paul Steffens⁸



arxiv 2203.15495

Perhaps the "quantum paraelectric" in STO and KTO is a classical nematic Evidence for soft mode (but no long-range order) at a few x 0.01 reciprocal lattice units Nematic requires that flexoelectric coupling must exceed some critical value Anisotropy (neglected here) will eventually win at low temperature and produce an ordered (striped) phase – large enough anisotropy will eat the quantum regime: Two crossovers and one phase transition on cooling:

Curie-Weiss paraelectric -> classical nematic liquid -> quantum nematic -> striped phase Electrostrictive coupling (neglected here) could preempt to force a first-order FE transition

Speculations and caveats

Have ignored anisotropy and electrostriction – both will in principle lead to 'stripe' order at low T

If flexoelectricity alone, incommensurate phase appears only if flexoelectric coupling f is large

 $(c - f^2/v_s^2) < 0$ c is quadratic dispersion of TO mode, v_s is sound velocity

"bare" coupling – at least as calculated from ab initio methods – is too small (Stengel PRB 93, 245107 (2016)

Electrostriction drives $v_s \rightarrow 0$ at the mean-field ferroelectric transition T_0 so this condition is relaxed

In STO is the "quantum paraelectric" phase for T < 30K instead dominated by classical incommensurate fluctuations?

Might one identify the "quantum polar-acoustic" phase as a regime where quantum fluctuations of incommensurate domains are important?

Role of electrostriction will be to sharpen the directionality of the polar fluctuations to become oriented nanodomains?

Some recent ab initio calculations have found STO to be disordered at T=0 (Zhao et al, arXiv:2105.05231)

Conclusion: Soft matter physics in hard materials

Maintaining structural integrity through changes of state matters

- The elastic strain field is literally the metric of space in which the electronic physics acts
- Can subvert this rigidity by using local degrees of freedom (tilts, rotations) sometimes even zero-point motion can be relevant

The need for soft correlated materials

Solid state refrigeration Battery electrodes

Electro-caloric effect and refrigeration

Electrocaloric effect = change in temperature in response to an electric field

Largest entropy changes if dipoles weakly coupled could then be of order k_B per unit cell ... but in practice much smaller



Entropy change depends on correlation length ξ in an electric field E_0

$$\frac{|\Delta S(T, E_0)|}{Nk_B} = \frac{3\zeta a^2}{8\pi} \left[\xi^{-2}(T, E_0) - \xi^{-2}(T, 0)\right],$$

Elastic strain enforces long correlation lengths which reduce fluctuations need a/ ξ ~ 1

Can we find materials that are close to that limit?



Guzman-Verri et al APL Mater, 4, 064106 (2016)

From ferroelectrics to relaxors? - broadening and modest enhancement at low fields Relaxor



Patent 2017008234 GG Verri and PBL

2.5

1.0

T-410 K

600

800 1000 1200 1400

1.2×10⁵ K 0.5

700

2.0 ღ

PMN $\bullet 10^2 \,\mathrm{Hz}$

• 10^{3} Hz

• 10⁵ Hz

• 10⁹ Hz

400

T[K]

Isothermal entropy change

500

200

0.0000

0

200

400

600

T[K]

300

Battery basics: electronic DOS vs. energy



Tuning ionic mobilities in battery electrodes



Rahnejat et al 2011 DOI: 10.1038/ncomms1574

Some research targets that need softer correlated materials

Solid electrolyte materials

We would like insulating oxides(?) with high ionic mobility

There are many superionic conductors with Na, Hg, Ag, but not Li. Why?

Cathode materials for intercalation chemistry tend to be strongly correlated and have insulating phases driven by Coulomb interaction

A consequence in general of using deep d-levels. Can one find phases that are continuously metallic with substantial changes in doping?

THANK YOU