Non-Equilibrium Dynamics and Nanofriction in Ion Coulomb Crystals

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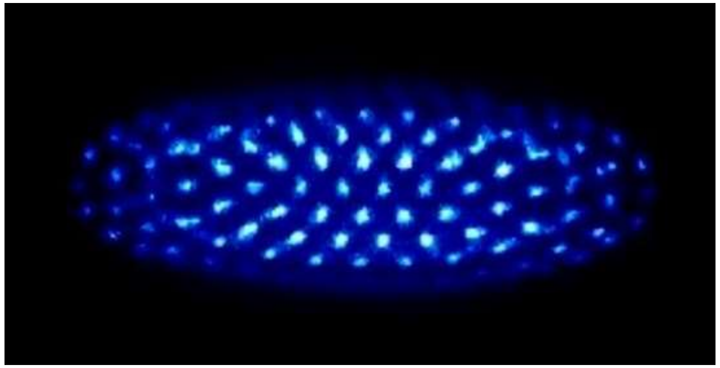


11. August 2022 – ECRYS Cargese

Physikalisch-Technische Bundesanstalt
Braunschweig und Berlin

PTB – National Metrology Institute

PTB / Braunschweig + Berlin

ca. 2200 employees

Founded 1887 by Werner von Siemens & Hermann von Helmholtz

The Team “Quantum Clocks and Complex Systems”

exchange with
Osaka and Tokyo

Int. Collaborations:
NICT Toyko (J)
University of Osaka (J)
CMI (Prag, Cz)
NPL (London, UK)
W. Zurek (Los Alamos NL)
R. Nigmatullin (Uni Sydney, Au)
Haggai Landa (IBM, IL)
…

Visiting scientists:
S. Ignatovich (ILP, Novosibirsk)
N. Ohtsubo (NICT, Tokyo)
M. Kitao (Osaka University)
M. Doležal (CMI, Prag)
L. Ye (JPL/CALTECH)
…

Industry Partners:
Grintech (Jena)
Naneo (Lindau)
D&G (Stuttgart)
Toptica (München)
Vacom (Jena)
Infineon (Munich)
…

TRiAC

International Joint Laboratory for
Trapped-Ion Integrated Atomic-Photonic Circuits

DFG

EURAMET
European Association of National Metrology Institutes

TC FS

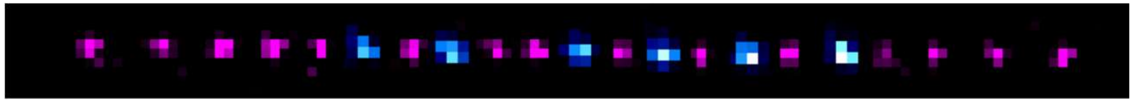
Bundesministerium
für Bildung
und Forschung

DQ mat

DLR

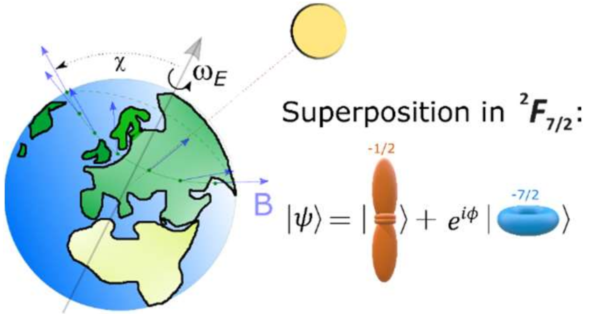
“Quantum Clocks and Complex Systems”

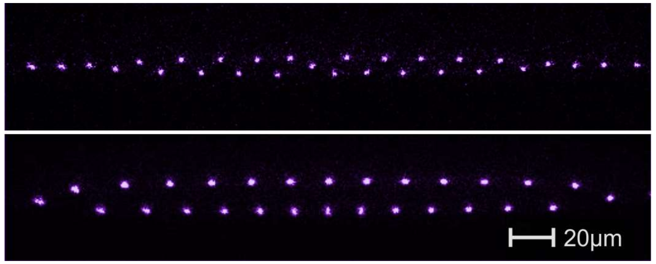
1. $^{115}In^+$ Multi-Ion Clock
2. Precision Spectroscopy in $^{172}Yb^+$ Ions
3. Dynamics of Coulomb-Crystals
4. Integrated Ion Traps
5. QTZ User Facility „Ion Traps“



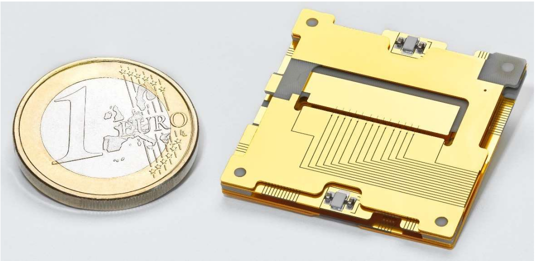
In^+ /Yb^+ crystals – optical clock

Test of Local Lorentz Invariance
e^-/photon sector **at 10^{-21}**



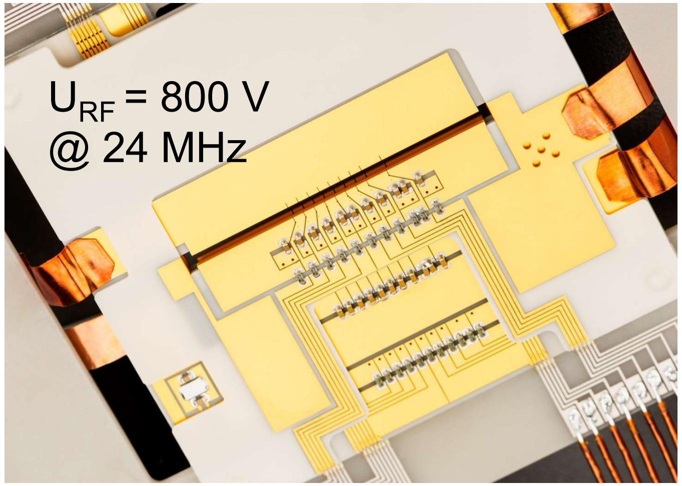


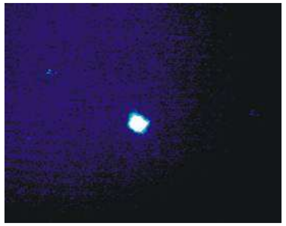
Topological Defects & Transport



Integrated & Scalable Traps
→ Nanophotonics
→ Quantum Information

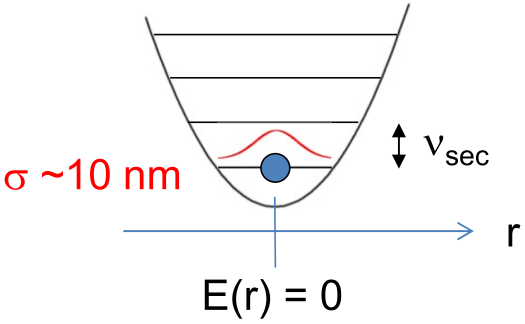
Trapped Ions… (in our case $^{172}\mathrm{Yb}^{+}$)

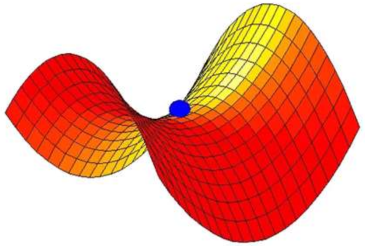




single Yb^{+}-ion

Trap Depth ~ 10^4 K





ponderomotive potential

$$\Psi = \frac{e^2 |E|^2}{4m\Omega^2}$$

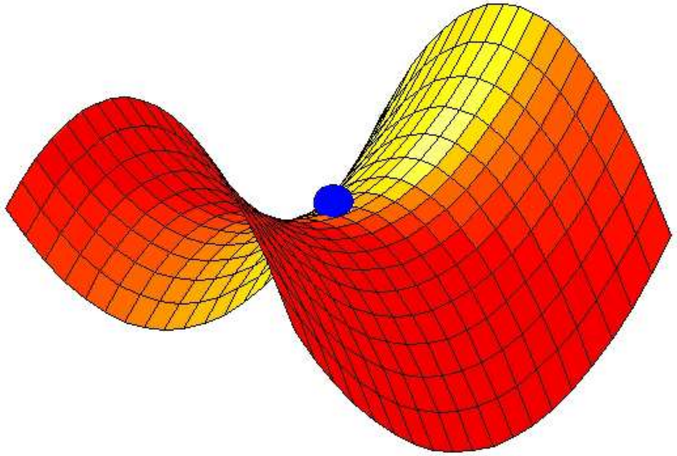
- 10^4 K deep traps → **long trapping times** for single ion (**up to months**)
- ions are trapped at **E = 0** → no systematic shifts to 1st order
- strong trap potential → strong localization (**σ ≈ nm**)
- high level control of internal (**pseudo-spin**) & external degrees of freedom (**bosonic degree of motion**)
- Laser-cooling to mK → resolved sideband-cooling to quantum mechanical ground-state of motion!

Current world record in clock accuracy: *Brewer et al., PRL 2019:* *accuracy* **$\Delta\nu/\nu$ = 9.4 x 10^{-19}**

Micromotion in RF ion traps

Quadrupol Potential

$$\Phi(r,z,t)=\frac{U_{AC}}{2r_0}\cos\Omega t\ (x^2+y^2-2z^2)$$



Mathieu equation
(writing $\tau=\Omega\, t/2$)

$$\frac{d^2}{d\tau^2}\begin{bmatrix}x\\y\\z\end{bmatrix}+(\underline{a_i}+2q_i\cos 2\tau)\begin{bmatrix}x\\y\\z\end{bmatrix}=0$$

Ion motion in trap

$$x_i(t)\cong[\underline{x_{0i}}+x_{1i}\cos(\omega_i t+\varphi_i)]\left[1+\frac{q_i}{2}\underline{\cos(\Omega t)}\right]+C\varphi_{AC}\sin(\Omega t)$$

micromotion
(rf driven motion)

secular frequency
(laser cool)

Micromotion in RF ion traps

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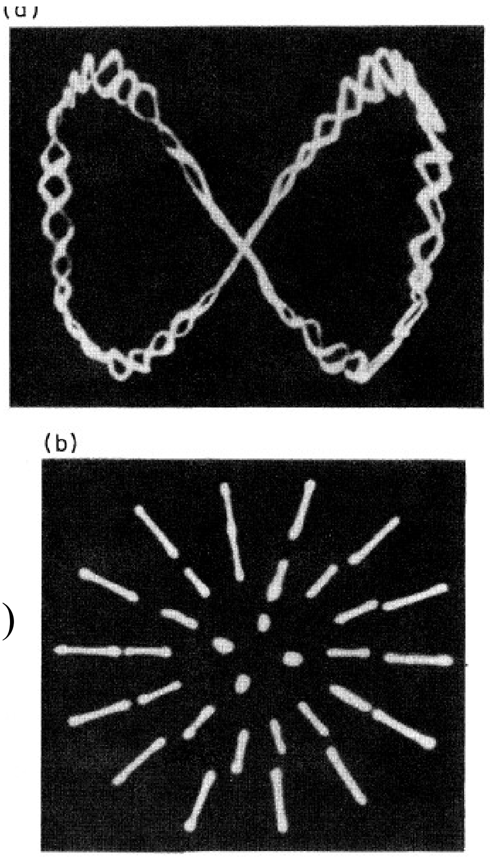
micromotion
(rf driven motion)

secular frequency
(laser cool)

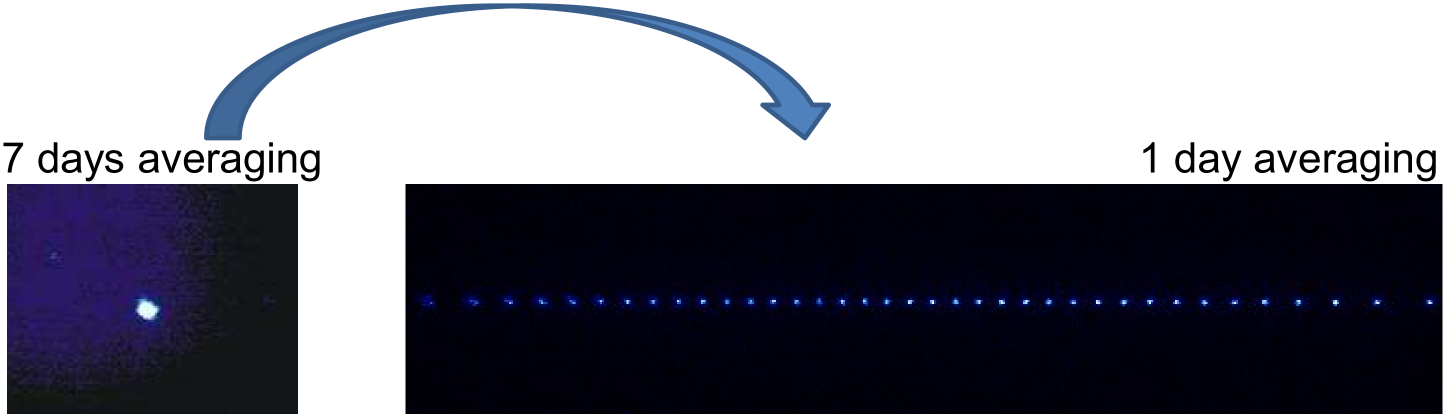
FIG. 9. (a) Photomicrograph of a Lissajous orbit in the r-z plane of a single charged particle of aluminum powder. The micromotion is visible. (b) Pattern of "condensed" Al particles (Wuerker and Langmuir, 1959).

Rev. Mod. Phys., Vol. 62, No. 3, July 1990



New Quantum Clocks ?

Now needed: „experimental methods, that allow to manipulate and measure many-body quantum systems.”



37 days averaging

1 day averaging

single Yb$^+$ ion

Coulomb crystal of 37 Yb$^+$ ions

Quantum Metrology ↔ Quantum Simulation & Information

Herschbach et al., *Appl. Phys. B* 107, 891 (**2012**)

2019: Measurement based multi-ion uncertainty budget

Confinement A

Confinement B

A) Direct cooling

B) Sympathetic cooling

Effect	Max shift $\Delta\nu/\nu_0$	$u(\Delta\nu/\nu_0)$	Max shift $\Delta\nu/\nu_0$	$u(\Delta\nu/\nu_0)$
Time dilation (thermal)	-2	0.4	-19	4
Heating (per second)	-3.1	0.2	-0.6	0.02
Time dilation (EMM)	-1.8	0.8	-1.3	0.6
AC Stark (thermal MM)	-0.003		-0.03	
AC Stark (EMM)	-0.2	0.1	-0.2	0.1
El. quadrupole shift	-0.02	<0.01	-1.1	0.02
BBR at 300 K temperature uncertainty	-137	0.15	-137	0.54
Total	-141.3	0.9	-158.7	4.1

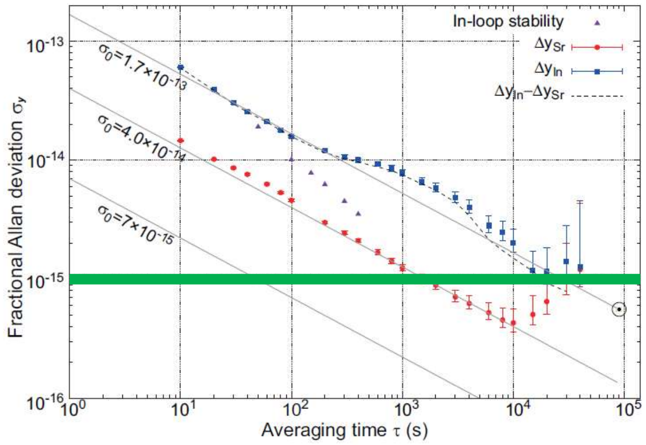
Units: 10^{-19}

Keller et al., "**Controlling systematic frequency uncertainties at the 10^{-19} level in linear Coulomb crystals**", PRA 99, 1 **(2019)**

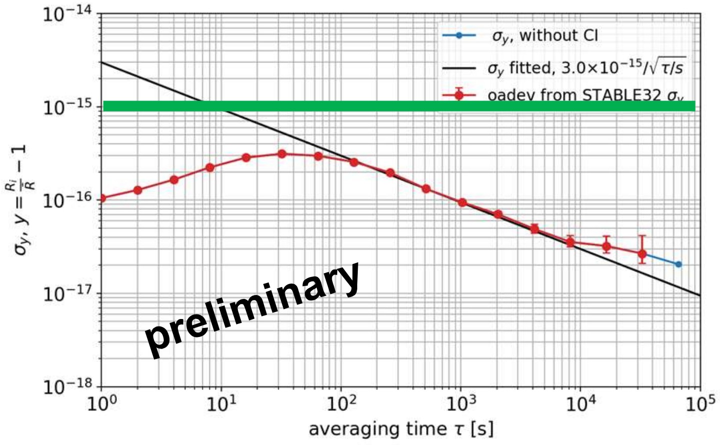
Summer 2021: First frequency ratio ^{115}In$^+$ - ^{87}Sr - ^{171}Yb$^+$

In$^+$ Yb$^+$

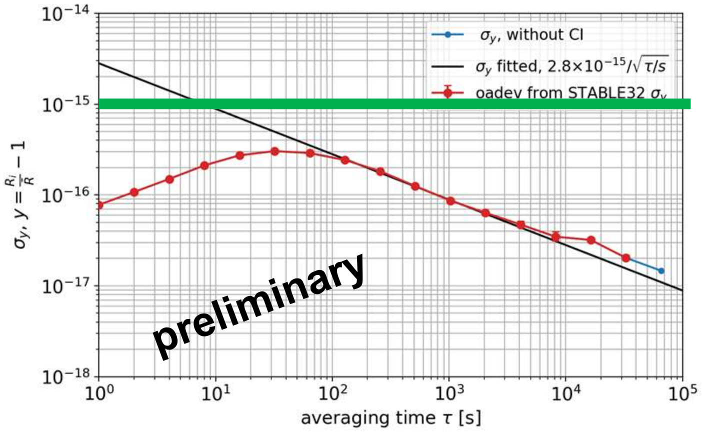
Frequency ratio measurement of ^{115}In$^+$ versus ^{87}Sr at NICT in Japan in 2020[1]



^{115}In$^+$/^{172}Yb$^+$ clock versus ^{87}Sr lattice clock[2]



^{115}In$^+$/^{172}Yb$^+$ clock versus ^{171}Yb$^+$ (E3) single ion clock[3]



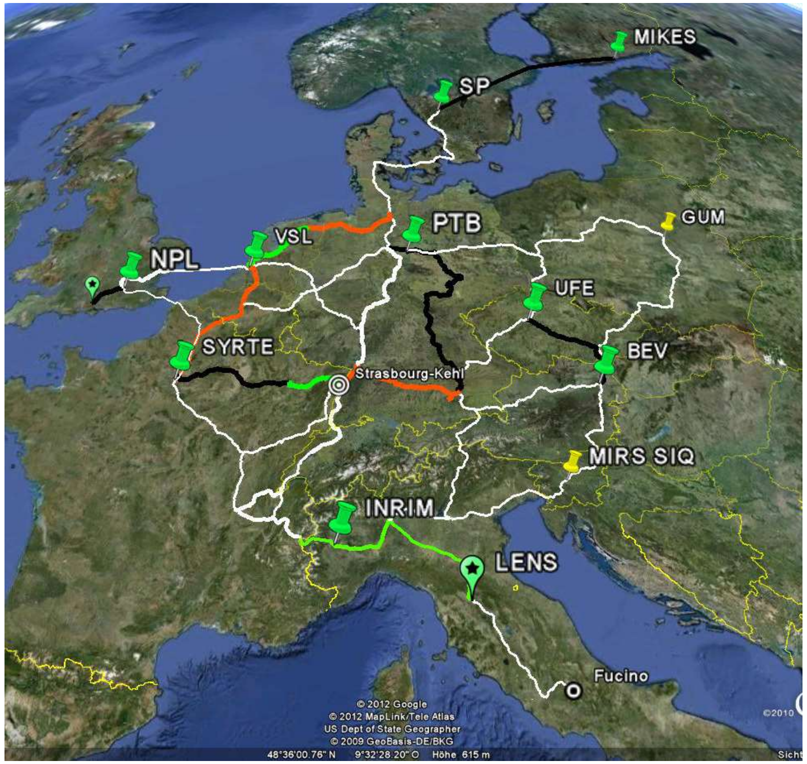
[1] Ohtsubo et al., *Opt. Lett.* **45**, 5950 (2020)
[2] Dörscher et al., *Metrologia* **58** 015005 (2021)
[3] Huntemann et al., *Phys. Rev. Lett.* **116**, 063001

→ 2 orders of magnitude improvement

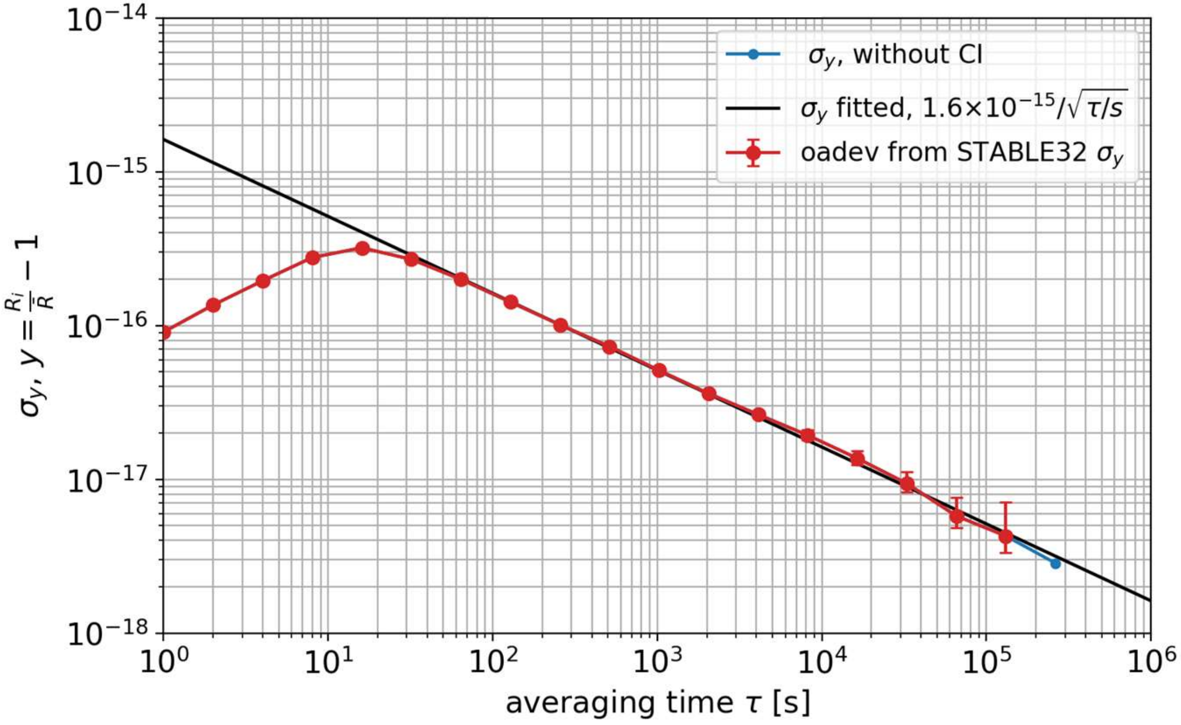
Februar / March 2022 – International Campaign

Yb$^+$ In$^+$ Yb$^+$ Yb$^+$

Clock comparison **Braunschweig** versus **Paris and Torino** via **European Fiber Network**

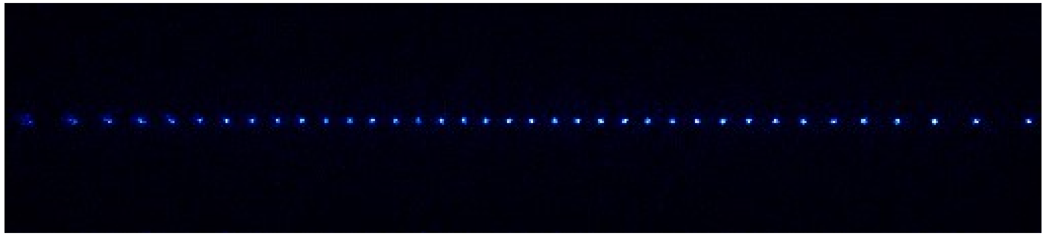


74 % uptime, ^{115}In$^+$ vs. ^{171}Yb$^+$

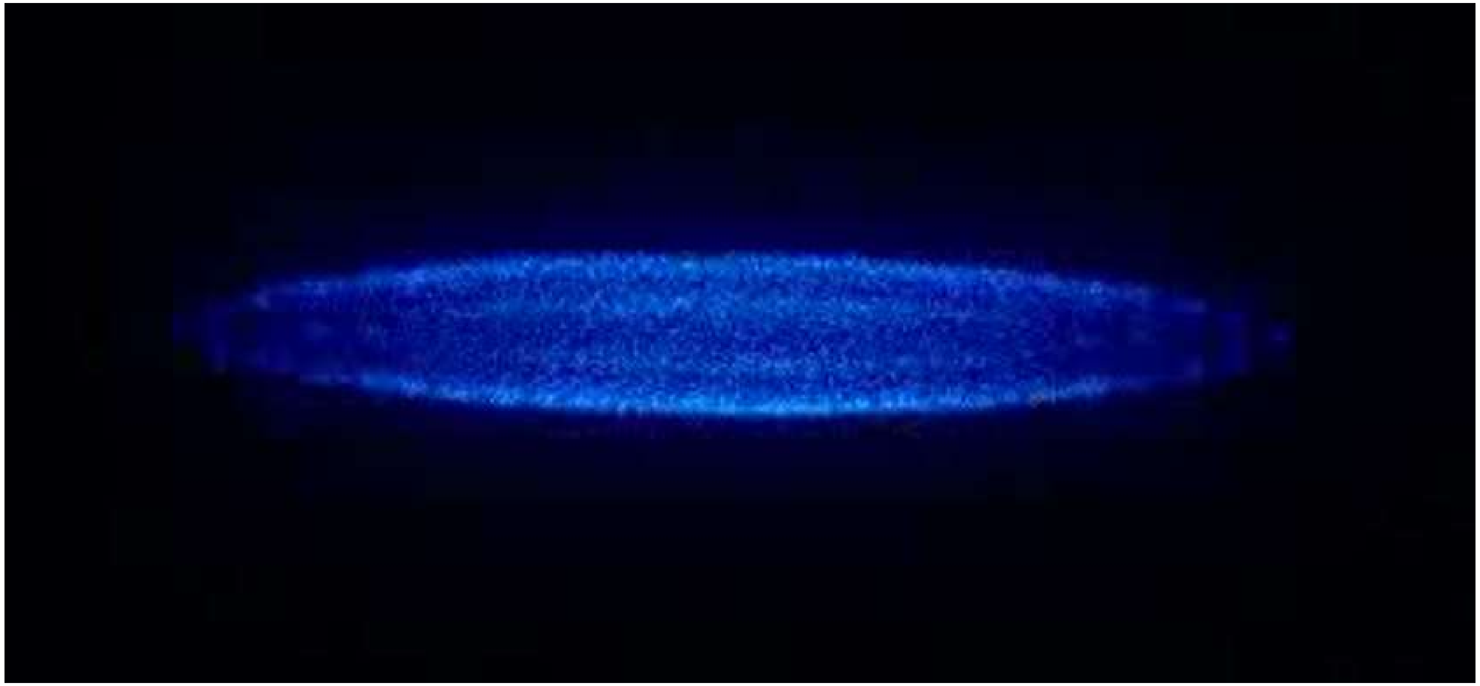


10^{-18}

Ion Coulomb Crystals



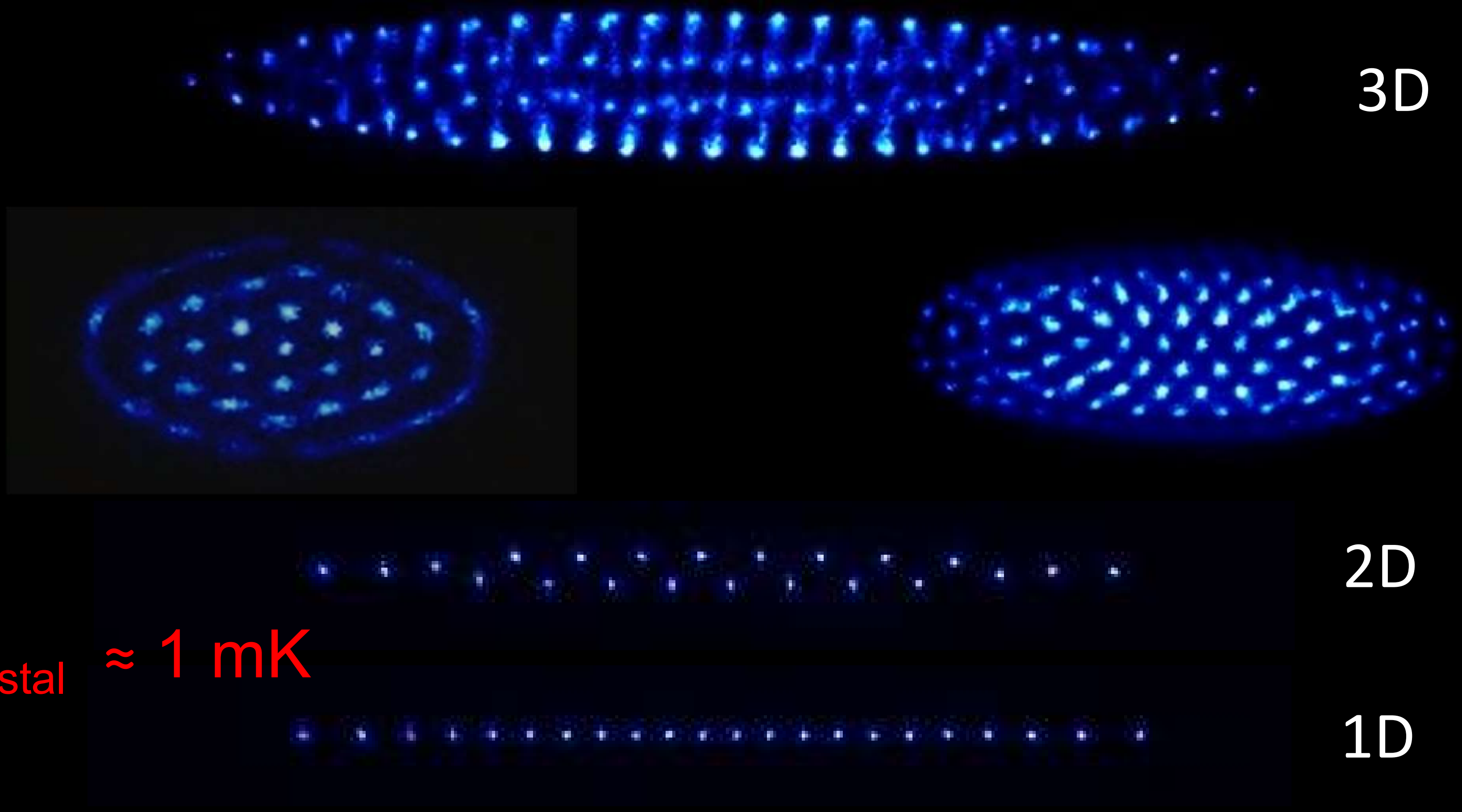
Hot Ion Plasma/Liquid $E_{kin} > E_{pot}$!



Self-organized system! Nonlinear chaotic dynamics

Ion Coulomb Crystals, $E_{pot} > E_{kin}$

$T < 20$ mK



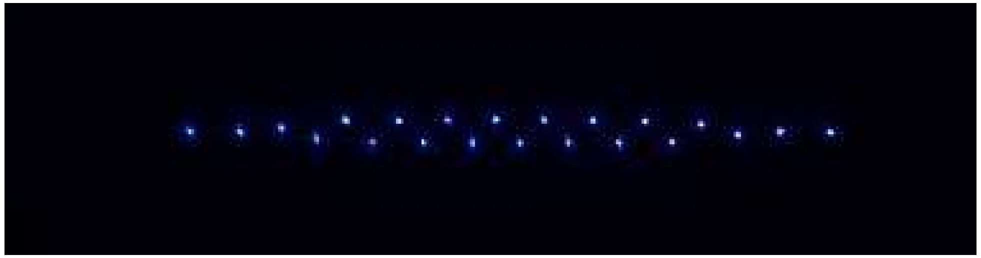
$T_{crystal} \approx 1$ mK

Phases in Ion Coulomb Crystals

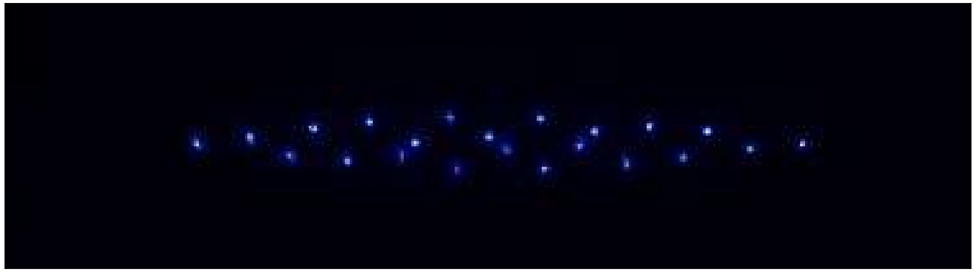
@ T = 1 mK

1 D

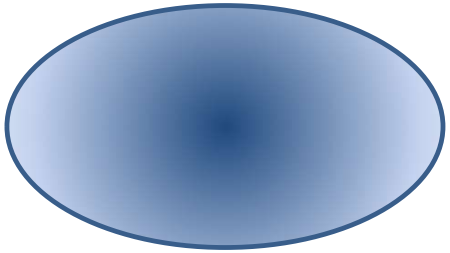
2 D



3 D



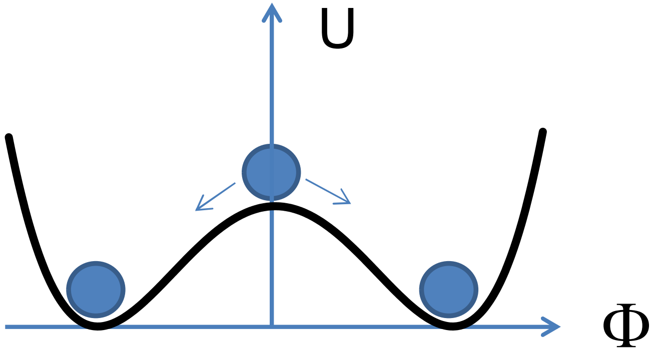
Control Parameter α:
Aspect Ratio of Trapping
Potential



Symmetry breaking phase transitions

What happens when a system changes from one equilibrium condition to another?

- Examples for 2nd-order phase transitions:
    - ferro-magnetism → para-magnetism
    - metal → superconductor
    - early universe



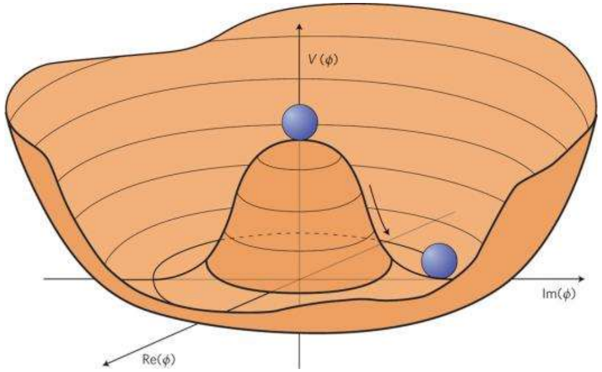
Φ = order parameter

Symmetry breaking phase transitions

What happens when a system changes
from one equilibrium condition to another?

- Examples for 2nd-order phase transitions:
  - ferro-magnetism → para-magnetism
  - metal → superconductor
  - early universe

Spontaneous symmetry breaking of Higgs field →

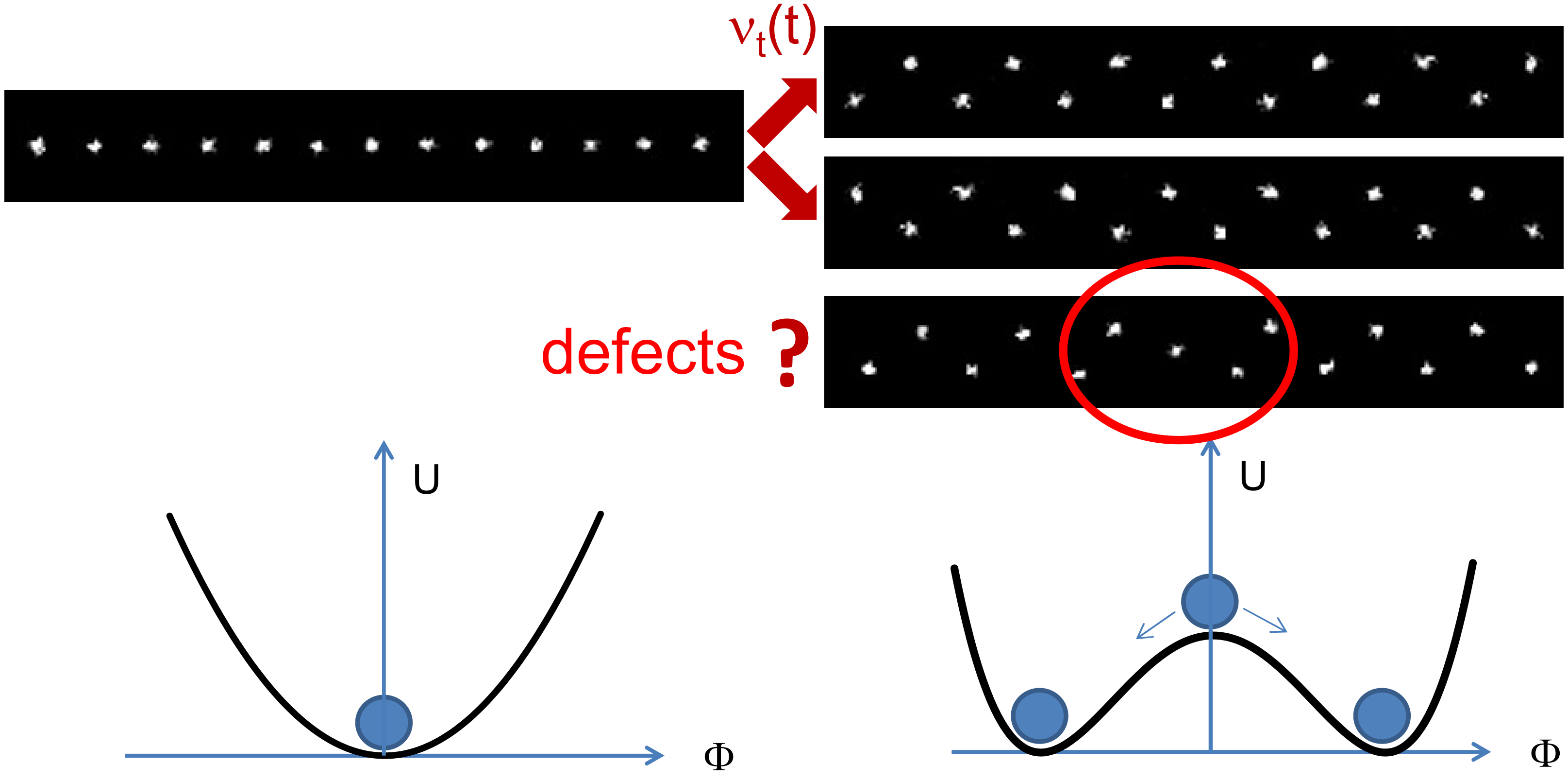


Nature Physics 7, 2 (2011)

Symmetry breaking in ion Coulomb crystals

Rotational symmetry

Mirror symmetry



2nd order phase transition[1]

1: Fishman *et al.*, PRB 77, 064111 (2008)

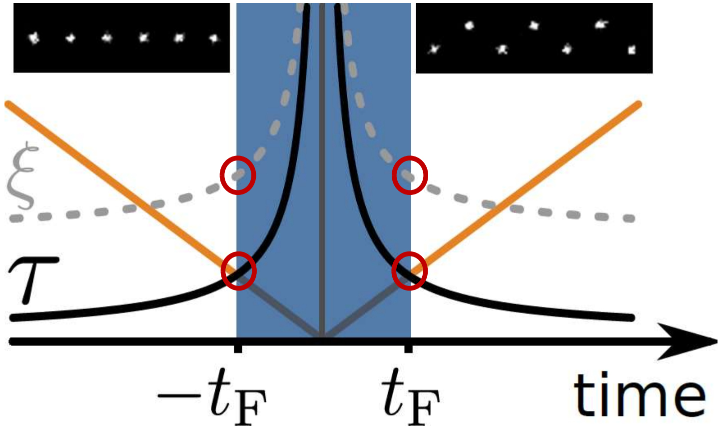
The Kibble-Zurek Mechanism

- ξ: correlation length
- τ: relaxation time
- defect density: $d = \dfrac{\xi}{system\ size}$

- $\tau \sim \left|\dfrac{\tau_Q}{t}\right|^{\upsilon z}$
- $\xi \sim \left|\dfrac{\tau_Q}{t}\right|^{\upsilon}$

- KZM: d from ξ at freezeout
- friction (laser cooling) negligible $\Longrightarrow$ $\nu = ½$, $z = 1$

Quenching the control field



test of KZM with defined ν, z

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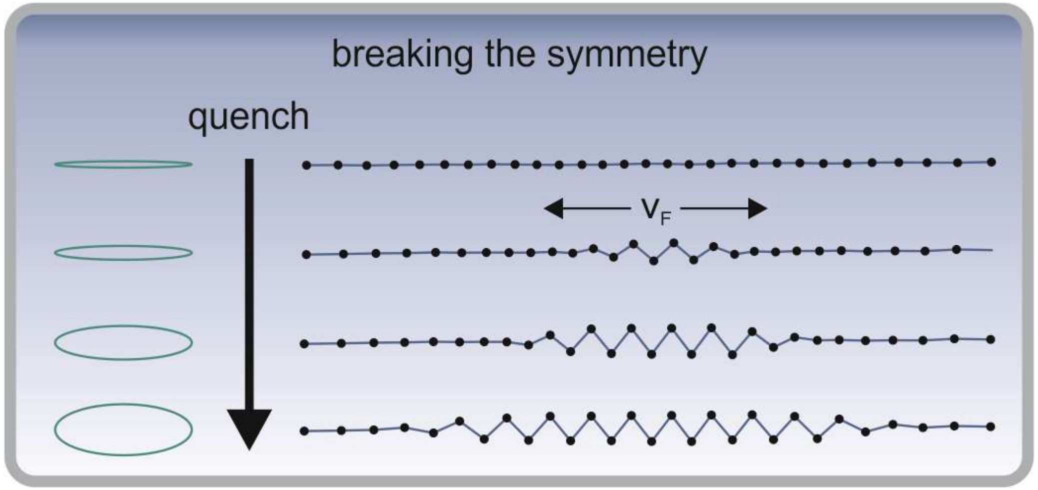
del Campo *et al.,* PRL 105, 075701 (2010)   Fishman *et al.,* PRB 77, 064111 (2008)

Harmonic Ion Traps – Inhomogeneous Case

- Ions in harmonic potential:
  phase transition spreads out from center!



- Phase front faster than speed of sound!

A. Del Campo et al., Phys. Rev. Lett. 105, 075701 (2010)

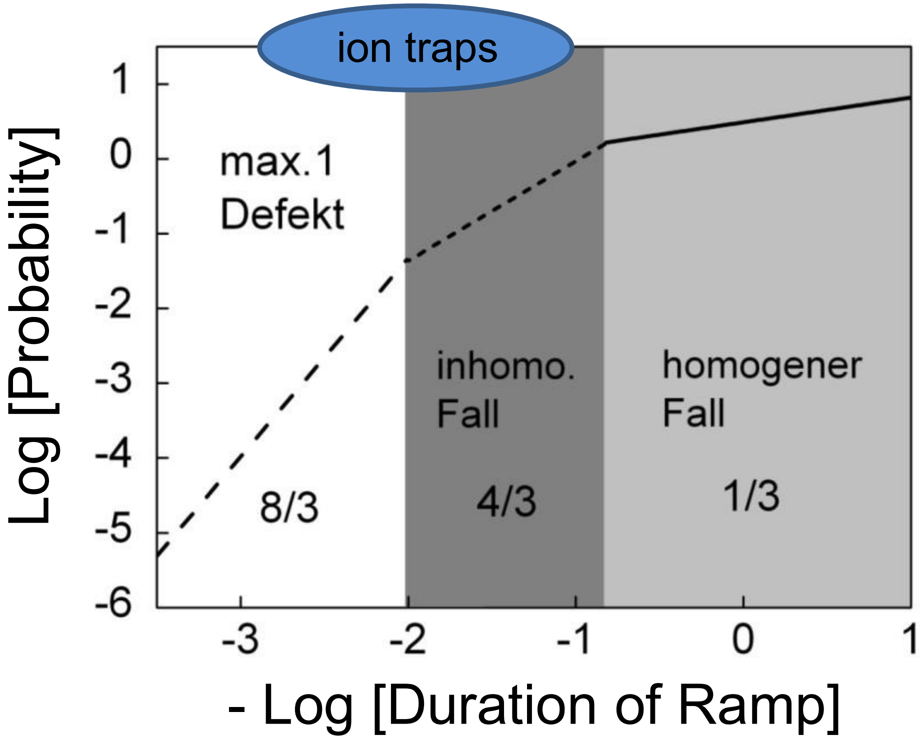
The Kibble-Zurek Mechanism

Prediction of KZM

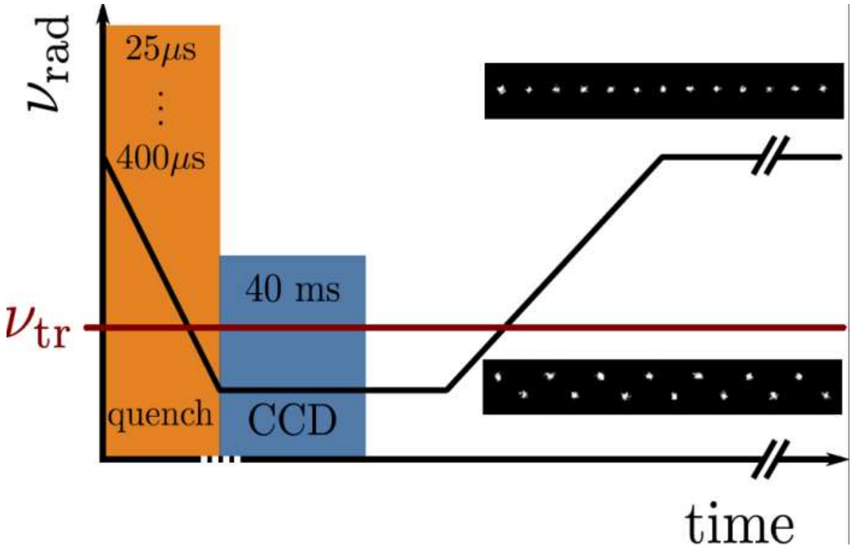
Power law scaling of defect density:

$$d \sim \left|\frac{1}{\tau_Q}\right|^{\upsilon/(1+\upsilon z)}$$

test of KZM with defined ν, z



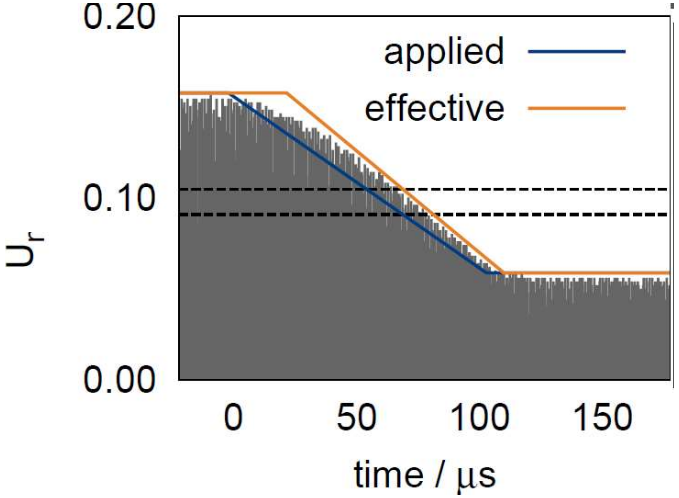
Experiment: fast, non-adiabatic radial quenches



ν_{axial} = 24.6 kHz

$\nu_{rad} \approx$ 500 – 140 kHz

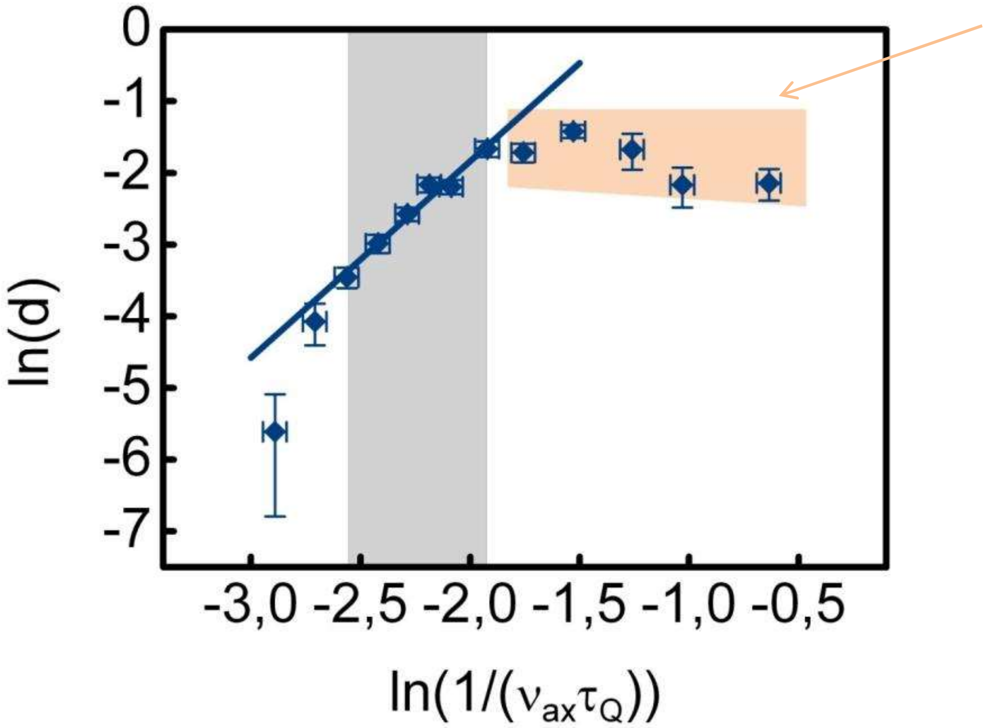
Monitor rf trapping voltage → quench rate



Experimental details in:
Pyka *et al.,* Nat. Commun. **4**, 2291 (2013)

Scaling of Defect Creation

Defect Probability as Function of Ramp Velocity:



excluded by simulations

- Theory: 8/3 ≈ 2.67
- Experiment: 2.7 ± 0.3

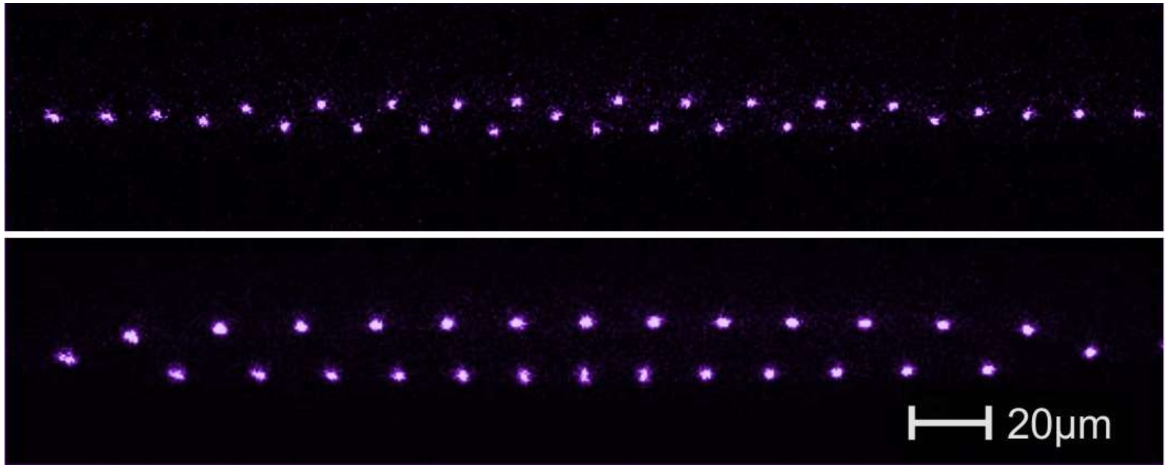
Pyka *et al.* **Nat. Commun**. 4, 2291 (**2013**)
Ulm *et al.* **Nat. Commun**. 4, 2290 (**2013**)

Nikoghosyan *et al.,* Universality in the dynamics of second-order phase transitions, **PRL** 116, 080601 (**2016**)

Puebla *et al.,* Fokker-Planck formalism approach to Kibble-Zurek scaling laws and nonequilibrium dynamics, **PRB** 95, 134104 (**2017**)

Topological defects in ion Coulomb crystals

Localized Defect $\nu_{rad}/\nu_z \approx 8$

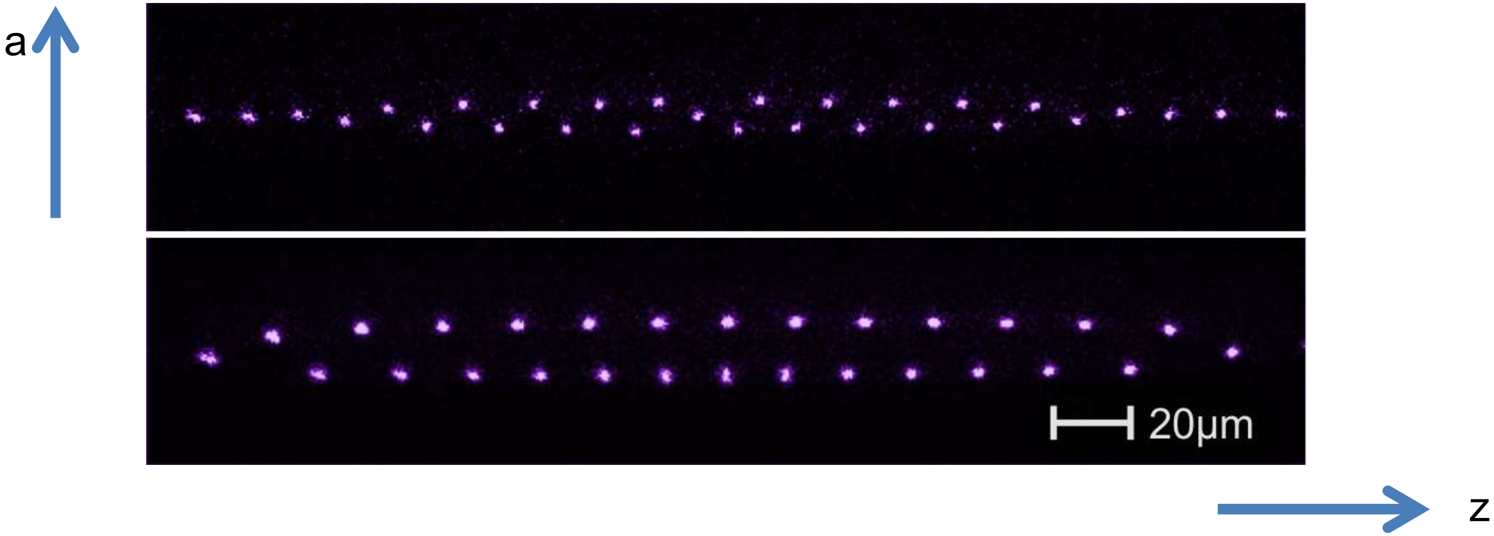


Extended Defect $\nu_{rad}/\nu_z \approx 5.5$

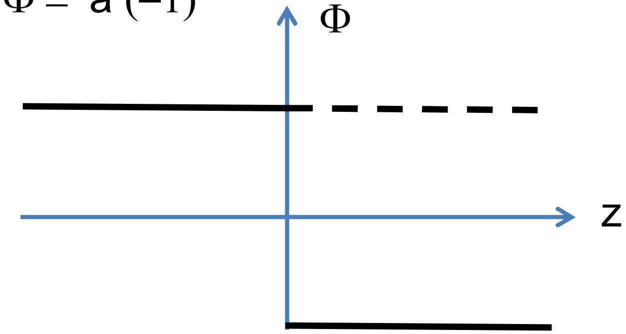
Pyka et al., Nat. Commun. 4, 2291 (**2013**)

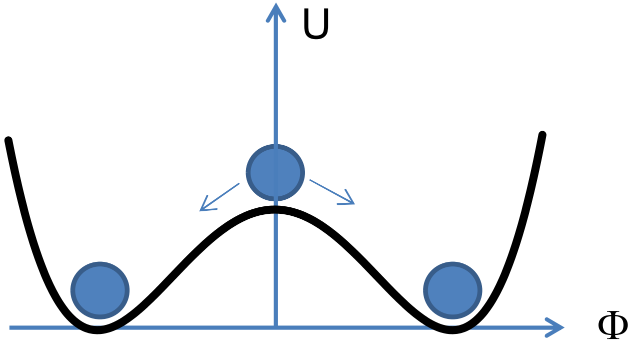
Order Parameter Φ

radial ion separation a:



Discrete field: $\Phi = a\,(-1)^k$





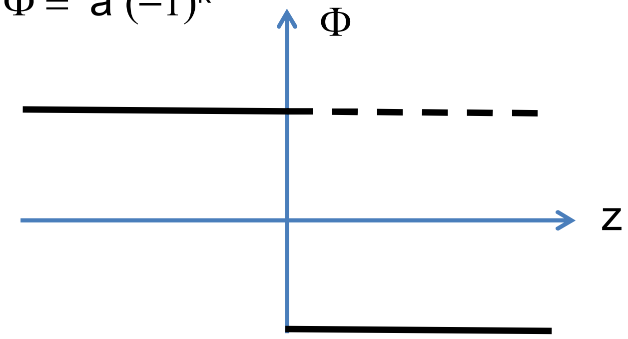
Topological Soliton:

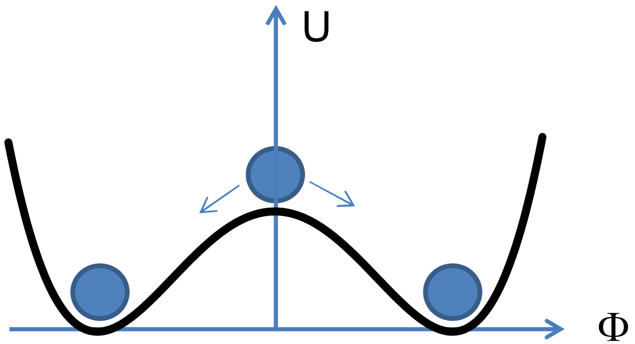
Langragian
of Scalar Field:

$$\boxed{L = \frac{1}{2}\left(\frac{\partial \Phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial \Phi}{\partial z}\right)^2 + \lambda \Phi^2 + A\Phi^4}$$

- field configurations, such that their presence can be detected by looking at the values of the field far away from the defect
- cannot be removed by local deformations of the field

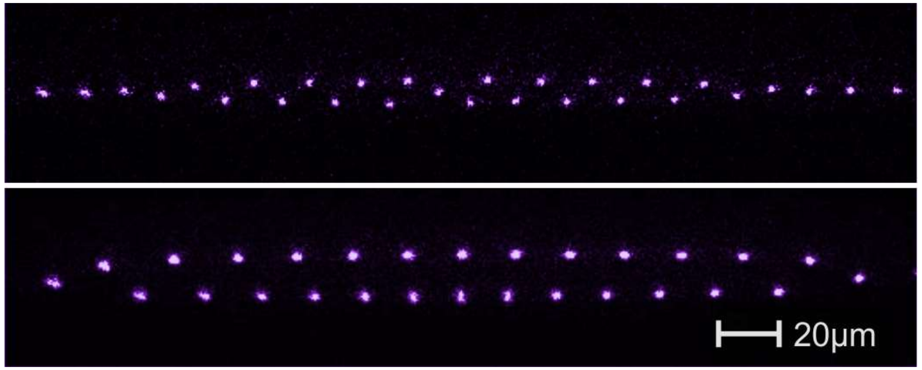
$\Phi =$ a $(-1)^k$



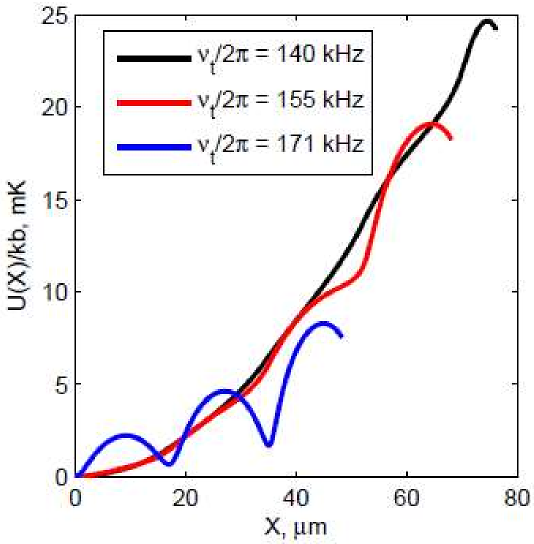


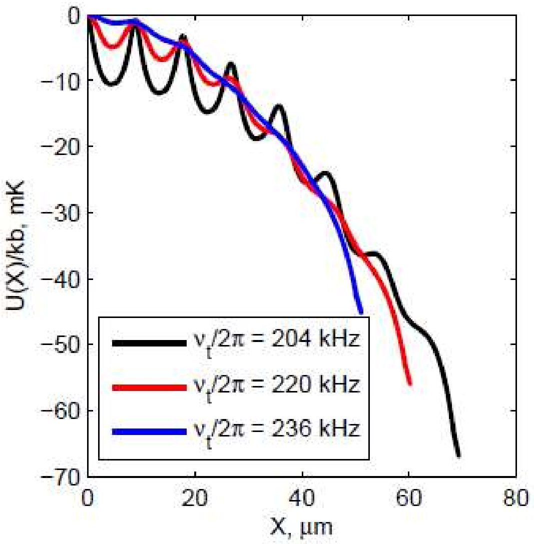
Some features of topological defects

Stability of topological defects



Peierls-Nabarro Potentials:

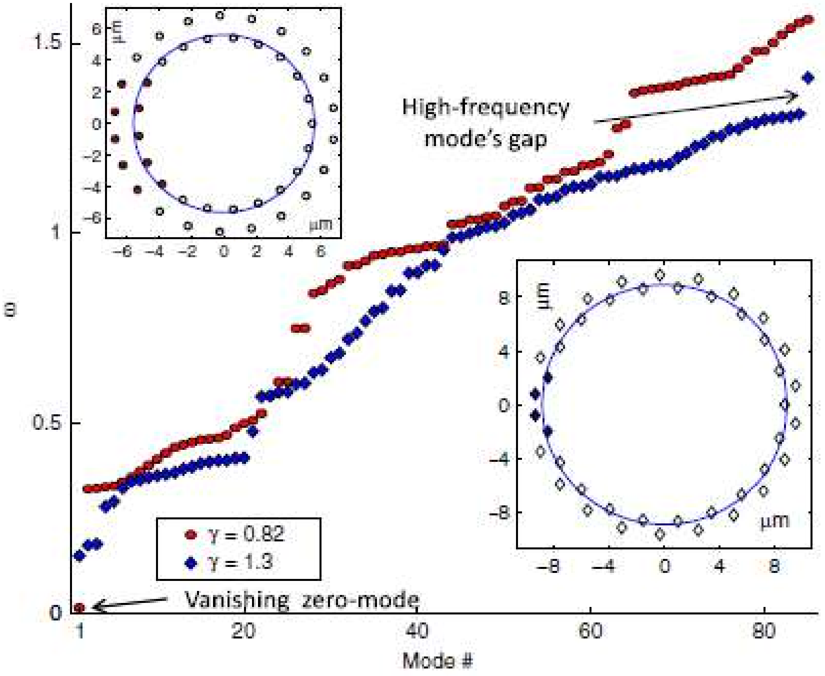




Partner et al.,
New J. Phys. **15**,
103013 (2013)

Kinks for quantum information - Theory

Soliton physics with laser cooled ions:
defects behave like quasi-particles – addressable via gapped mode!

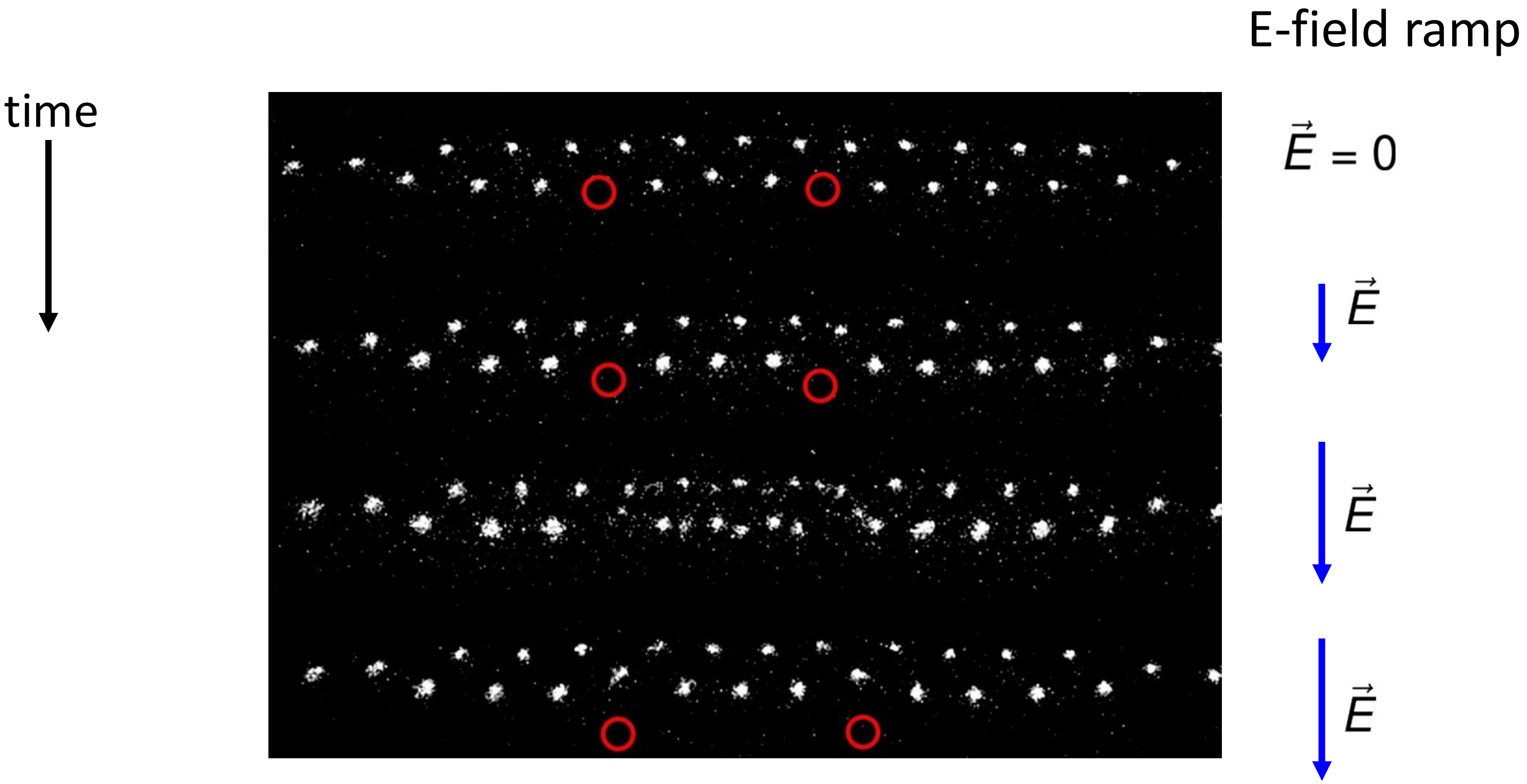


Long coherence times of
localized internal modes:

Storage of Q-Info &
Transport of Entanglement:

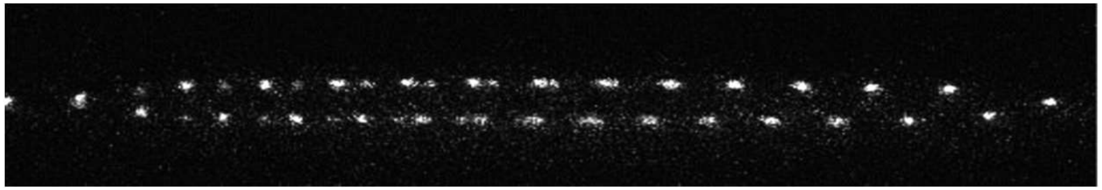
Landa et al., PRL ***104****, 043004 (2010)*
Landa et al., PRL ***113****, 053001 (2014)*

E-field Creating Kink & Anti-Kink



Partner et al., New J. Phys. **15**, 103013 (2013)

The Aubry Phase Transition
-
Friction at the Atomic Scale



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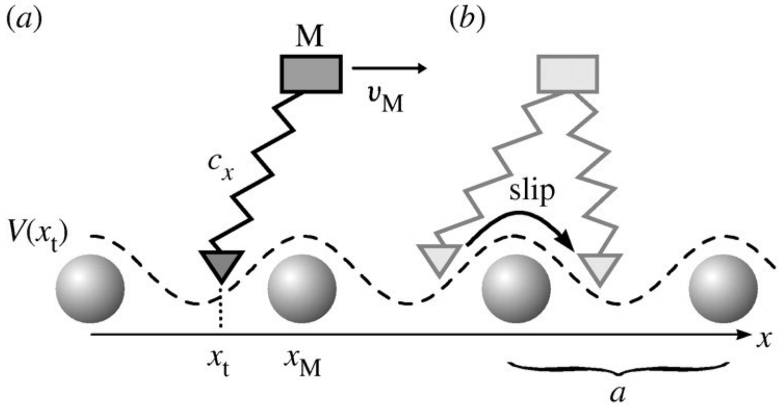
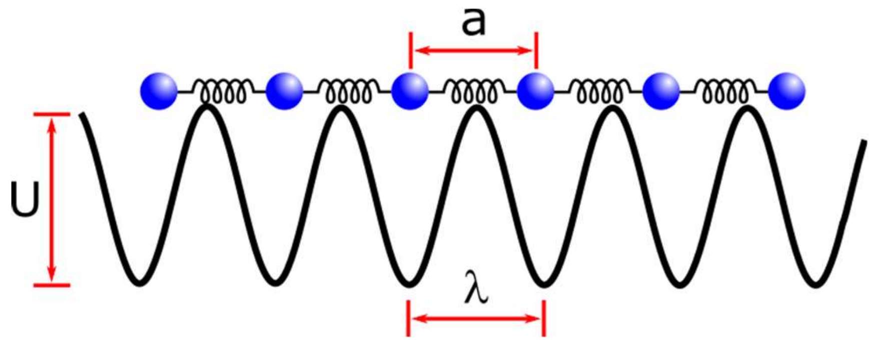
Complex, Self-Organized System with Back-Action

Theoretical models



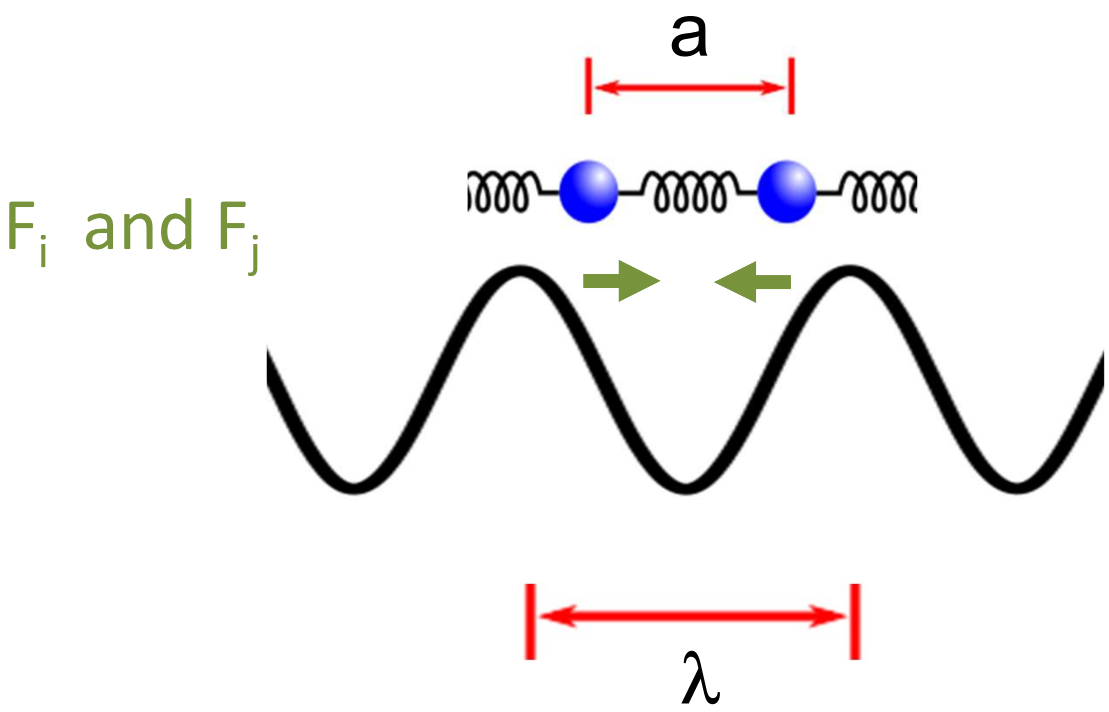
Prandt-Tomlinson (1928)



Many-body effects:

Frenkel-Kontorova (1938)

Many-body system: lattice forces can cancel



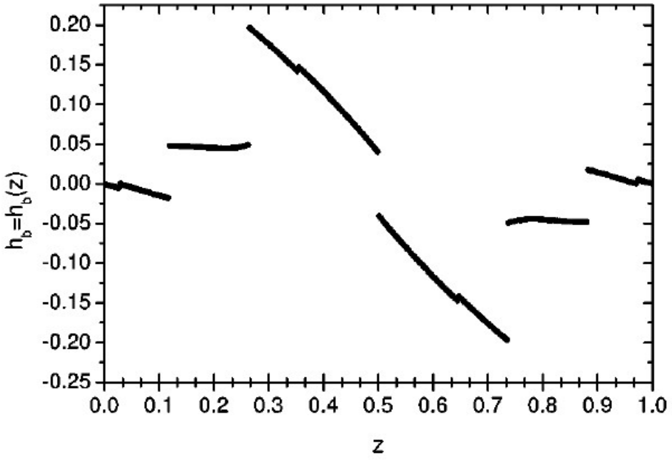
If **a** and λ are incommensurate? → **superlubricity! (1983)**

Aubry phase transition (1983)

If **a** and λ are incommensurate:

- systems becomes **superlubric**
- **analyticity breaking of hull function**

Hull function



ccoordinate of particle with perturbing corrugation potential

coordinate of particle without perturbing corrugation potential

a.k.a „the devil‘s stair case“

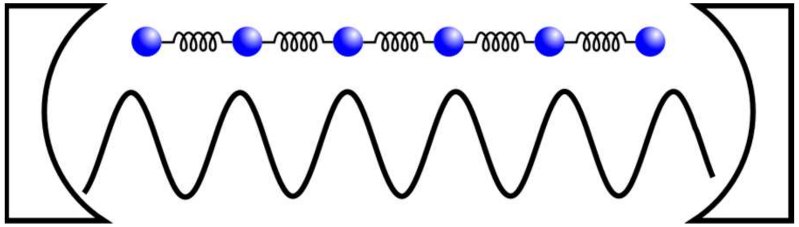
Einax *et al.*, PRE **70**, 046113 (2004)

How can we experimentally realize nanofriction?

Vanossi *et al.,* Modeling friction: From nanoscale to mesoscale, Rev. Mod. Phys. 85 (2013)

Proposal: use linear ion chain in optical corrugation potential

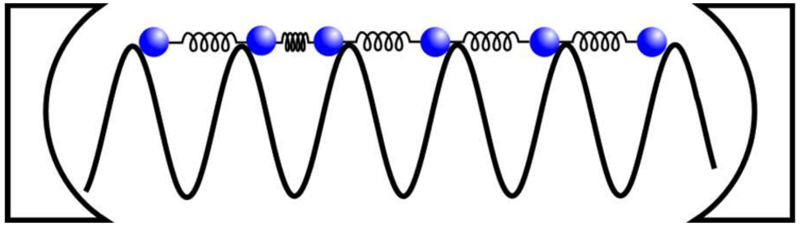
Theory: Benassi et al., *Nature Commun* 2, 236 (2011)
Puttivarasin et al., *New J. Phys*. 13, 075012 (2011)
Fogarty et al., *Phys. Rev. Lett.* 115, 233602 (2015) ...



Standing optical wave in cavity

Theory prediction for finite systems (qualitative) !

1.

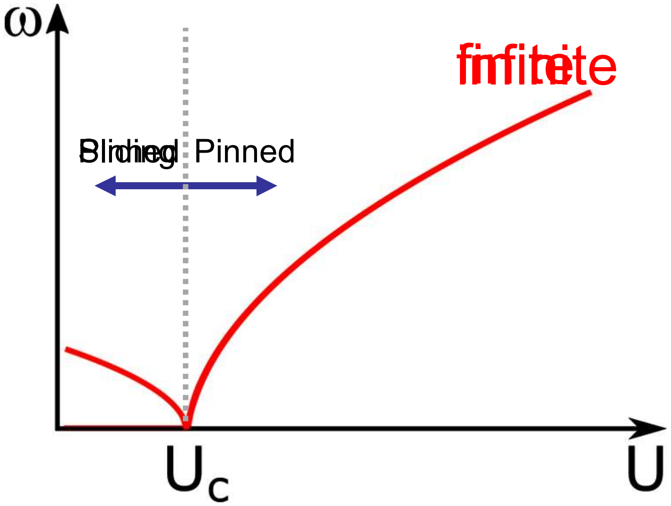


Symmetry Breaking
from sliding to
pinned phase[1]

2.

Vanishing vibrational „soft mode“
drives the transition from
sticking to sliding regime[2]:

Soft Mode !



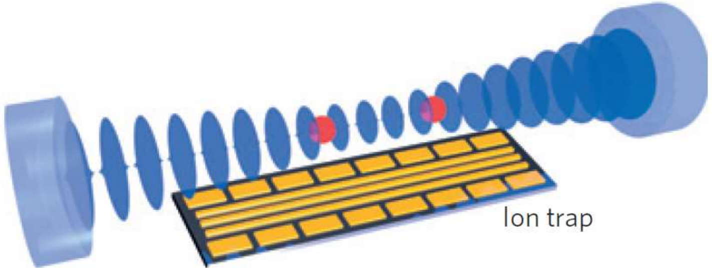
[1] Benassi et al., *Nature Commun* 2, 236 (2011)
[2] Puttivarasin et al., *New J. Phys*. 13, 075012 (2011)

Experiments with ions in standing optical lattice:

Bylinskii *et al., Science* 348, 1115 (2015)

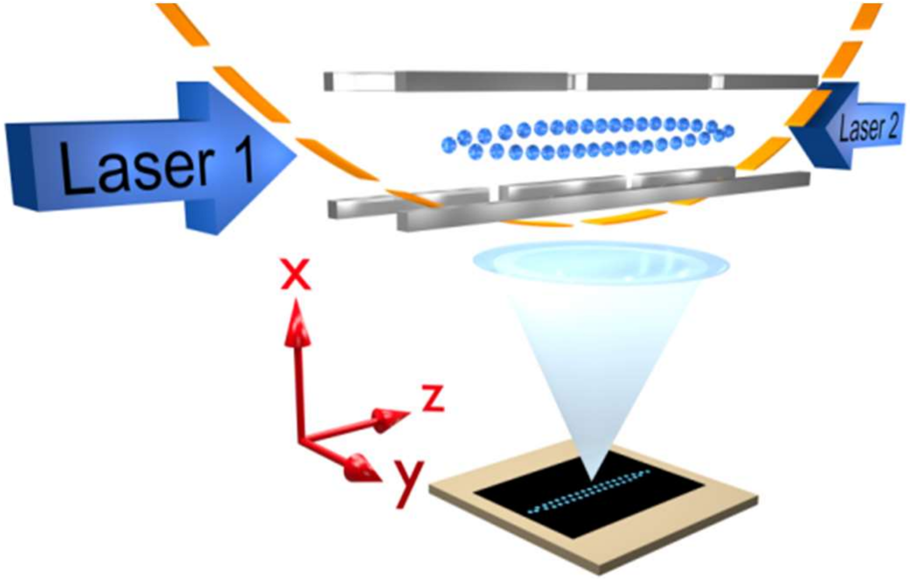
Gangloff *et al.,*Nature Physics 11, 915 (2015)

Bylinskii *et al.,* Nature Materials 15, 717 (2016)



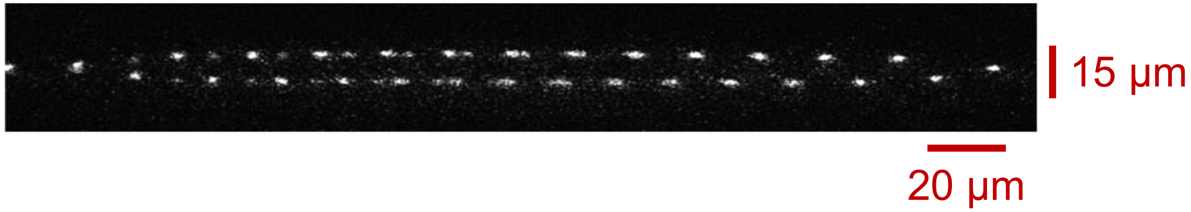
Gangloff et al., *Nature Physics*, **11** (2015)

Our System: self-organized crystal with back-action



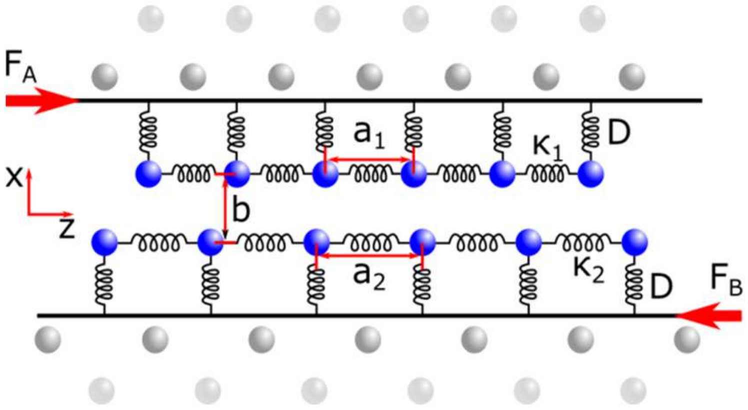
Spatial resolution
of ions positions
= 40 nm

$T_{crystal}$ ≈ 1 mK



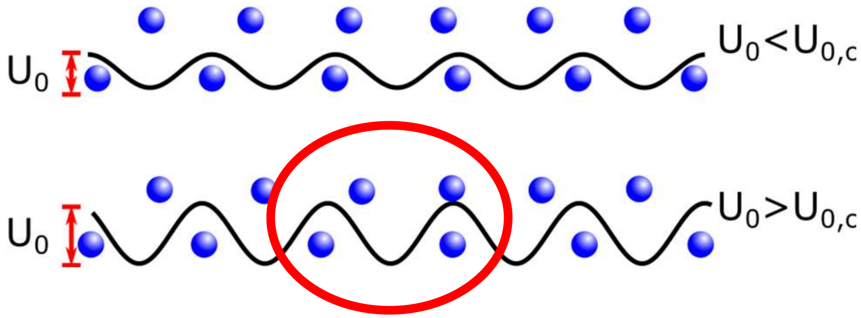
Self-organized crystal with back-action

Locally the kink disturbes the quasi-periodicity of the two atomic layers



$$D = m\,\omega_{ax}^2$$: ion trap

$$\kappa \sim \frac{e^2}{\pi\epsilon_0}\frac{1}{ma^3}$$: ion interaction

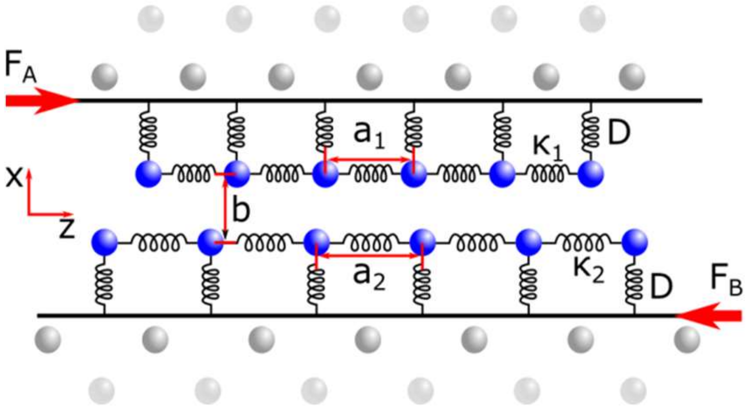


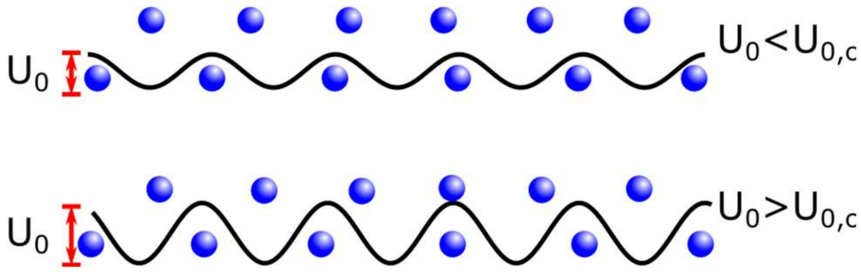
Symmetry breaking?

Interaction between the ion layers gives corrugation potential!

Self-organized crystal with back-action

Locally the kink disturbes the quasi-periodicity of the two atomic layers





control parameter
in experiment:

$$\alpha = \omega_{rad}/\omega_{axial}$$

Experimental Observations

Kiethe *et al.*, ***Probing Nanofriction and Aubry-type Signatures in a Finite Self-Organized System,*** Nat. Comm. 8, 15364 (2017)

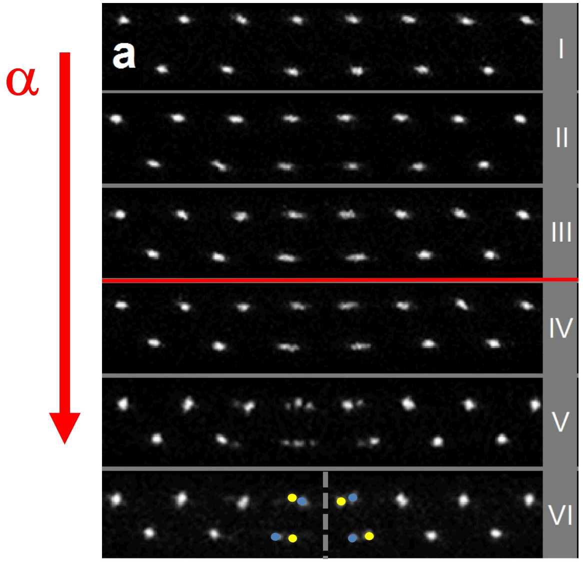
Kiethe *et al.*, ***Nanofriction and Motion of Topological Defects in Coulomb Crystals,*** New J. Phys. 20, 123017 (2018)

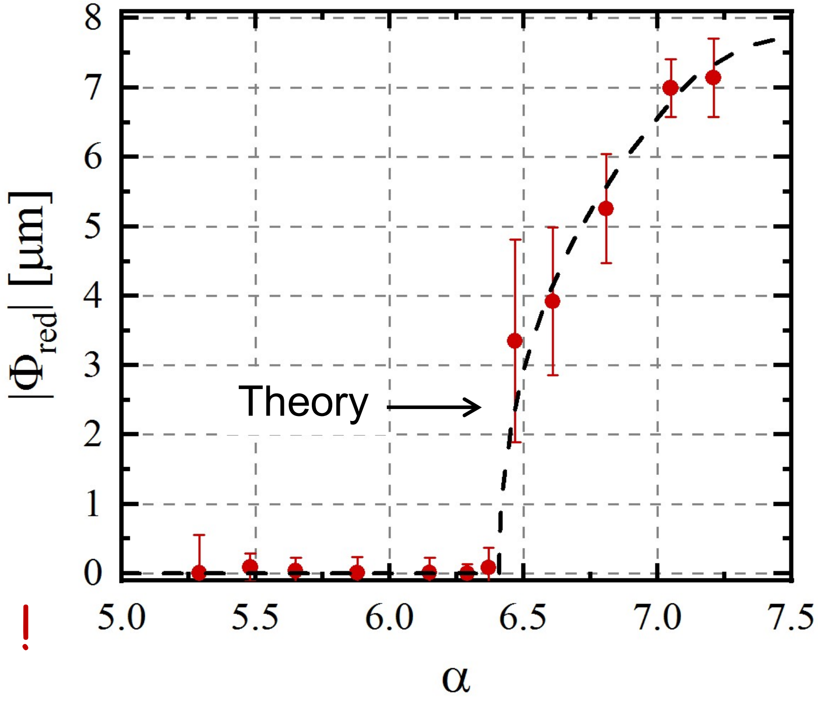
Order parameter → parametrizes symmetry breaking

Order parameter Φ := relative distance to closest ion in other layer

$$\boldsymbol{\Phi} = \sum_{i \in Chain\,A} \mathrm{sgn}(z_i) \cdot \min_{j \in Chain\,B} |z_i - z_j|$$

Experimental observation:





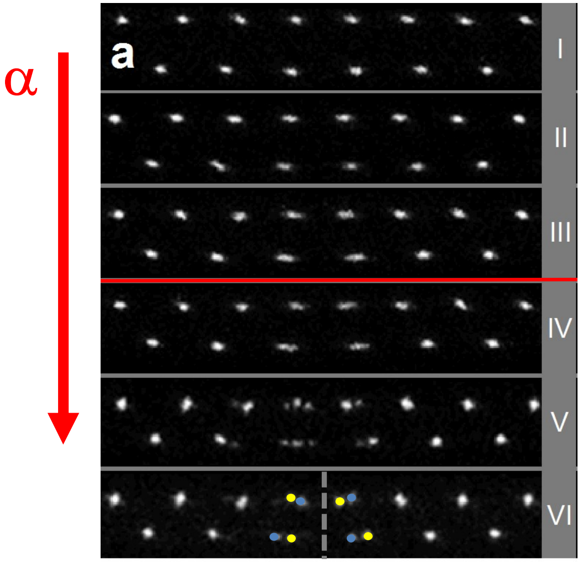
2nd-order phase transition !

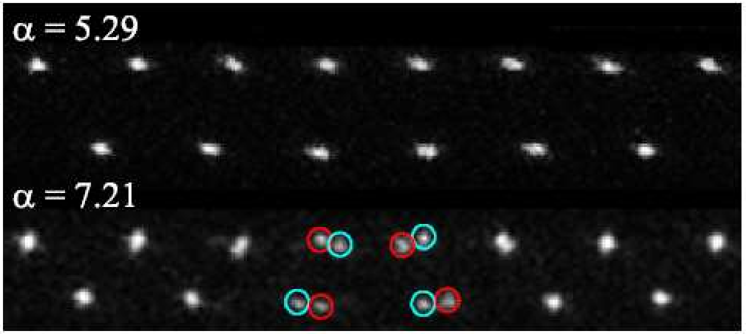
Order parameter → parametrizes symmetry breaking

Order parameter Φ := relative distance to closest ion in other layer

$$\Phi = \sum_{i \in Chain\,A} \mathrm{sgn}(z_i) \cdot \min_{j \in Chain\,B} |z_i - z_j|$$

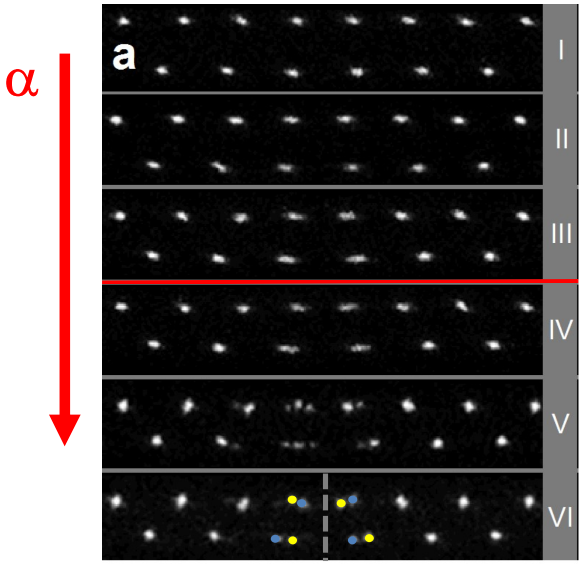
Experimental observation:





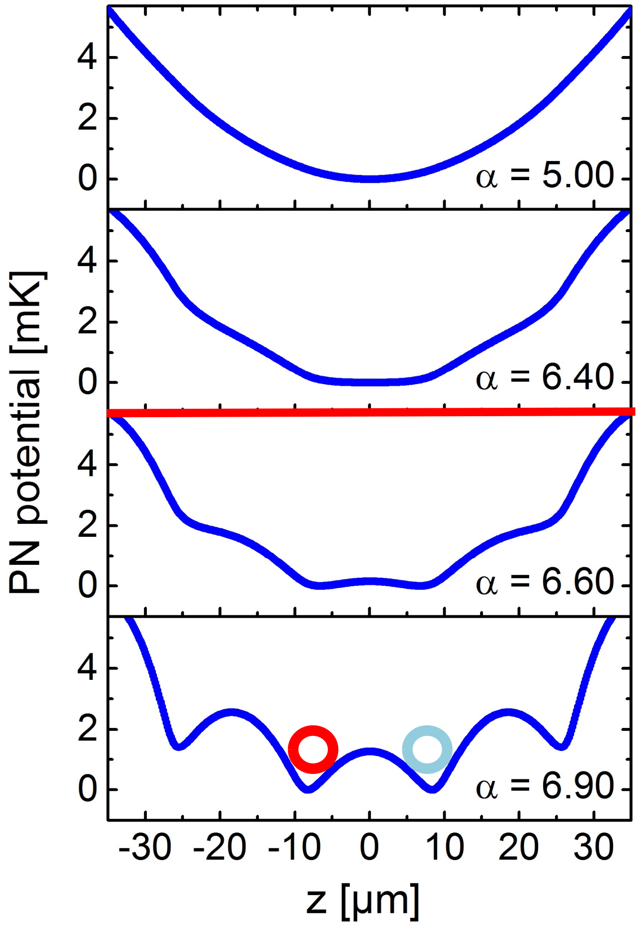
Order parameter → parametrizes symmetry breaking

Experimental observation:

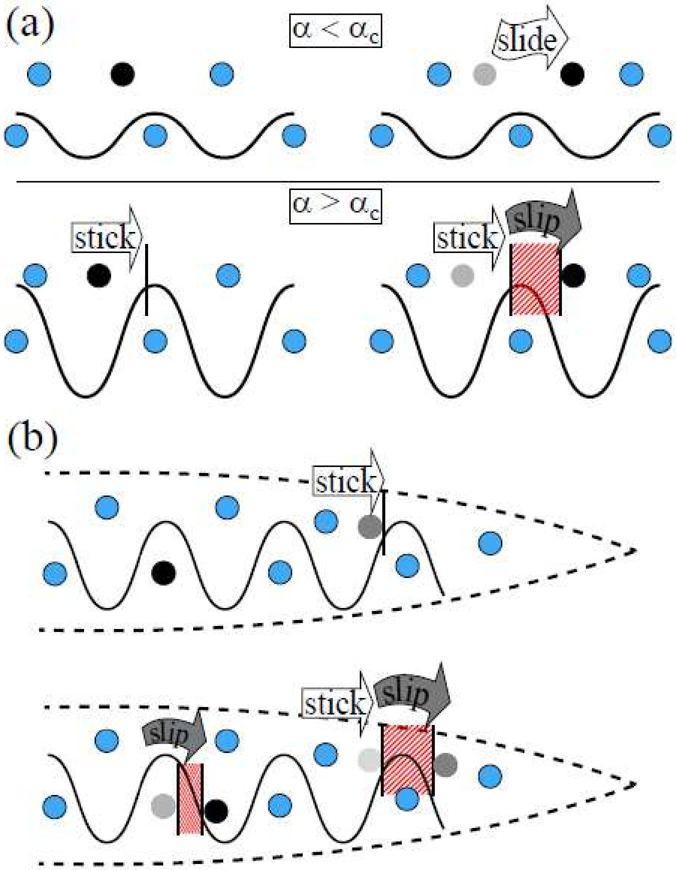


Peierls-Nabarro Barriers

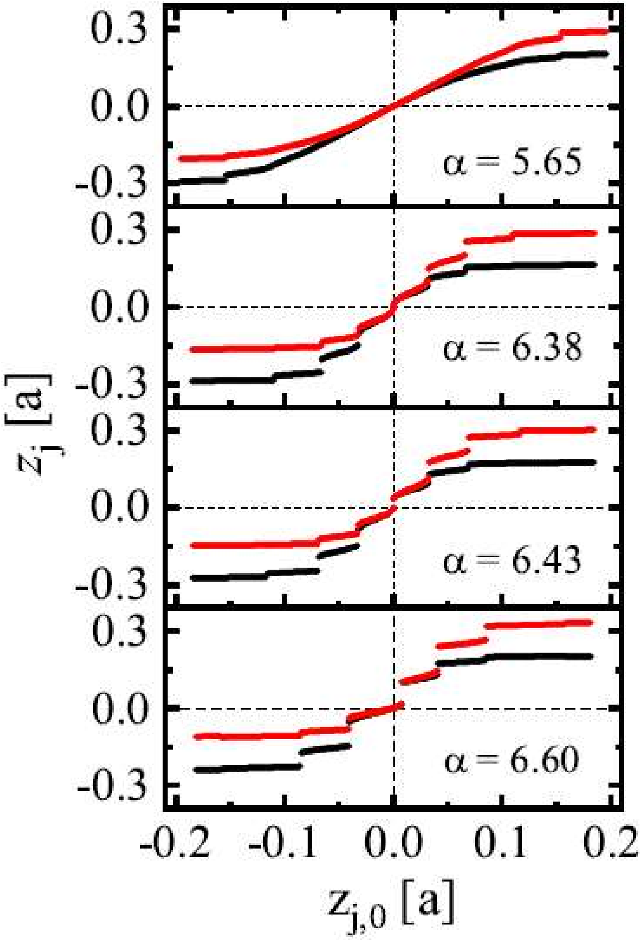
= potential energy of kink soliton



Order parameter → parametrizes symmetry breaking



Kiethe et al., NJP (2018)



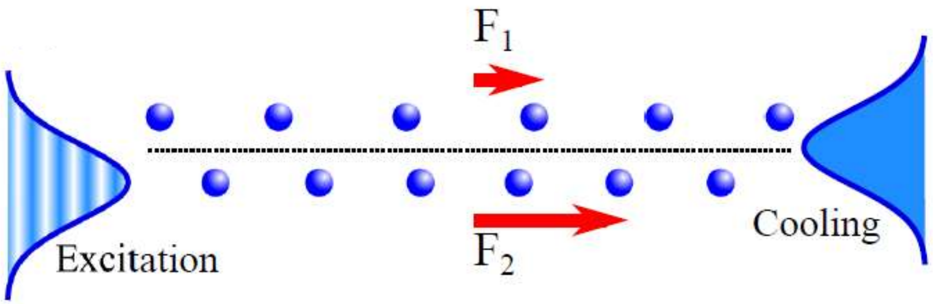
Hull function

Can we observe the Sliding Mode ?

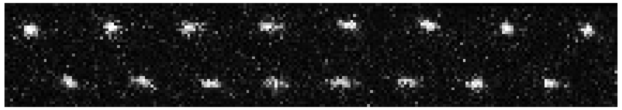
Vibrational Shear Mode should drive the Phase Transition

Amplitude modulated cooling laser
→ **periodic, differential light force** $F = F_0 \cos [2\pi\, \nu_{exc}\, t]$

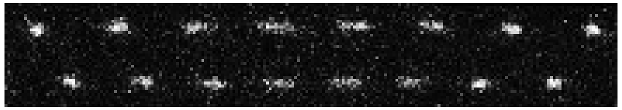
Focused
Laser Beam



off resonant excitation

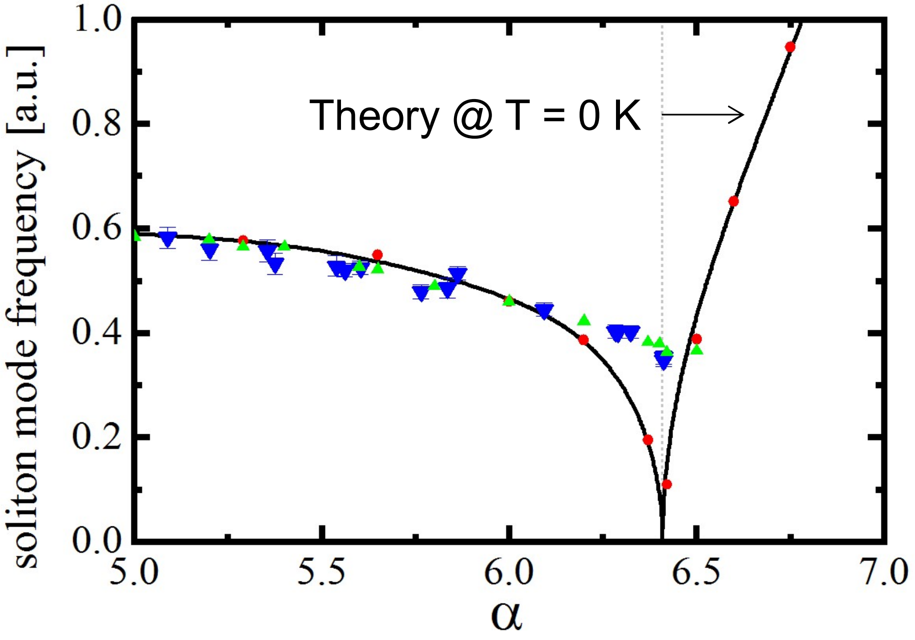


on resonant excitation



First observation of the localized kink mode!

Observation of the Sliding Mode



legend:

— Hessian matrix $T = 0\mathrm{K}$

● simulations $T = 5\mu\mathrm{K}$

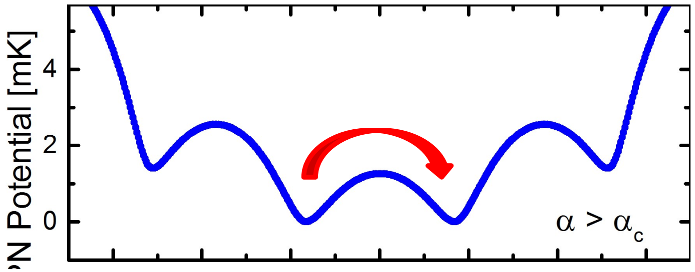
▲ simulations $T = 1\mathrm{mK}$

▼ **exp. data**

Yes, but strong non-linearities due to finite temperature!

PN-energy barriers of kink soliton:

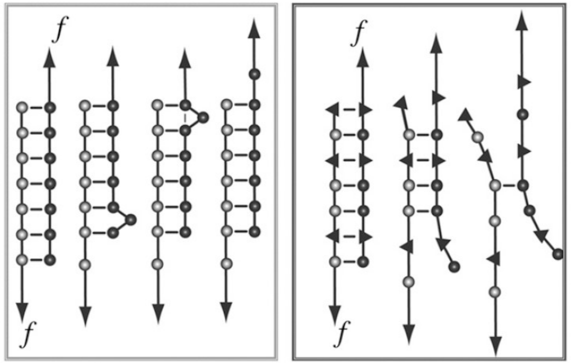
→ thermal energy allows for switching



Friction Summary

- Symmetry breaking & non-analytic hull function for the first time observed in self-organized Coulomb crystal
- Temperature induced non-linearities impact soft-mode
- 2nd stage cooling to **T = µK** via In$^+$ ions → **go quantum**
- **Defect reduces friction by more than an order of magnitude**

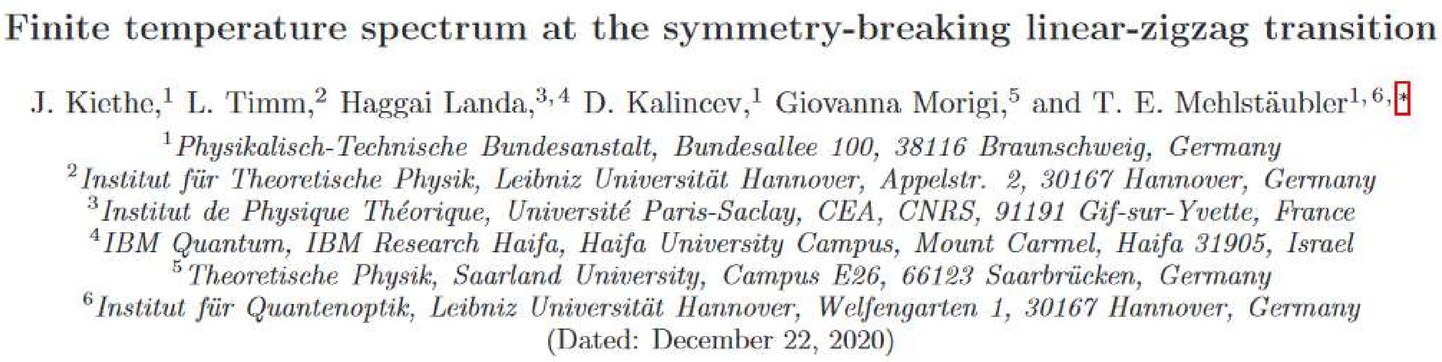
Kink assisted DNA unfolding



Kühner et al., Biophysical Journal 92, 2491 (2007)

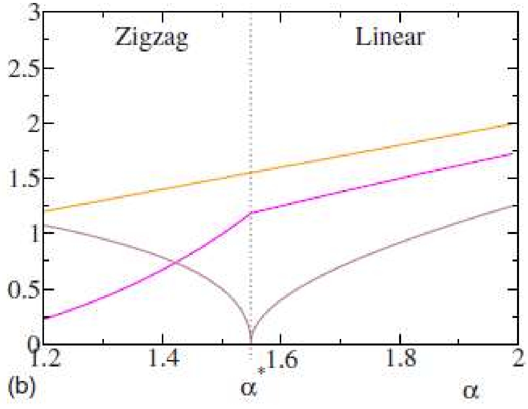
Follow-up work …

How does a soft mode couple to thermal phonon environment?



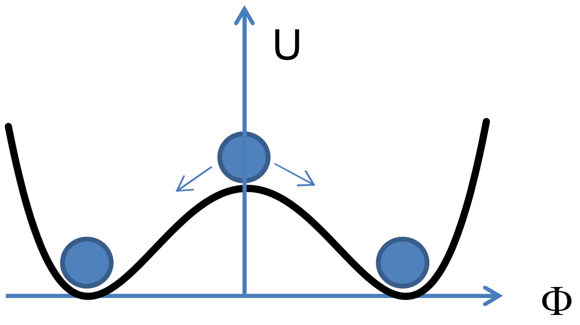
Finite temperature spectrum at the symmetry-breaking linear-zigzag transition

J. Kiethe,[1] L. Timm,[2] Haggai Landa,[3,4] D. Kalincev,[1] Giovanna Morigi,[5] and T. E. Mehlstäubler[1,6,*]
[1] *Physikalisch-Technische Bundesanstalt, Bundesallee 100, 38116 Braunschweig, Germany*
[2] *Institut für Theoretische Physik, Leibniz Universität Hannover, Appelstr. 2, 30167 Hannover, Germany*
[3] *Institut de Physique Théorique, Université Paris-Saclay, CEA, CNRS, 91191 Gif-sur-Yvette, France*
[4] *IBM Quantum, IBM Research Haifa, Haifa University Campus, Mount Carmel, Haifa 31905, Israel*
[5] *Theoretische Physik, Saarland University, Campus E26, 66123 Saarbrücken, Germany*
[6] *Institut für Quantenoptik, Leibniz Universität Hannover, Welfengarten 1, 30167 Hannover, Germany*
(Dated: December 22, 2020)



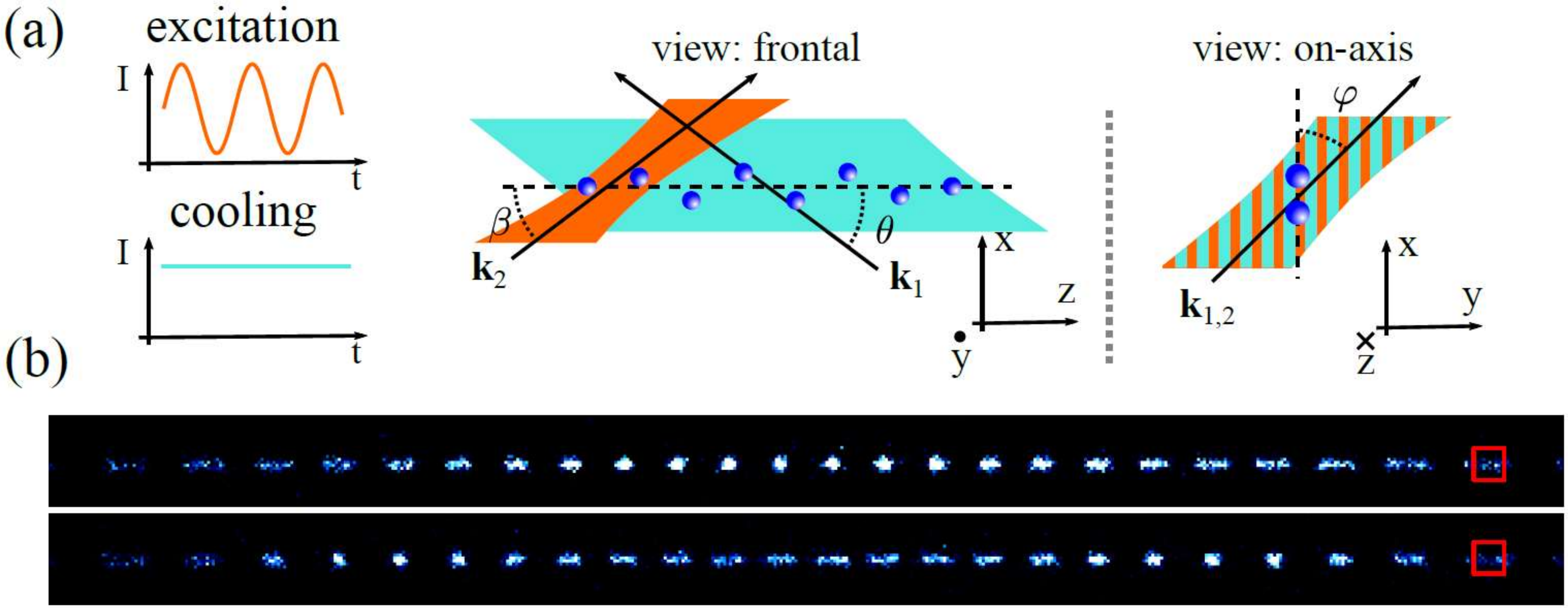
Also at linear-to-zigzag transition:
System changes between potential minima
→
similar at Aubry-type transition

effective potential



Fishman *et al.,* PRB 77, 064111 (2008)

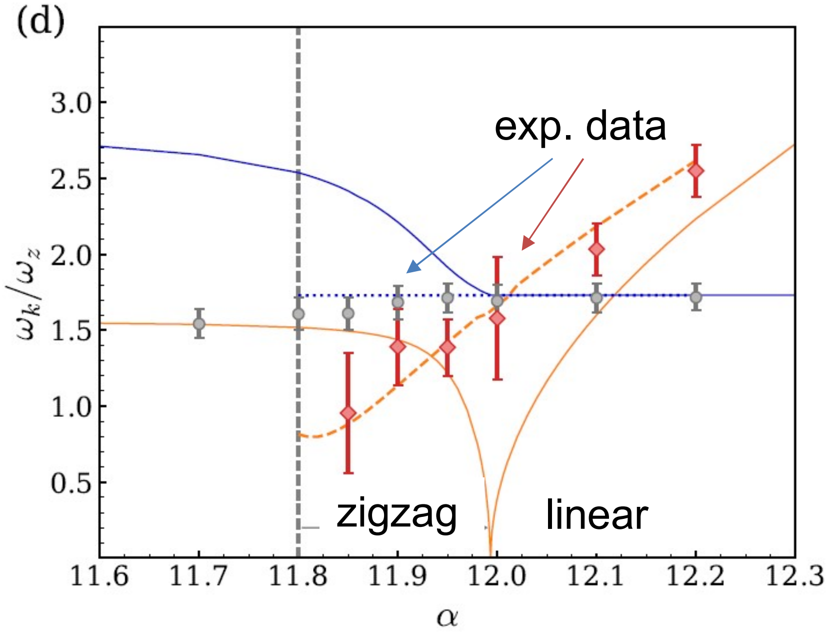
Spectroscopy of phonons via resonant light forces



Photos of excitation of lowest energy phonons (1 node, 2 nodes, …)

Coupling of soft mode to thermal phonon environment

Theory for two lowest frequency modes:



dashed lines = new theory

Equation of motion of phonon *i* under non-linear interaction:

$$V_4 = \frac{1}{2!}\sum_{i=1}^{3N} m\omega_i^2\Theta_i^2 + \frac{1}{3!}\sum_{ijk=1}^{3N} L_{ijk}\Theta_i\Theta_j\Theta_k + \frac{1}{4!}\sum_{ijkl=1}^{3N} M_{ijkl}\Theta_i\Theta_j\Theta_k\Theta_l$$

$$\ddot{\Theta}_i = -\frac{1}{m}\frac{\partial V_4}{\partial \Theta_i} - \gamma_i\Theta_i + \Xi_i(t)$$

Time average over higher frequency phonons:

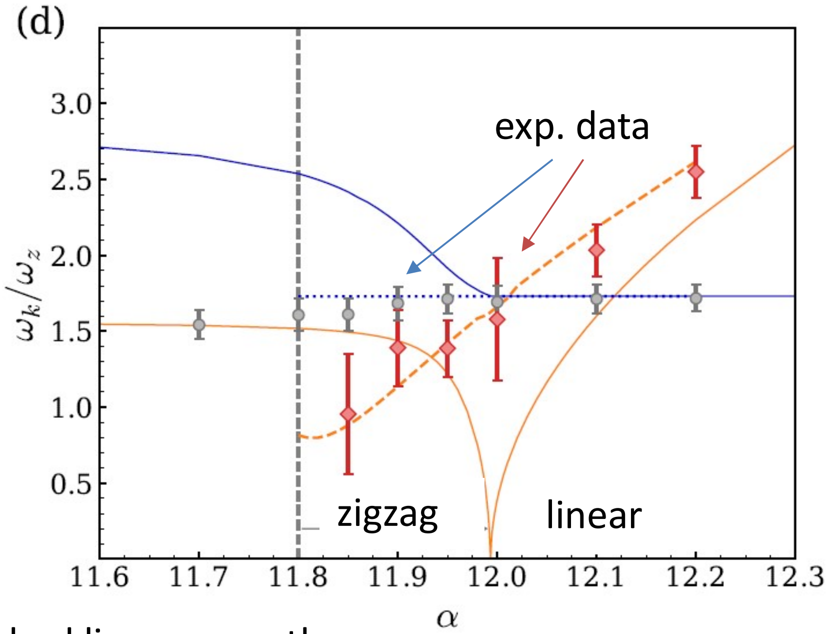
$$\ddot{\Theta}_1 = -\tilde{\omega}_1^2\Theta_1 - \frac{1}{2}\nu_{12}^2\Theta_2 + \eta_1 - \gamma_1\Theta_1 + \Xi_1$$

$$\ddot{\Theta}_2 = -\tilde{\omega}_2^2\Theta_2 - \frac{1}{2}\nu_{12}^2\Theta_1 + \eta_2 - \gamma_2\Theta_2 + \Xi_2 ,$$

with $\tilde{\omega}_i(T)^2 = \omega_i^2 + \nu_{\mathrm{eff},i}^2 T$ $\qquad \nu_{\mathrm{eff},i}^2 = \frac{1}{2m}\sum_{k\neq 1,2} M_{iikk}\frac{k_B}{m\omega_k^2}$

Coupling of soft mode to thermal phonon environment

Theory for two lowest frequency modes:



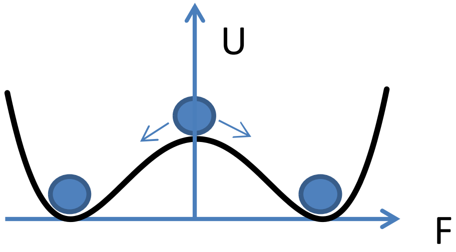
dashed lines = new theory

Conclusion:
also at Linear-to-Zigzag Transition…

→ soft mode is modified due to non-linear coupling to thermal phonon environment

→ soft-mode sees **modified Coulomb environment due to oscillating phonon environment**

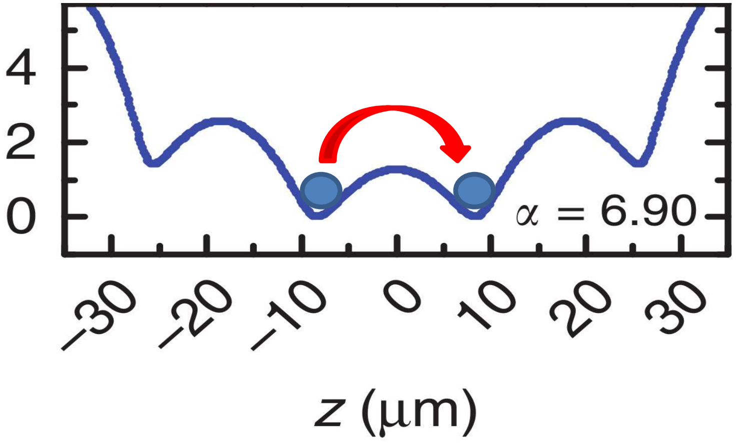
→ Floquet physics

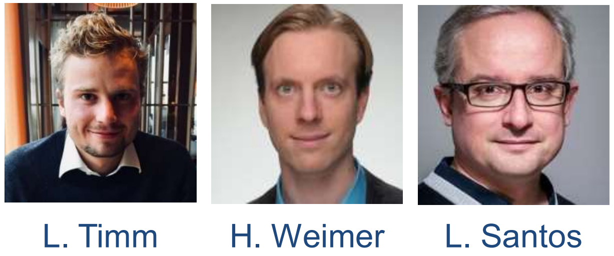


J. Kiethe, L. Timm, H. Landa, D. Kalincev, G. Morigi, T. E. Mehlstäubler
Finite-temperature spectrum at the symmetry-breaking linear to zigzag transition
Phys. Rev. B, **103**, 104106 (2021)

For really low temperatures: Is there quantum nanofriction ?

PN-energy barriers of kink soliton:



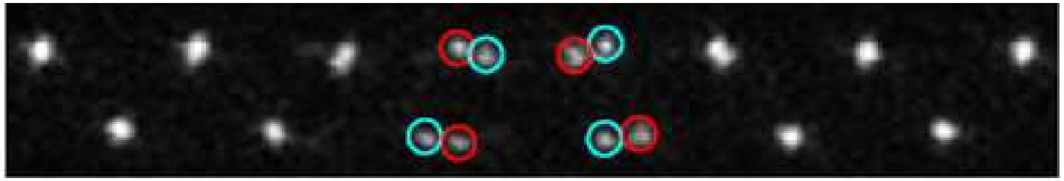


- In future: 2nd stage cooling to **T = μK** via polarization gradient cooling could be possible

Is tunneling over 10s of micrometer possible?

Is there quantum nanofriction ?

Tunneling over 10s of micrometer? → collective excitation!
→ ions move by only a few nm while top. defect moves by 10s µm



Define effective mass of the quasi particle:

$$M_{\text{eff}}(X) = m \sum_i \left(\frac{d\vec{r}_{i,C}(X)}{dX} \right)^2$$

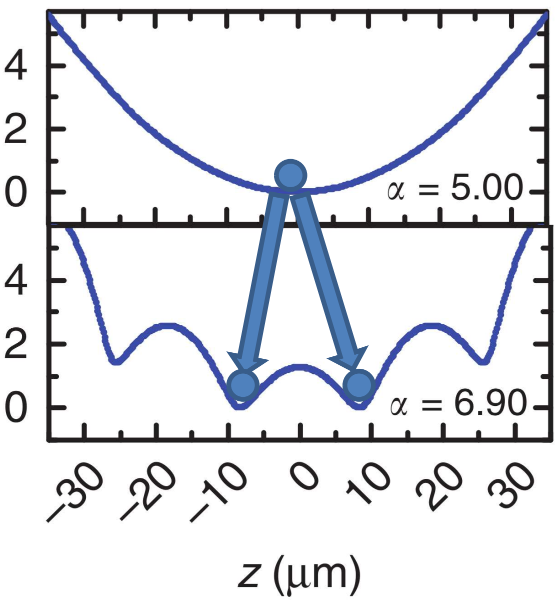
Solve Hamiltonian for quasi particle:

$$\hat{H}_s = \hat{P} \frac{1}{2M_{\text{eff}}(\hat{X})} \hat{P} + U(\hat{X})$$

L. Timm, L. A. Rüffert, H. Weimer, L. Santos, T. E. Mehlstäubler
Quantum nanofriction in trapped ion chains with a topological defect, Phys. Rev. Research, 3, 043141 (2021)

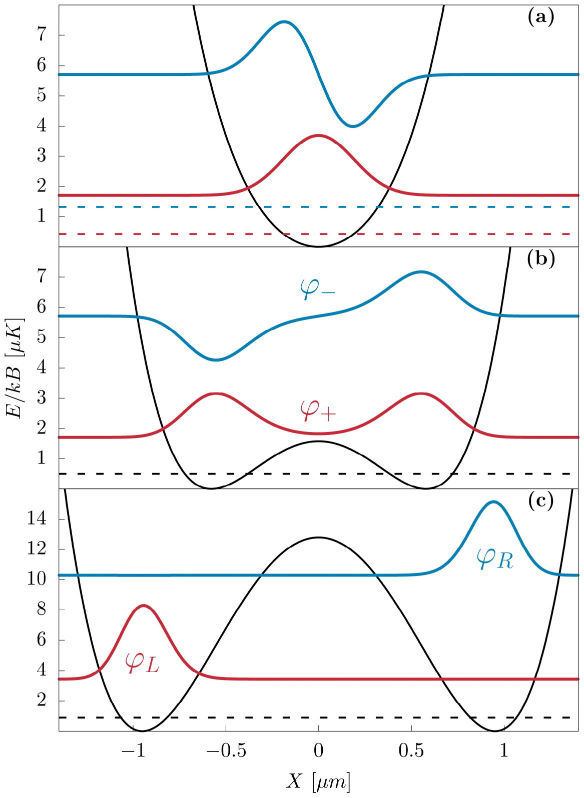
Quantum nanofriction

Critical α



QUANTUM WORLD

Assumption: Kink = quantum particle in classical PN potential



Sliding phase:

- Harmonic eigenstates
- Equidistant spectrum

Tunneling regime:

- Barrier splits ground state wavefunction
- Sym. and antisym. pairs

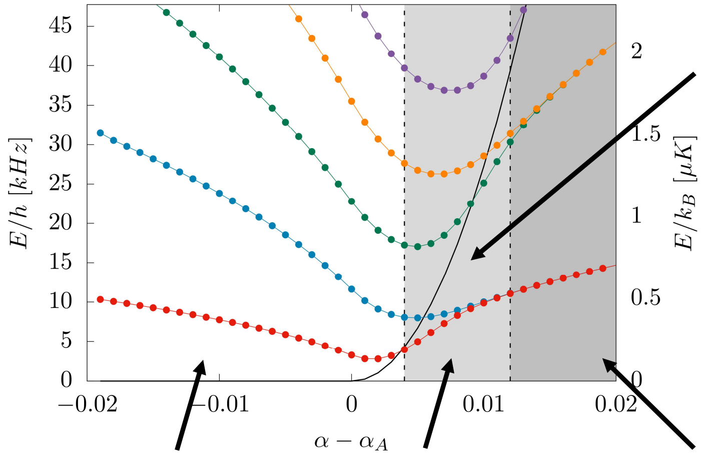
Quasi-classical regime:

- Large barrier →tunneling negligible
- Localized states

Consequences for observables in experiments?

Quantum nanofriction

ENERGY SPECTRUM



PN barrier height

Sliding

Tunneling regime

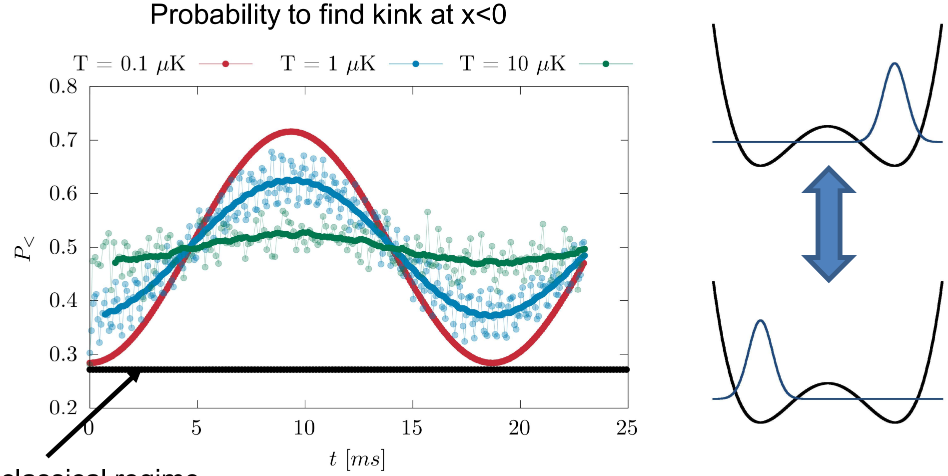
Quasi-classical regime

- Quantum fluctuations: No "softmode"
- Tunneling regime: Eigenstate energy below barrier height but tunneling causes energy gap
- Quasi-classical regime: Degenerate sets of eigenstates, localized left/right

→ **Spectroscopic measurement of the energy spectrum after ground-state cooling**

TUNNELING DYNAMICS

Probability to find kink at x<0



Quasi-classical regime
for T = 0.1 μK

- Initialize a localized state in one potential minimum and monitor evolution
- Oscillation between left and right minimum due to tunneling
- Not observable in quasi-classical regime

→ **At low μK temperatures tunneling dynamics is observed**

Work by other groups

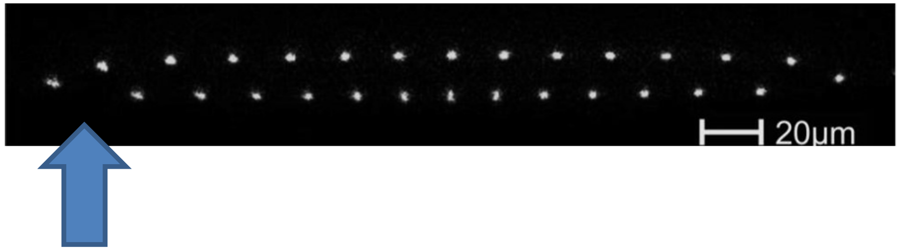
Kiethe *et al.*, ***Probing Nanofriction and Aubry-type Signatures in a Finite Self-Organized System,***
Nat. Communs. 8, 15364 (2017)

Brox *et al.*, ***Spectroscopy and Directed Transport of Topological Solitons in Crystals of Trapped Ions,*** PRL 119, 153602 (2017)

Kiethe *et al.*, ***Nanofriction and Motion of Topological Defects in Coulomb Crystals,*** New J. Phys. 20, 123017 (2018)

→ Symmetries in Ion Trap and Light Forces

Outlook: Energy Transport with Topological Defect



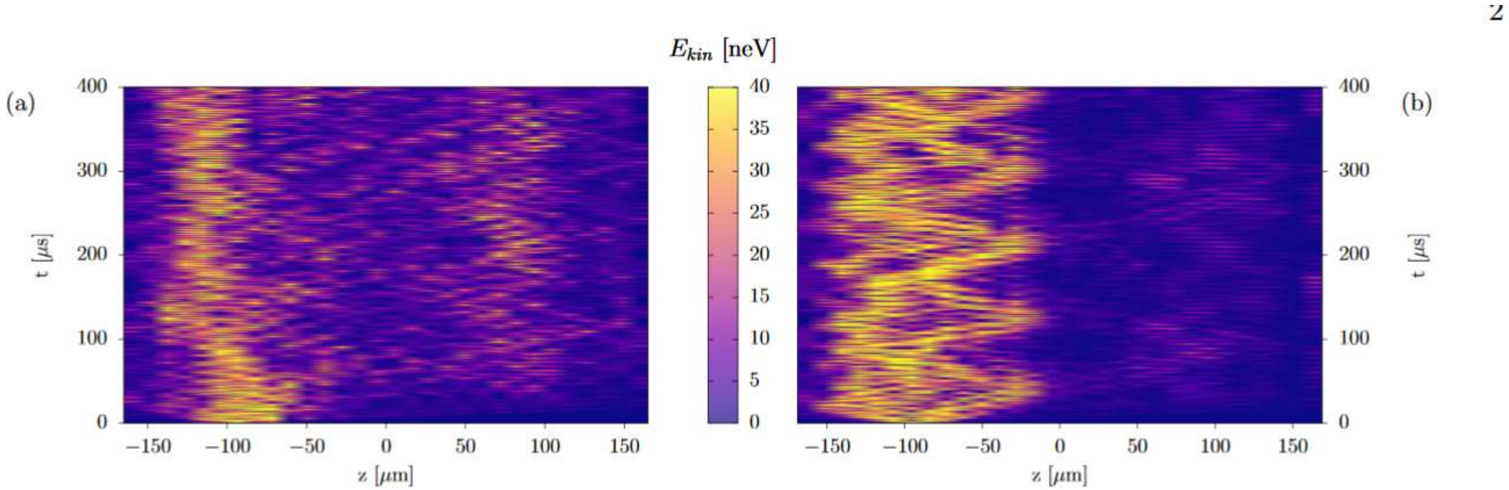


FIG. 2: Dynamics of the kinetic energy after a 1 μm displacement of ion 7 along z, calculated using Eq. (2), for (a) $\alpha = 5.5$ (sliding phase) and (b) $\alpha = 6.8$ (pinning phase).

Timm et al., ***Energy localization in interacting atomic chains with topological solitons,*** Phys. Rev. Res. (2020)

The Team “Quantum Clocks and Complex Systems”

Thank you for your attention!

Bac D. Kalincev, M. Brinkmann, H. A. Fürst, N. H. Hausser

K. Rossignol, A. P. Kulosa, J. Kiethe, T. Nordmann

New:

exchange with Osaka and Tokyo

Int. Collaborations:
NICT Toyko (J)
University of Osaka (J)
CMI (Prag, Cz)
NPL (London, UK)
W. Zurek (Los Alamos NL)
R. Nigmatullin (Uni Sydney, Au)
ILP and Uni Novosibirsk (R)
Haggai Landa (IBM, IL)
…

Visiting scientists:
S. Ignatovich (ILP, Novosibirsk)
N. Ohtsubo (NICT, Tokyo)
M. Kitao (Osaka University)
M. Doležal (CMI, Prag)
L. Ye (JPL/CALTECH)

Industry Partners:
Grintech (Jena)
Naneo (Lindau)
D&G (Stuttgart)
Toptica (München)
Vacom (Jena)
QUARTIQ (Berlin)
…

TRiAC

International Joint Laboratory for Trapped-Ion Integrated Atomic-Photonic Circuits

DFG

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European Association of National Metrology Institutes

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