Euclidean Q-balls of fluctuating SDW/CDW density waves as the pairing 'glue' in the pseudogap and superconducting phases of high-Tc cuprates

S.I. Mukhin

Theoretical Physics & Quantum Technologies Department,

[1]NUST "MISIS", Moscow, Russia

[2]Institute Lorentz, Leiden University, Leiden, Netherlands

- Peter Karpov – **MISIS & Max-Plank Inst., Dresden**
- Timur Galimzyanov - **MISIS**

- Serguei Brazovskii – **Univ. Paris-Sud, Orsay**
- Jan Zaanen - **Leiden University**
- Carlo Beenakker - **Leiden University**
- Konstantin Efetov- **RU Bochum**

S.I. Mukhin, Condens. Matter, 3, 39 (2018)

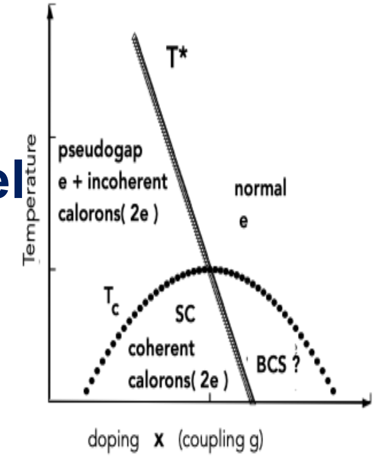
S.I. Mukhin, T.R. Galimzyanov, PRB 100, 081103 (R) (2019)

S.I. Mukhin, Condens. Matter, 7, 31 (2022)

S.I. Mukhin, arXiv:2108.10372v3 14 Feb (2022)

ECRYS 2022, IESC

1. Emergence at T* of Euclidean Q-balls of SDW/CDW with Cooper/local pair condensates (Q is conserved magnon number = Noether charge) in Hubbard model with 'nested' antinodal points : 1st order phase transition



2. Semiclassical Eliashberg equations: Q-ball local energy minimum at finite amplitude of 'pairing glue'

3. Q-ball properties:
   i) Q-ball superconducting density n_s scales linear with T_c
   ii) Q-balls 'gas' has diamagnetic moment at $T_c < T < T^*$
   iii) Q-ball fermionic spectral (pseudo)gap g_0 in the antinodal points of fermionic momenta $T_c < T < T^*$: $g_0 \sim \sqrt{n_s}$ at T*; $g_0 \sim n_s$ at T_c

4. Infinite percolating cluster of Q-balls vs infinite Q-ball radius: bulk superconducting transition scenarios

5. Euclidean Q-ball of CDW (SDW) fluctuations with wave-vector Q_{DW} causes superconducting s (d)-wave order in the Brillouin zone

Q-ball in the Eliashberg scheme

1. Euclidean action S_M of a scalar complex field M(τ,r) U(1) invariant to global phase rotation ϕ, $M \Rightarrow Me^{i\phi}$:

$$S_M = \int_0^\beta \int_V d\tau d^D \mathbf{r} \frac{1}{g}\left\{ |\partial_\tau M(\tau,\mathbf{r})|^2 + s^2 |\partial_\mathbf{r} M(\tau,\mathbf{r})|^2 + \mu_0^2 |M(\tau,\mathbf{r})|^2 + gU_f(|M(\tau,\mathbf{r})|^2) \right\}$$

2. Define D+1-dimensional 'current density' $\{j_\tau, \vec{j}\}$ of the scalar field $M(\tau,\mathrm{r})$:

$$j_\alpha = \frac{i}{2}\left\{ M^*(\tau,\mathbf{r})\partial_\alpha M(\tau,\mathbf{r}) - M(\tau,\mathbf{r})\partial_\alpha M^*(\tau,\mathbf{r}) \right\}$$

3. Conservation of the ‘Noether charge’ along the Matsubara time axis:

$$\frac{\partial}{\partial\tau}\int_V j_\tau d^D\mathbf{r} = -s^2\int_V div\vec{j}\, d^D\mathbf{r} = -s^2\oint_{S(V)} \vec{j}\cdot d\vec{S} = 0,$$ provided that:

$$M(\tau,\mathbf{r}) = e^{-i\Omega\tau} M\Theta\{\mathbf{r}\}; \quad \Theta(\mathbf{r}) \equiv 1; \mathbf{r}\in V; \Theta(\mathbf{r}) \equiv 0; \mathbf{r}\notin V$$

4. The Noether charge of the Q-ball is constant: $Q = \int_V j_\tau d^D\mathbf{r} = \Omega M^2 V = const$

The consequence is finite volume of Q-ball : $V_Q = \dfrac{Q}{\Omega M^2}$

Rosen (1968), S. Coleman (1985)

Q-ball in Eliashberg scheme

$$S_M^0 = \int_0^\beta \int_V d\tau d^D\mathbf{r}\frac{1}{g}\left\{|\partial_\tau M|^2 + s^2|\partial_\mathbf{r} M|^2 + \mu_0^2|M|^2\right\}, \; M \equiv M(\tau,\mathbf{r})$$

bare 'pairing boson' field D(ω, q) $\propto < \left|M_{\omega,q}\right|^2 >$

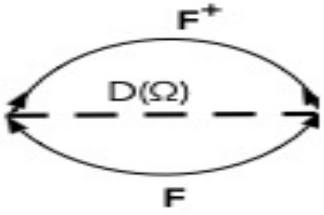
$$S_f = \int_0^\beta \int_V d\tau d^D\mathbf{r}\sum_{\mathbf{q},\sigma}\left[c^+_{\mathbf{q}\sigma}(\partial_\tau + \varepsilon_q)c_{\mathbf{q},\sigma} + \left(c^+_{\mathbf{q}+\mathbf{Q}_{\mathbf{DW}},\sigma}M(\tau,\mathbf{r})\sigma c_{\mathbf{q},\sigma} + H.c.\right)\right]$$

Gor′kov Green function $F_{q\omega} = < c_{q\sigma}c_{-q-\sigma} >_\omega$

$$S_M = \int_0^\beta \int_V d\tau d^D\mathbf{r}\frac{1}{g}\left\{|\partial_\tau M(\tau,\mathbf{r})|^2 + s^2|\partial_\mathbf{r} M(\tau,\mathbf{r})|^2 + \mu_0^2|M(\tau,\mathbf{r})|^2 + gU_f(|M(\tau,\mathbf{r})|^2)\right\}$$

energy of superconducting fluctuation inside Q-ball: $gU_f = \Delta\Omega_s/V$

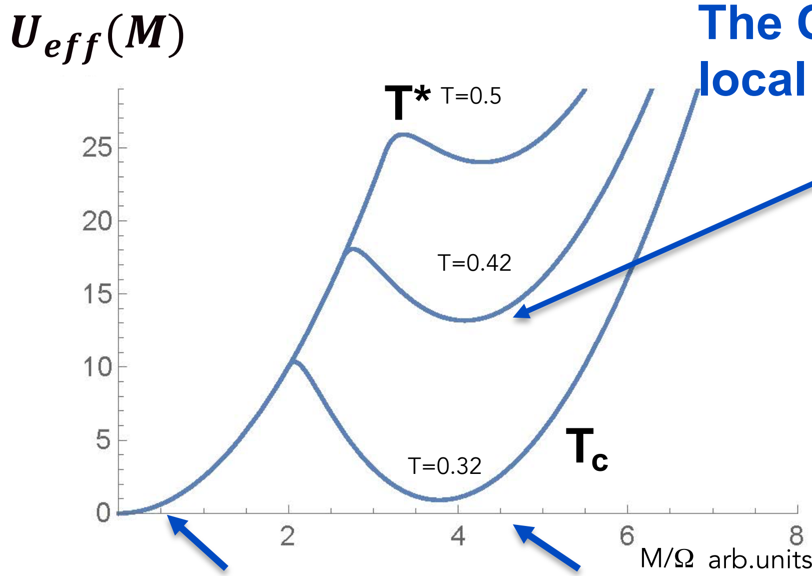
$$VU_f(|M(\tau,\mathbf{r})|) = \Delta\Omega_s = -T\ln\frac{Tr\left\{e^{-\int_0^\beta H_{int}(\tau)d\tau}\mathcal{G}(0)\right\}}{Tr\left\{\mathcal{G}(0)\right\}} \quad =$$



The Q-ball stability condition

$$\Omega_Q = TS_Q = \frac{1}{g}\left[V|\partial_\tau M|^2 + V\left[\mu_0^2 M^2 + gU_f\,(M)\right]\right]$$

$$\mu_0^2 M^2 + gU_f(M) \equiv U_{eff}(\mathrm{M})$$



The Q-ball stability condition :
local minimum of U_{eff}(M) at finite amplitude M

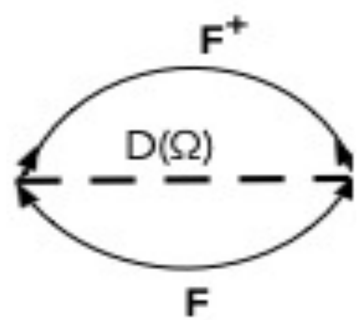
Rosen (1968), S. Coleman (1985)

spin-waves = weak coupling

Q-modes= strong coupling

Source of local minimum: binding energy of fermions in the SDW/CDW field

$$D(\Omega) = {M^2}/{T}$$



→ $gU_f = \Delta\Omega_s/V$

Finite Q-ball volume

$$\Omega_Q = TS_Q = \frac{1}{g}\left[V|\partial_\tau M|^2 + V\left[\mu_0^2 M^2 + gU_f\,(M)\right]\right] = \frac{1}{g}\left[\frac{Q^2}{VM^2} + VU_{eff}\right]$$

Q-ball volume minimizes its free energy Ω_Q:

$$\partial\,\Omega_Q/\partial V = 0 = -\,Q^2/_{V^2M^2} + U_{eff}$$

Conserved Q-ball charge

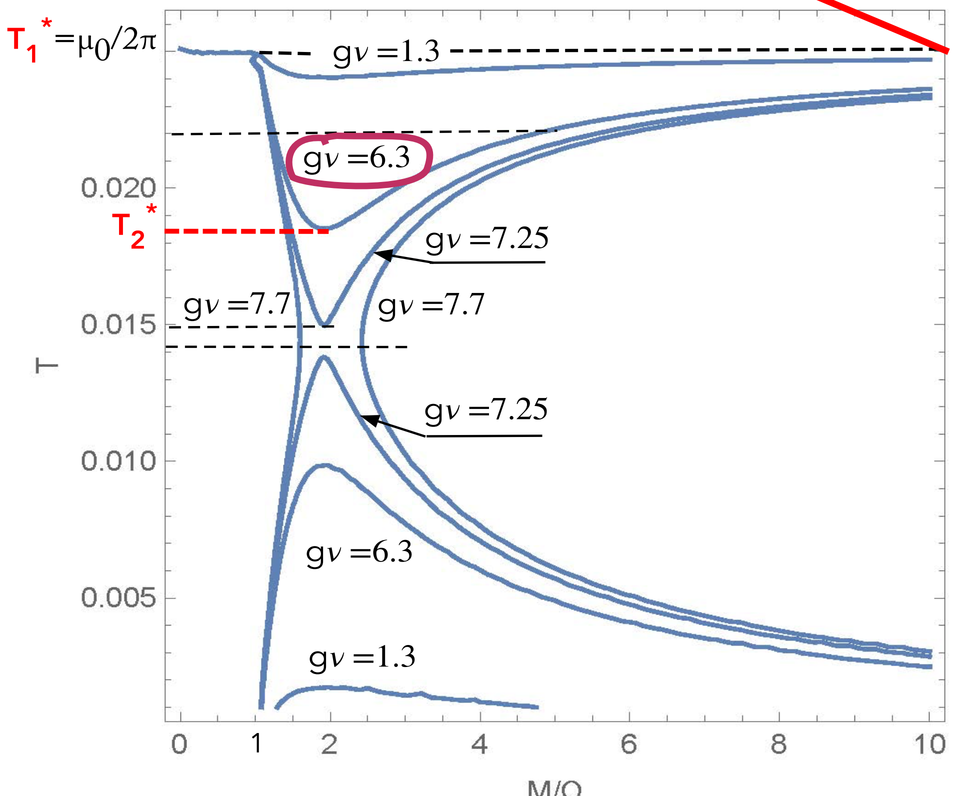
$$V|\partial_\tau M|^2 = V\Omega^2 M^2$$

$$Q = \int_V j_\tau d^D\mathbf{r} = \Omega M^2 V = const$$

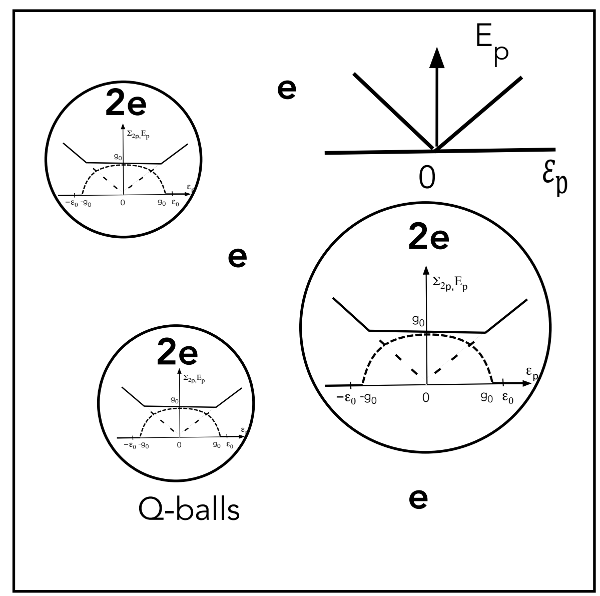
$$\widetilde{U}_{eff} = -\,\Omega M^2 + U_{eff}(M) = 0$$

$$\Omega = 2\pi T n$$

self-consistency condition for amplitude M

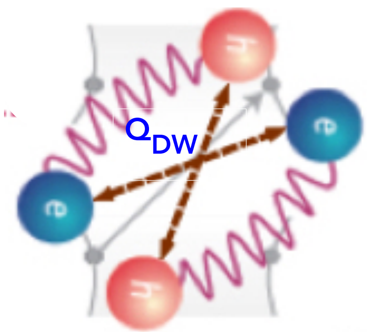


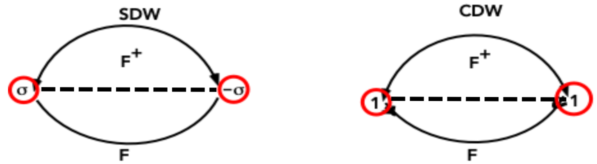
Pseudogap phase with Q-balls

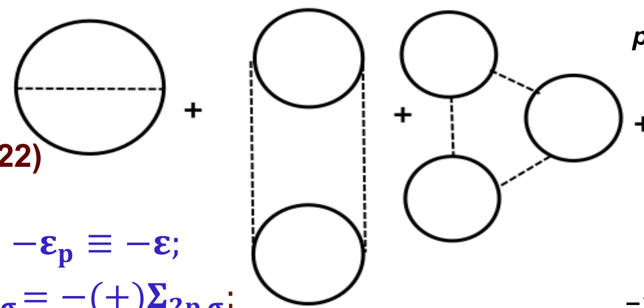


Contours $\widetilde{U}_{eff} = 0$; coordinates: $\left\{\frac{M}{\Omega}, T\right\}$ for different values of the coupling constant $g\nu$

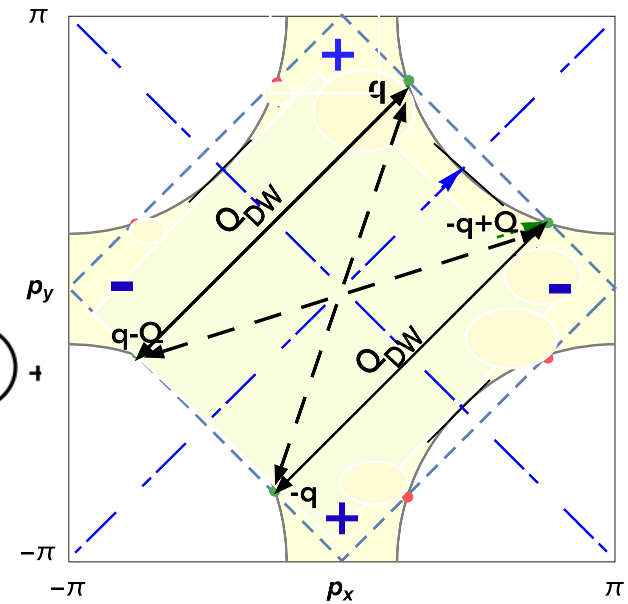
The nature of the local minimum of the function $U_{eff}(M)$







The case of Q-ball SDW



S.I. Mukhin, Condens. Matter, 7, 31 (2022)
S.I. Mukhin, arXiv:2108.10372v3 14 Feb (2022)
S.I. Mukhin, Condens. Matter, 3, 39 (2018)

nesting condition: $\varepsilon_{p-Q_{DW}} = -\varepsilon_p \equiv -\varepsilon;$

d(s) − wave condition: $\Sigma_{2p-Q_{DW},\sigma} = -(+)\Sigma_{2p,\sigma};$

Interfermion exchange by Q-ball SDW/CDW mode, with nesting wave-vector Q_{DW}, as a mechanism of superconducting pairing

$$H_{int} = \int_V d^D r \sum_{q,\sigma} \left(c^+_{q+Q_{DW},\sigma} M(\Omega, Q_{DW}) \sigma c_{q,\sigma} + \mathrm{H.c.}\right)$$

$$M(\tau, r) = M\left(e^{-i\Omega\tau} e^{iQ_{DW}\cdot r} + e^{i\Omega\tau} e^{-iQ_{DW}\cdot r}\right)\Theta(r); \qquad \Theta(r) \equiv 1;\ r \in V;\ \Theta(r) \equiv 0;\ r \notin V$$

$$VU_f(|M(\tau,r)|) = \Delta\Omega_s = -T \ln \frac{Tr\left\{e^{-\int_0^\beta H_{int}(\tau)d\tau} G(0)\right\}}{Tr\left\{G(0)\right\}} \equiv \Omega_s - \Omega_0; G(0) \equiv e^{-\beta H_0};$$

Self-consistent solution for the amplitude of the Q-ball SDW/CDW mode and wave function of the superconducting condensate F

ECRYS 2022, IESC

Eliashberg Equations and Bound States Along the Axis of Matsubara Time

$$\Sigma_{2p,\sigma}(\omega) = -T\sum_{\pm\Omega} \frac{D_{Q_{DW}}(\Omega)\Sigma_{2,p-Q_{DW},\sigma}(\omega-\Omega)}{\left|i(\omega-\Omega) - \varepsilon_{p-Q_{DW}} - \Sigma_{1p-Q_{DW},\sigma}(\omega-\Omega)\right|^2 + \left|\Sigma_{2p-Q_{DW},\sigma}(\omega-\Omega)\right|^2}$$

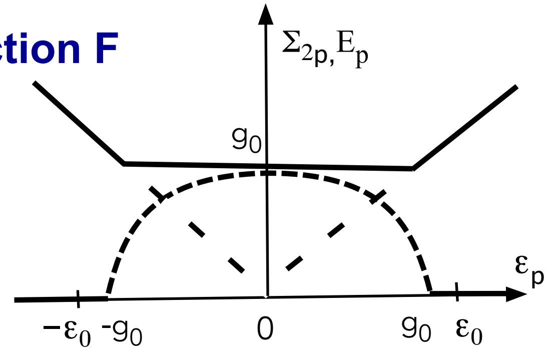
$$D_{Q_{DW}}(\Omega) \equiv \frac{M^2}{T}; \quad D_{Q_{DW}}(\tau) = 2M^2\cos(\Omega\tau); \quad \Sigma_{2p-Q_{CDW},\sigma} = -\Sigma_{2p,\sigma}$$

$$F_{p,\sigma}(\omega) = \frac{-\Sigma_{2p,\sigma}}{|i\omega - \varepsilon_p - \Sigma_{1p,\sigma}(\omega)|^2 + |\Sigma_{2p,\sigma}(\omega)|^2}, \quad \omega = \pi(2n+1)T; \quad n = 0, \pm 1, \ldots$$

The exact consequence of the above is the Mathieu equation (!):

$$-\partial_\tau^2 F_{p,\sigma}(\tau) - 2M^2\cos(\Omega\tau)\,F_{p,\sigma}(\tau) = -g_0^2 F_{p,\sigma}(\tau); \qquad F_{p,\sigma}\left(\tau + \frac{1}{T}\right) = -F_{p,\sigma}(\tau)$$

To be solved under odd parity condition for the Gor'kov function F



The gap g_0 in the fermion spectrum E_p of Q-balls – solid line; self-energy Σ_(2p,σ) dependence on ε_p – short dashed line

$$g_0^2 \equiv \varepsilon_p^2 + \left|\Sigma_{2p,\sigma}(\omega)\right|^2$$

Hence we are looking for the eigenvalue of first excited state of the Mathieu equation, leading to self-consistent solution for the superconducting gap:

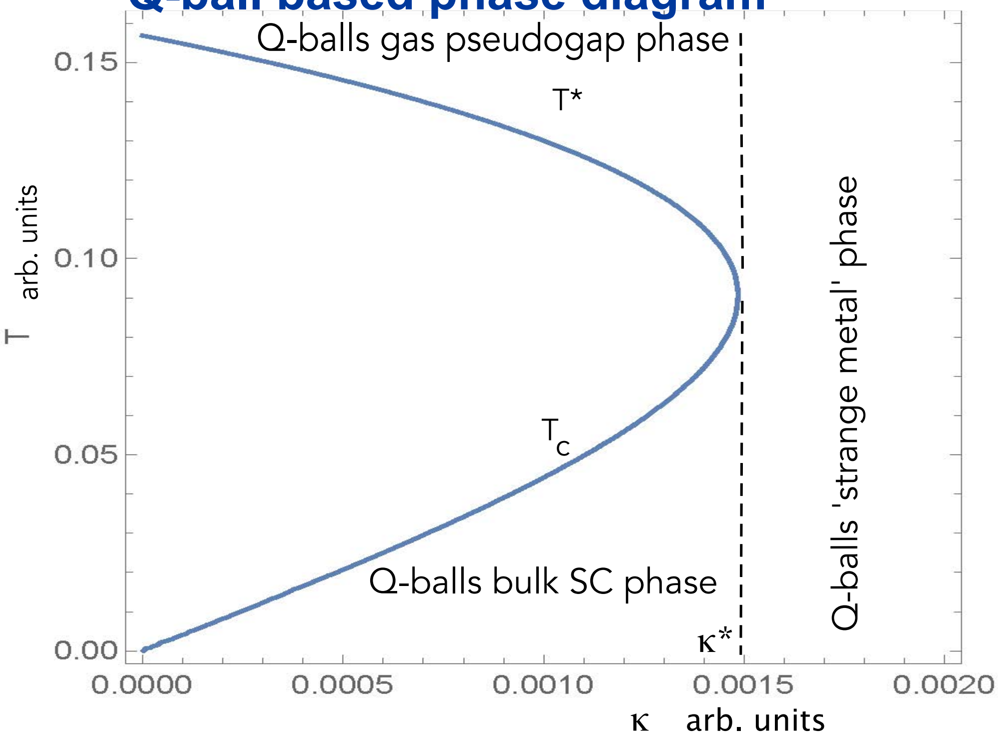
$$g_0^2 \approx 2M(M-\Omega) \;; \quad \Omega = 2\pi T n; \; n=1,2,\ldots$$

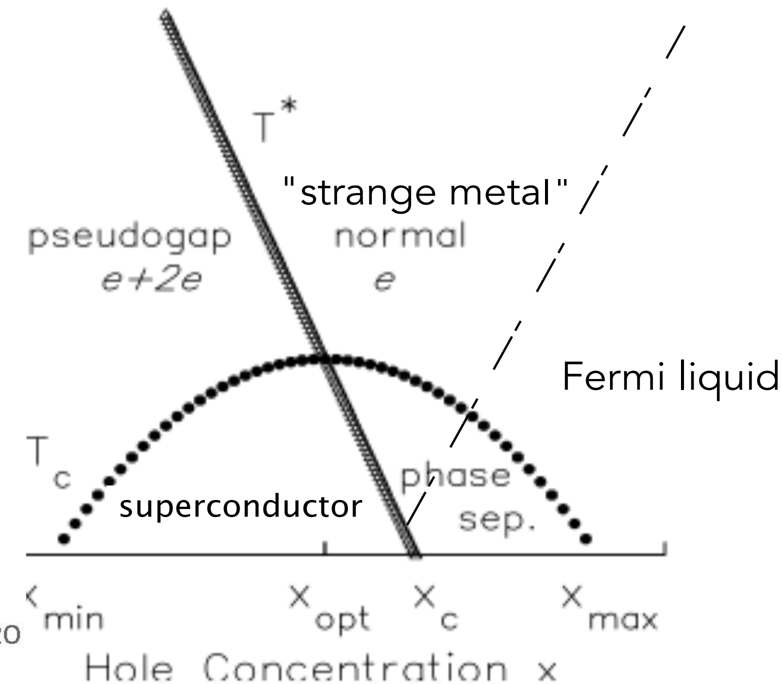
Contour plots of the self-consistency equation and HTSC phase diagram in Q-ball model

$U_{eff} - \Omega M^2 = 0$ $\Rightarrow$ **self-consistency equation (*) in the numerically obtained approximation:**

$\mu_0^2 M^2 + gU_f(M) \equiv U_{eff}$ (*) ; $\left(\mu_0^2 - \Omega^2\right) - \frac{\kappa}{\Omega} = 0$; $\kappa \equiv c\frac{4g\nu\varepsilon_0}{3}$; $c \approx 0.01$

Q-ball based phase diagram





$\kappa^* = \frac{2\mu_0^3}{3^{3/2}}$; $T_c = T^* = \frac{\mu_0}{2\pi\sqrt{3}}$; $\mu_0 \sim 100 - 200$ meV ; $\Rightarrow T_c(opt) \sim 100 \div 200$ K

Tranquada, J.M.et al. Nature, 429, 534 (2004)

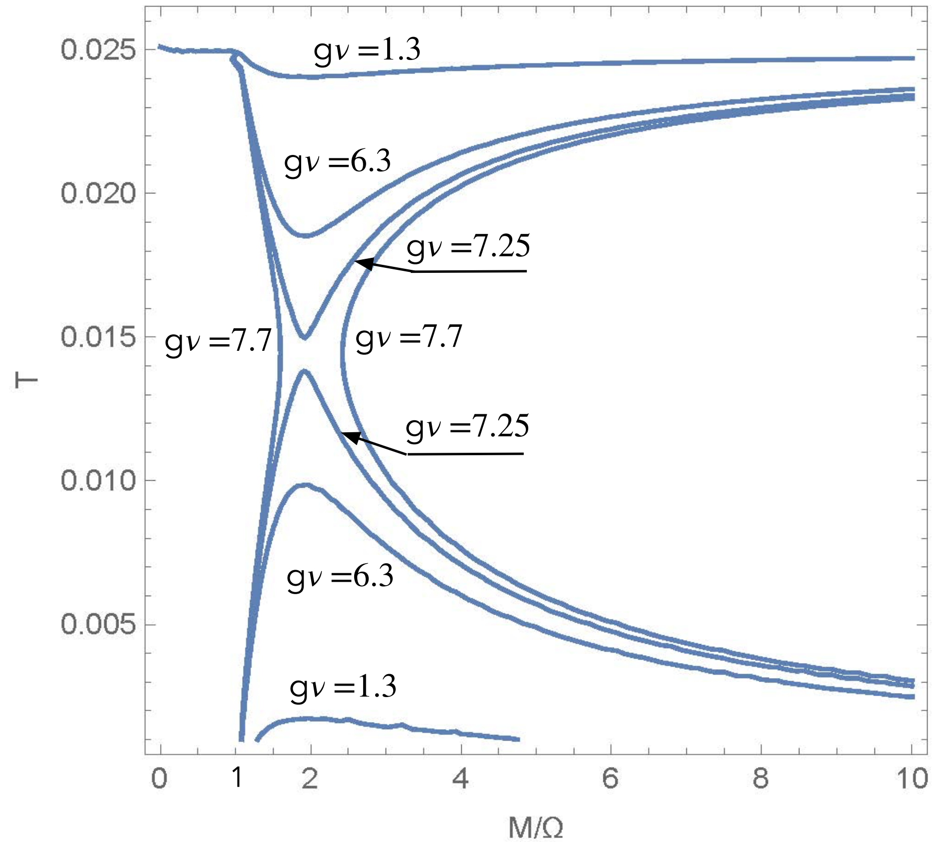
$\kappa = \to 0$; $T_c(\kappa) = \frac{\kappa}{2\pi\mu_0^2}$; $T^*(\kappa) \to T^*_{max} = \frac{\mu_0}{2\pi}$ $\Rightarrow T^*_{max} \sim 200 \div 400$K

Contour plots of the self-consistency equation

$$U_{eff} - \Omega M^2 = 0$$

$$\mu_0^2 M^2 + gU_f(M) \equiv U_{eff}$$

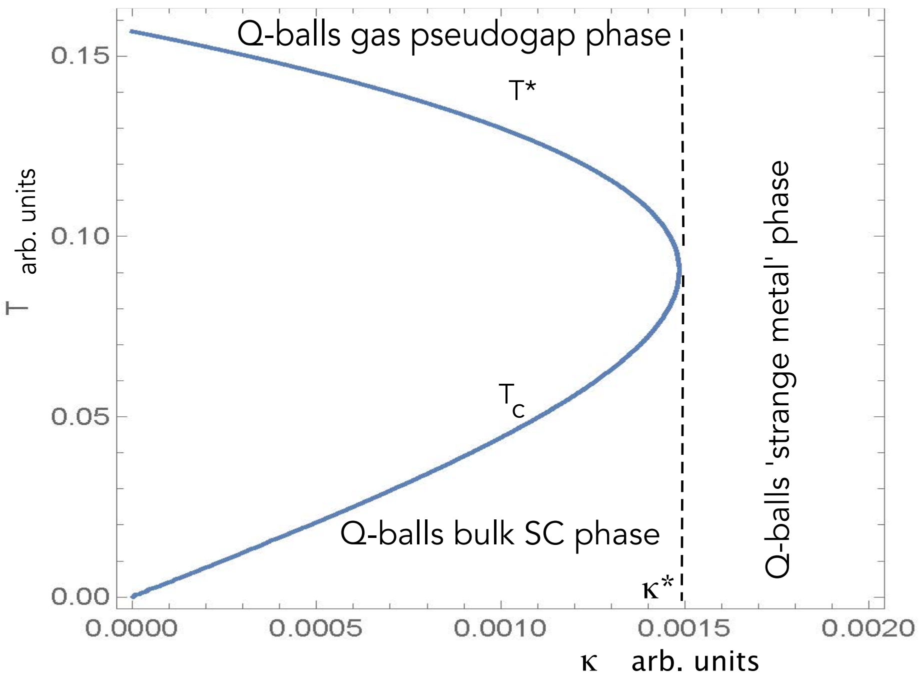
$\Rightarrow$



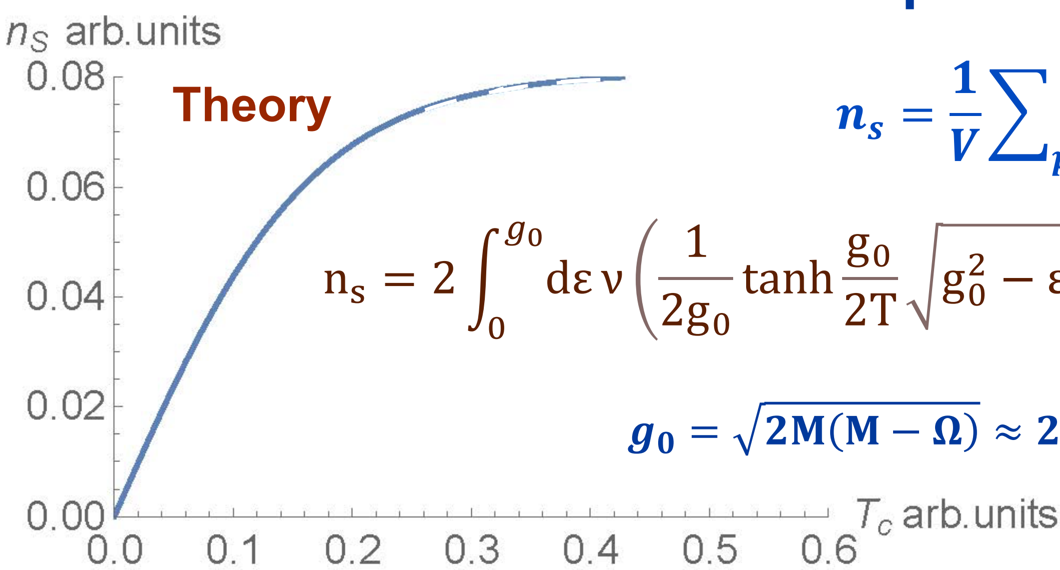
$$(\mu_0^2 - \Omega^2) - c\frac{4g\nu\varepsilon_0}{3\Omega} = 0$$

self-consistency equation

Q-ball based phase diagram



The 'Uemura plot'



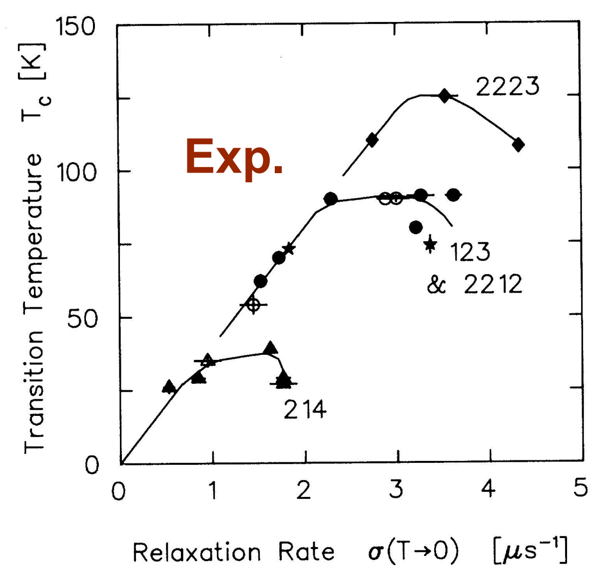
$$n_s = \frac{1}{V}\sum_p \left| T \sum_\omega F_p(\omega) \right|^2$$

$$n_s = 2\int_0^{g_0} d\varepsilon\, \nu \left(\frac{1}{2g_0} \tanh \frac{g_0}{2T} \sqrt{g_0^2 - \varepsilon^2} \right)^2 \Theta(\varepsilon_0 - \varepsilon) \approx \frac{\nu\varepsilon_0}{2} \tanh \frac{4\pi T_c}{3\varepsilon_0}$$

$$g_0 = \sqrt{2M(M-\Omega)} \approx 2\Omega_c; n_s \approx \frac{2\pi\nu}{3} T_c$$

$$M = \Omega\left(1 + \left(\frac{T_n^* - T}{\mu_0}\right)^{\frac{2}{5}} \left(\frac{15\mu_0^2}{4\sqrt{2}g\nu}\right)^{\frac{2}{5}}\right), \quad T_n^* = \frac{\mu_0}{2\pi n};$$

$$g_0^2 = \left(T_n^* - T\right)^{\frac{2}{5}} \Omega^2 \left(\frac{15\mu_0}{g\nu}\right)^{\frac{2}{5}} \Rightarrow n_S \propto g_0^2$$

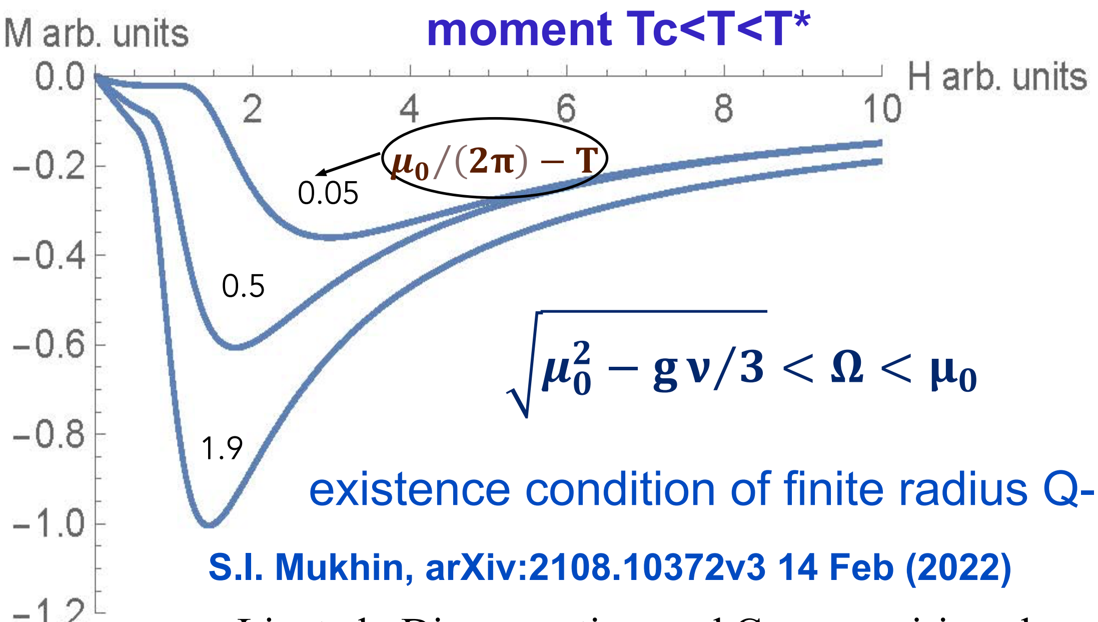


Y. J. Uemura, et al., Universal correlations between Tc and n_s/m^* in high-Tc cuprate superconductors, PRL **62**, 23 (1989).

Q-ball solution M(r)

$$M(r) \equiv \frac{\chi(r)}{r} = \begin{cases} \dfrac{\sin kr}{r}\dfrac{\Omega R}{\sin kR};\ 0 < r \le R; \\ \dfrac{\Omega R}{r}\exp\{\lambda(R-r)\};\ R < r < \infty; \end{cases}$$

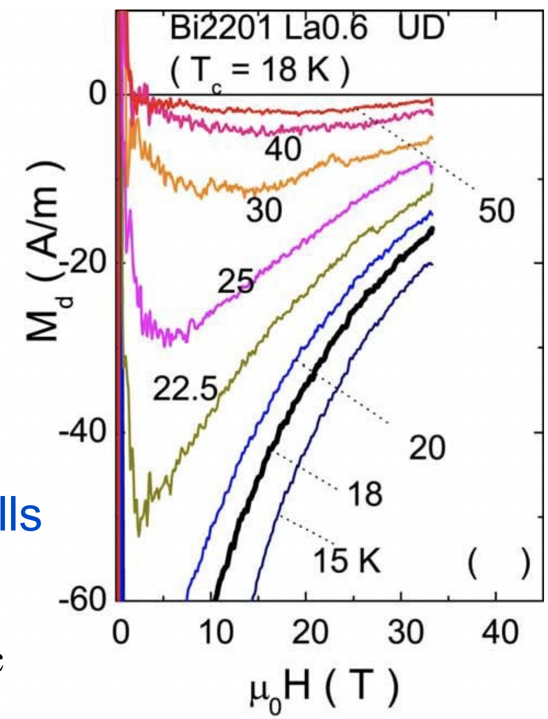
Q-ball gas diamagnetic moment Tc<T<T*



$$\sqrt{\mu_0^2 - g\,\nu/3} < \Omega < \mu_0$$

existence condition of finite radius Q-balls

S.I. Mukhin, arXiv:2108.10372v3 14 Feb (2022)

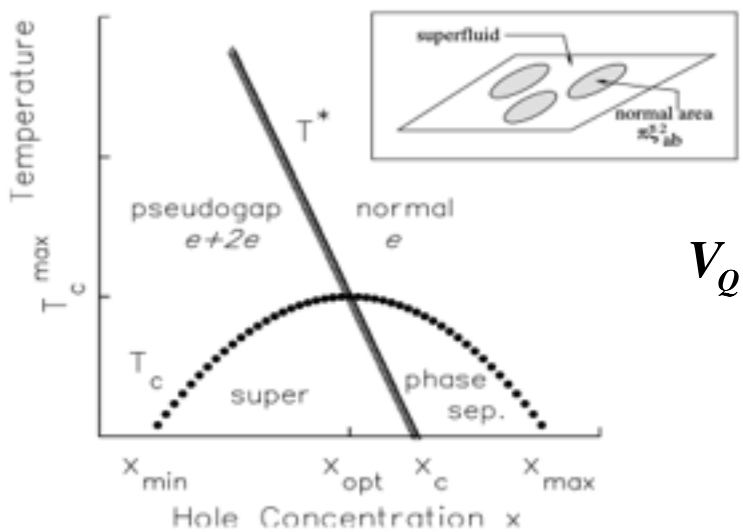


Li, et al., Diamagnetism and Cooper pairing above Tc in cuprates, Phys. Rev. B 81, 054510 (2010).

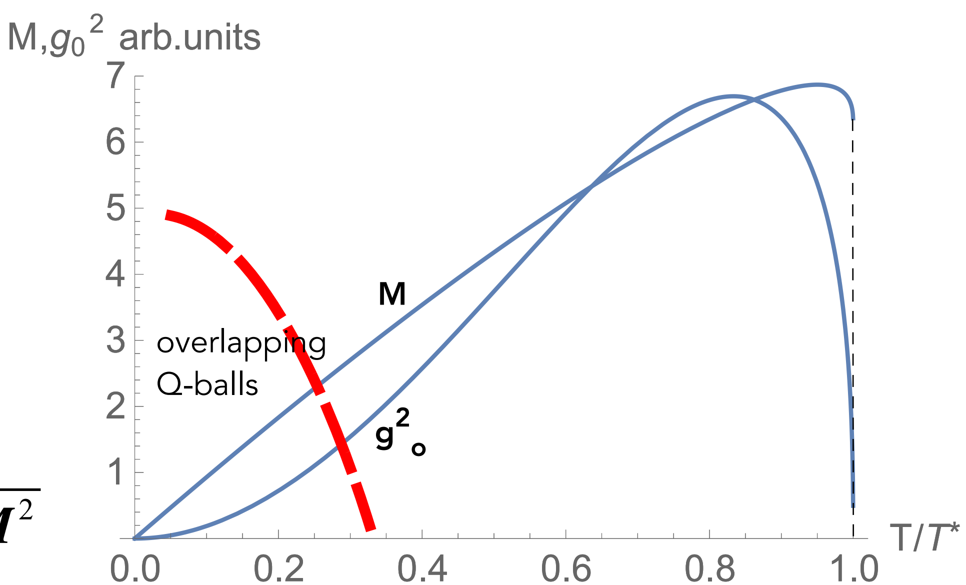
T* and Pseudogap Phase
Analytical solutions of the self-consistency equations

$$M=\Omega\left(1+\left(\frac{T_n^*-T}{\mu_0}\right)^{\frac{2}{5}}\left(\frac{15\mu_0^2}{4\sqrt{2}g\nu}\right)^{\frac{2}{5}}\right),\quad T_n^*=\frac{\mu_0}{2\pi n};\qquad \left(\frac{15\mu_0^2}{4\sqrt{2}g\nu}\right)<<1$$

$$g_0^2=\left(T_n^*-T\right)^{\frac{2}{5}}\Omega^2\left(\frac{15\mu_0}{g\nu}\right)^{\frac{2}{5}}$$



$$V_Q=\frac{Q}{\Omega M^2}$$



Finite size constraints on the minimal 'Noether charge' and specific heat of Q-ball 'gas'

Linearized Ginzburg-Landau (GL) equation for the superconducting order Parameter Ψ for a Q-ball of radius R in the spherical coordinates:

$$-\frac{\hbar^2}{4m}\ddot{\chi} = bg_0^2\chi; \quad \Psi(\rho) = \frac{C\chi(\rho)}{\rho}; \quad \Psi(R) = 0$$

Solution: $\chi \propto \sin(k_n\rho); \quad Rk_n = \pi n,; \quad n = 1, 2, ...,$

Hence the smallest radius R_m of a Q-ball and corresponding volume $V_{\{Q\_m\}}$ should obey the following conditions:

$$\frac{\hbar^2}{4m}\left(\frac{\pi}{R_m}\right)^2 \le bg_0^2, \rightarrow V_Q^{1/3} \ge V_{Q_m}^{1/3} = \left(\frac{Q_m}{\Omega M^2}\right)^{1/3} \equiv R_m = \pi\sqrt{\frac{\hbar^2}{4mbg_0^2}}$$

But at T* g_0 vanishes and ,hence, the minimal size of Q-ball diverges:

$$g_0^2 = \left(T_n^* - T\right)^{\frac{2}{5}}\Omega^2\left(\frac{15\mu_0}{g\nu}\right)^{\frac{2}{5}}$$

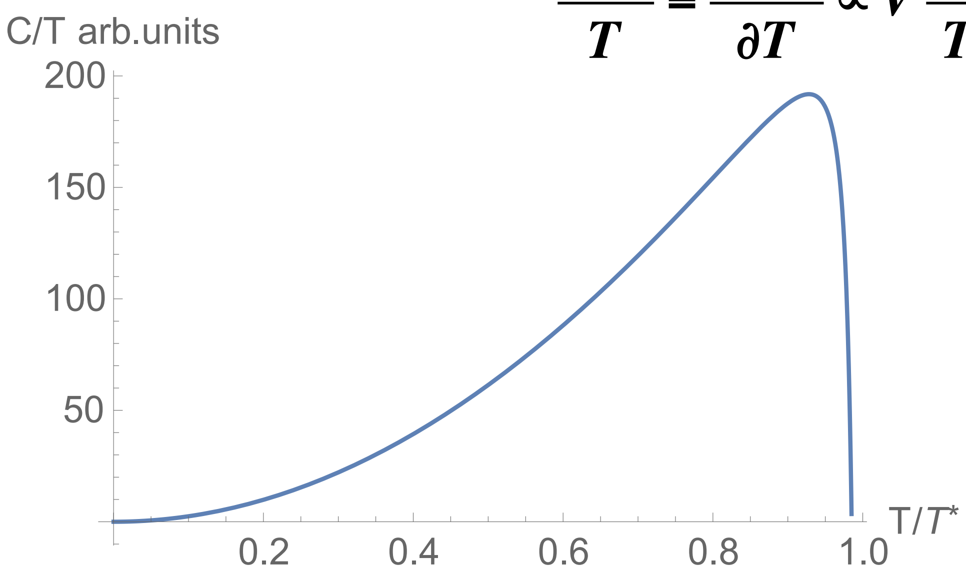
Finite size constraints on the minimal 'Noether charge' and specific heat of Q-ball 'gas'

Find Q-balls contribution to specific heat using for their entropy the thermodynamic expression for the Boltzmann 'gas' :

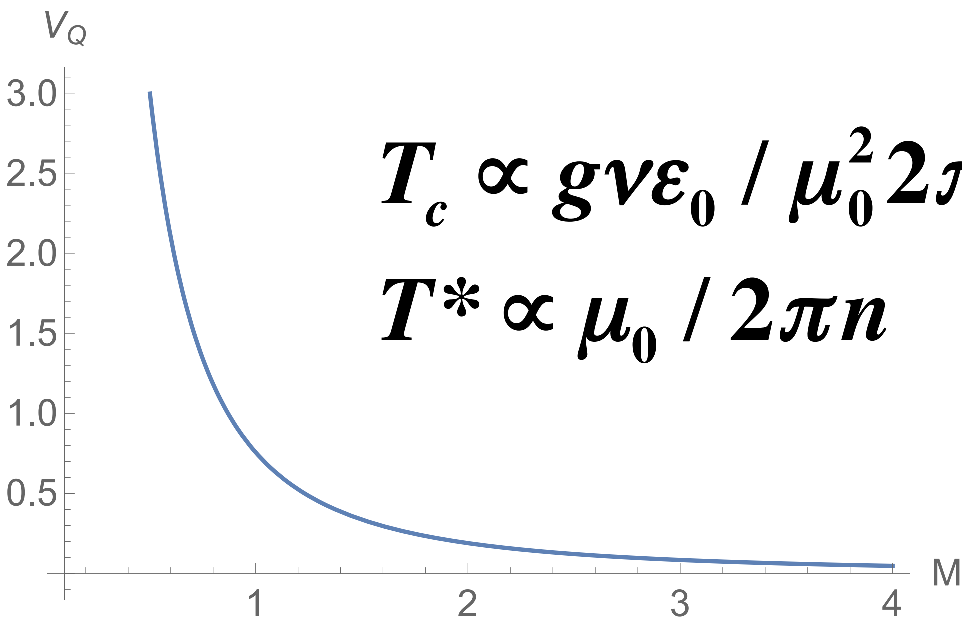
$$S_Q = \sum_{Q,n} G_{Q,n} \overline{n_{Q,n}} \ln \frac{e}{\overline{\overline{n_{Q,n}}}}; \quad \overline{n_{Q,n}} = \exp\left\{-\frac{E_{Q,n}}{k_B T}\right\} = \exp\left\{-\frac{2Q\Omega}{g k_B T}\right\}, G_{Q,n} = \frac{V}{V_Q},$$

Find contribution of Q-balls to the entropy and specific heat of the system after summation over charge $Q>Q_{min}$:

$$\frac{C_{V,n}}{T} = \frac{\partial S_{Q,n}}{\partial T} \propto V \frac{\Omega^3}{T}\left(1 - \frac{C\Omega}{5g(T_n^* - T)^{8/5}}\right)\exp\left\{-\frac{C 2\pi n}{g(T_n^* - T)^{3/5}}\right\}$$



Euclidean Q-balls: "first date"

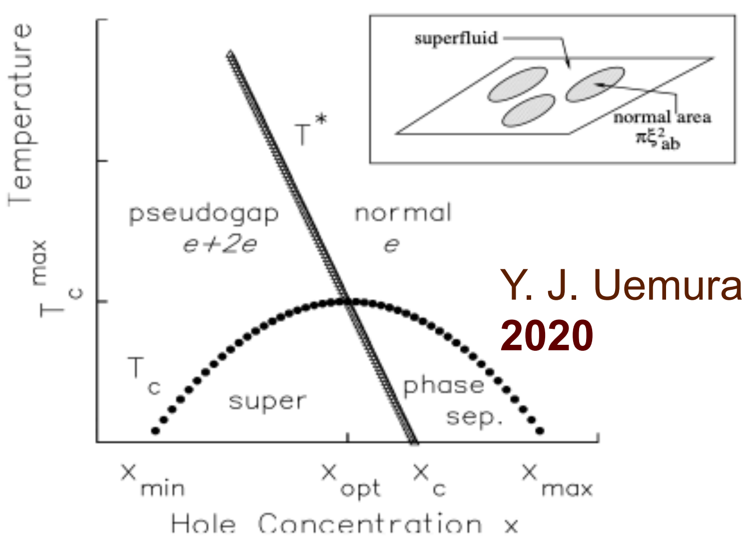


$$T_c \propto g\nu\varepsilon_0 / \mu_0^2 2\pi n$$

$$T^* \propto \mu_0 / 2\pi n$$

$$V_Q = \frac{Q}{\Omega M^2}$$

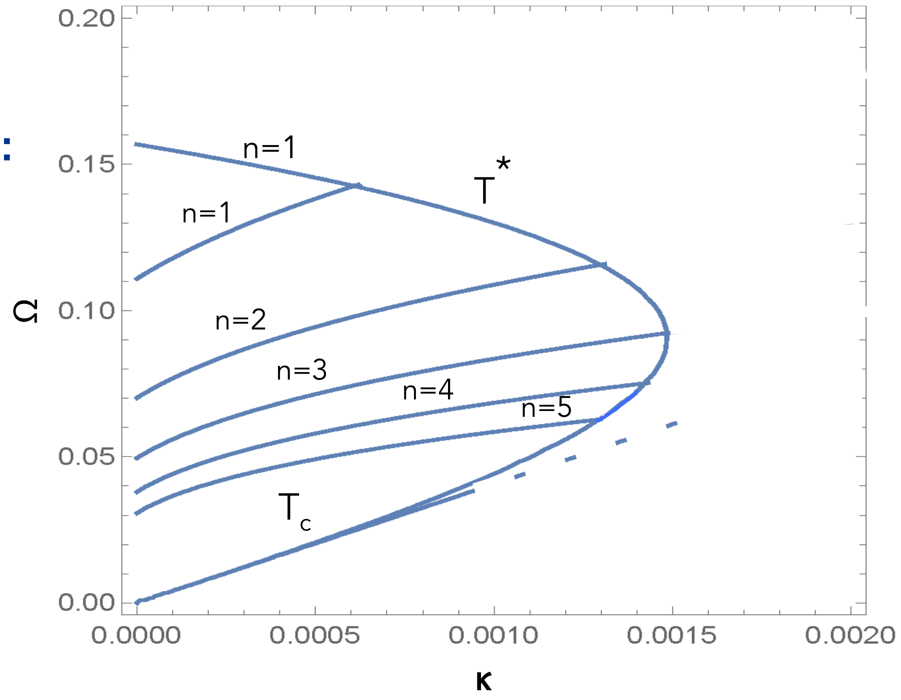
Superconducting transition temperature T_c is one, at which Q-ball radius becomes infinite , or at which different Q-balls with Cooper pairs form infinite percolating cluster.



Y. J. Uemura
2020

Oscillating solutions of the self-consistency equation for a Q-ball in the phase-space plane : coupling constant κ – Matsubara frequency Ω.

Discrete lines marked with integers n = 1, 2, 3, 4, 5 contain points on the phase-space plane where oscillations of the spin/charge density modulus M occur inside the Q-balls. The straight line, denoted by the symbol T_c is transition to the bulk superconductivity.



$$\frac{M(\tau)}{\Omega} = \frac{z_0^2 - c^2}{z_0 + c\cos\Omega_n\tau}, c^2 \equiv \frac{1}{\gamma}\left(1 - \frac{\mu_0^2\Omega - \Omega^3}{\kappa}\right)$$

$$\mu_0^2 - \Omega^2 - \frac{\kappa}{\Omega}\left(1 - \gamma z_0^2\right) = \Omega_n^2, \ n = 1, 2, \ldots$$

$$\gamma \equiv -\frac{f''(z_0)}{2f(z_0)} \approx 2.47, z_0 \approx 1.38,$$

$$\kappa = g\nu\varepsilon_0 f(z_0) \approx c\frac{4g\nu\varepsilon_0}{3}; c \approx 0.01$$

CONCLUSIONS

The "gas" of Q-balls with Cooper pairs condensates can emerge at T* as a fluctuating "short-range order" that self-consistently condenses into a stable superconducting condensate at T_c, while the Q-balls become "vacuum gluons" providing the "glue" for Cooper pairs.

PERSPECTIVES

1. Investigation of the transport properties of the phase with “gas” of Q-balls containing condensates of Cooper pairs (s/d-wave nematic)
2. Investigation of elementary fermionic and bosonic excitations in phase with Q-balls (Quantization of the spectrum of Q-ball states)
3. Applicability check for quantum computing
4. Verification of applications in cosmology ("dark matter")

Supermassive dark-matter Q-balls in galactic centers?
Sergey Troitsky JCAP11, 027 (2016)

THANK YOU !