Bath induced phase transition in the XXZ chain

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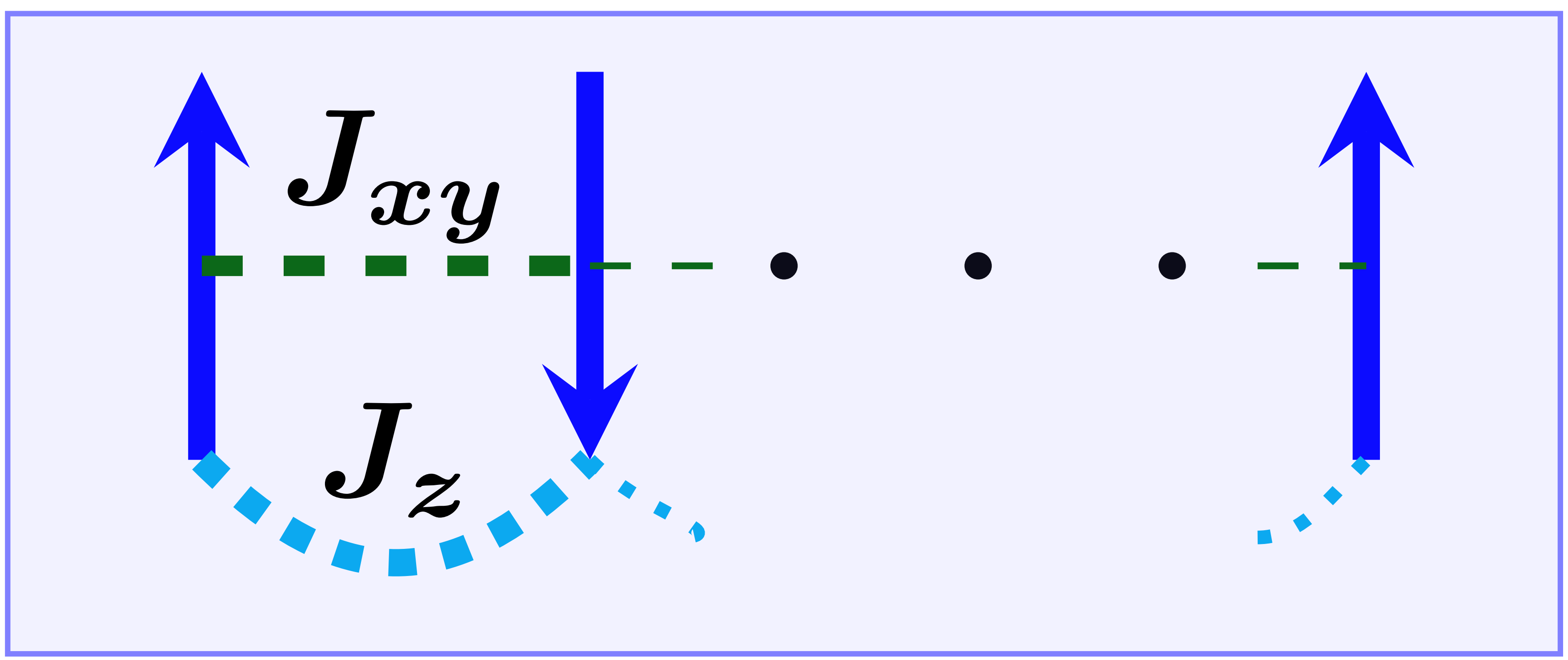
The XXZ spin chain

$$H_S = \sum_{j=1}^{N} J_z S_j^z S_{j+1}^z + J_{xy}\left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y\right)$$



XXZ spin chain

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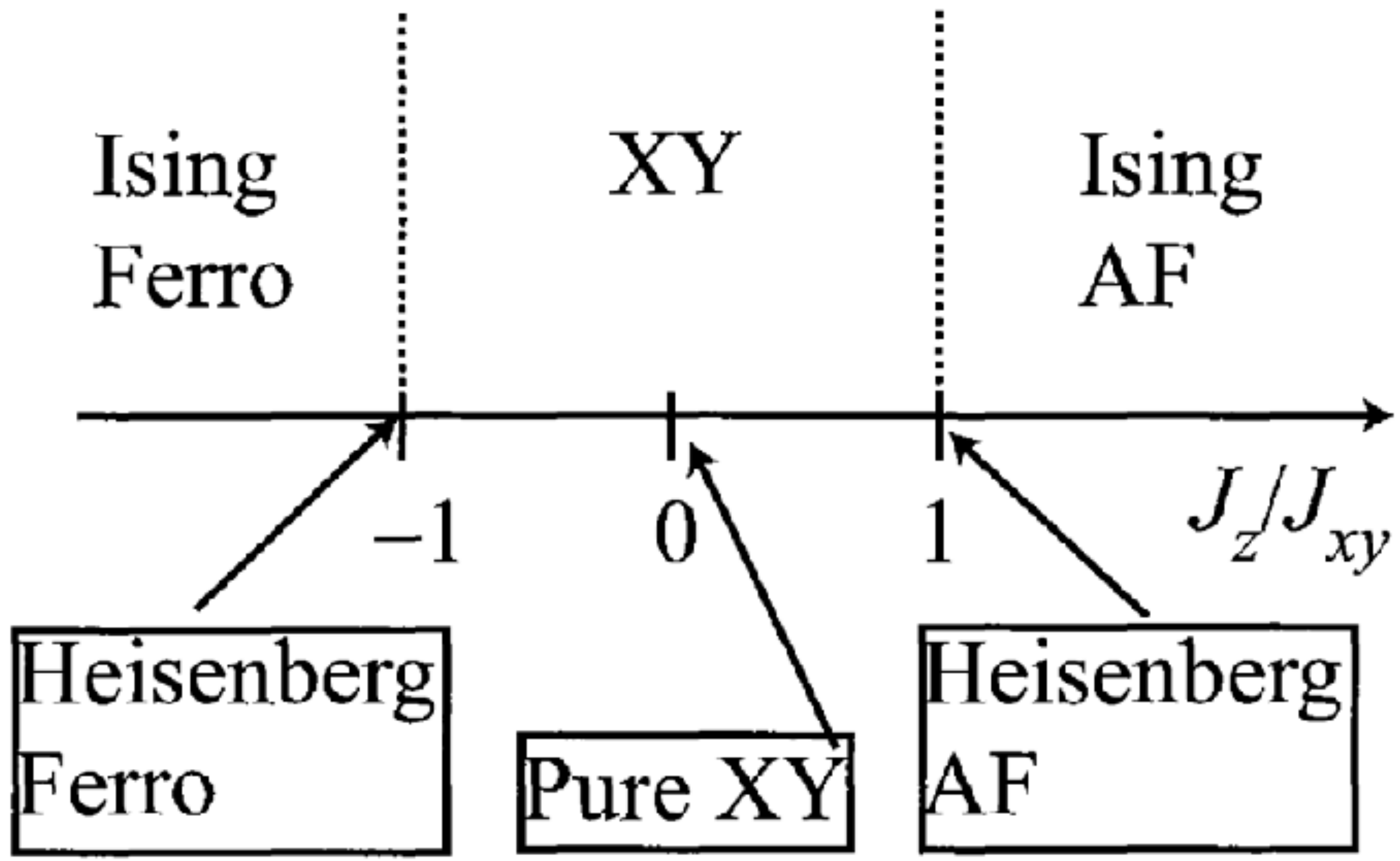
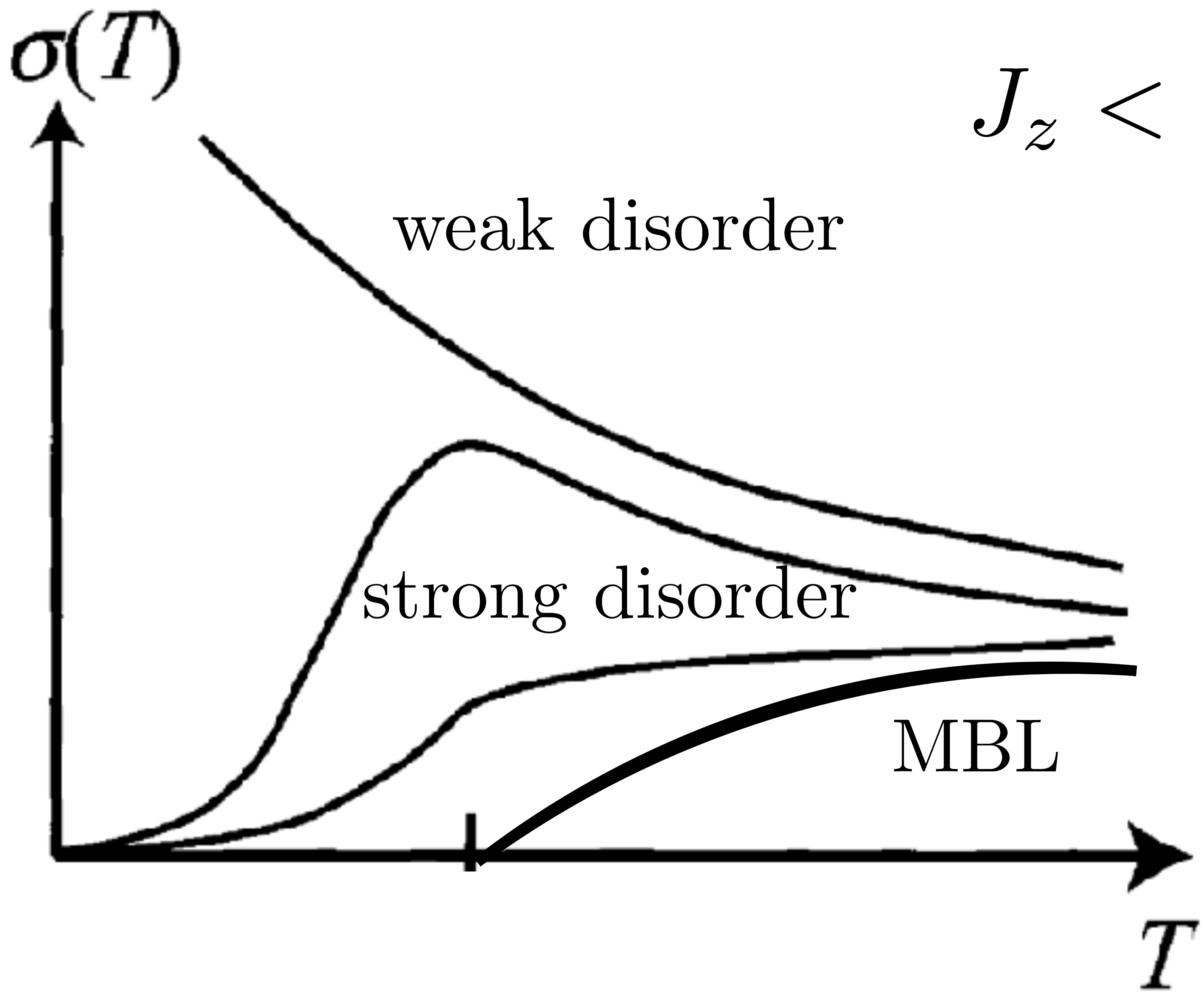
$$H_{J-W} = -\frac{J_{xy}}{2}\sum_{j=1}^{N}\left(c_{j+1}^\dagger c_j + h.c.\right) + J_z \sum_{j=1}^{N}\left(c_{j+1}^\dagger c_{j+1} - \frac{1}{2}\right)\left(c_j^\dagger c_j - \frac{1}{2}\right)$$



The disordered XXZ spin chain

$$H_{dis} = \sum_{j=1}^{N} J_z S_j^z S_{j+1}^z + J_{xy} \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y \right) + h_j S_j^z$$



The XXZ spin chain couples with baths

$$H = H_{\rm S} + H_{\rm B} + H_{\rm SB}$$

$$H_S = \sum_{j=1}^{N} J_z S_j^z S_{j+1}^z + J_{xy}\left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y\right)$$

$$H_{\rm B} = \sum_{jk} \frac{P_{jk}^2}{2m_k} + \frac{m_k \Omega_k^2}{2} X_{jk}^2$$

$$H_{\rm SB} = \sum_{j=1}^{N} S_j^z \sum_k \lambda_k X_{jk}$$

The XXZ spin chain couples with baths

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$$H_S = \sum_{j=1}^{N} J_z S_j^z S_{j+1}^z + J_{xy}\left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y\right)$$

$$H_\mathrm{B} = \sum_{jk} \frac{P_{jk}^2}{2m_k} + \frac{m_k \Omega_k^2}{2} X_{jk}^2$$

$$h_j(t)$$

$$H_\mathrm{SB} = \sum_{j=1}^{N} S_j^z \sum_k \lambda_k X_{jk}$$

Nature of the baths

$$J(\Omega) = \frac{\pi}{2} \sum_k (\lambda_k^2 / m_k \Omega_k) \delta(\Omega - \Omega_k)$$

$$J(\Omega) = \pi \alpha \Omega^s \quad \text{for} \quad \Omega \in (0, \Omega_D)$$

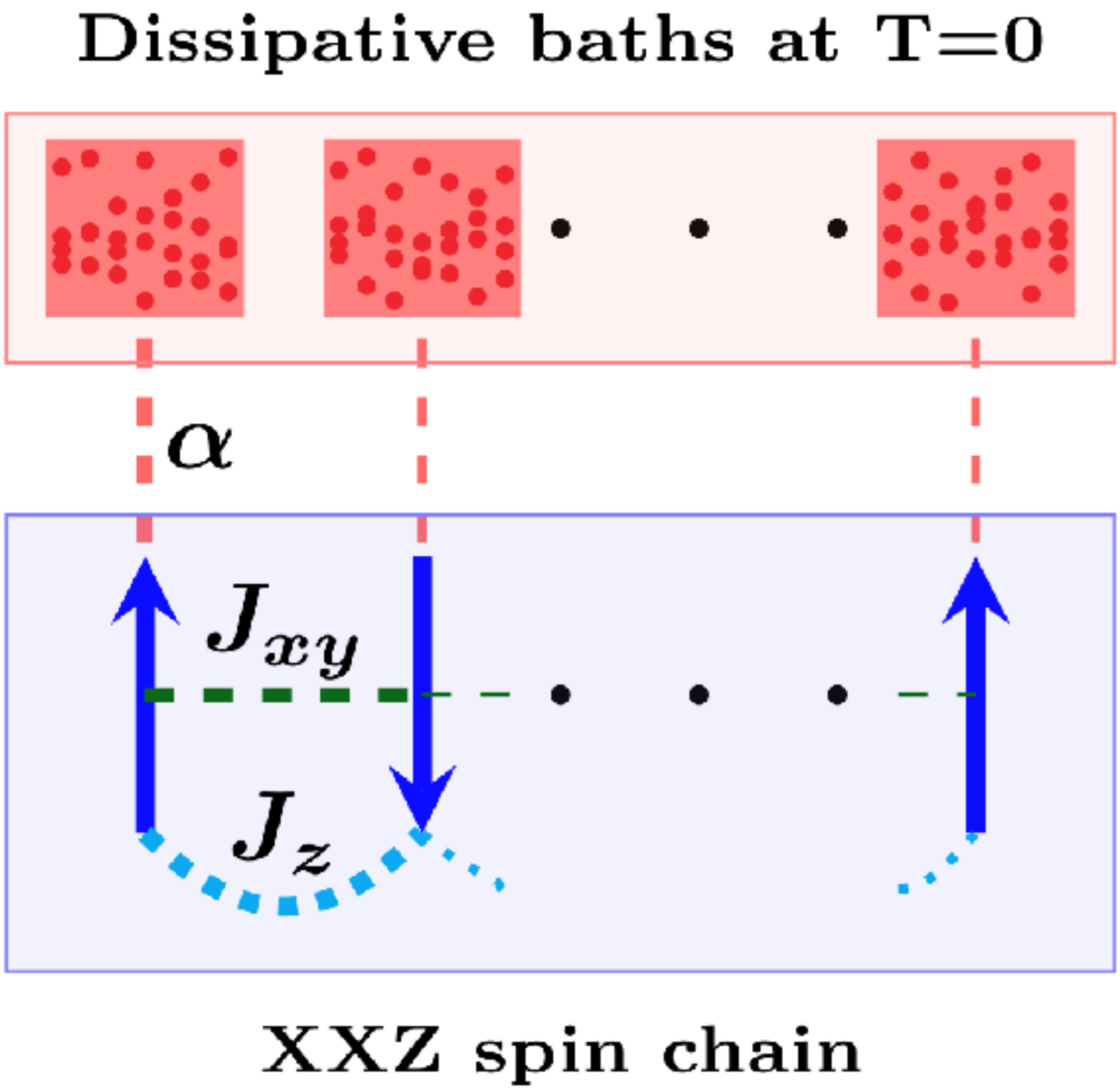
$s < 1$ sub Ohmic baths

$s = 1$ Ohmic baths

$s > 1$ super Ohmic baths

Sketch of our model

$$H = H_{\mathrm{S}} + H_{\mathrm{B}} + H_{\mathrm{SB}}$$



Mapping to a 2D classical field (Bosonization)

$$\mathrm{P}rob[\phi] \propto \exp\left(-S_{\mathrm{tot}}[\phi]\right) \qquad S_{\mathrm{tot}} = S_{\mathrm{LL}} + S_{\mathrm{int}}$$

$$S_{\mathrm{LL}} = \frac{1}{2\pi K}\int_0^L dx \int_0^\beta d\tau \left[\frac{1}{u}\left(\partial_\tau \phi(x,\tau)\right)^2 + u\left(\partial_x \phi(x,\tau)\right)^2\right]$$

Interaction with the bath as a non quadratic term

$$\mathrm{P}rob[\phi] \propto \exp\left(-S_{\mathrm{tot}}[\phi]\right) \qquad S_{\mathrm{tot}} = S_{\mathrm{LL}} + S_{\mathrm{int}}$$

$$S_{\mathrm{LL}} = \frac{1}{2\pi K}\int_0^L dx \int_0^\beta d\tau \left[\frac{1}{u}\left(\partial_\tau \phi(x,\tau)\right)^2 + u\left(\partial_x \phi(x,\tau)\right)^2\right]$$

$$S_{\mathrm{int}} = -\frac{\alpha}{4\pi^2}\int_0^L dx \int_0^\beta d\tau \int_0^\beta d\tau' \, \frac{\cos\left(2\left(\phi(x,\tau) - \phi(x,\tau')\right)\right)}{|\tau - \tau'|^2}$$

Dictionary Classical model / Quantum chain

$$G(q,\omega_n) = \langle \phi(q,\omega_n)\phi(-q,-\omega_n)\rangle \qquad \omega_n = \frac{2\pi n}{\beta}$$

$$\chi = \lim_{q\to 0}\frac{q^2}{\pi^2}G(q,\omega_n=0)$$

$$\rho_S = \lim_{\omega_n\to 0}\frac{\omega_n^2}{\pi^2}G(q=0,\omega_n)$$

$$\sigma(\omega\to 0) = \frac{e^2}{\pi^2\hbar}\mathrm{Re}\left[\omega_n G(q=0,\omega_n)\right]_{i\omega_n\to\omega+i\epsilon}$$

Results for a Luttinger liquid (LL)

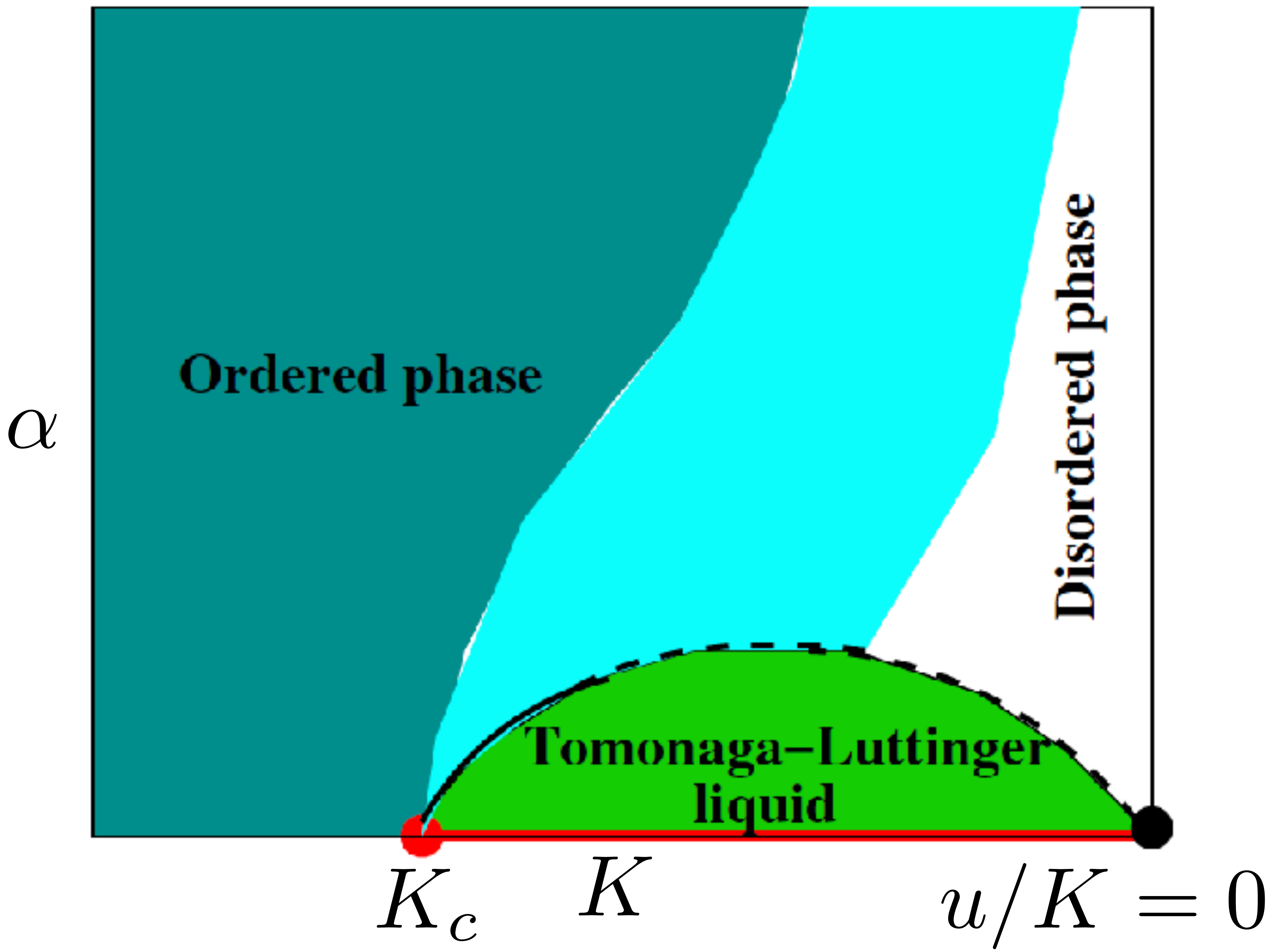
$$G(q,\omega_n) = \langle \phi(q,\omega_n)\phi(-q,-\omega_n)\rangle = \frac{\pi K}{\omega_n^2/u + uq^2}$$

$$\chi = \lim_{q\to 0}\frac{q^2}{\pi^2}G(q,\omega_n=0) = \frac{1}{\pi}\frac{K}{u}$$

$$\rho_S = \lim_{\omega_n\to 0}\frac{\omega_n^2}{\pi^2}G(q=0,\omega_n) = \frac{1}{\pi}uK$$

$$\sigma(\omega\to 0) = \frac{e^2}{\pi^2\hbar}\mathrm{Re}\left[\omega_n G(q=0,\omega_n)\right]_{i\omega_n\to\omega+i\epsilon} = \frac{e^2}{\hbar}uK\delta(\omega)$$

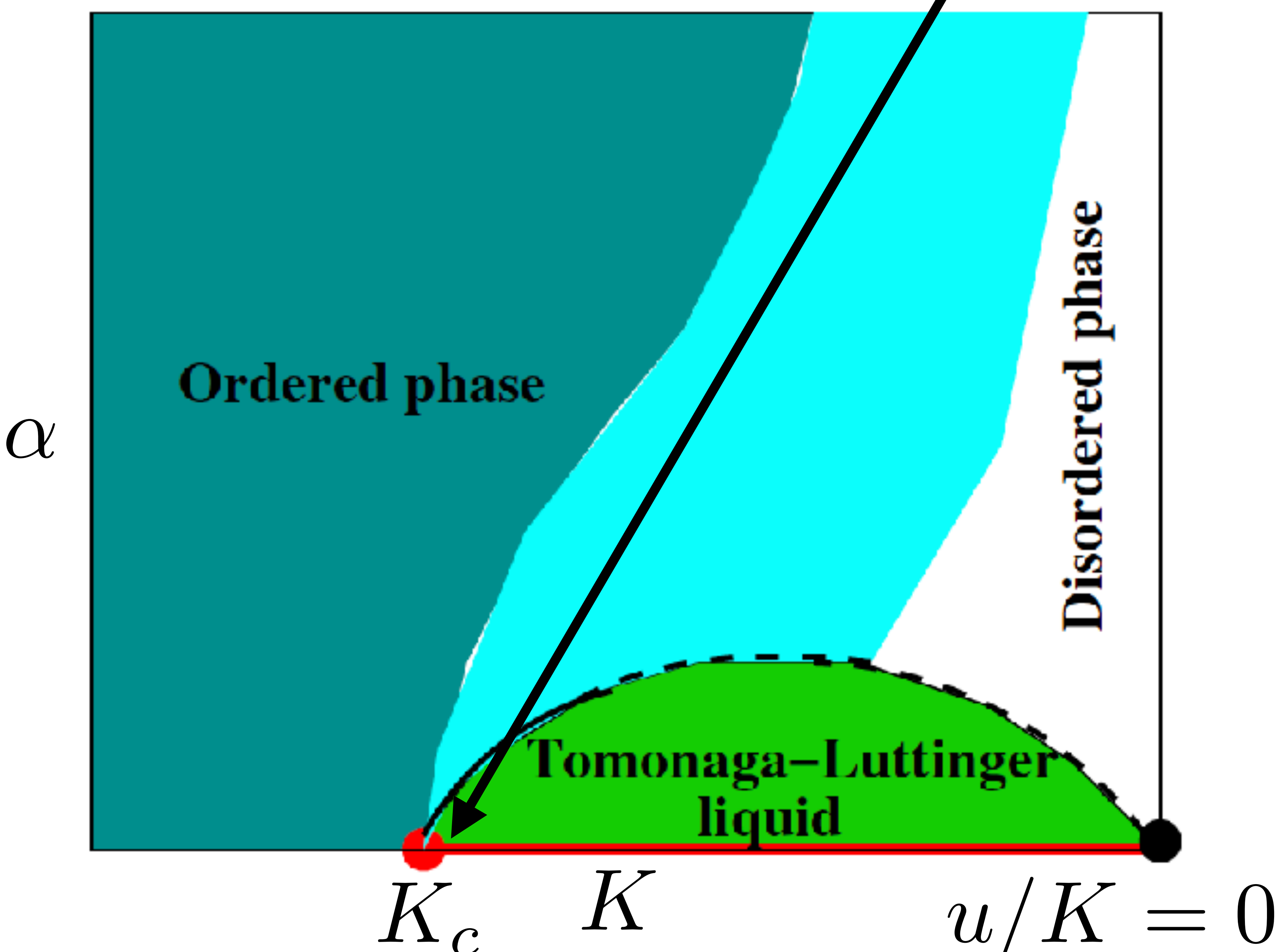
Perturbative RG study



by M. Cazalilla, F. Sols, F. Guinea PRL 2006

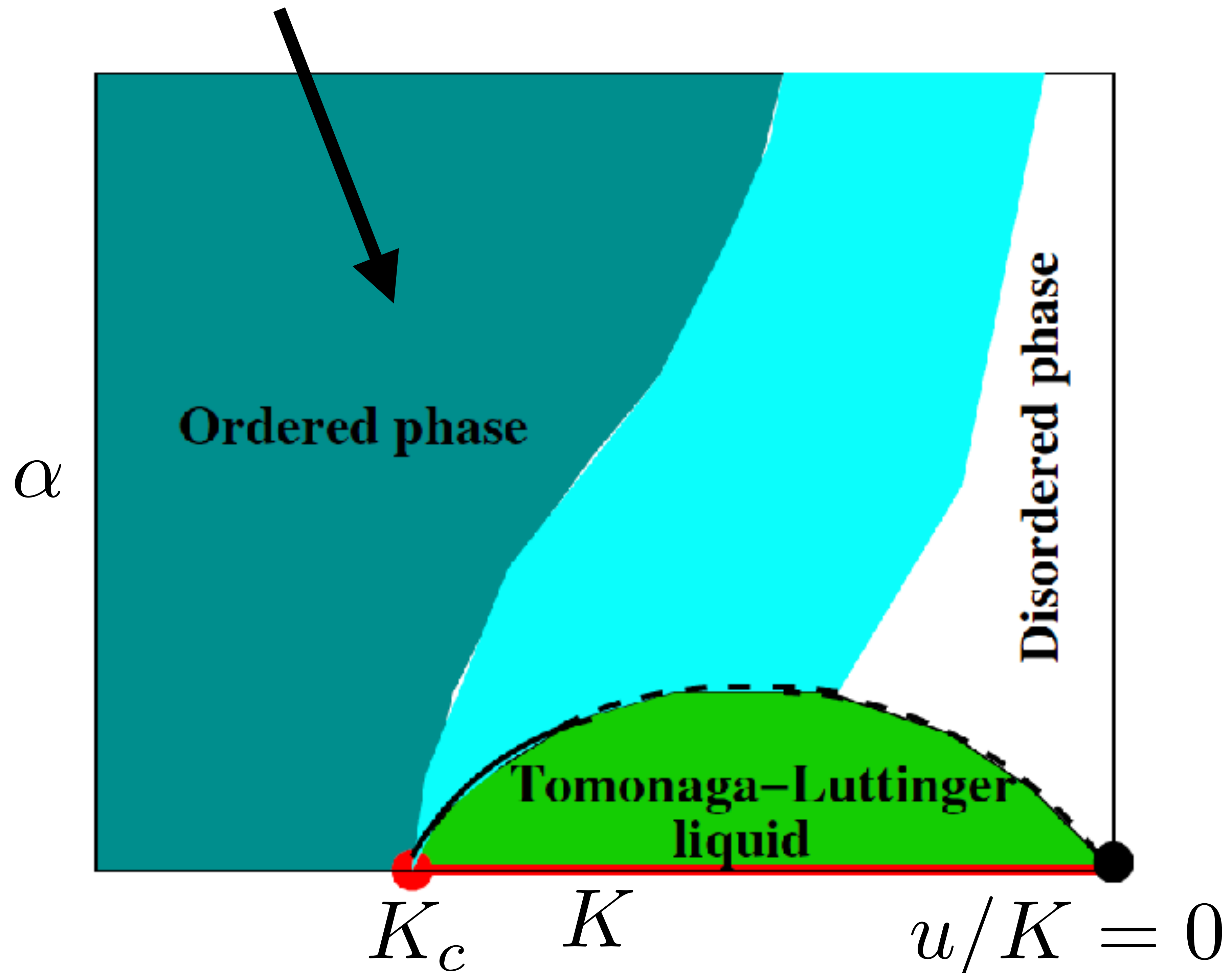
Perturbative RG study

$$\alpha, u, K \quad \overset{RG}{\Longrightarrow} \quad \alpha_r = 0, u_r, K_r$$



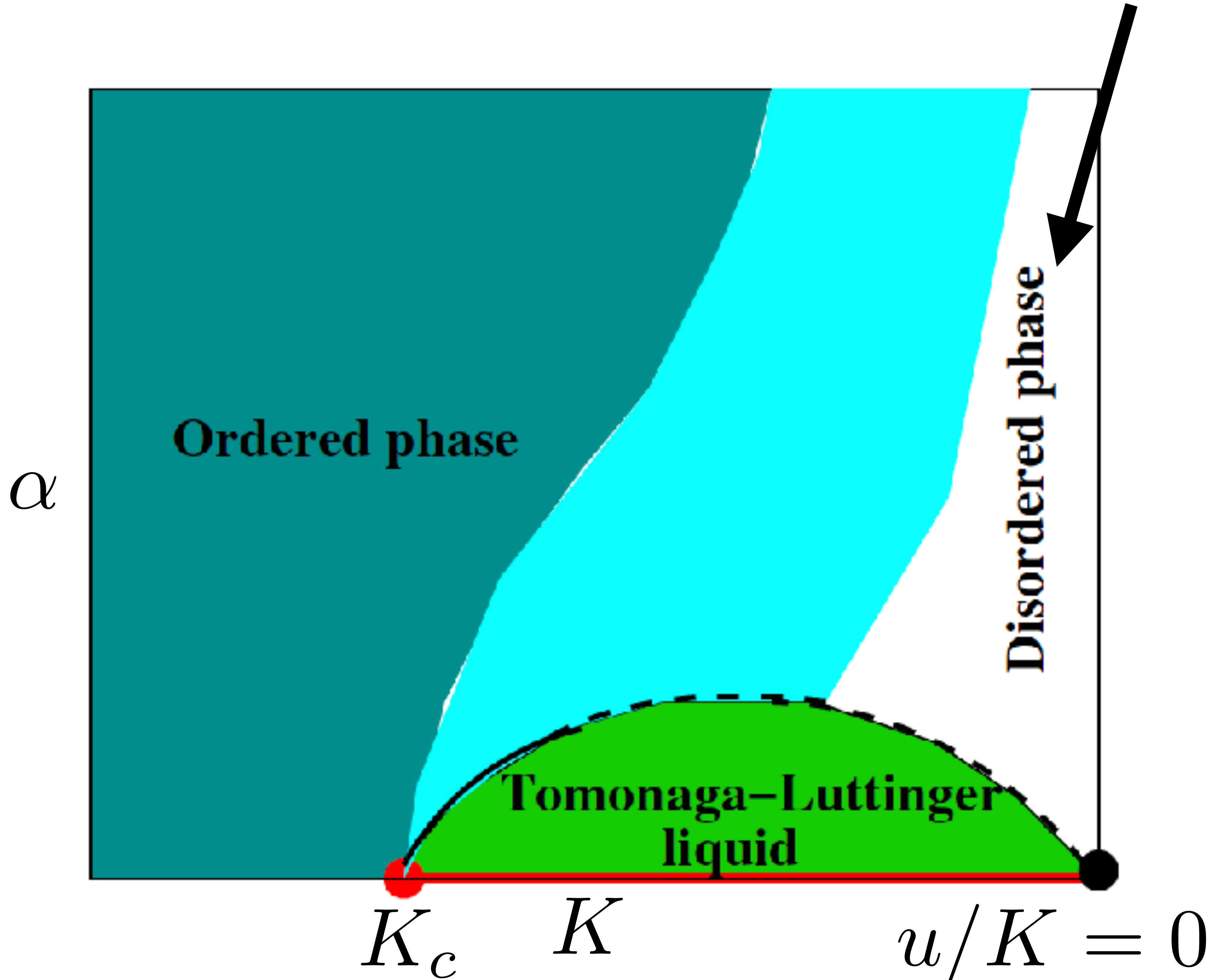
Perturbative RG study

$$G(q,\omega_n) = \frac{1}{\frac{u}{\pi K}q^2 + \frac{\alpha}{\pi^2}|\omega_n|}$$

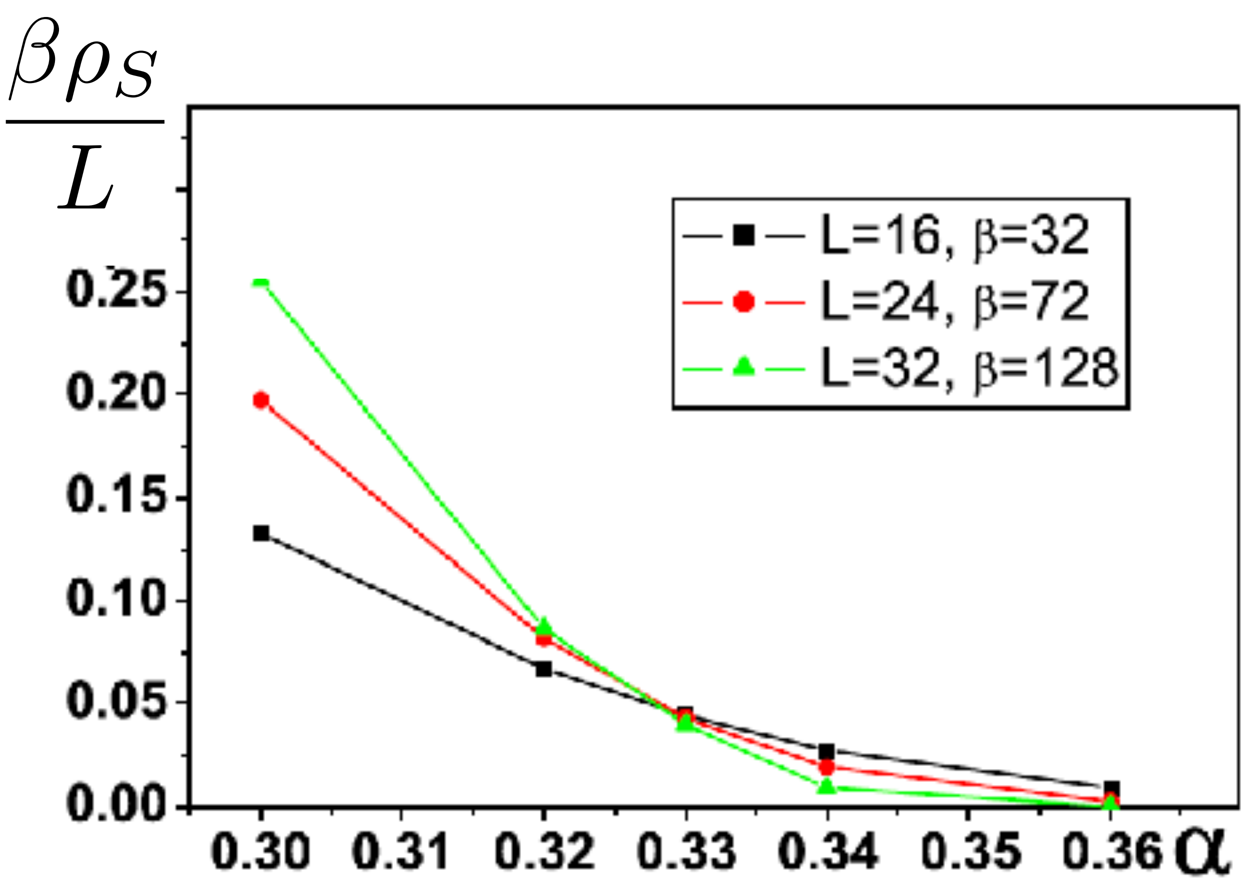


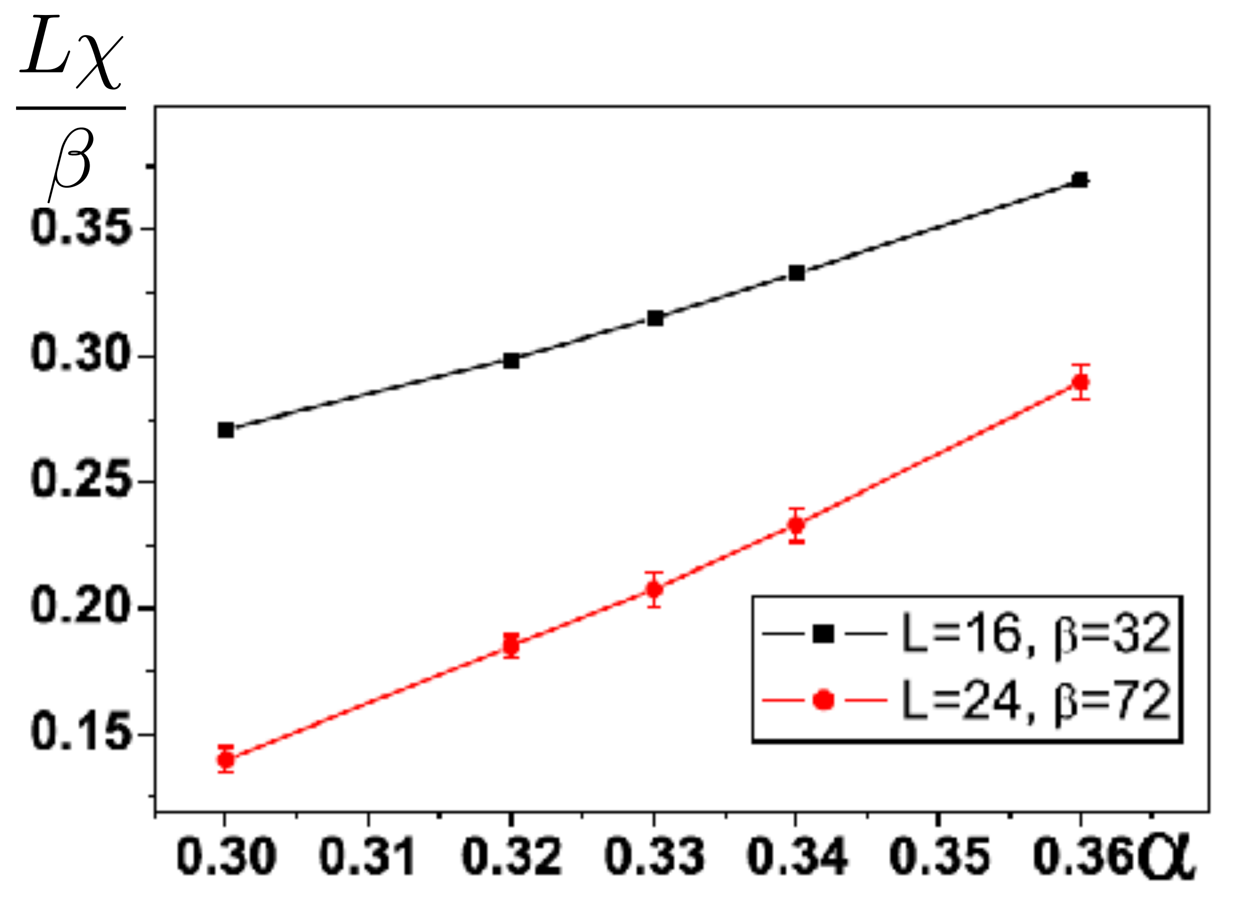
Perturbative RG study

$$G(q,\omega_n) = \frac{1}{\frac{u}{\pi K}(q^2+\xi^{-2}) + \frac{\alpha}{\pi^2}|\omega_n|}$$



Quantum Monte Carlo hard bosons



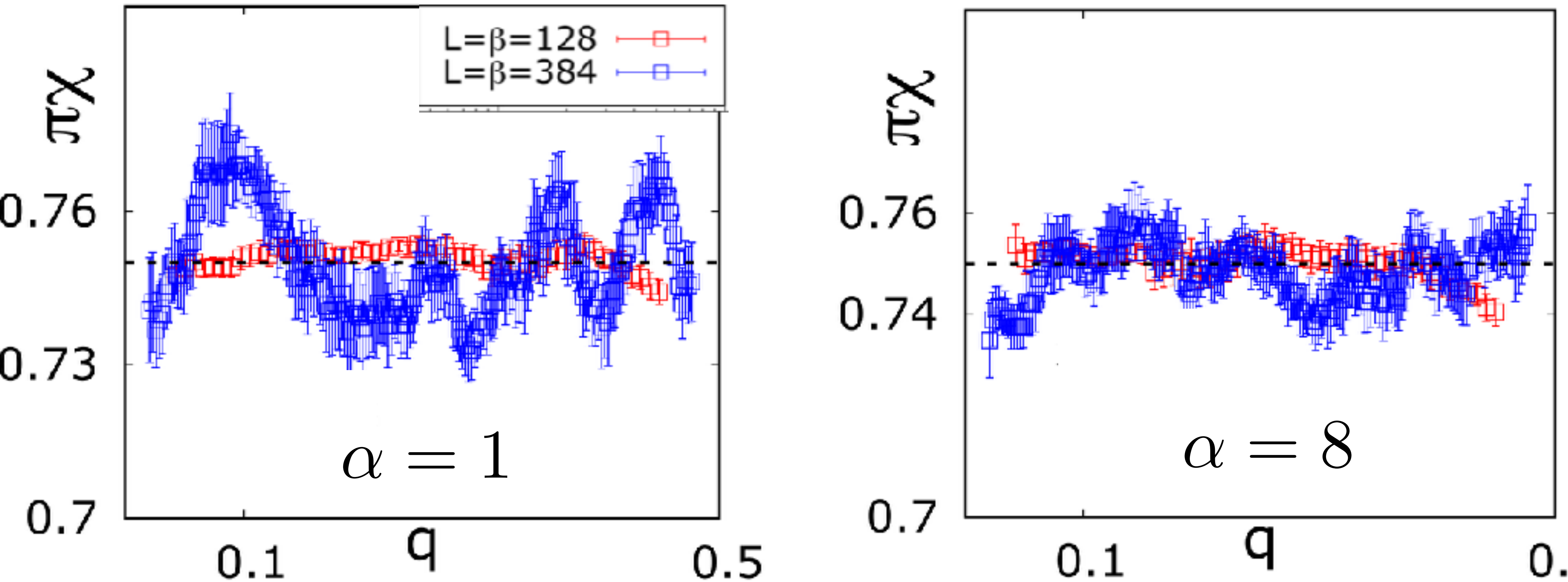


- Spin stiffness vanishes at the critical point
- dissipative phases high suscetibility

by Z. Cai, F. Schollwock, L. Pollet PRL 2014

Our results 1: susceptibility

$$K = 0.75 \quad u = 1 \quad \beta = L$$



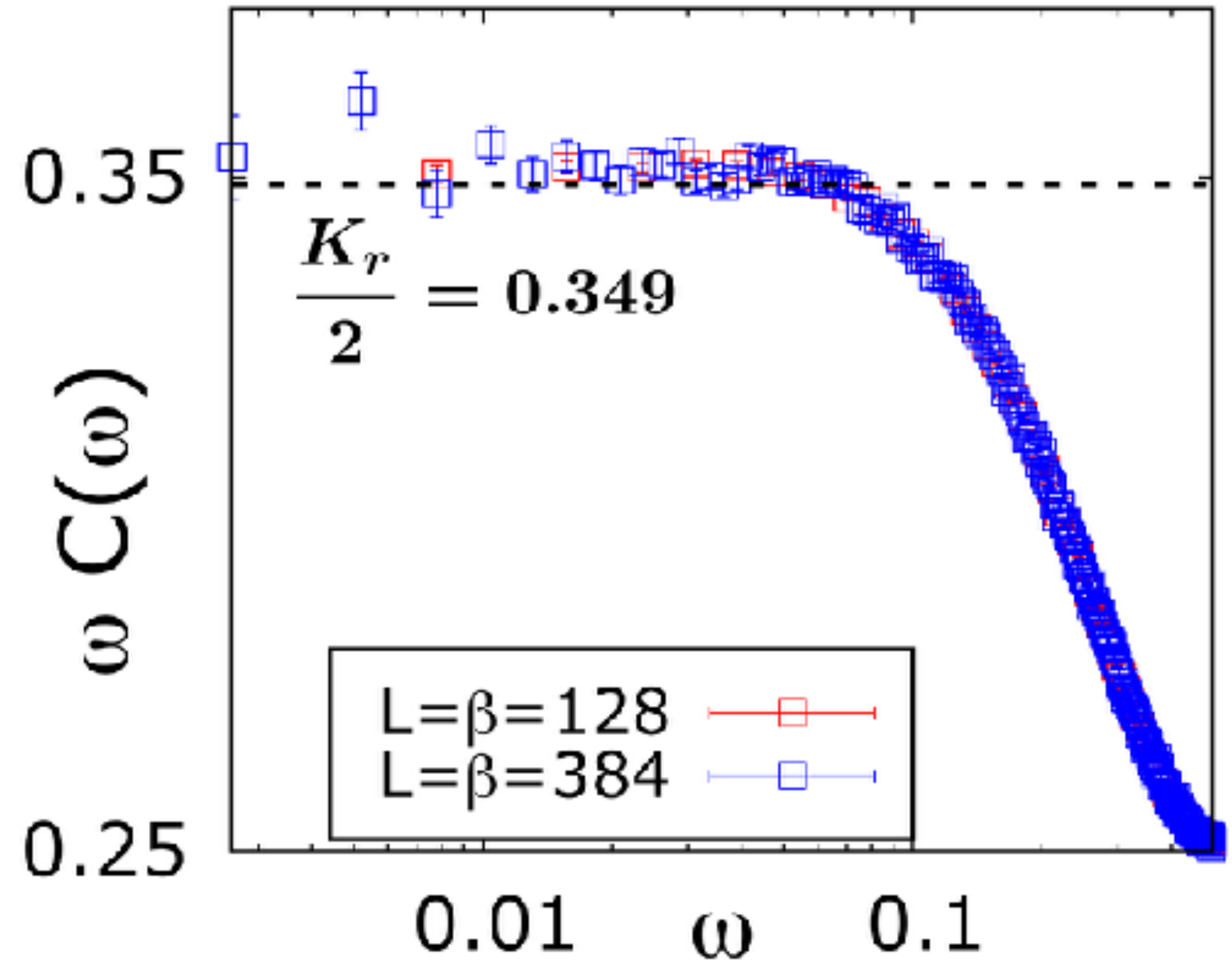
$$\pi\chi = \frac{K_r}{u_r} = \frac{K}{u} = 0.75$$

Our results 2: “spin stiffness”

$$C(\omega_n) = \frac{1}{\pi^2 L\beta}\int_0^\infty dq\,|\phi(q,\omega_n)|^2$$

$$C_{LL}(\omega_n) = \frac{K_r}{2\omega_n}$$

$\alpha = 1$



LL: finite spin stiffness

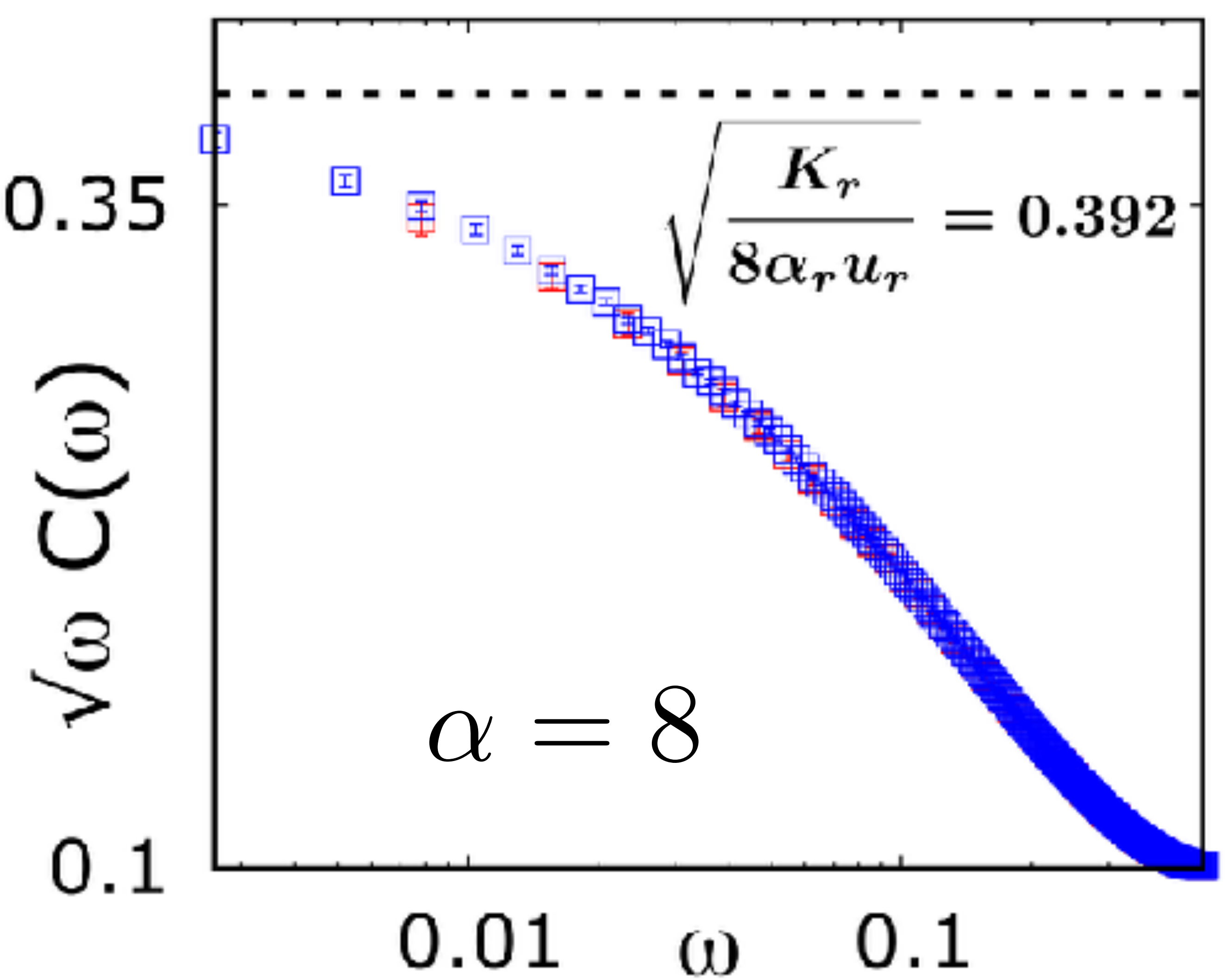
Variational method

$$\mathrm{P}rob[\phi] \propto \exp\left(-S_{\mathrm{tot}}[\phi]\right) \qquad S_{\mathrm{tot}} = S_{\mathrm{LL}} + S_{\mathrm{int}}$$

$$S_{var} = \sum_n \int dq\ \phi(q,\omega_n)\, G^{-1}_{\mathrm{var}}(q,\omega_n)\, \phi(-q,-\omega_n)$$

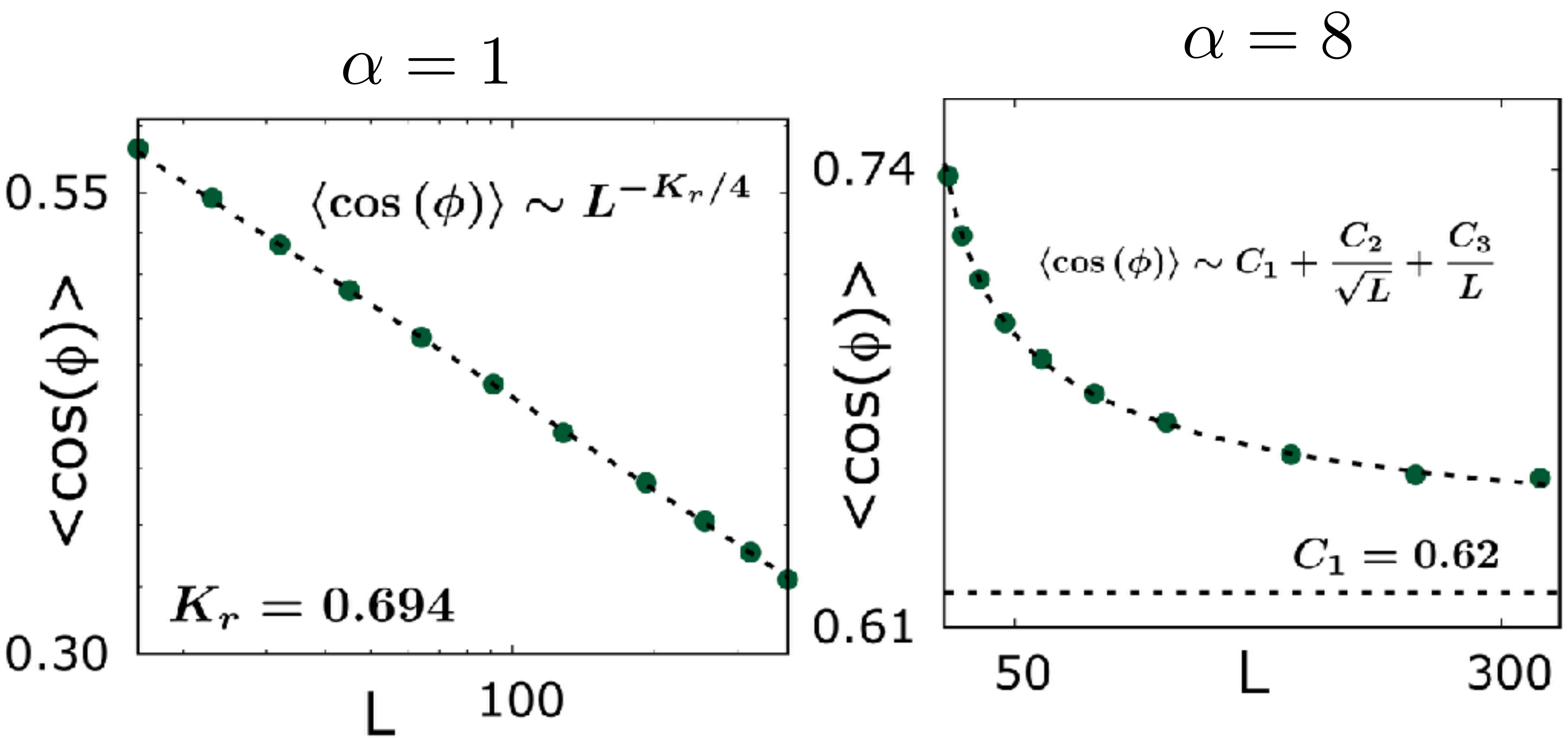
$$G^{-1}_{\mathrm{var}}(q,\omega_n) = \frac{uq^2}{2\pi K} + \frac{\alpha_r}{\pi^2}\left|\omega_n\right| + a_1\left|\omega_n\right|^{\frac{3}{2}} + a_2\omega_n^2$$

$$C_{var}(\omega_n) = \sqrt{\frac{K_r}{8\pi u_r(\alpha_r\omega_n/\pi^2 + a_1\omega_n^{3/2} + a_2\omega_n^2)}} \sim \sqrt{\frac{K_r}{8u_r\alpha_r\omega_n}}$$

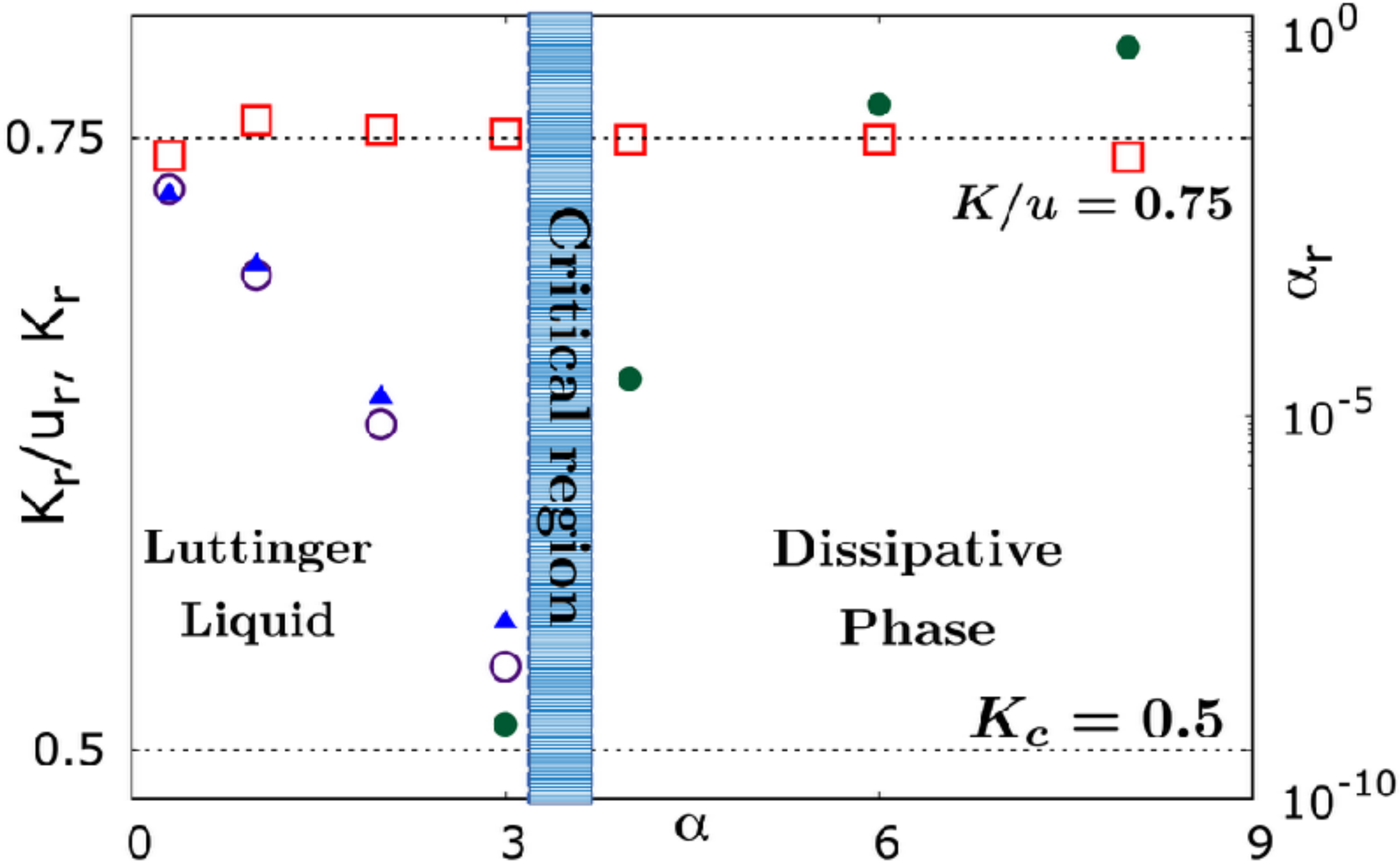


Dissipative: zero spin stiffness

Our results 3: independent check



Summary of the results



Transport properties the results

$$\sigma_{\mathrm{DC}}^{LL} \equiv \mathrm{Re}\left[\sigma(\omega \to 0)\right] = (e^2 uK/\hbar)\delta(\omega)$$

$$G_{var}(q=0,\omega_n) \sim \frac{1}{\alpha_r |\omega_n|^s} \quad \Longrightarrow \quad \mathrm{Re}[\sigma(\omega)] = \frac{e^2}{\hbar \alpha_r} \frac{\epsilon}{(\omega^2+\epsilon^2)^{s/2}}$$

$$\sigma_{\mathrm{DC}}^{s=1} = \frac{e^2}{\hbar \alpha_r}$$

$$\sigma_{\mathrm{DC}}^{s<1} = 0$$

Conclusion

- Low dissipation we found a LL. High dissipation consistent with a variational ansatz with zero spin stiffness
- KT transition: the critical point is LL with Kr=Kc=1/2
- Dissipative phase is more localized : standard conductor for s=1; insulator for subohmic baths