*Quantum **smectic** fracton order*

Boulder

Center for Theory of Quantum Matter

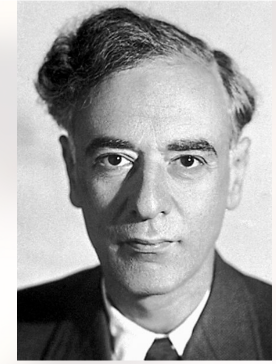
CTQM

M. Pretko and L.R., PRLs 2018
Z. Zhai and L.R., PRB 2019
L.R. and M. Hermele, PRL 2020
L.R., PRL 2020
Z. Zhai and L.R., AOP 2021
A. Gromov and L.R., RMP in preparation

$: Simons Investigator

ECRYS 2022, 17 August, 2022

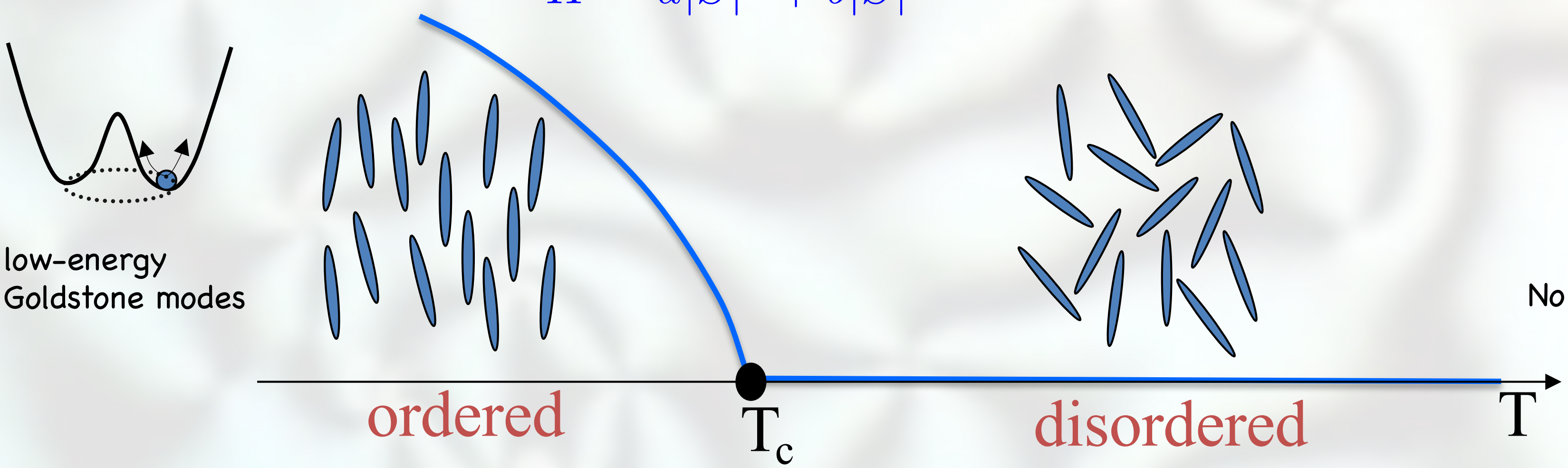
States of (bosonic) matter: Landau paradigm



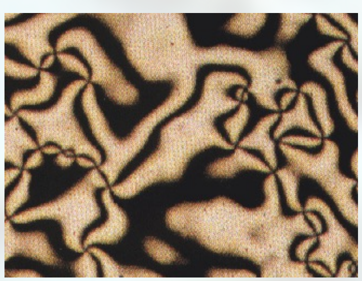
Lev Landau

- "conventional" *ordered* states: *CDW, AFM, SF, liquid crystals…*
  - *local* order parameter, *S(r)*
  - classified by patterns of *spontaneously broken symmetry*
  - *short*-range entangled

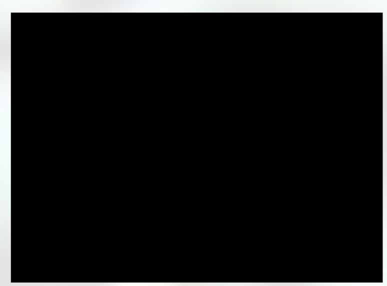
$$H = a|\vec{S}|^2 + b|\vec{S}|^4$$



(ferromagnet, nematic)



(paramagnet, isotropic)

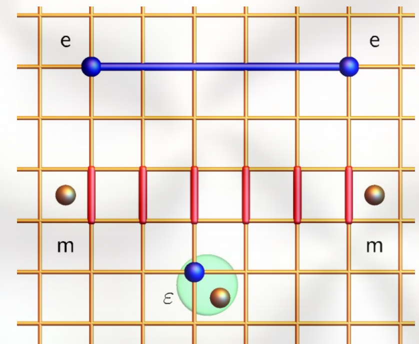


Anderson
Laughlin
Wen
Kitaev
Sachdev
Fisher
…

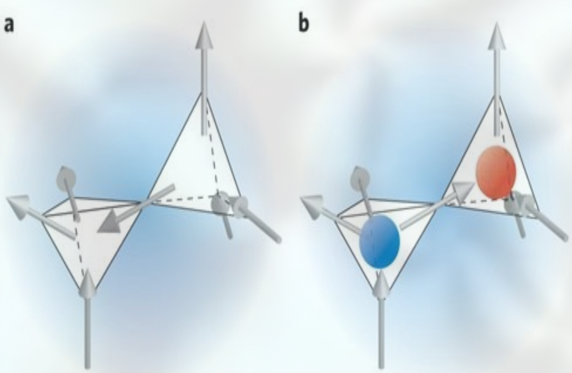
States of quantum matter

(beyond symmetry breaking)

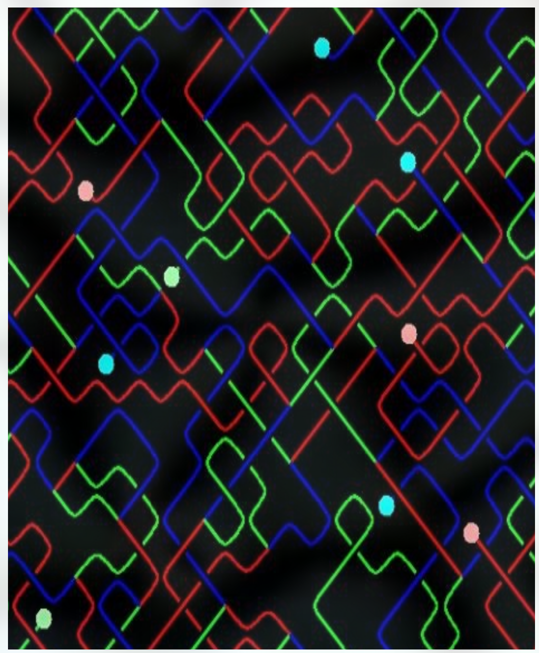
- “conventional” quantum *‘liquid’* states, e.g., FQHE, spin ice, toric code, …
  - Gauge theory (Z_2, U(1), …) description



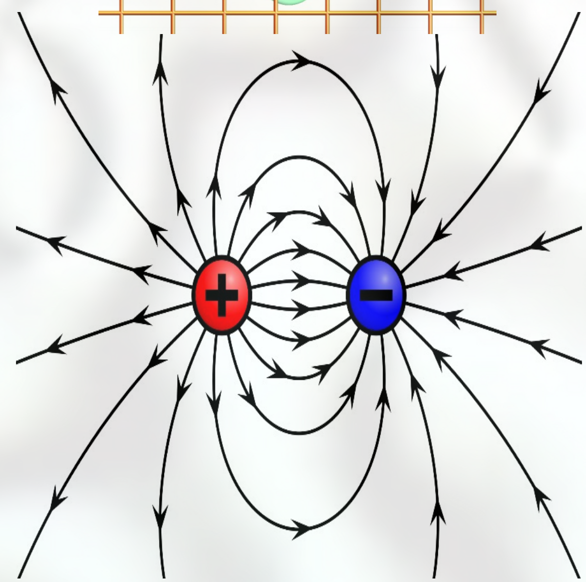
Example: U(1) Spin Liquid



Geometric frustration
(e.g. “spin-ice rules”)



String condensate (*Wen*)
vanishing line tension



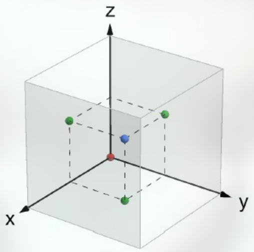
$$\mathcal{L}_{\mathcal{EM}} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu}$$

Emergent
electromagnetism!

C. Chamon, 2005
A. Rasmussen, et al., 2016
J. Haah, 2011, '13
S. Bravyi, et al., 2011
B. Yoshida, 2013
S. Vijay, L. Fu, 2015, '16
...

Fracton quantum matter

- *new class of* quantum *'liquids': Z_2 fractons*, e.g., Haah's code, X-cube, lattice rotors,...

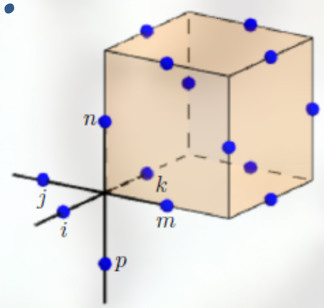


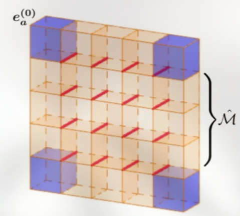
$$A_c = \prod_{n\in\partial c} \sigma_n^x$$

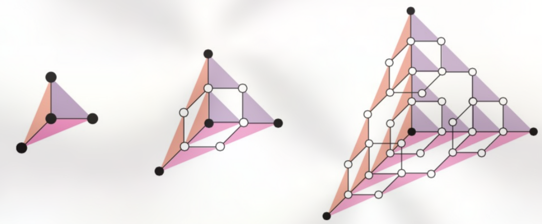
$$B_s^{(xz)} = \sigma_i^z \sigma_p^z \sigma_n^z \sigma_k^z$$

$$B_s^{(xy)} = \sigma_i^z \sigma_j^z \sigma_k^z \sigma_m^z$$

$$B_s^{(yz)} = \sigma_j^z \sigma_n^z \sigma_m^z \sigma_p^z$$



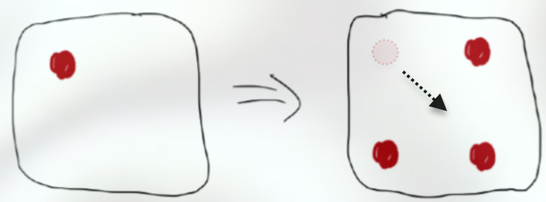




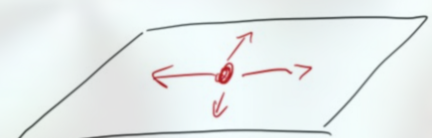
Fractal operator

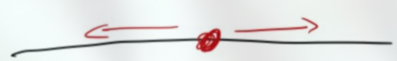
  - Non-local, fractionalized excitations with **restricted mobility** and **exponential topological degeneracy**, beyond TQFT description,...

    - -> at corners of extended objects: fractons – *immobile* in isolation



    - -> at ends of undeformable string: dipoles – *subdimensional*



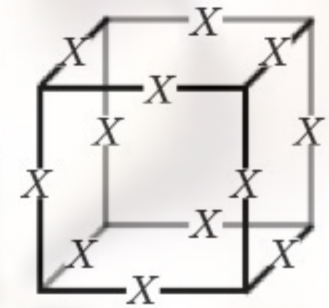


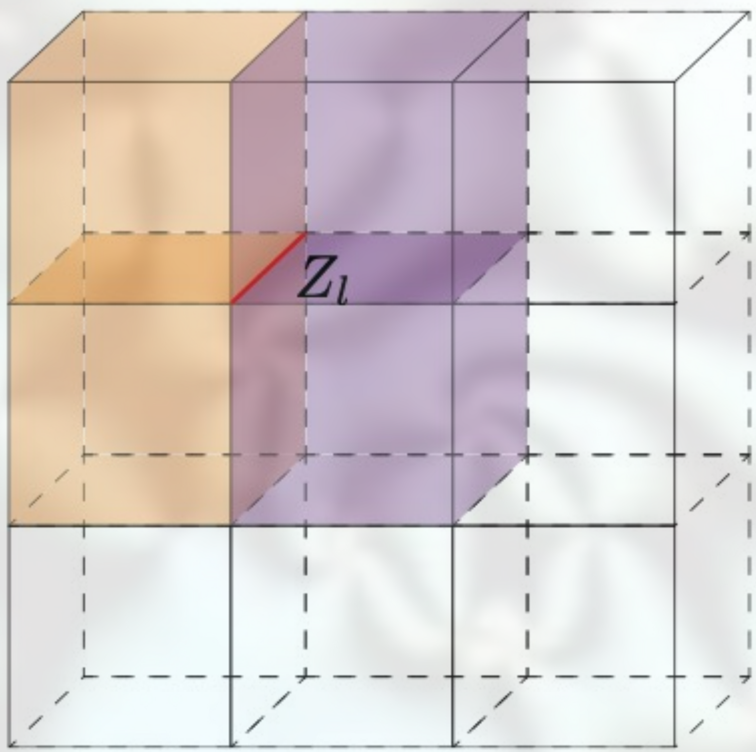
X-cube model

Vijay, Haah, Fu, 2015, 2016

$$H_{XC} = -\sum_{\mathcal{C}} \prod_{l \in \mathcal{C}} X_l - \sum_{\mu = x,y,z} \sum_{v_\mu} \prod_{l \in v_\mu} Z_l$$

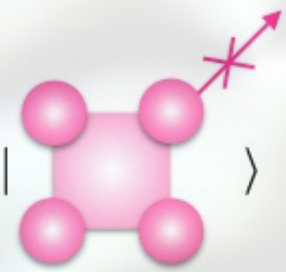
$\mathcal{C}$:



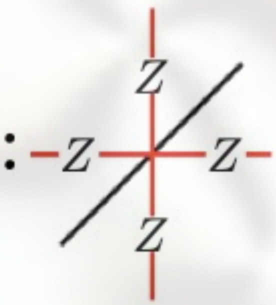


$$\Big(\prod_{l \in v_\mu} Z_l\Big)|GS\rangle = |GS\rangle$$

$$\Big(\prod_{l \in \mathcal{C}} X_l\Big)|GS\rangle = |GS\rangle$$

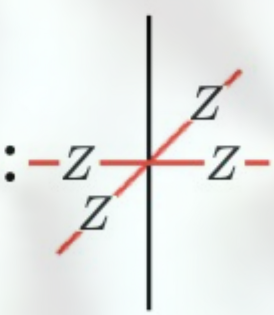
$Z_l|GS\rangle = |$  $\rangle$

v_x :



v_y :

v_z :



(slide from H. Ma)

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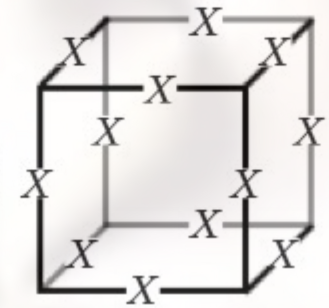

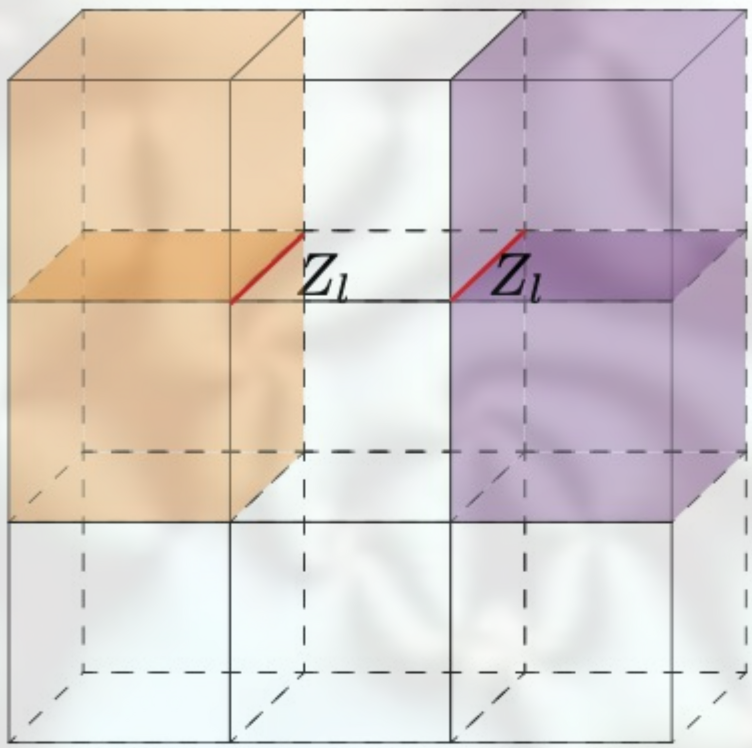

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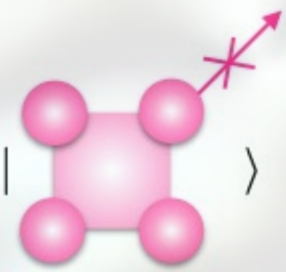
$Z_l|GS\rangle = |$  $\rangle$

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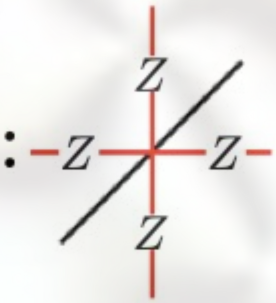
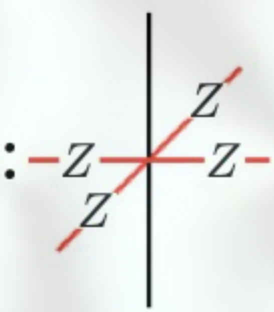


v_y :

v_z :



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Vijay, Haah, Fu, 2015, 2016

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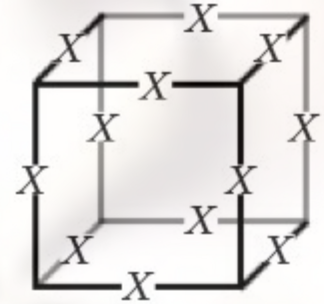

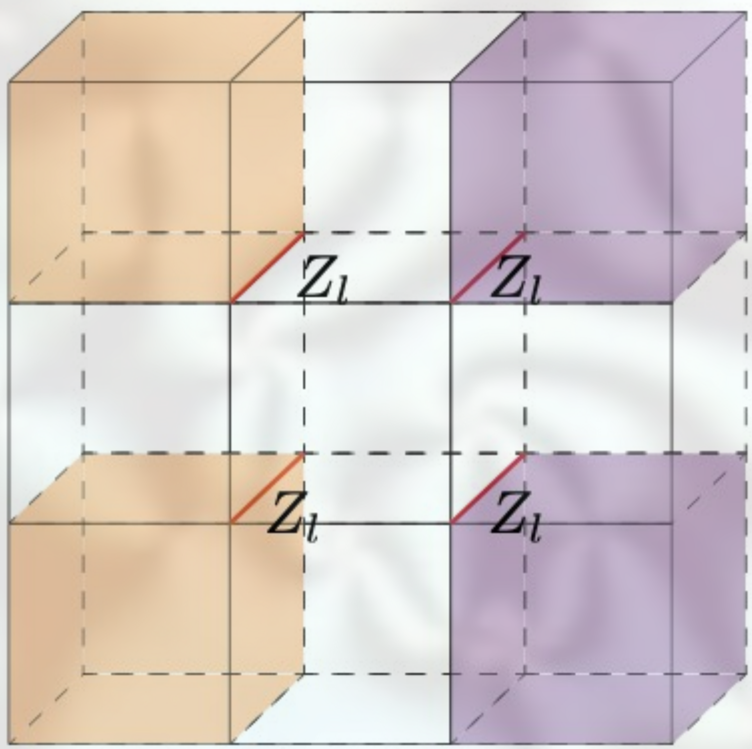

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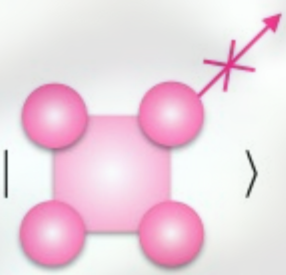
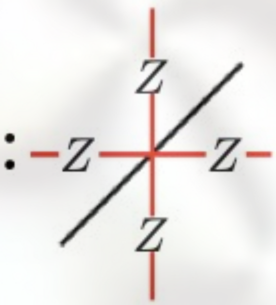
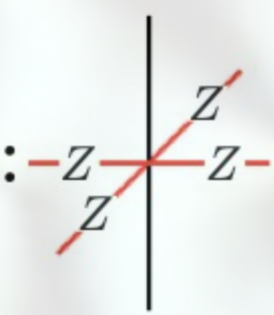
$Z_l|GS\rangle = |$  $\rangle$

v_x :



v_y :

v_z :



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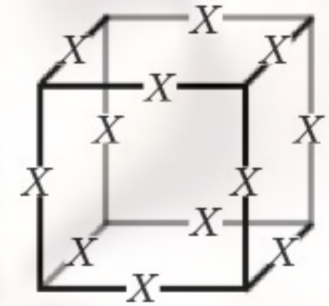

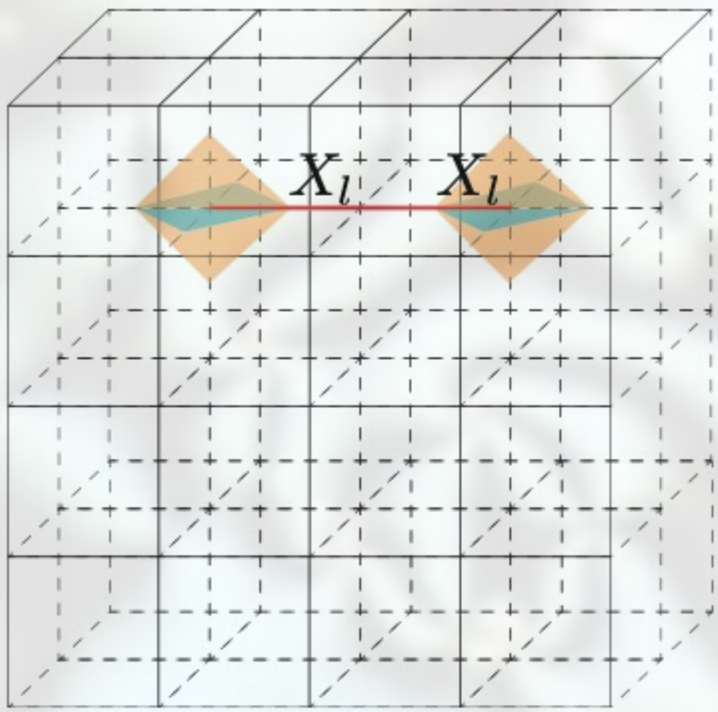

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$X_l|GS\rangle = |$  $\rangle$

v_x :, v_y :, v_z :

(slide from H. Ma)

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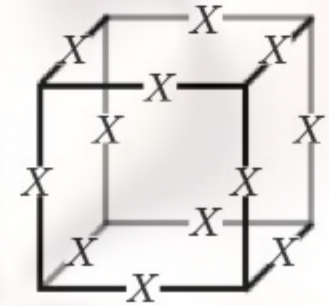

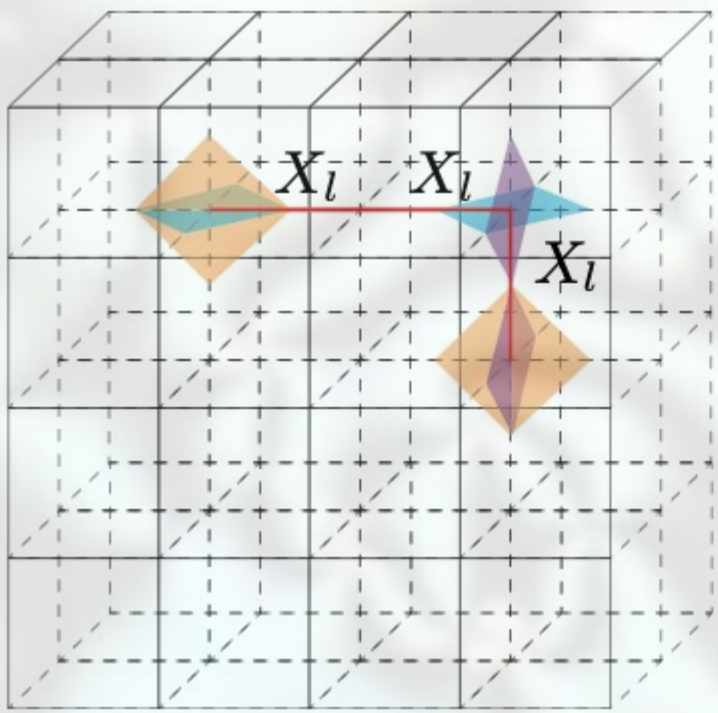

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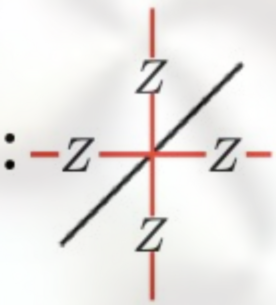
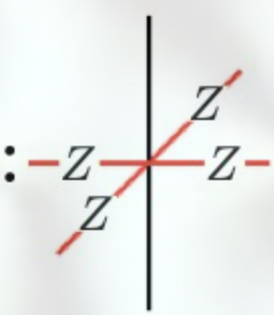


v_y :

v_z :



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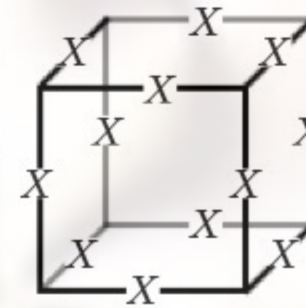

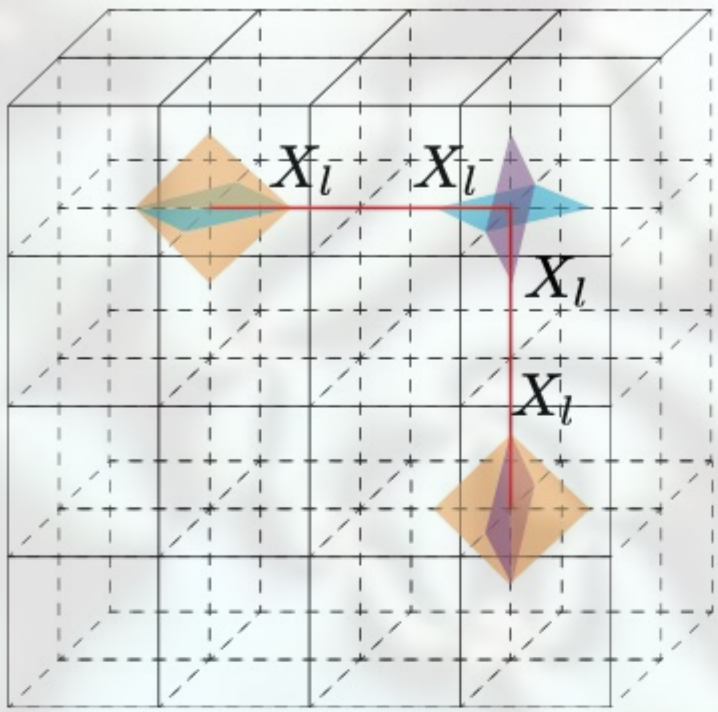

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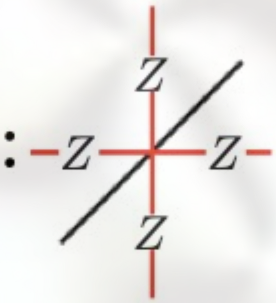
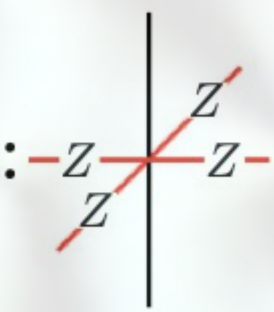
v_x :



v_y :

v_z :

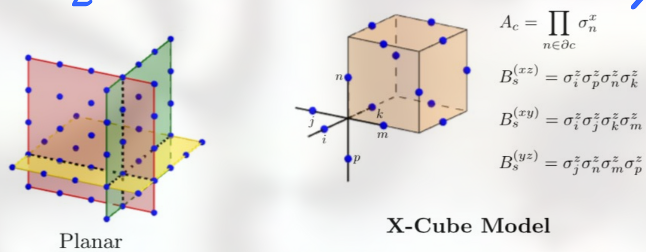


(slide from H. Ma)

S. Vijay, et al., 2015, '16
M. Pretko, 2016, '17
T. Hsieh, et al., 2017
K. Slagle, Y. B. Kim, 2017
H. Ma, et al., 2017
X. Chen, et al., 2017, '18

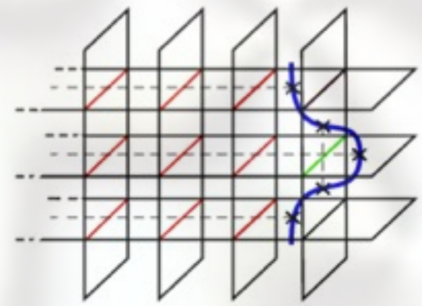
Fracton developments

- *"gauging" global Z_2 subdimensional symmetry spin model*



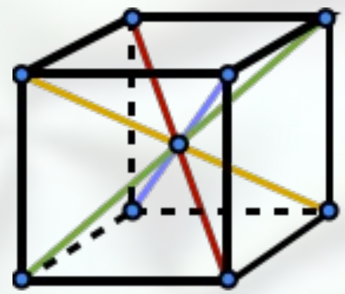
S. Vijay, J. Haah, L. Fu, 2016

- *Coupled-layers construction*



Ma, Lake, Chen, Hermele, 2017

- *Coupled-chains construction*



Halasz, Hsieh, et al., 2017

- *Parton construction*

- *Higher rank tensor gauge theory* $\partial_i \partial_j E_{ij} = \rho_f$

M. Pretko, 2016

Outline

- Crystal elasticity – ***tensor*** gauge theory duality
  (with Pretko, PRLs 2018, 2018)

- Fractons in ***vector*** gauge theories
  (with Hermele, PRL 2020)

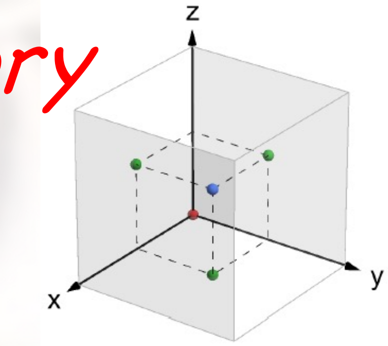
- Quantum ***smectic*** and its vector gauge dual

- Higgs crystal to smectic gauge dual transition
  (L.R. PRL 2020, and with Zhai PRB 2020)

Rasmussen, You, C. Xu, 2016

Pretko, 2016

*Fractons via **tensor** gauge theory*



- U(1) symmetric tensor gauge theory (2+1D):

$$\mathcal{H} = \frac{1}{2} E_{ij} E_{ij} + \frac{1}{2} B_i B_i \qquad [E^{ij}, A_{ij}] = i\delta^{(2)}(\mathbf{x}) \qquad B^i = \epsilon_{jk}\partial^j A^{ki}$$

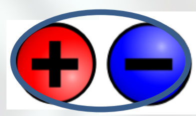
- Gauss' law: $\partial_i \partial_j E^{ij} = \rho$

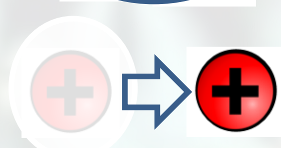
Pretko, 2016

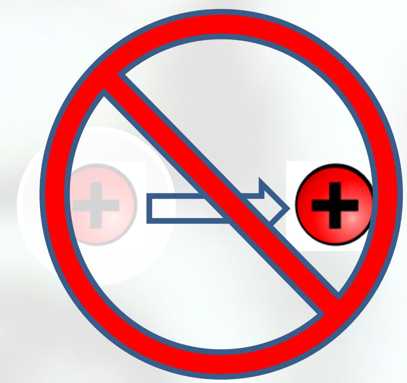
- Conservation of charges and of *dipoles* ---> *fractons* phenomenology!

-> moving charge changes dipole moment -> forbidden by dipole conservation

- immobile

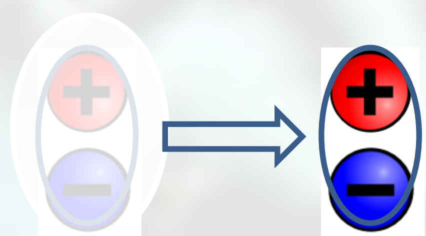






-> dipole motion constrained

- subdimensional



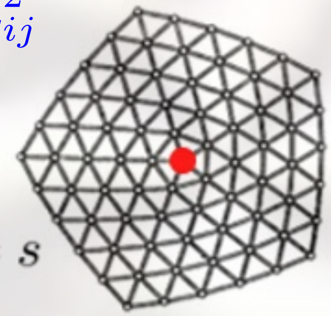
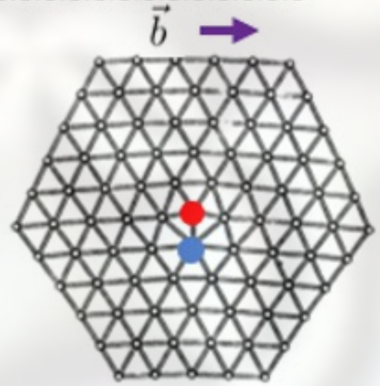
Fractons

¿ any physical realizations ?

***YES*: 2D quantum crystal!**

M. Pretko,
L. R., 2017
PRLs 2018

Fracton-elasticity duality

$\mathcal{H} = \frac{1}{2}B_i^2 + \frac{1}{2}E_{ij}^2$	$\mathcal{H} = \frac{1}{2}\pi_i^2 + \frac{1}{2}u_{ij}^2$
Fracton $\partial_i \partial_j E^{ij} = \rho$	Disclination $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s$ 
Dipole	Dislocation 
Gauge Modes	Phonons
Electric Field E_{ij}	Strain Tensor u_{ij}
Magnetic Field B_i	Lattice Momentum π_i

$$\partial_t B^i + \epsilon_{jk}\partial^j E_\sigma^{ki} = 0. \quad \Longleftrightarrow \quad \partial_t \pi^i - \partial_j \sigma^{ij} = 0$$

Faraday <-> Newton

L. R. Hermele, PRL 2020

Fractons via **vector** gauge theory

- Reformulate elasticity into 3 flavored coupled xy models:

$$\mathcal{H} = \frac{1}{2}\pi_k^2 + \frac{1}{2}C(\partial_i u_k - \theta\epsilon_{ik})^2 + \frac{1}{2}L^2 + \frac{1}{2}K(\nabla\theta)^2$$

- Dualize to 3 coupled U(1) vector gauge theory: $(A_{[ij]} \equiv \epsilon_{ik}A_{ik})$

$$\tilde{\mathcal{H}} = \frac{1}{2}C|\mathbf{E}_k|^2 + \frac{1}{2}(\nabla\times\mathbf{A}_k)^2 + \frac{1}{2}K|\mathbf{e}|^2 + \frac{1}{2}(\nabla\times\mathbf{a} - A_{[ij]})^2 - \mathbf{A}_k\cdot\mathbf{J}_k^b - \mathbf{a}\cdot\mathbf{j}^s$$

- Gauss' law: $\nabla\cdot\mathbf{e} = s \qquad \nabla\cdot\mathbf{E}_k = \tilde{p}_k \qquad (\tilde{p}_k = p_k - e_k)$

- Gauge redundancy: $\mathbf{A}_k \to \mathbf{A}_k + \nabla\chi_k, \quad A_{0k} \to A_{0k} + \partial_t\chi_k$

  $a_k \to a_k + \partial_k\phi - \chi_k, \quad a_0 \to a_0 + \partial_t\phi$

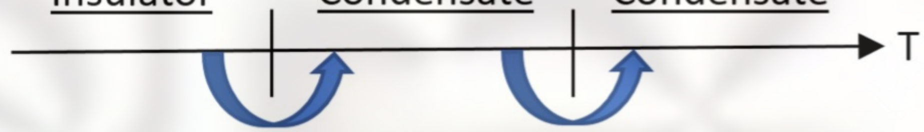
- fractons:

  gauge invariance demands $\partial_t p_k + \nabla\cdot\mathbf{J}_k = j_k \longrightarrow \mathbf{j} = 0$

Zhai, L. R., 2018

Fracton condensation transition

2D

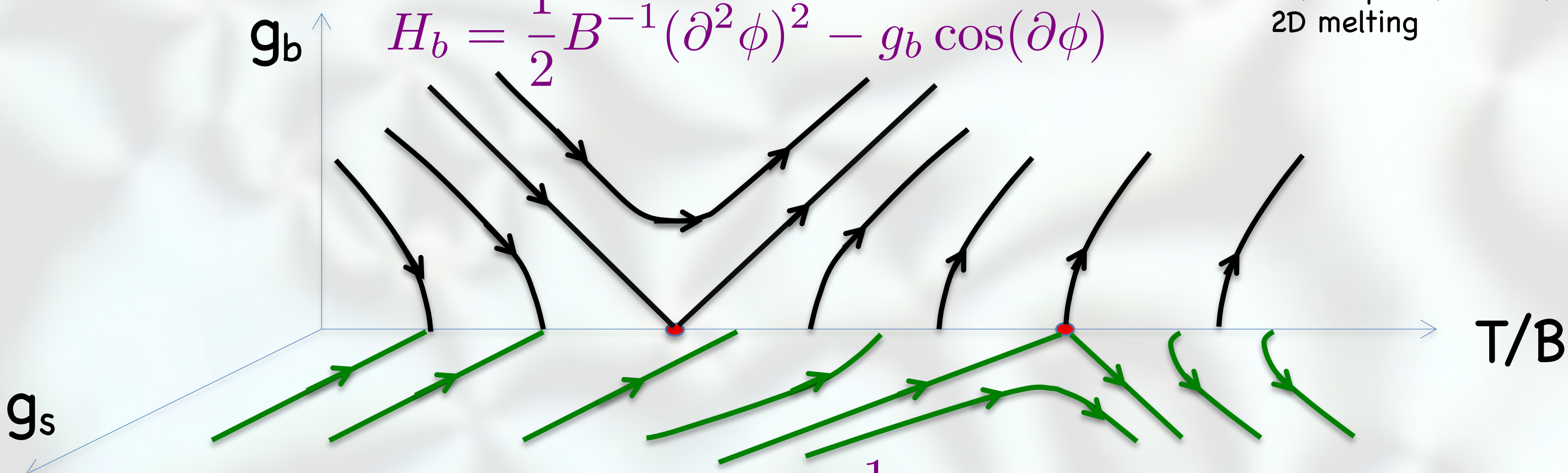


2D scalar fracton model:

$$\tilde{\mathcal{H}} = \frac{1}{2}B^{-1}(\nabla^2\phi)^2 - \underbrace{g_s \cos\left(\frac{2\pi}{6}\phi\right)}_{\text{charges}} - g_b \sum_{n=1,2,3} \underbrace{\cos\left(\mathbf{b}_n \cdot \hat{z} \times \nabla\phi\right)}_{\text{dipoles}}$$

KT, Halperin, Nelson, Young '79
2D melting

$$H_b = \frac{1}{2}B^{-1}(\partial^2\phi)^2 - g_b\cos(\partial\phi)$$



$$H_s = \frac{1}{2}g_b(\partial\phi)^2 - g_s\cos(\phi)$$

M. Pretko, L. R., PRL2018

2+1D *Quantum melting fractons*

Kumar Potter, 2018

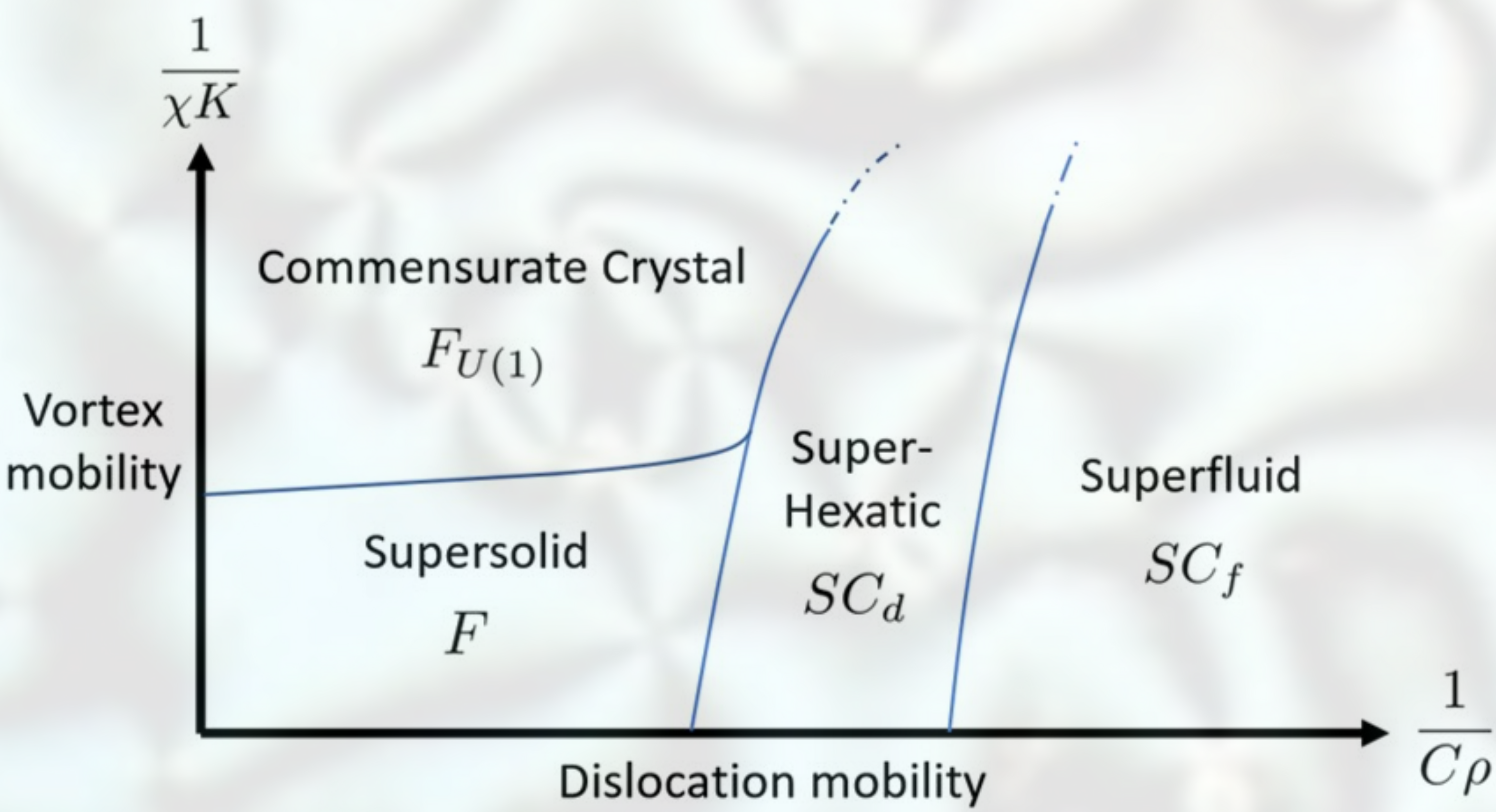
- Fracton dipoles $b_n = \sqrt{n_d} e^{i\theta_n}$ condense: -> ***super**-hexatic*

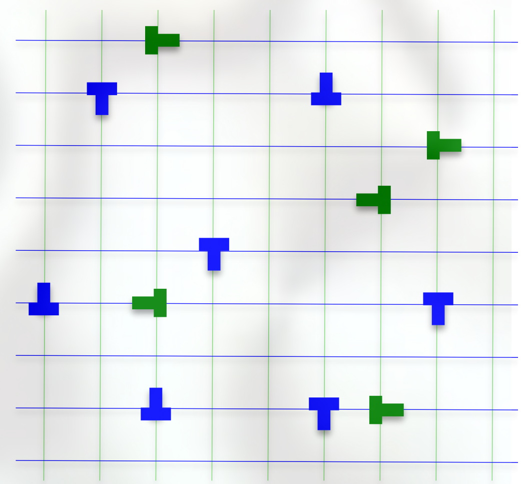
$$\tilde{\mathcal{H}}_{cr} = |(i\nabla - p_k \mathbf{A}_k)\psi_k|^2 + V(\psi_k) + \tilde{\mathcal{H}}_{Max}[\mathbf{A}_k, \mathbf{E}_k]$$

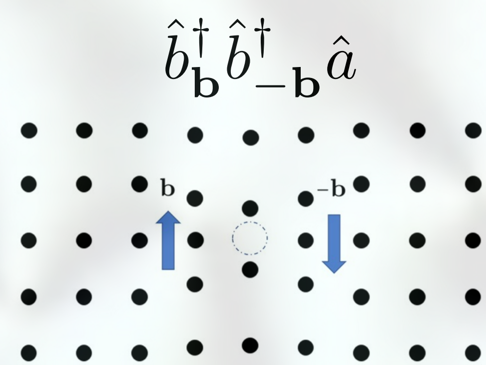
Higgs transition - condensation of x,y-dipoles (dislocations)

$$\psi_x \neq 0, \quad \psi_y \neq 0 \quad \rightarrow \quad \mathbf{A}_{x,y} \approx 0 \ \text{Higgs'ed/gapped}$$

Quantum melting: *crystal → superfluid*

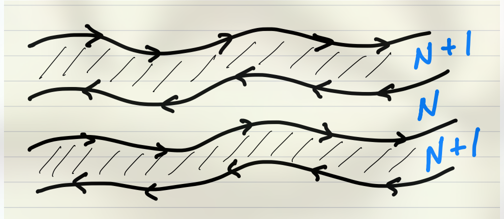


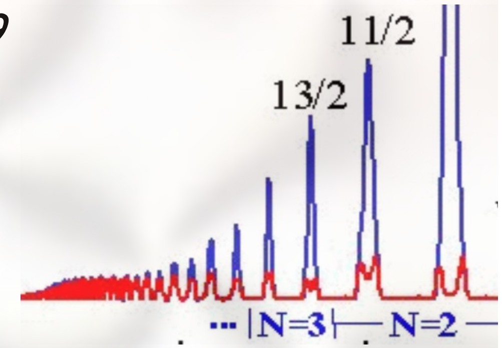




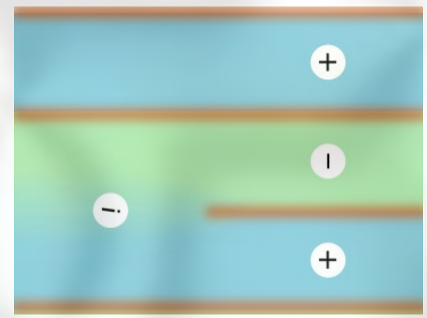
Quantum liquid crystals

- Quantum Hall *Fogler, et al. '96, Moessner, Chalker '96, Fradkin, Kivelson '99, MacDonald, Fisher '99, L.R., Dorsey '02,…Eisenstein, et al. '99*

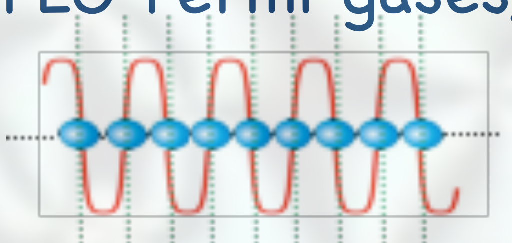




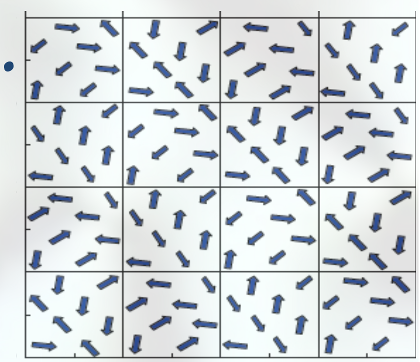
- SDW, CDW, PDW in doped Mott insulators
  Tranquada, et al., '95, Kivelson, Fradkin, Emery '98, Sachdev,…



- Imbalanced FFLO Fermi gases, SOC Bose gases, *L.R. et al. '09,'11, Zhai '15*

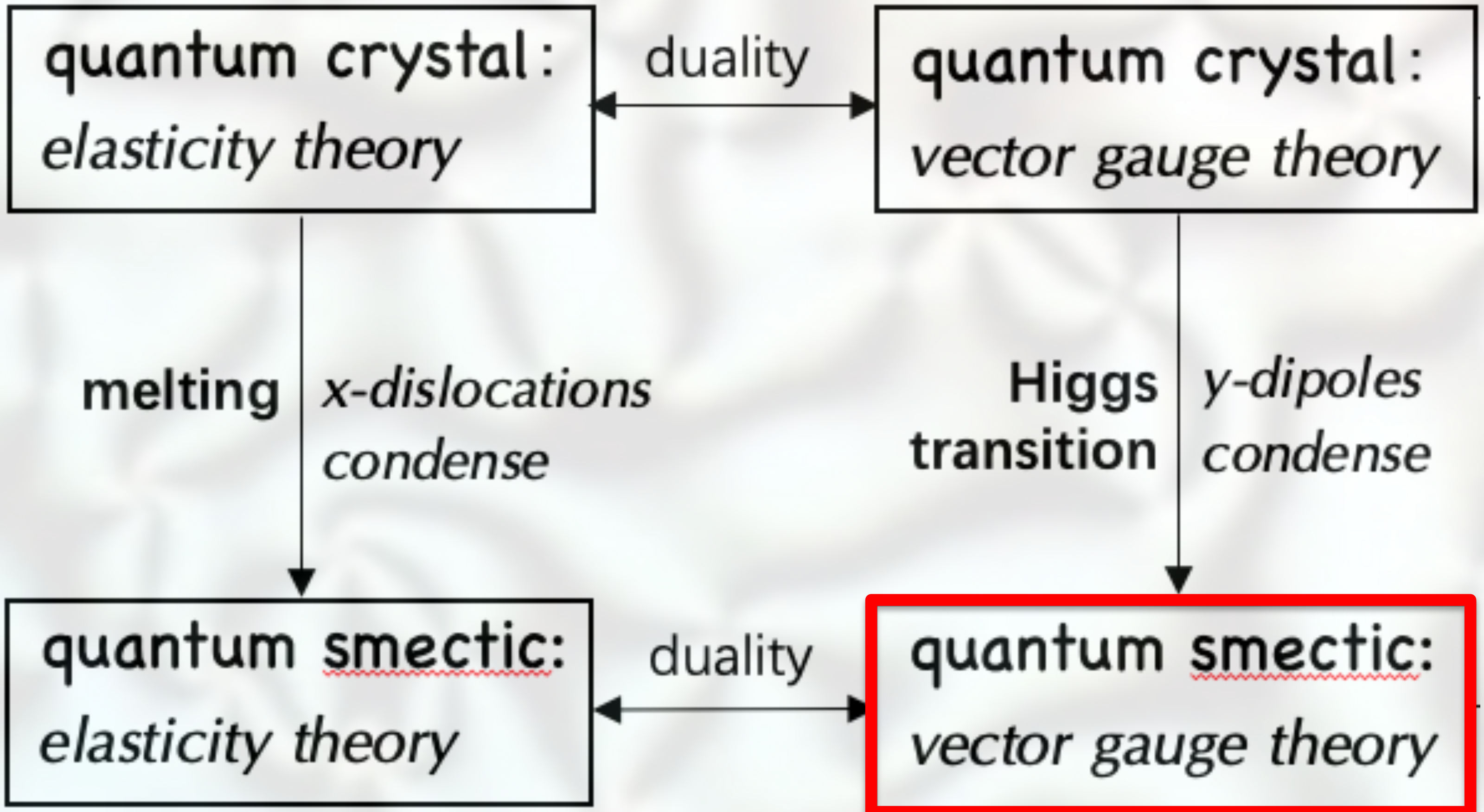


- Helical, frustrated magnets, e.g., MnSi, FeGe, AB_2X_4,…
  Pfleiderer, et al.' 09, Bergman, et al. '07



L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Crystal – smectic – gauge duality



L.R PRL 2020
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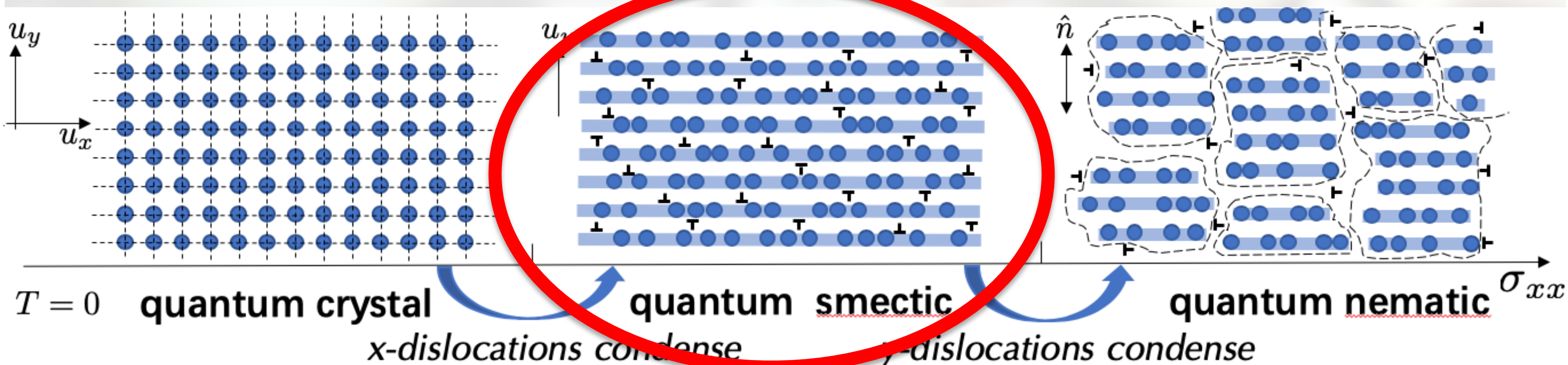
AOP 2021

Anisotropic quantum melting

Ostlund, Halperin '81

- Crystal: $\mathcal{H}_{cr} = \frac{C}{2} u_{ij}^2 + \frac{1}{2}\pi^2$
- Condense x-dislocations: $b_x = \sqrt{n_d} e^{i\theta_x}$ -> ***super*-smectic**



- Super-smectic:

$$\mathcal{H}_{sm} = \underbrace{\frac{C}{2}(\nabla u_y - \theta\hat{\mathbf{x}})^2 + \frac{K}{2}(\nabla\theta)^2 + \frac{1}{2}\pi^2 + \frac{1}{2}L^2}_{\text{quantum smectic elasticity}} + \underbrace{\frac{1}{2}(\nabla\phi_s)^2 + \frac{U}{2}n^2}_{\text{bosonic atoms}}$$

Z. Zhai,
L. R.,2020

Smectic elasticity – gauge duality

- Super-smectic:

$$\mathcal{H}_{sm} = \underbrace{\frac{C}{2}(\nabla u_y - \theta\hat{\mathbf{x}})^2 + \frac{K}{2}(\nabla\theta)^2 + \frac{1}{2}\pi^2 + \frac{1}{2}L^2}_{\text{quantum smectic elasticity}} + \underbrace{\frac{1}{2}(\nabla\phi_s)^2 + \frac{U}{2}n^2}_{\text{bosonic atoms}}$$

- Dualize to U(1) vector gauge theory:

$$\tilde{\mathcal{H}}_{sm} = \frac{1}{2}C\mathbf{E}^2 + \frac{1}{2}(\nabla\times\mathbf{A})^2 + \frac{1}{2}K\mathbf{e}^2 + \frac{1}{2}(\nabla\times\mathbf{a} - \hat{\mathbf{y}}\cdot\mathbf{A})^2 - \mathbf{A}\cdot\mathbf{J} - \mathbf{a}\cdot\mathbf{j}$$

- Gauss' law: $\nabla\cdot\mathbf{E} = n_b + \hat{\mathbf{x}}\cdot\mathbf{e}$ $\quad\quad$ $\nabla\cdot\mathbf{e} = n_s$

- Gauge redundancy: $\mathbf{A}\rightarrow\mathbf{A}+\nabla\chi, \quad A_0\rightarrow A_0+\partial_t\chi$

  $\mathbf{a}\rightarrow\mathbf{a}+\nabla\phi-\chi\hat{\mathbf{x}}, \quad a_0\rightarrow a_0+\partial_t\phi$

- restricted mobility:

  gauge invariance demands $\partial_t n_b + \nabla\cdot\mathbf{J} = \hat{\mathbf{x}}\cdot\mathbf{j} \longrightarrow j_x = 0$

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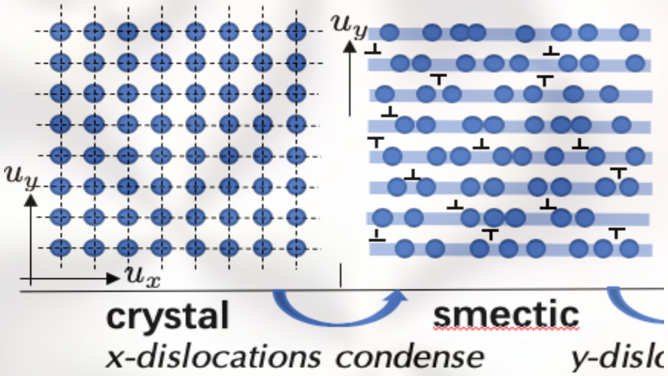
Higgs'ing crystal gauge dual -> smectic gauge dual



- Crystal gauge dual:

$$\tilde{\mathcal{H}} = \frac{1}{2}C|\mathbf{E}_k|^2 + \frac{1}{2}(\nabla\times\mathbf{A}_k)^2 + \frac{1}{2}K|\mathbf{e}|^2 + \frac{1}{2}(\nabla\times\mathbf{a} - A_{[ij]})^2 - \mathbf{A}_k\cdot\mathbf{J}_k^b - \mathbf{a}\cdot\mathbf{j}^s$$

$$\tilde{\mathcal{H}}_{cr} = |(i\nabla - p_k\mathbf{A}_k)\psi_k|^2 + V(\psi_k) + \tilde{\mathcal{H}}_{Max}[\mathbf{A}_k, \mathbf{E}_k]$$

  - Higgs transition - condensation of y-dipoles (x-dislocations)

$$\psi_x = 0, \quad \psi_y \neq 0 \quad \rightarrow \quad \mathbf{A}_y \approx 0 \ \text{ gapped}$$

- Smectic gauge dual: $\tilde{\mathcal{H}}_{sm}[\mathbf{A}^x, \mathbf{a}] \approx \tilde{\mathcal{H}}_{cr}[\mathbf{A}^x, \mathbf{A}^y = 0, \mathbf{a}]$

$$\tilde{\mathcal{H}}_{sm} = \frac{1}{2}C\mathbf{E}^2 + \frac{1}{2}(\nabla\times\mathbf{A})^2 + \frac{1}{2}K\mathbf{e}^2 + \frac{1}{2}(\nabla\times\mathbf{a} - \hat{\mathbf{y}}\cdot\mathbf{A})^2 - \mathbf{A}\cdot\mathbf{J} - \mathbf{a}\cdot\mathbf{j}$$

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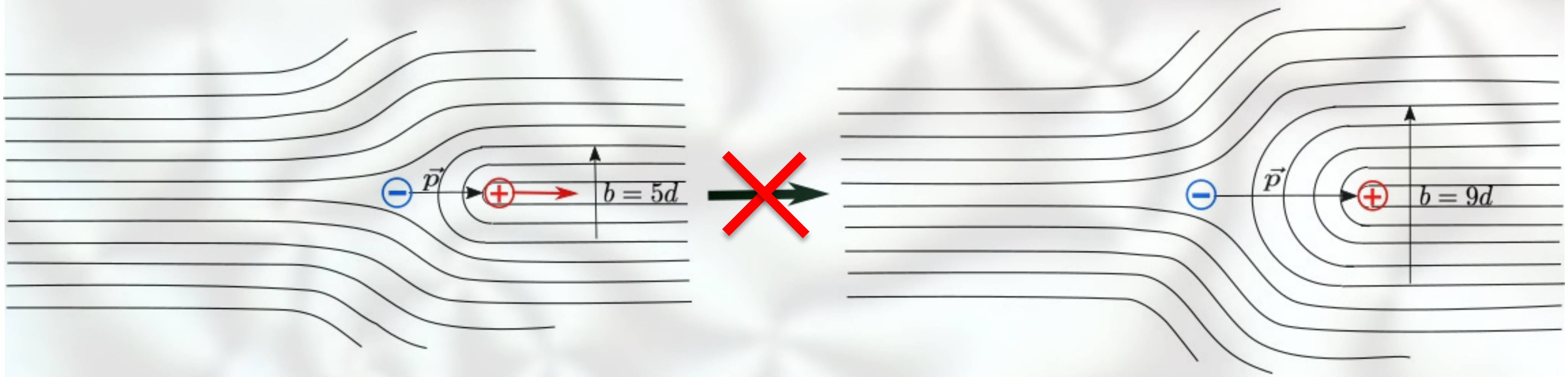
Restricted disclination mobility

gauge invariance demands $\partial_t p + \nabla \cdot \mathbf{J} = \hat{\mathbf{x}} \cdot \mathbf{j} \longrightarrow j_x = 0$

- Fractonic restricted dynamics *via disclination microscopics:*



requires a nonlocal process of adding a pair smectic half-layer per lattice constant of disclination separation

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Anomalous 'diffusion'

Gromov, Lucas, Nandkishore 2020

- Fractonic restricted dynamics -> *anomalous hydrodynamics*

-> continuity: $\partial_t p + \nabla \cdot \mathbf{J} = j_x$ $\quad \partial_t n + \nabla \cdot \mathbf{j} = 0$

-> Fick's law for dipoles: $\mathbf{J} = -\gamma \nabla p = \gamma \nabla \partial_x n$

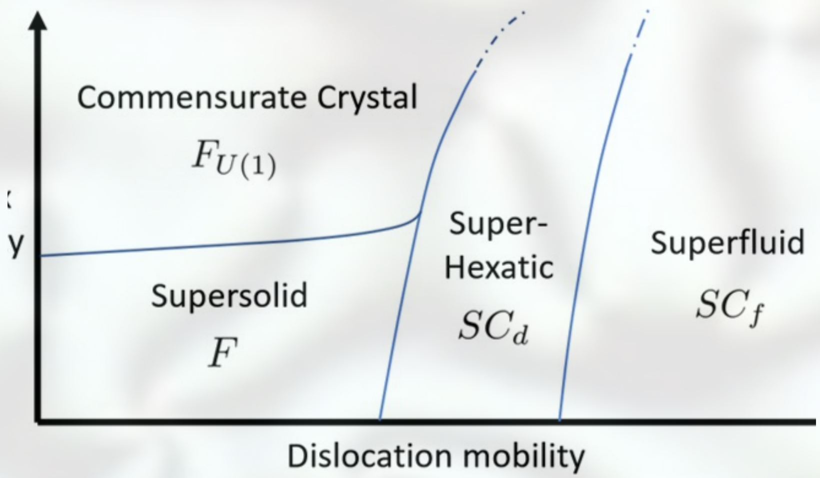
-> $j_x = \nabla \cdot \mathbf{J} = \gamma \nabla^2 \partial_x n$

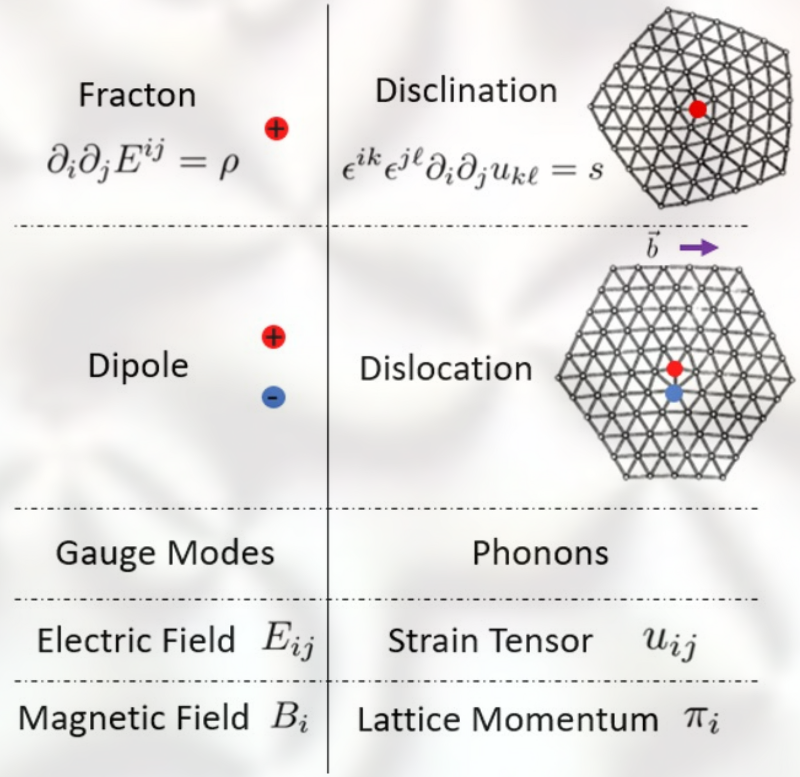
-> $\partial_t n + \hat{\Gamma} n = 0$ → $\Gamma_k = D k_y^2 + \gamma k_x^4$

$$n(t) \sim t^{-3/4}$$

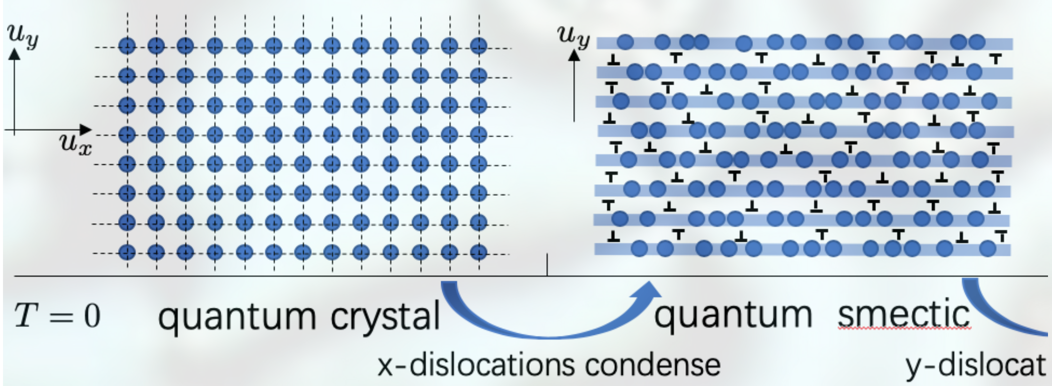
Summary and conclusions

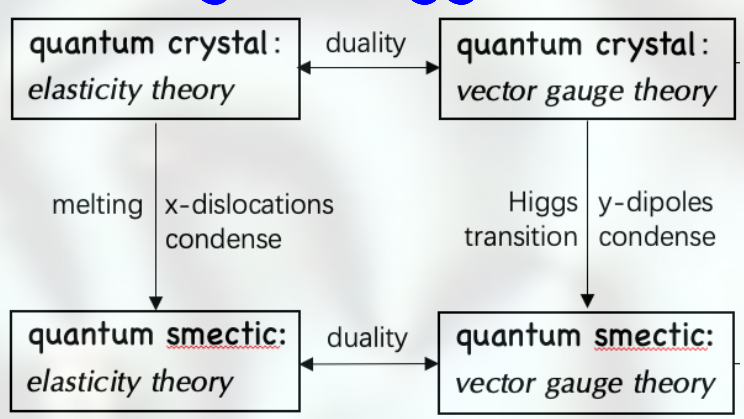
- ***Fractons – elasticity duality***
  realized as defects in quantum crystal
- **Fractonic phases and transitions:**





- ***Fractonic smectic*** via anisotropic melting -> Higgs, dualize





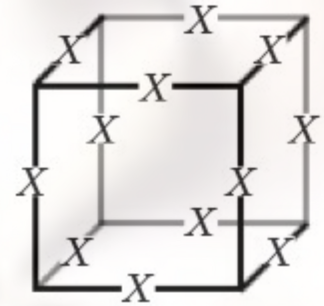
Thank you

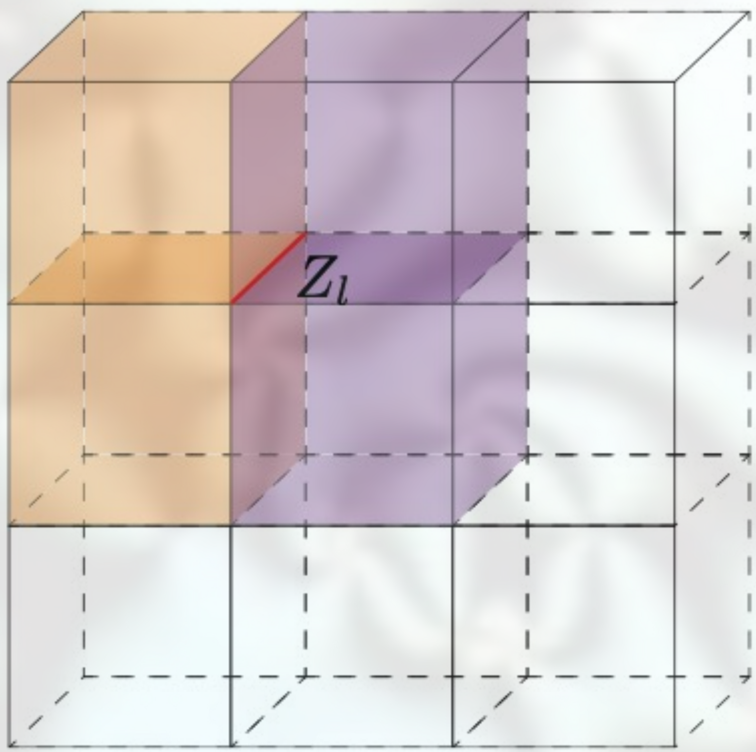
X-cube model

Vijay, Haah, Fu, 2015, 2016

$$H_{XC} = -\sum_{\mathcal{C}} \prod_{l \in \mathcal{C}} X_l - \sum_{\mu = x,y,z} \sum_{v_\mu} \prod_{l \in v_\mu} Z_l$$

$\mathcal{C}$:





$$\left(\prod_{l \in v_\mu} Z_l\right)|GS\rangle = |GS\rangle$$

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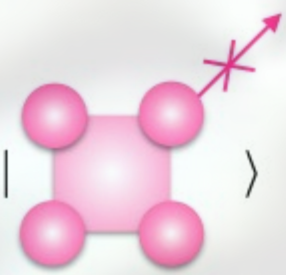
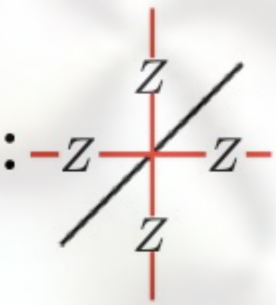
$Z_l|GS\rangle = |$



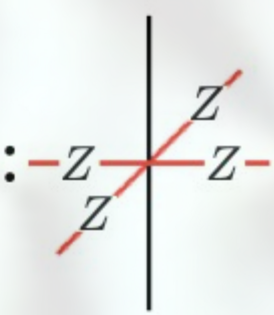
$\rangle$

v_x :



v_y :

v_z :



(slide from H. Ma)

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Vijay, Haah, Fu, 2015, 2016

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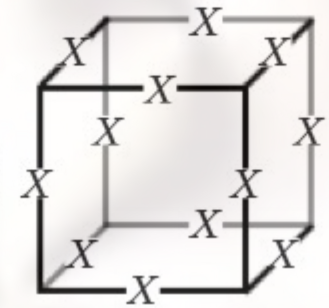

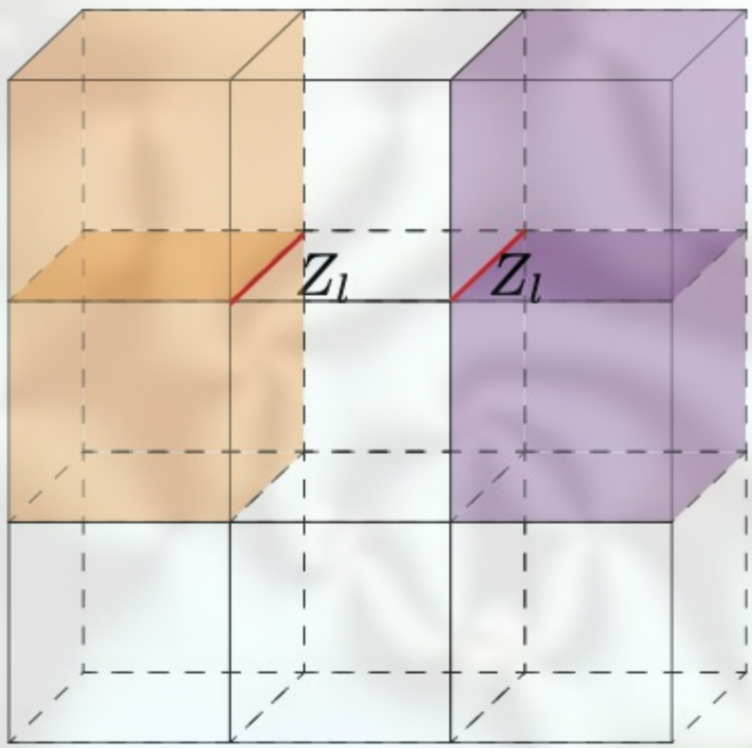

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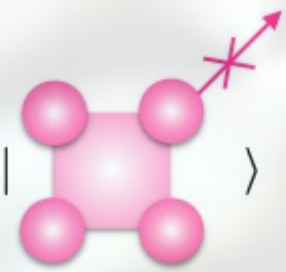
$Z_l|GS\rangle = |$  $\rangle$

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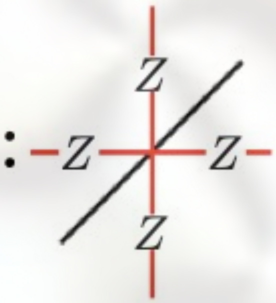

v_y :


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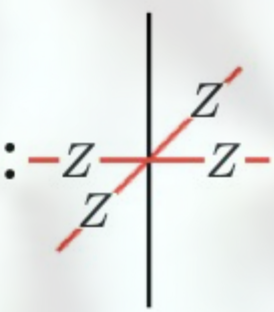

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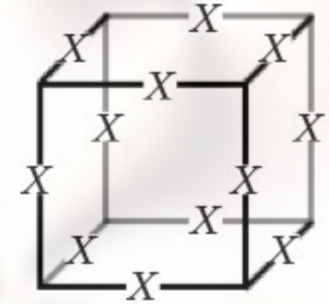

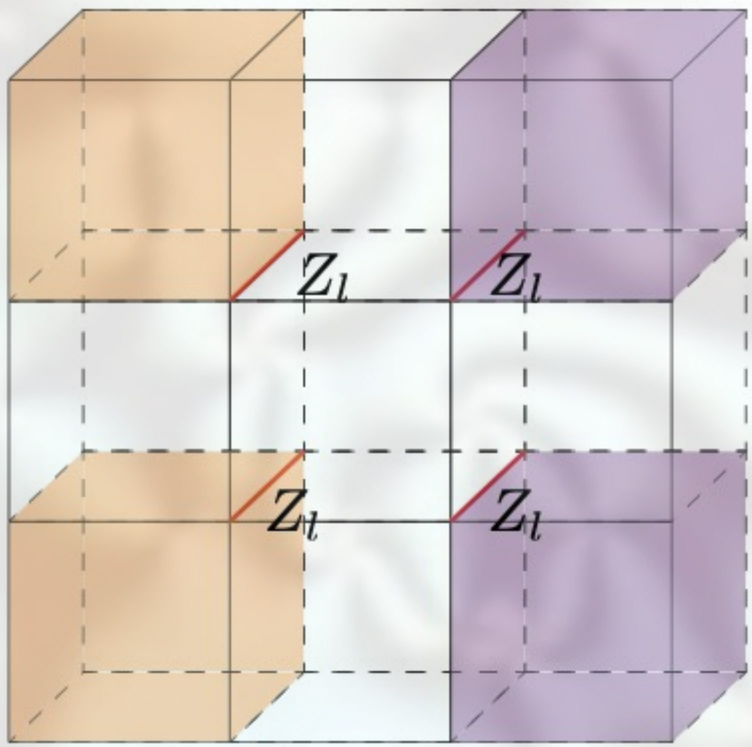

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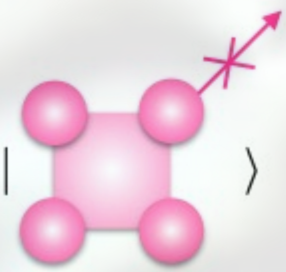
$Z_l|GS\rangle = |$  $\rangle$

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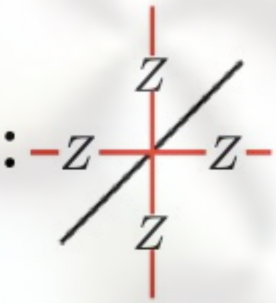

v_y :

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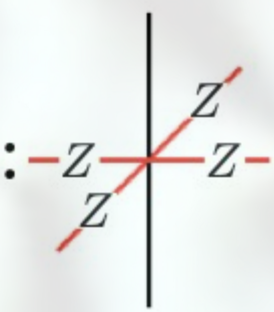

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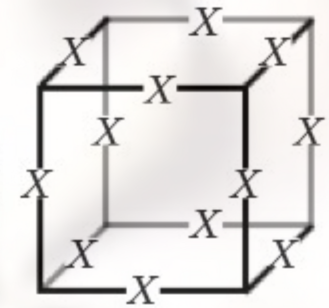

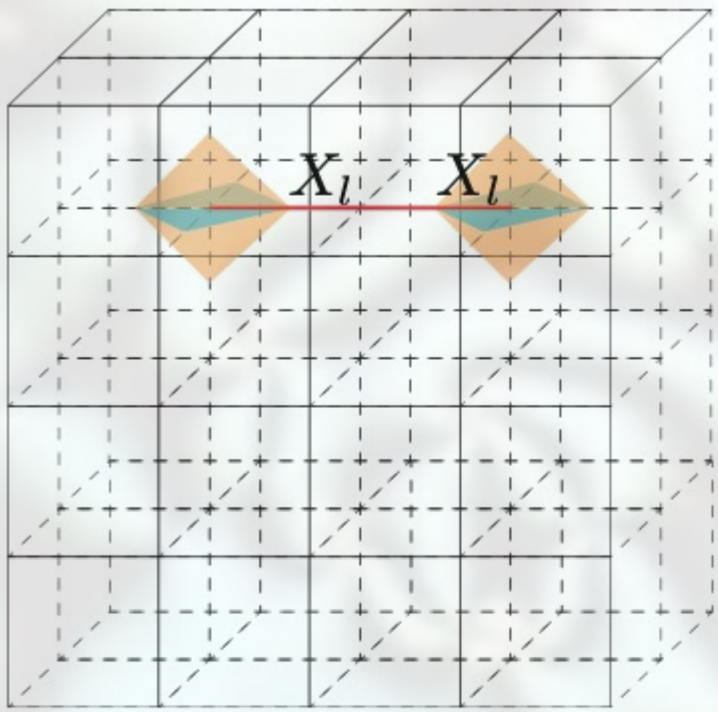

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$X_l|GS\rangle = |$  $\rangle$

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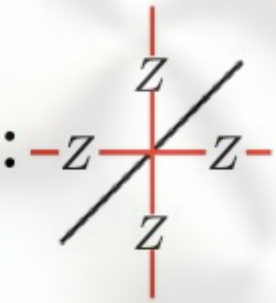

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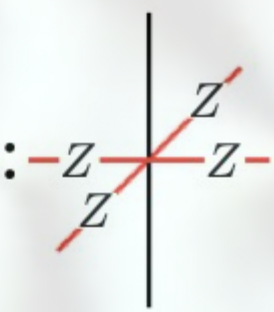

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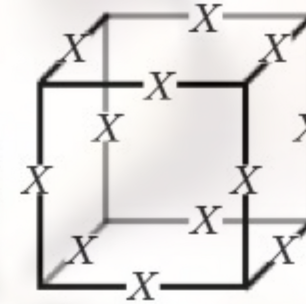

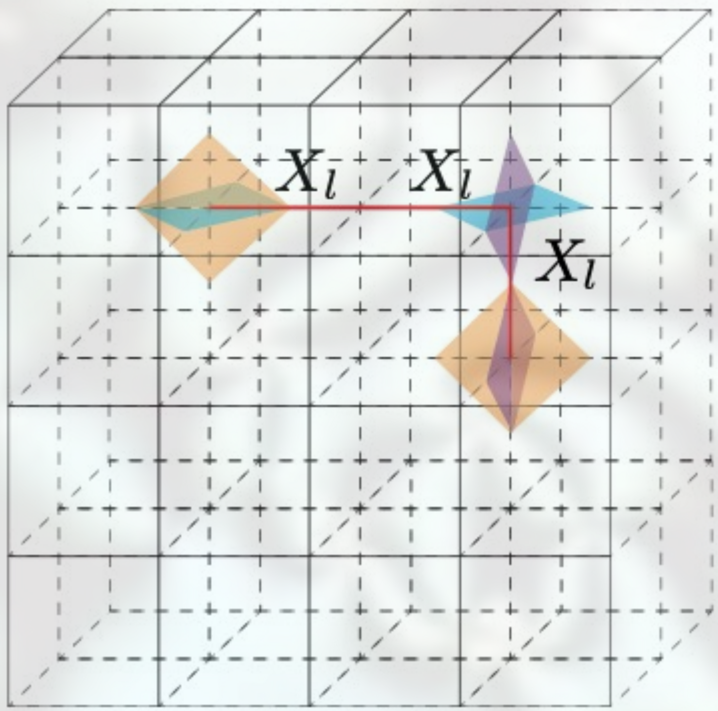

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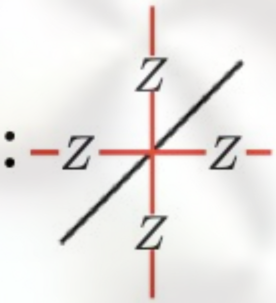

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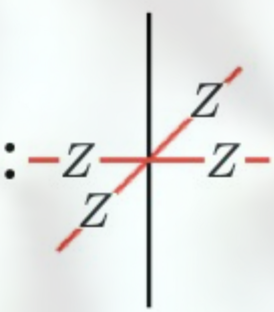

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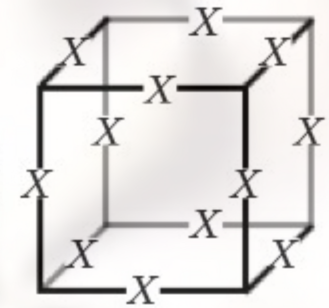

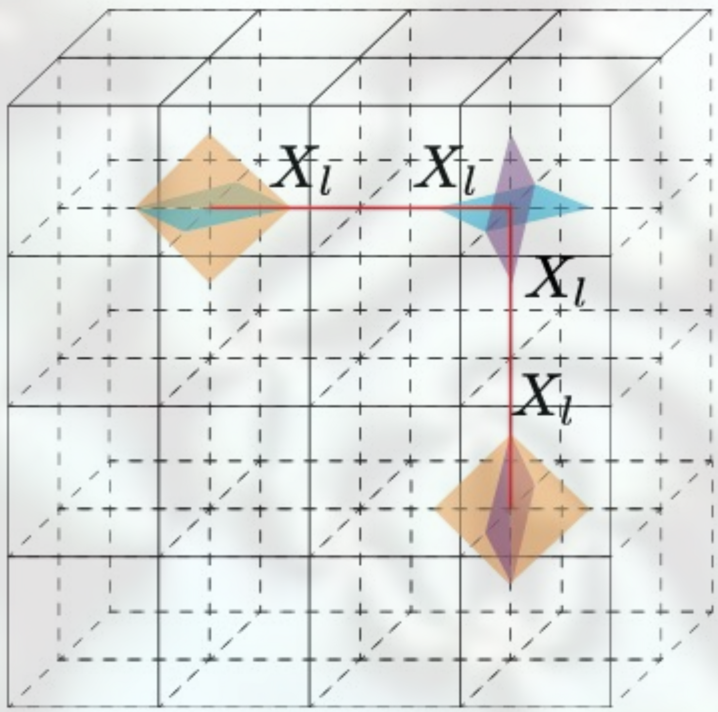

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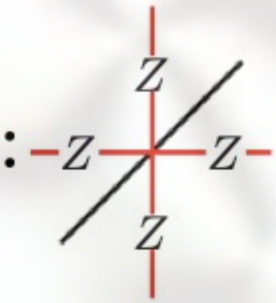
$$\left(\prod_{l\in \mathcal{C}} X_l\right)|GS\rangle = |GS\rangle$$

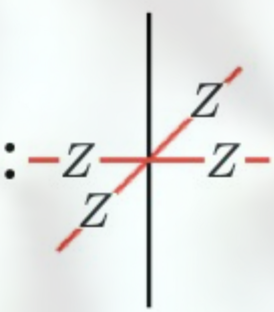
$X_l|GS\rangle = |$  $\rangle$

v_x :



v_y :

v_z :

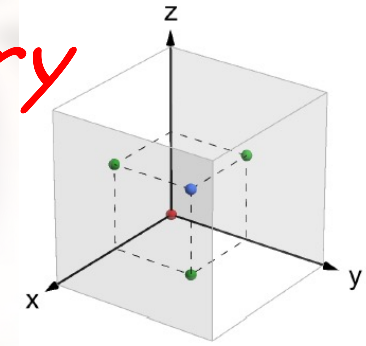


(slide from H. Ma)

Rasmussen, You, C. Xu, 2016

Pretko, 2016

Fractons via ***tensor*** gauge theory



- U(1) symmetric tensor gauge theory (2+1D):

$$\mathcal{H} = \frac{1}{2} E_{ij} E_{ij} + \frac{1}{2} B_i B_i \qquad [E^{ij}, A_{ij}] = i\delta^{(2)}(\mathbf{x}) \qquad B^i = \epsilon_{jk} \partial^j A^{ki}$$

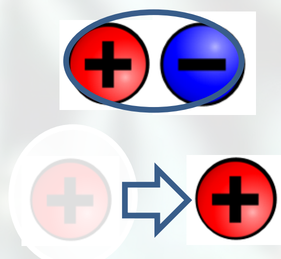
- Gauss' law: $\partial_i \partial_j E^{ij} = \rho$

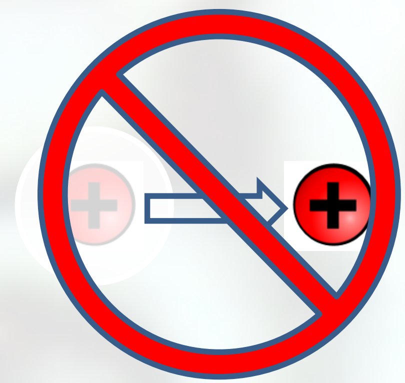
Pretko, 2016

- Conservation of charges and of *dipoles* ---> *fractōn* phenomenology!

-> moving charge changes dipole moment -> forbidden by dipole conservation

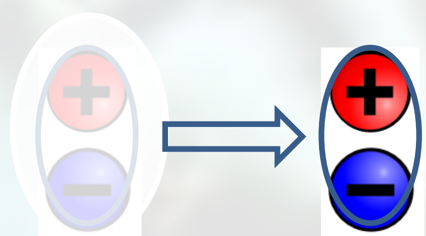
- immobile





-> dipole motion constrained

- subdimensional



Fractons

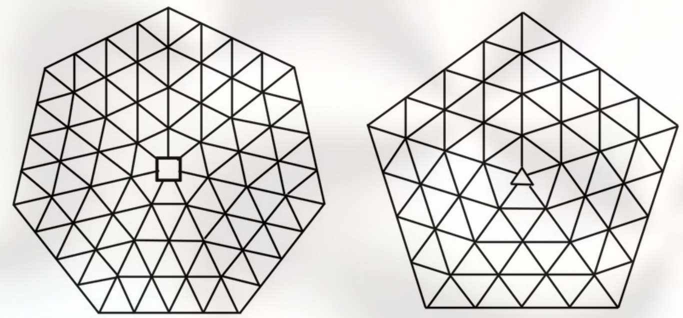
¿ any physical realizations ?

***YES*: 2D quantum crystal!**

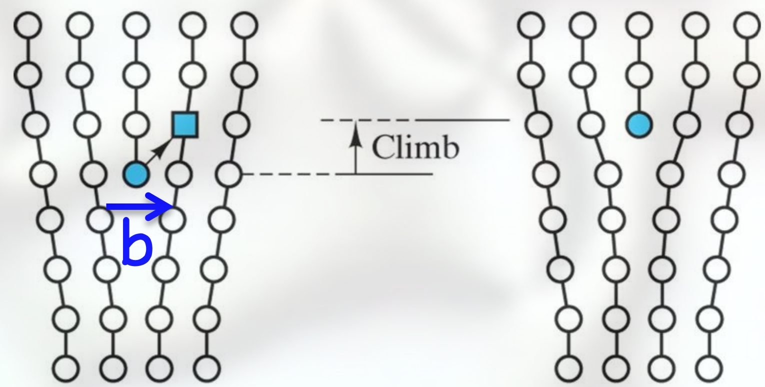
L. R., 2016
classical duality
vector gauge theory

Topological defects in a crystal

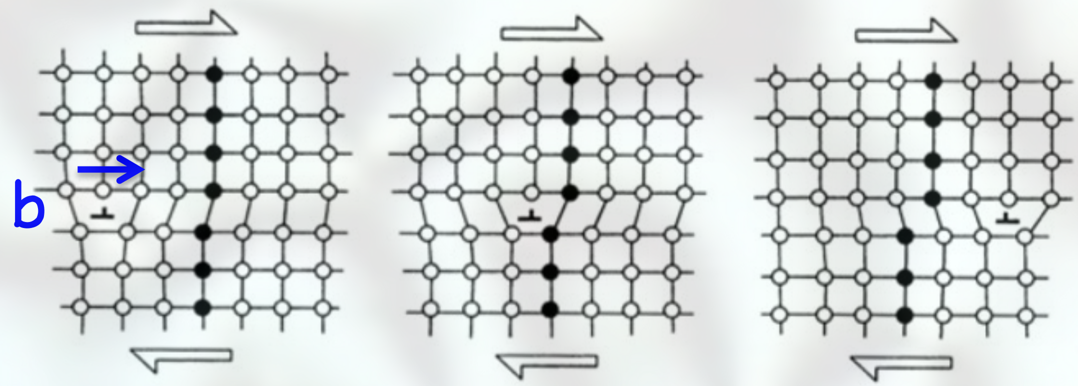
- *Disclination:*
  immobile



- *Dislocation climb:*
  constrained by
  v/i diffusion



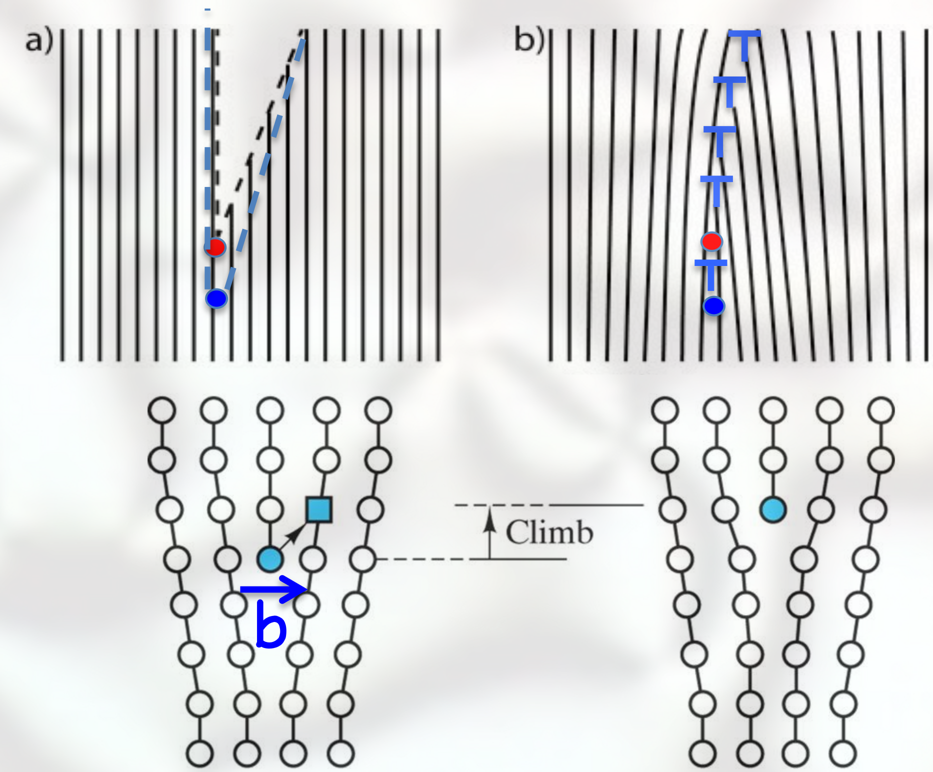
- *Dislocation glide:*
  subdimension (d-1)
  motion



L. R., 2016
classical duality

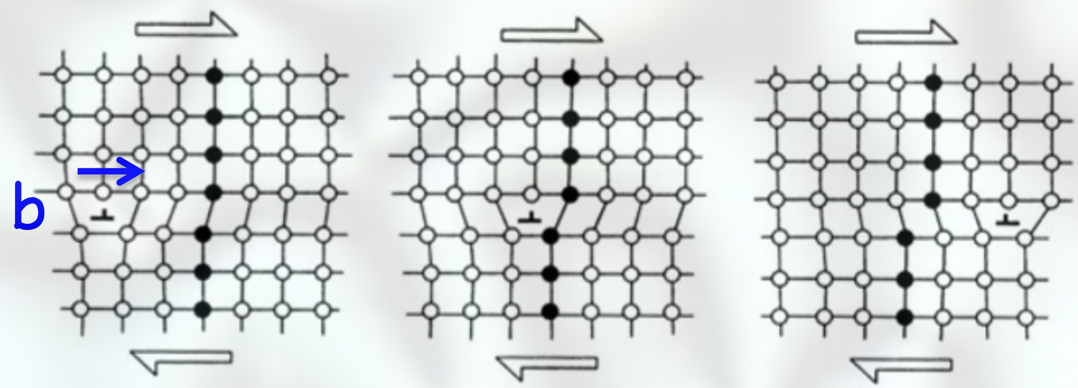
Topological defects in a crystal

- *Disclination:*
  immobile

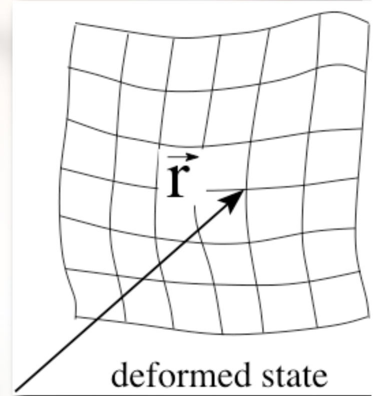


- *Dislocation climb:*
  constrained by
  v/i diffusion

- *Dislocation glide:*
  subdimension (d-1)
  motion

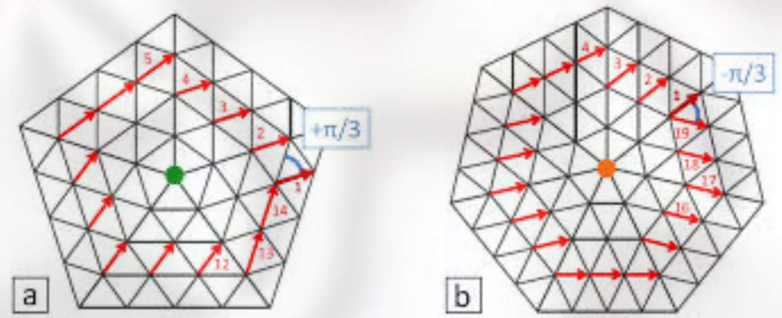


Elasticity theory and defects

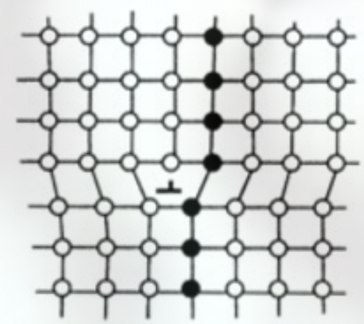


- Eulerian phonons: $\vec{r} = \vec{R} + \vec{u}(\vec{r})$
- Strain: $u_{ij} = \frac{1}{2}(\partial_i \vec{R} \cdot \partial_j \vec{R} - \delta_{ij}) \approx \frac{1}{2}(\partial_i u_j + \partial_j u_i)$
- Hamiltonian: $\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2} C_{ij,kl} u_{ij} u_{kl}$
- Topological defects
  - Disclinations: $\nabla \times \nabla \theta = s\,\delta^2(\vec{r}) \equiv s(\vec{r})$

    (bond angle: $\theta = ½\, \varepsilon_{ij} \partial_i u_j$)

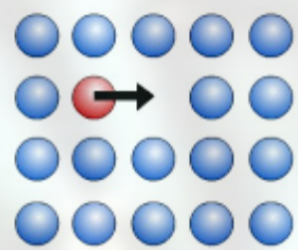


  - Dislocations: $\nabla \times \nabla u_i = b_i \delta^2(\vec{r}) \equiv b_i(\vec{r})$.



  - Vacancies/interstitials:

$$n_d = n_v - n_i$$

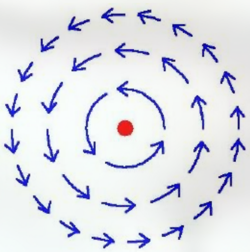


Dasgupta, Halperin
Fisher, Lee

Boson-vortex duality

Superfluid

vortices



$\mathbf{j} = \nabla\phi$

topological winding

$$\nabla \times \mathbf{j} = \rho$$

Maxwell Gauge Theory (with matter)

particles

Gauss's law:

$$\nabla \cdot \mathbf{E} = \rho$$

Goldstone mode

$$H = \frac{1}{2}\int d^2x\,\left[|\nabla\phi|^2 + n^2\right]$$

$$[n, \phi] = i$$

photon

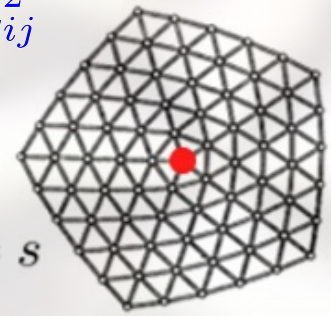
$$H = \frac{1}{2}\int d^2x\,\left[|\mathbf{E}|^2 + (\nabla \times \mathbf{A})^2\right]$$

$$[A_i, E_j] = i\delta_{ij}$$

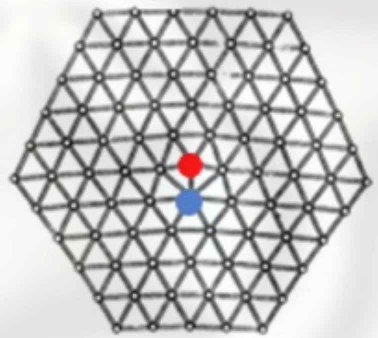
M. Pretko,
L. R., 2017
PRLs 2018

Fracton-elasticity duality

$\mathcal{H} = \frac{1}{2}B_i^2 + \frac{1}{2}E_{ij}^2$	$\mathcal{H} = \frac{1}{2}\pi_i^2 + \frac{1}{2}u_{ij}^2$
Fracton $\partial_i\partial_j E^{ij} = \rho$	Disclination $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s$
Dipole	Dislocation
Gauge Modes	Phonons
Electric Field E_{ij}	Strain Tensor u_{ij}
Magnetic Field B_i	Lattice Momentum π_i



$\vec{b}$



$$\partial_t B^i + \epsilon_{jk}\partial^j E^{ki}_{\sigma} = 0. \quad \Longleftrightarrow \quad \partial_t \pi^i - \partial_j \sigma^{ij} = 0$$

Faraday <-> Newton

L. R. 2016
L. R., Hermele
PRL 2020

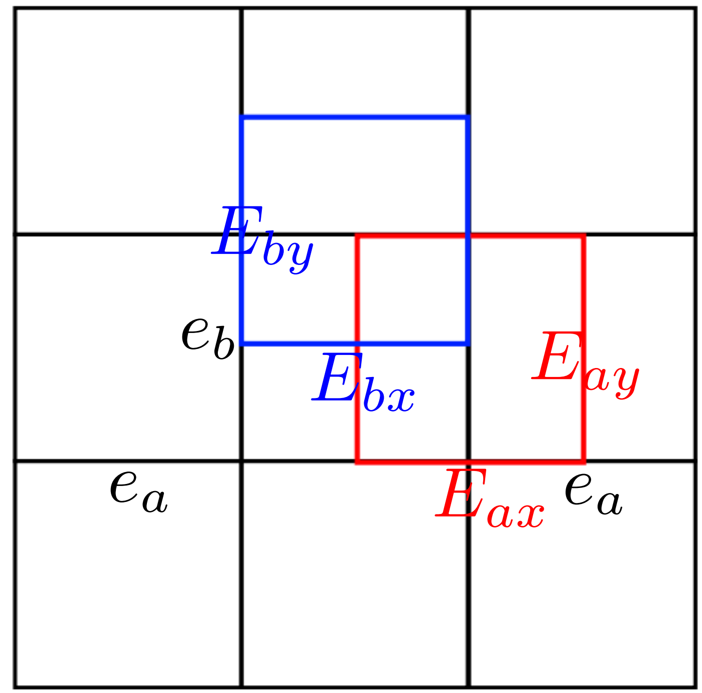
Fractons via **vector** gauge theory

- Lattice fractonic *vector* gauge theory:

E$_a$ **E**$_b$ ***e***

$$[\hat{A}_{ik}, \hat{E}_{jk'}] = -i\delta_{ij}\delta_{kk'}\delta^2(\mathbf{x}-\mathbf{x}'),$$

$$[\hat{a}_i, \hat{e}_j] = -i\delta_{ij}\delta^2(\mathbf{x}-\mathbf{x}')$$



– Gauss' law:

$$\nabla\cdot\mathbf{e} = s \qquad \nabla\cdot\mathbf{E}_k = e_k$$

(k = a, b)

$$\tilde{\mathcal{H}} = \frac{1}{2}C|\mathbf{E}_k|^2 + \frac{1}{2}(\nabla\times\mathbf{A}_k)^2 + \frac{1}{2}K|\mathbf{e}|^2 + \frac{1}{2}(\nabla\times\mathbf{a} - A_{[ij]})^2 - \mathbf{A}_k\cdot\mathbf{J}_k^b - \mathbf{a}\cdot\mathbf{j}^s$$

gauge invariance demands $\partial_t p_k + \nabla\cdot\mathbf{J}_k = j_k \longrightarrow \mathbf{j} = 0$

"Fractons via higher-form symmetry breaking"
Qi, L. R., Hermele, AOP 2020

L. R. 2016
L. R., Hermele
PRL 2020

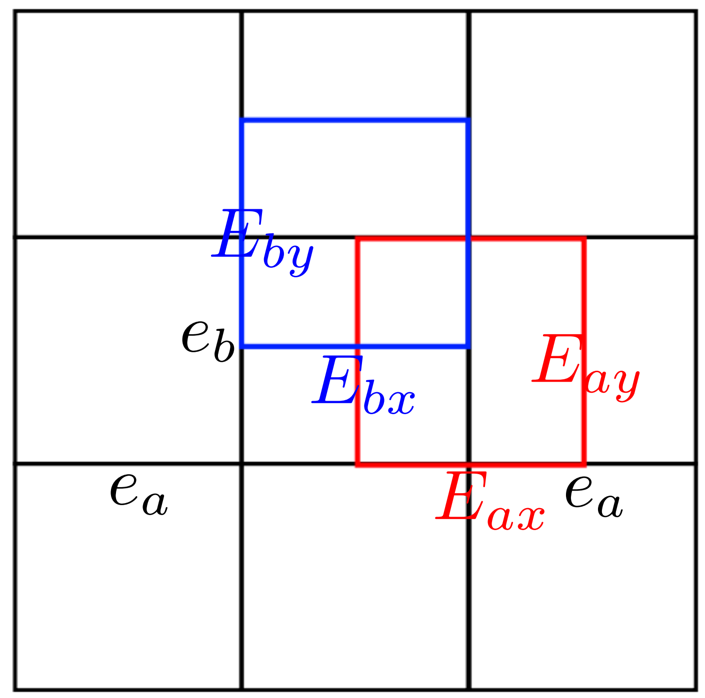
Fractons via **vector** gauge theory

- Lattice fractonic vector gauge theory:

E_a E_b e

$$[\hat{A}_{ik}, \hat{E}_{jk'}] = -i\delta_{ij}\delta_{kk'}\delta^2(\mathbf{x}-\mathbf{x}'),$$

$$[\hat{a}_i, \hat{e}_j] = -i\delta_{ij}\delta^2(\mathbf{x}-\mathbf{x}')$$



- Gauss' law:

$$\nabla \cdot \mathbf{e} = s \qquad \nabla \cdot \mathbf{E}_k = e_k$$

(k = a, b)

$$\tilde{H} = \frac{U_E}{2}\sum_{\ell \in L_k} E_{k\ell}^2 + \frac{U_e}{2}\sum_{\ell \in L} e_\ell^2 + \frac{K_E}{2}\sum_{\square_k}(\nabla \times \mathbf{A}_k)^2$$

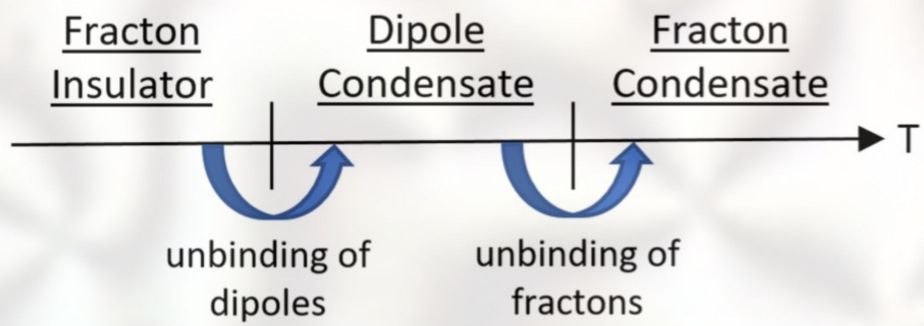
$$-K_e \sum_{\square} \cos\left[(\nabla \times \mathbf{a})_\square + A_{xy}(\ell_x) - A_{yx}(\ell_y)\right]$$

"Fractons via higher-form symmetry breaking"
Qi, L. R., Hermele, AOP 2020

Zhai, L. R., 2018

Fracton condensation transition

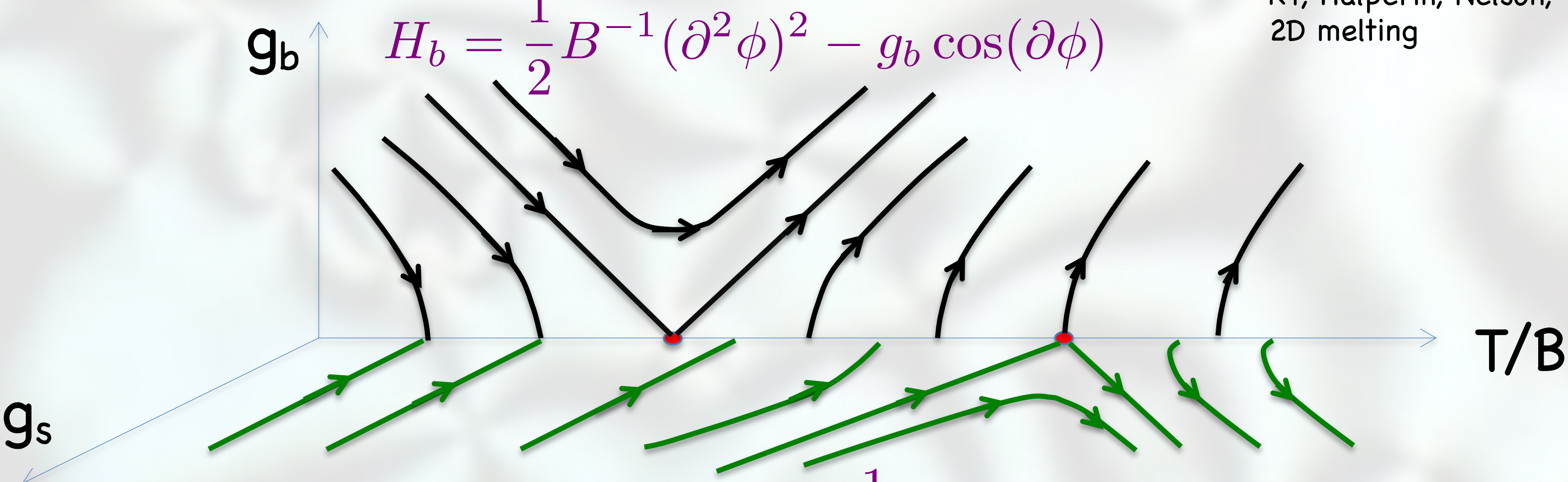
2D



2D scalar fracton model:

$$\tilde{\mathcal{H}} = \frac{1}{2}B^{-1}(\nabla^2\phi)^2 \underbrace{- g_s \cos\left(\frac{2\pi}{6}\phi\right)}_{\text{charges}} - g_b \sum_{n=1,2,3} \underbrace{\cos\left(\mathbf{b}_n \cdot \hat{z} \times \nabla\phi\right)}_{\text{dipoles}}$$

KT, Halperin, Nelson, Young '79
2D melting



$$H_s = \frac{1}{2}g_b(\partial\phi)^2 - g_s\cos(\phi)$$

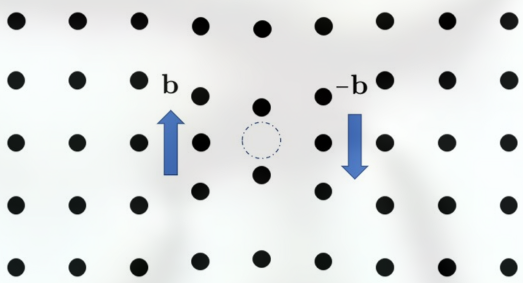
M. Pretko, L. R., 2018

2D *Quantum melting fractons*

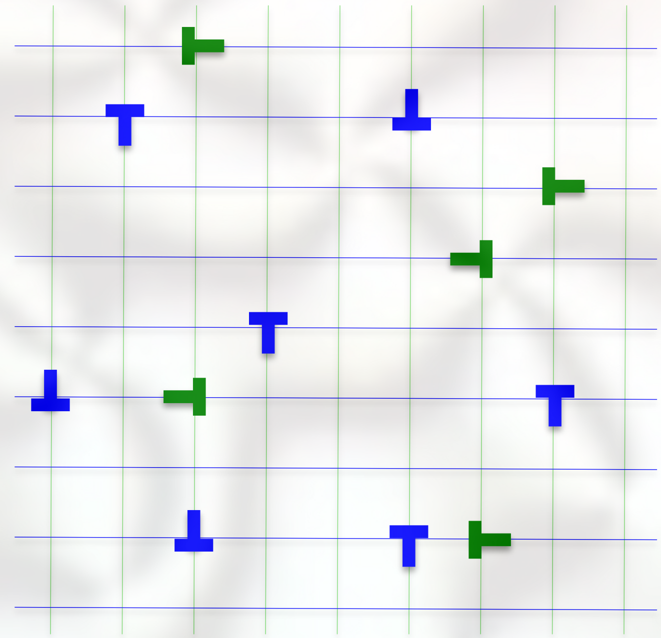
Kumar Potter, 2018

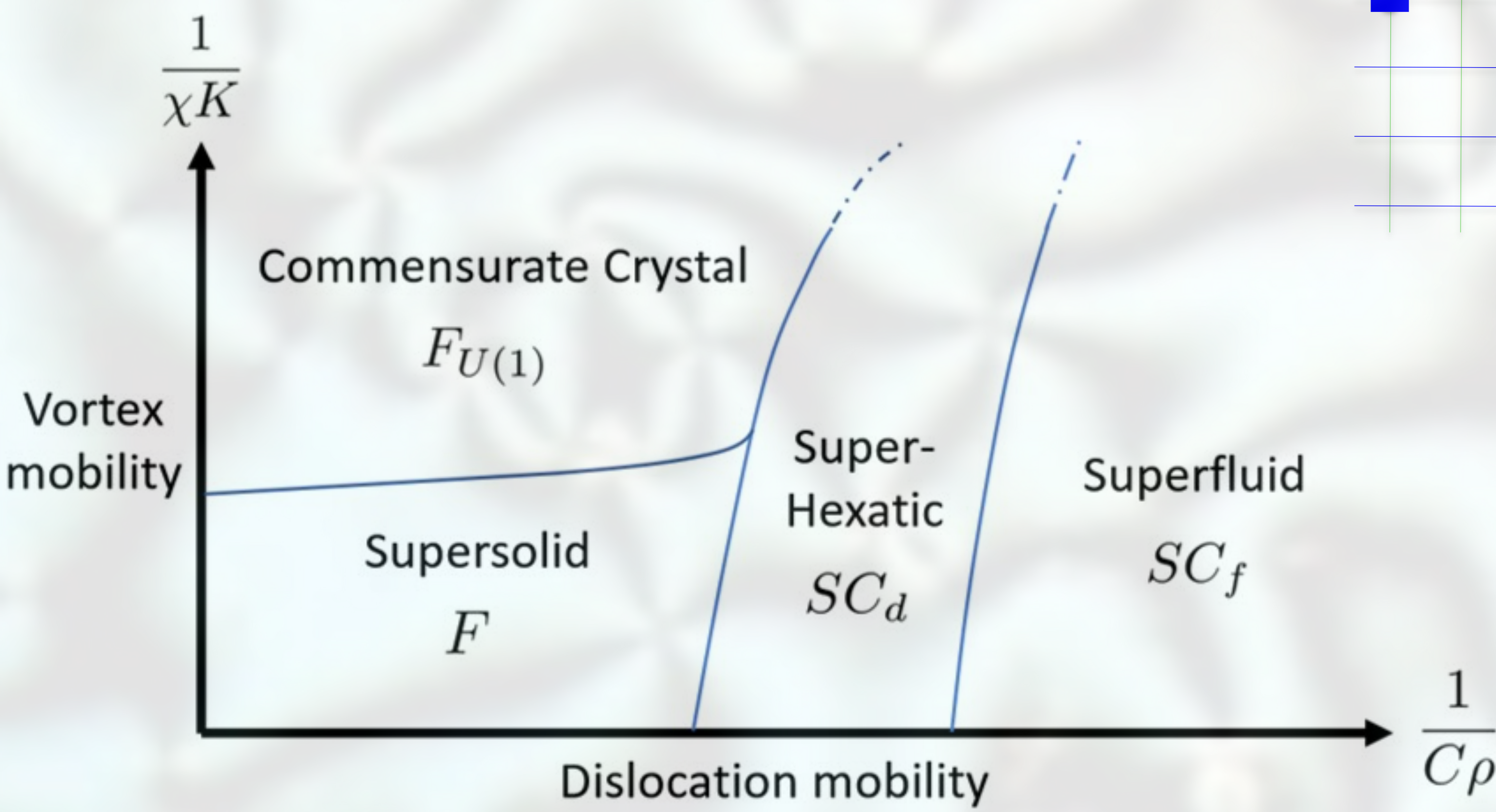
- Fracton dipoles $b_n = \sqrt{n_d} e^{i\theta_n}$ condense: -> ***super**-hexatic*

$$\mathcal{L} = \frac{1}{2} E_{ij}^2 - \frac{1}{2} B_i^2 - \cos(\partial_t \theta - A_0) + g \cos(\partial_i \partial_j \theta - A_{ij})$$



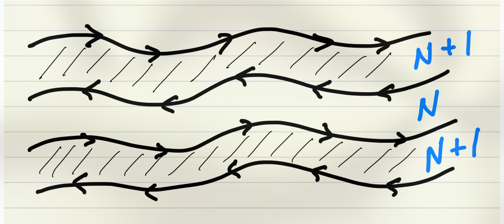
$$\hat{b}^\dagger_{\mathbf{b}} \hat{b}^\dagger_{-\mathbf{b}} \hat{a}$$

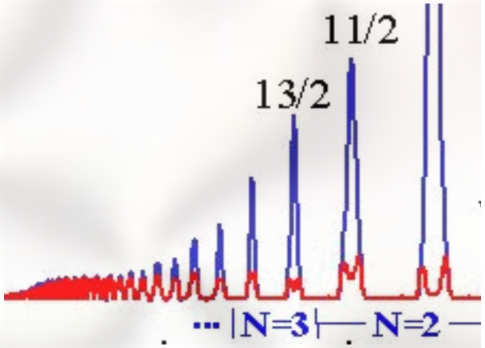




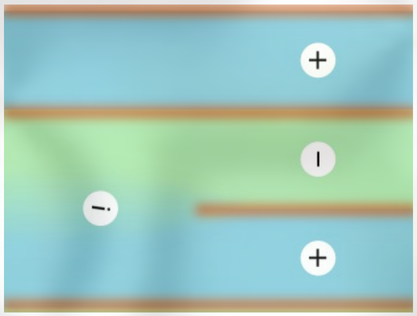
Quantum liquid crystals

- Quantum Hall *Fogler, et al. '96, Moessner, Chalker '96, Fradkin, Kivelson '99, MacDonald, Fisher '99, L.R., Dorsey '02,…Eisenstein, et al. '99*

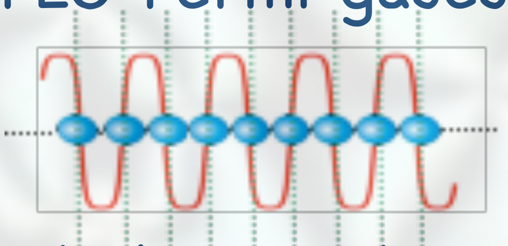




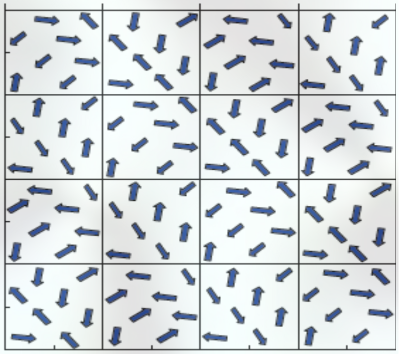
- SDW, CDW, PDW in doped Mott insulators
  Tranquada, et al., '97, Kivelson, Fradkin, Emery '98, Sachdev,…



- Imbalanced FFLO Fermi gases, SOC Bose gases, *L.R. et al. '09,'11, Zhai '15*



- Helical, frustrated magnets, e.g., MnSi, FeGe, AB_2X_4,…
  Pfleiderer, et al.' 09, Bergman, et al. '07

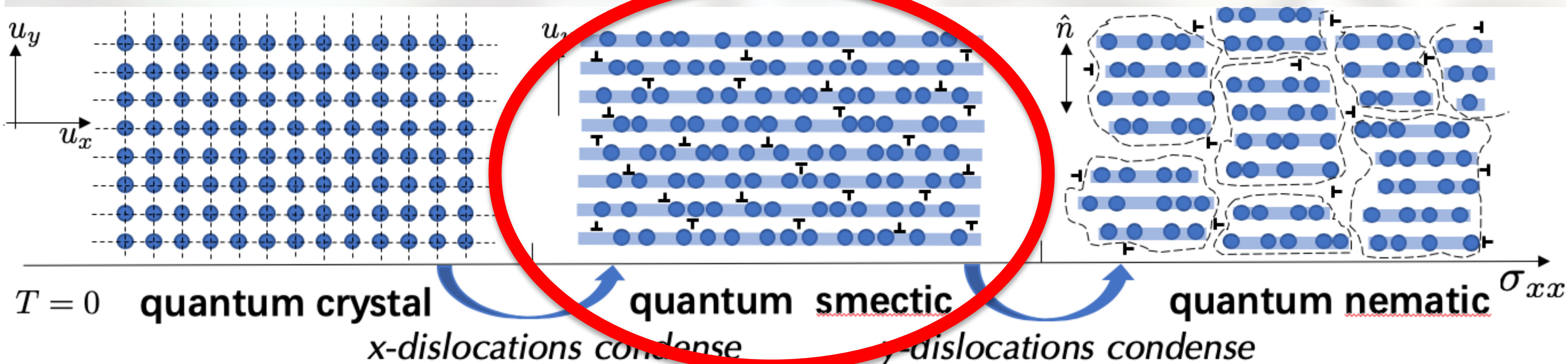


L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Anisotropic quantum melting

Ostlund, Halperin '81

- Crystal: $\mathcal{H}_{cr} = \frac{C}{2} u_{ij}^2 + \frac{1}{2}\pi^2$
- Condense x-dislocations: $b_x = \sqrt{n_d} e^{i\theta_x}$ -> ***super**-smectic*

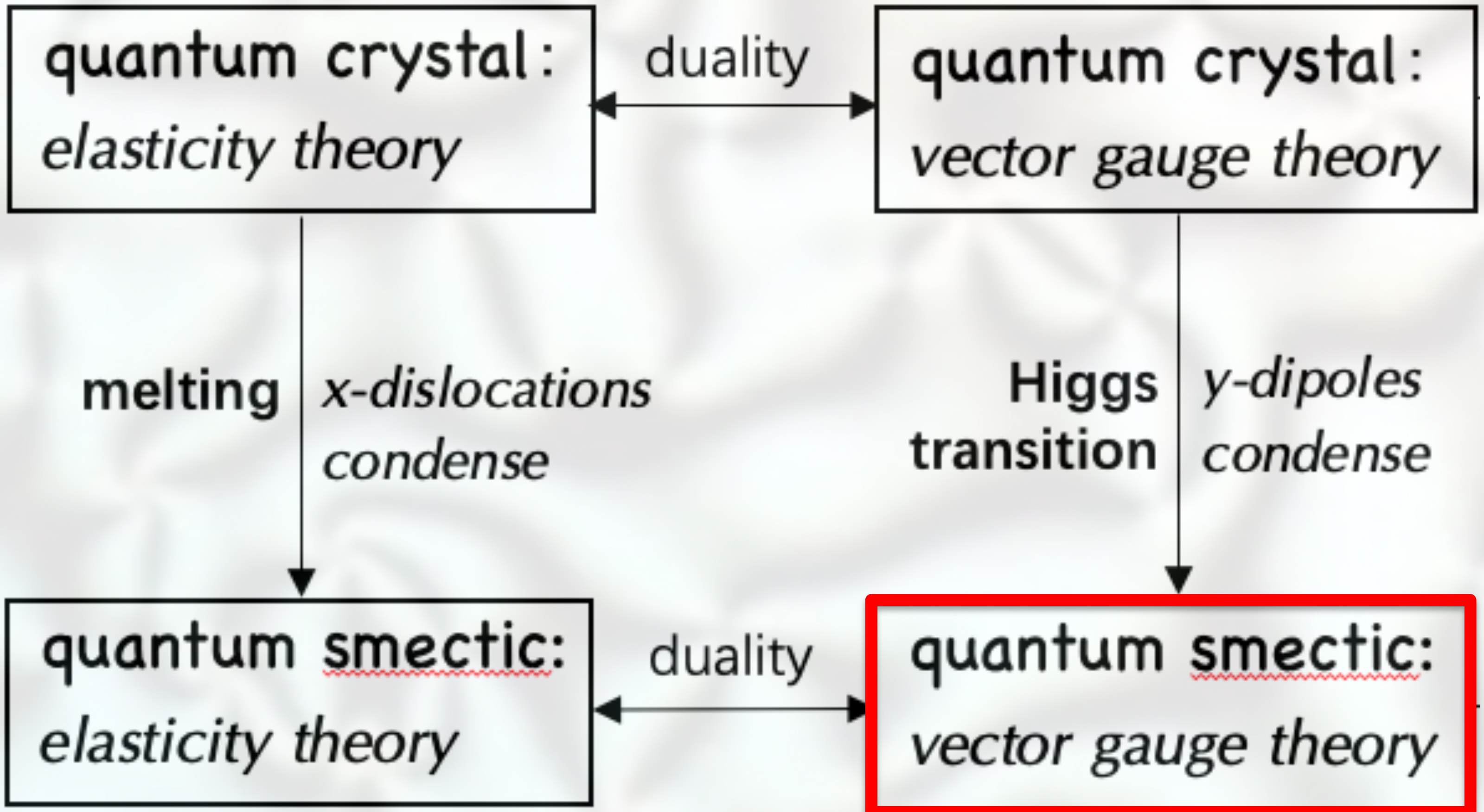


- Super-smectic:

$$\mathcal{H}_{sm} = \underbrace{\frac{C}{2}(\nabla u_y - \theta\hat{\mathbf{x}})^2 + \frac{K}{2}(\nabla\theta)^2 + \frac{1}{2}\pi^2 + \frac{1}{2}L^2}_{\text{quantum smectic elasticity}} + \underbrace{\frac{1}{2}(\nabla\phi_s)^2 + \frac{U}{2}n^2}_{\text{bosonic atoms}}$$

L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Crystal – smectic – gauge duality



L.R PRL 2020
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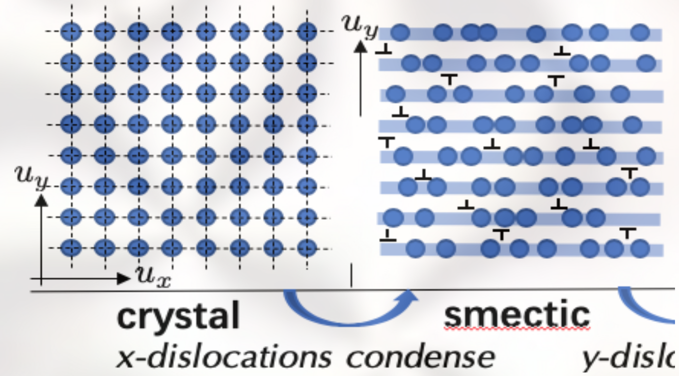
Higgs'ing crystal gauge dual -> smectic gauge dual



- Crystal gauge dual:

$$\tilde{\mathcal{H}} = \frac{1}{2}C|\mathbf{E}_k|^2 + \frac{1}{2}(\nabla\times\mathbf{A}_k)^2 + \frac{1}{2}K|\mathbf{e}|^2 + \frac{1}{2}(\nabla\times\mathbf{a} - A_{[ij]})^2 - \mathbf{A}_k\cdot\mathbf{J}_k^b - \mathbf{a}\cdot\mathbf{j}^s$$

$$\tilde{\mathcal{H}}_{cr} = |(i\nabla - p_k\mathbf{A}_k)\psi_k|^2 + V(\psi_k) + \tilde{\mathcal{H}}_{Max}[\mathbf{A}_k, \mathbf{E}_k]$$

- Higgs transition - condensation of y-dipoles (x-dislocations)

$$\psi_x = 0, \quad \psi_y \neq 0 \quad \rightarrow \quad \mathbf{A}_y \approx 0 \;\; \text{gapped}$$

- Smectic gauge dual: $\tilde{\mathcal{H}}_{sm}[\mathbf{A}^x, \mathbf{a}] \approx \tilde{\mathcal{H}}_{cr}[\mathbf{A}^x, \mathbf{A}^y = 0, \mathbf{a}]$

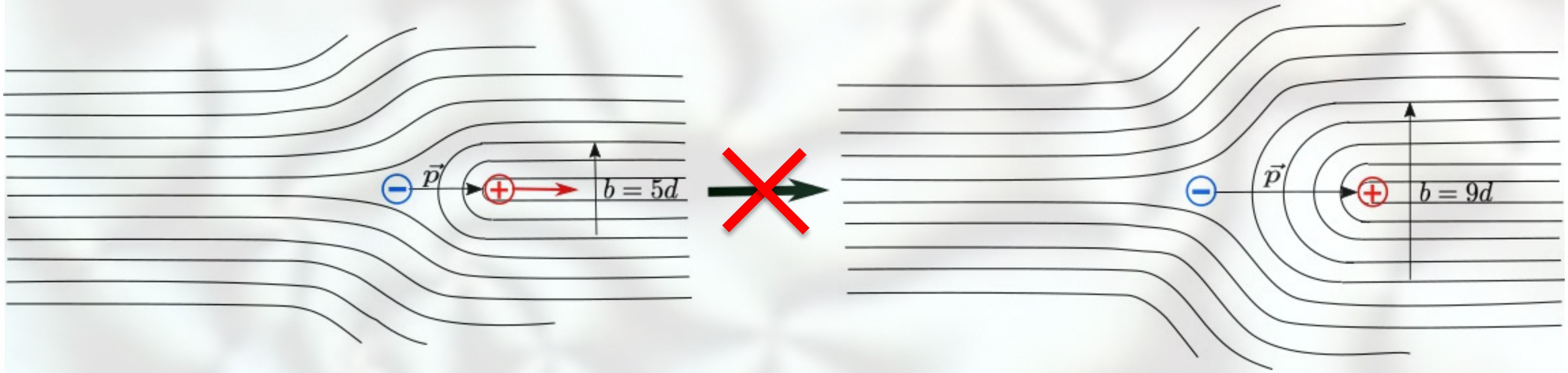
$$\tilde{\mathcal{H}}_{sm} = \frac{1}{2}C\mathbf{E}^2 + \frac{1}{2}(\nabla\times\mathbf{A})^2 + \frac{1}{2}K\mathbf{e}^2 + \frac{1}{2}(\nabla\times\mathbf{a} - \hat{\mathbf{y}}\cdot\mathbf{A})^2 - \mathbf{A}\cdot\mathbf{J} - \mathbf{a}\cdot\mathbf{j}$$

L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Restricted disclination mobility

gauge invariance demands $\partial_t p + \nabla \cdot \mathbf{J} = \hat{\mathbf{x}} \cdot \mathbf{j} \longrightarrow j_x = 0$

- Fractonic restricted dynamics *via disclination microscopics:*



requires a nonlocal process of adding a pair smectic half-layer per lattice constant of disclination separation

L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Anomalous 'diffusion'

Gromov, Lucas, Nandkishore 2020

- Fractonic restricted dynamics -> *anomalous hydrodynamics*

-> continuity: $\partial_t p + \nabla \cdot \mathbf{J} = j_x$ $\quad \partial_t n + \nabla \cdot \mathbf{j} = 0$

-> Fick's law for dipoles: $\mathbf{J} = -\gamma \nabla p = \gamma \nabla \partial_x n$

-> $j_x = \nabla \cdot \mathbf{J} = \gamma \nabla^2 \partial_x n$

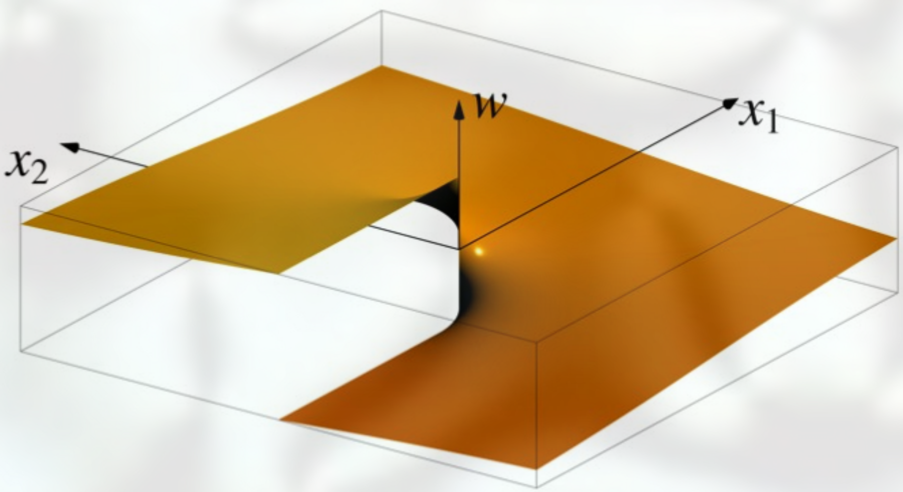
-> $\partial_t n + \hat{\Gamma} n = 0$ → $\Gamma_k = D k_y^2 + \gamma k_x^4$

$$n(t) \sim t^{-3/4}$$

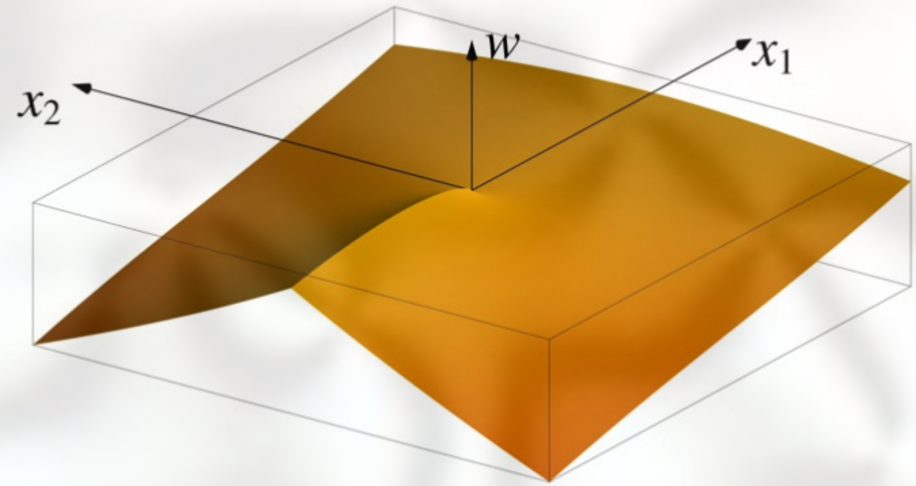
Manoj, et al, 2020

Tearing fractons in membranes

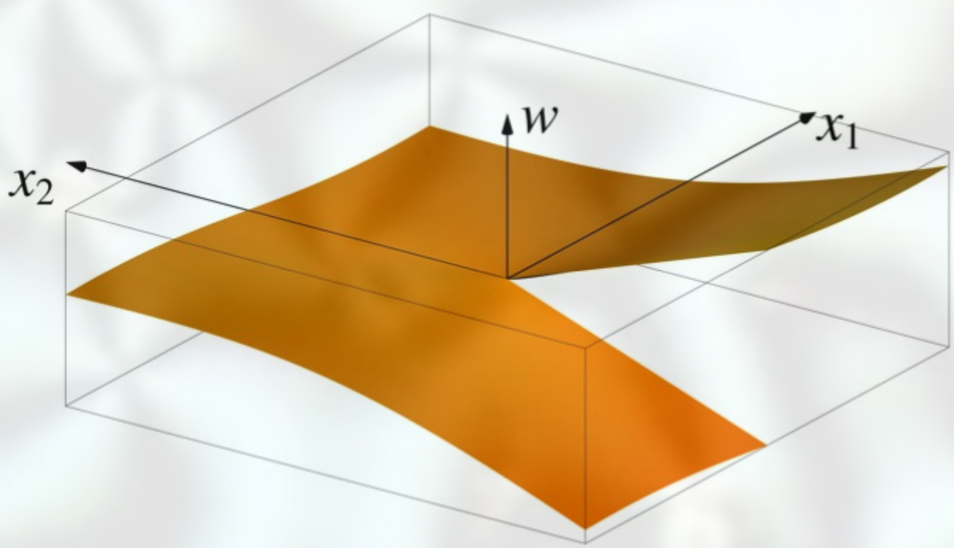
- Vector-charge rank-2 tensor gauge theory



(a) Tear defect



(b) Fold defect

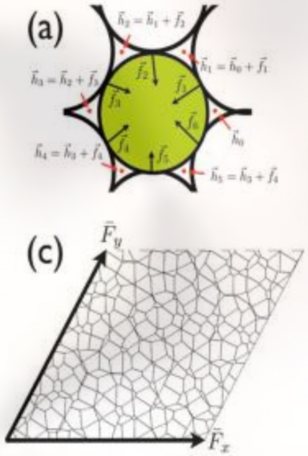


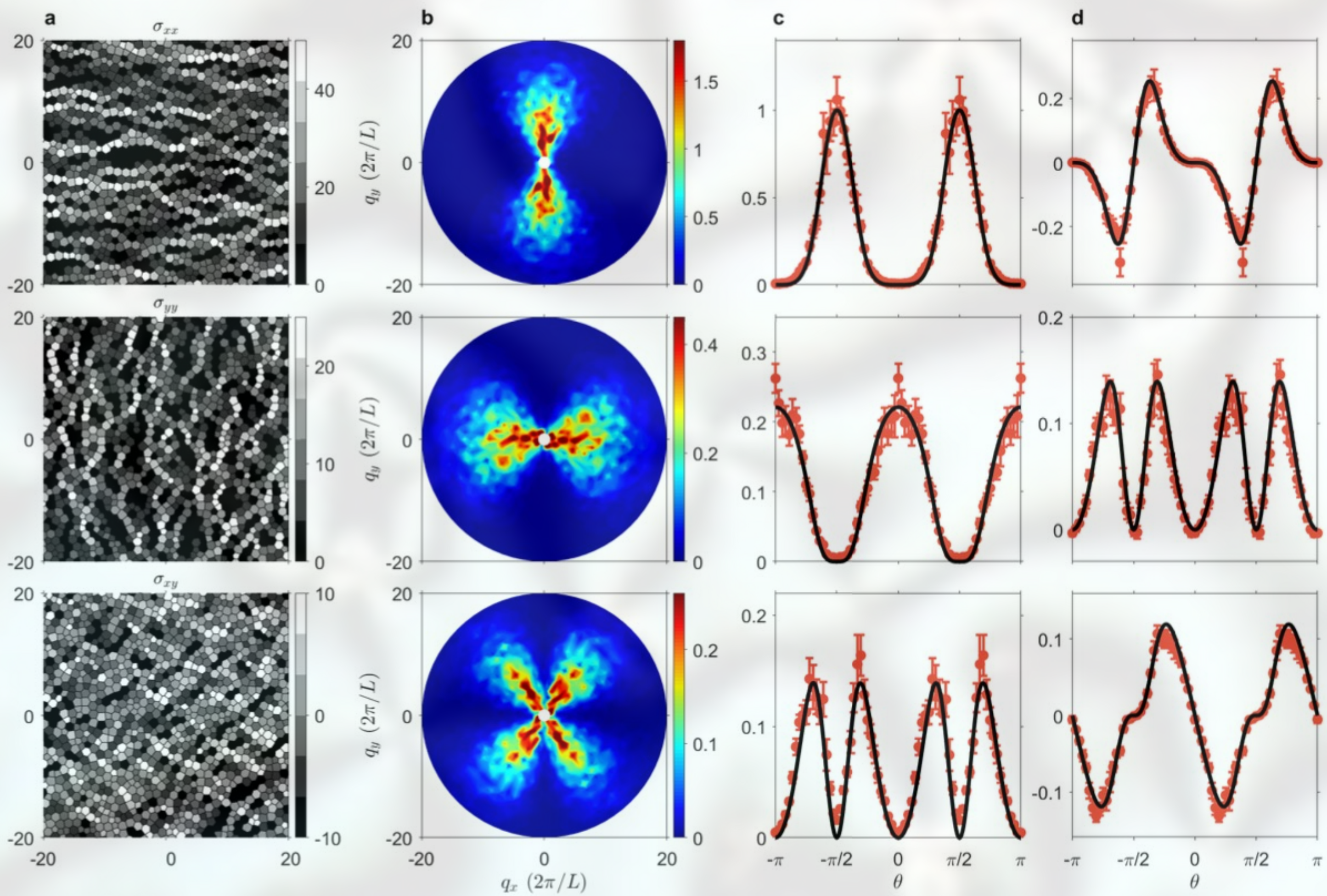
Bulbul Chakraborty, et al 2020

Fragile solids

- Stress-stress correlation function $\langle \sigma_{ij}\sigma_{kl} \rangle$
  -> vector-charge rank-2 tensor gauge theory

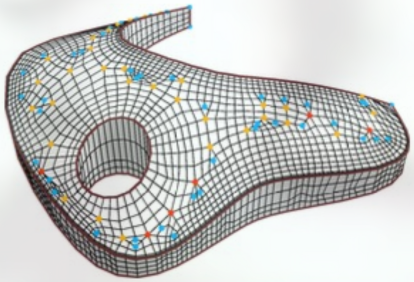
$$\partial_i \sigma_{ij}(\mathbf{r}) = f_j(\mathbf{r}) \longleftrightarrow \partial_i E_{ij} = \rho_j,$$

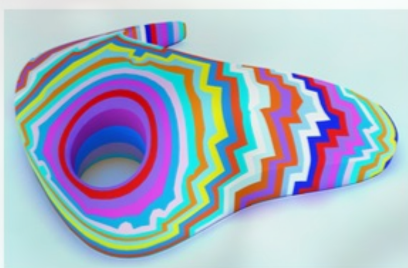


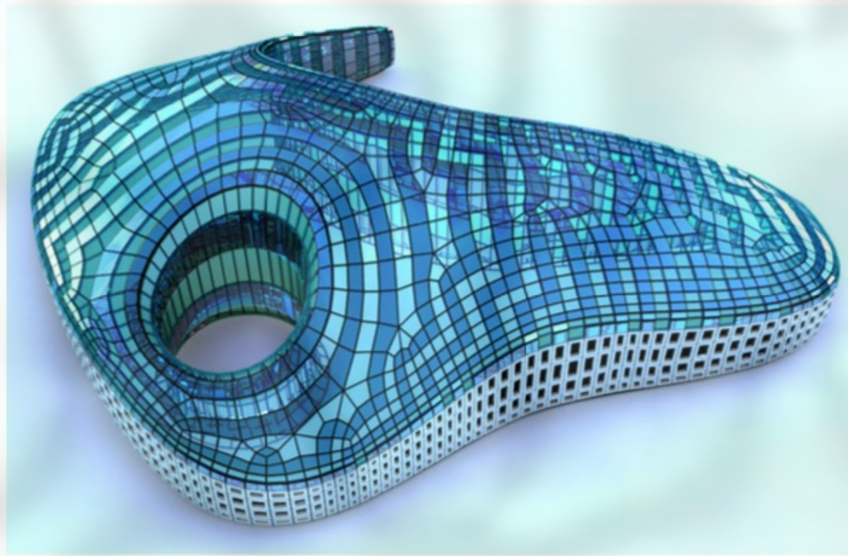


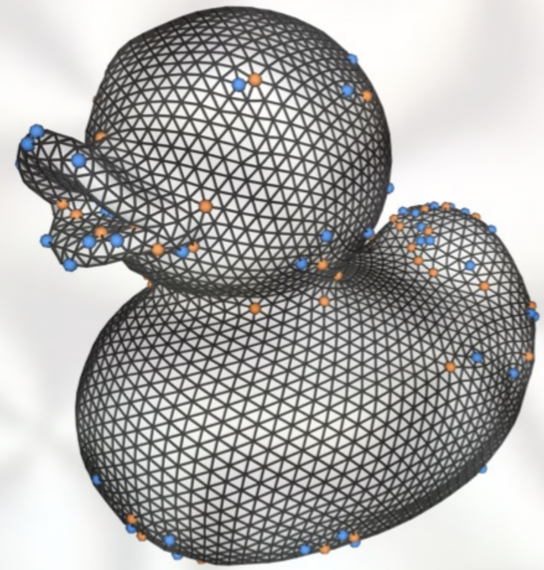
Wonka,
et al 2010, '11

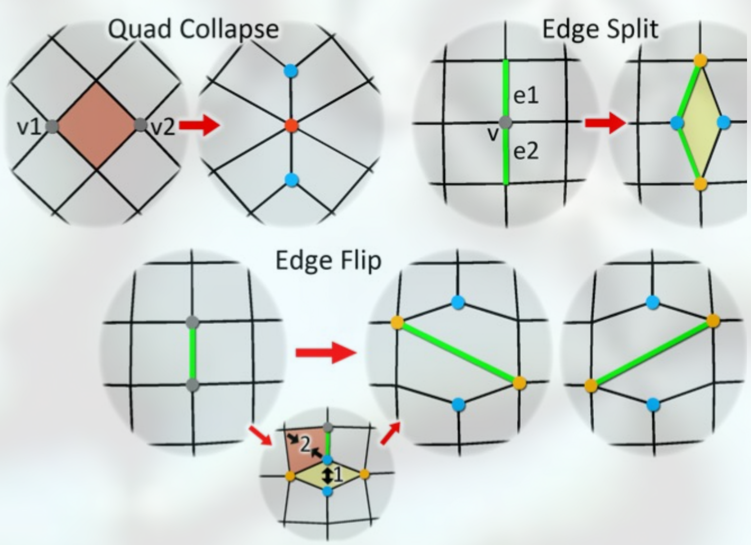
Evolution of triangulated meshes

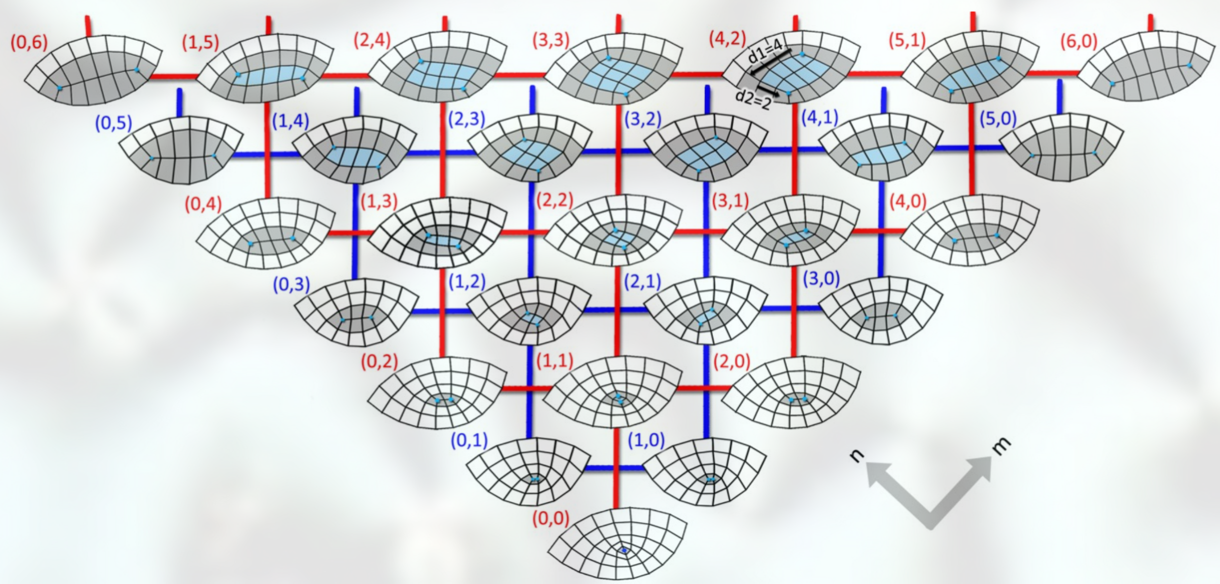








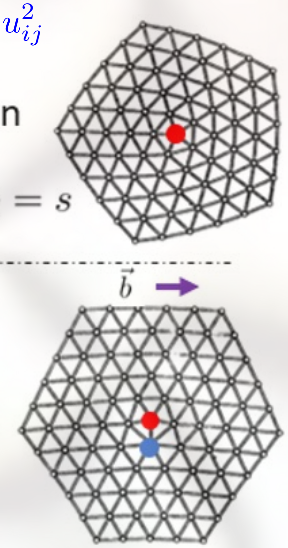




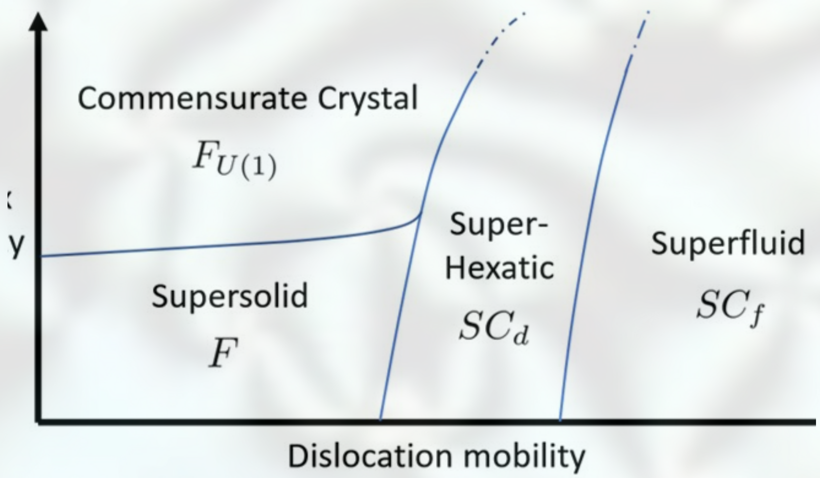
Summary and conclusions

- *New class of fractonic quantum liquids*
  excitations w/ restricted/fractionalized mobility

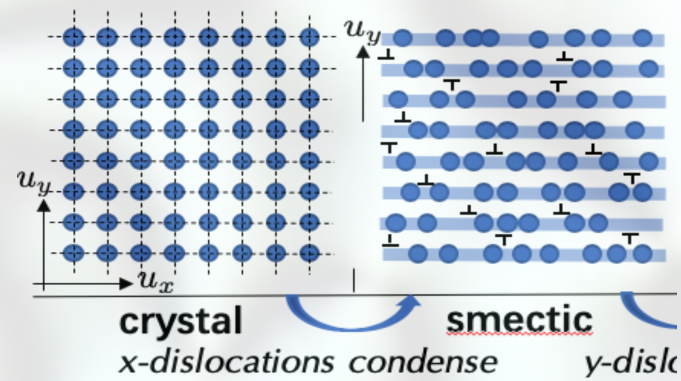
$\mathcal{H}=\frac{1}{2}B_i^2+\frac{1}{2}E_{ij}^2$	$\mathcal{H}=\frac{1}{2}\pi_i^2+\frac{1}{2}u_{ij}^2$
Fracton $\partial_i\partial_j E^{ij}=\rho$	Disclination $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell}=s$
Dipole	Dislocation
Gauge Modes	Phonons
Electric Field E_{ij}	Strain Tensor u_{ij}
Magnetic Field B_i	Lattice Momentum π_i



- *Fractons – elasticity duality*
  realized as defects in quantum crystal
- *Fractonic phases and transitions:*



- *Quantum melting criticality?*
- *QH smectic ?* (anisotropic melting, via Higgs transitions, LR 'PRL20)
- *Elastic nonlinearities ?*
- *Classification, relation to Z_2 models, higher form symmetries, …?*
- *Animation dynamics and editing of tessellated surfaces ?*



L. R., 2018

2D *T-R breaking fractons*

Kumar
Potter, 2018

- Wigner crystal in B-field elasticity (also vortex lattice)

$$\hat{\mathcal{H}} = \frac{1}{2} C^{ijk\ell} \hat{u}_{ij} \hat{u}_{k\ell}$$

$$[u_x(\mathbf{r}), u_y(\mathbf{r}')] = i\ell^2 \delta^2(\mathbf{r} - \mathbf{r}').$$

- T-R breaking fracton phase

$$\hat{\mathcal{L}} = \frac{1}{2} \mathbf{B} \times \partial_t \mathbf{B} - \frac{1}{2} C^{ijk\ell} E_{ij} E_{k\ell}$$

$$\rightarrow \quad \omega \sim q^2$$

M. Pretko, L. R.,2017
PRLs 2018

Fracton-elasticity duality

- Elastic Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_t u^i)^2 - \frac{1}{2}C^{ijk\ell}u_{ij}u_{k\ell}$

---> $\mathcal{L} = \frac{1}{2}C^{-1}_{ijk\ell}\sigma^{ij}\sigma^{k\ell} - \frac{1}{2}\pi^i\pi_i - \sigma^{ij}(\partial_i\tilde{u}_j + u^{(s)}_{ij}) + \pi^i\partial_t(\tilde{u}_i + u^{(s)}_i)$

- Disclinicity: $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s(\mathbf{x}) + \hat{\mathbf{z}}\cdot\nabla\times\mathbf{b}(\mathbf{x})$

- Momentum conservation (Newton) constraint: $\partial_t\pi^i - \partial_j\sigma^{ij} = 0$

- Electric, magnetic fields: $B^i = \epsilon^{ij}\pi_j$ $\quad E^{ij}_\sigma = \epsilon^{ik}\epsilon^{j\ell}\sigma_{k\ell}$

-> Faraday law: $\partial_t B^i + \epsilon_{jk}\partial^j E^{ki}_\sigma = 0$

-> Gauge fields: $B^i = \epsilon_{jk}\partial^j A^{ki}$ $\quad E^{ij}_\sigma = -\partial_t A^{ij} - \partial_i\partial_j\phi$

-> Gauge freedom: $A_{ij} \to A_{ij} + \partial_i\partial_j\alpha$ $\quad \phi \to \phi + \partial_t\alpha$

-> Peach-Koehler force: $F_i = E_{ij}\, p_j$

Fracton-Elasticity Duality

Boulder

Center for Theory of Quantum Matter

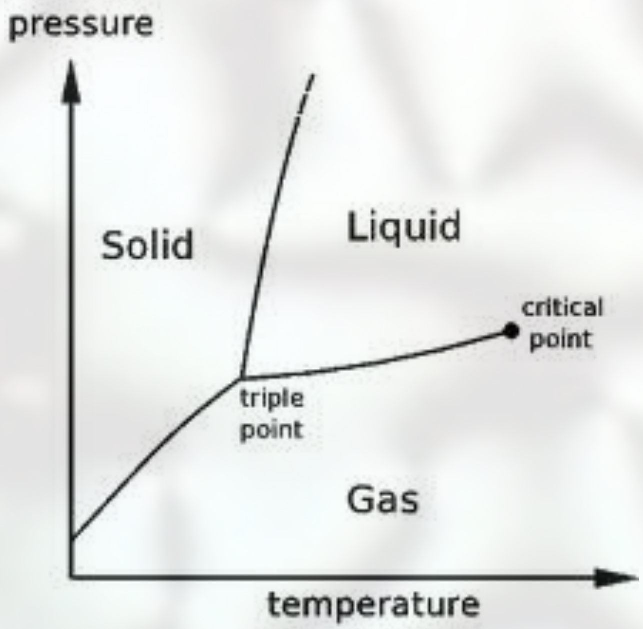
CTQM

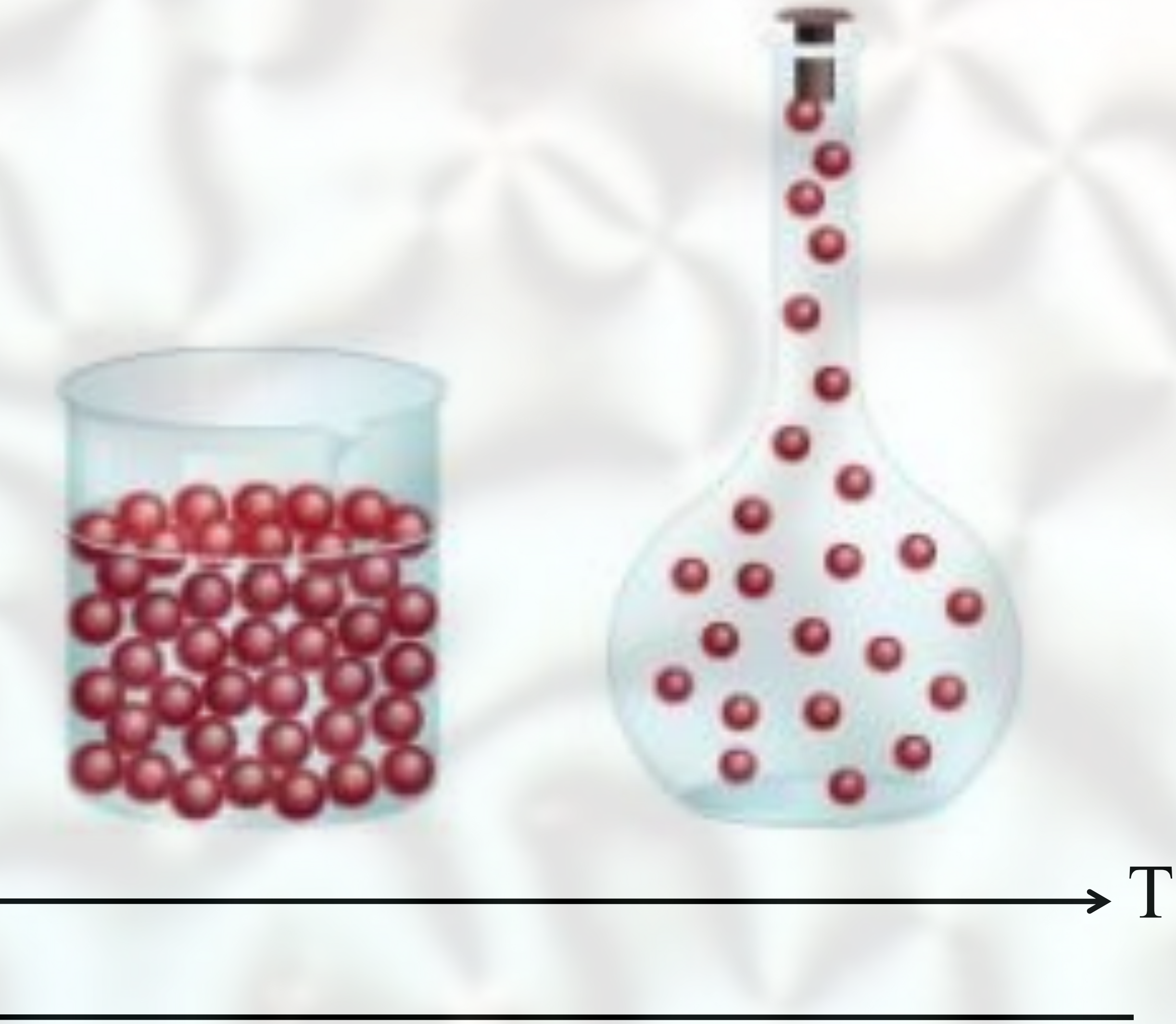
M. Pretko and L.R., PRLs 2018,
Z. Zhai and L.R. arXiv 2019
L.R. and Hermele, arXiv 2019
M. Pretko, Z. Zhai, and L.R. arXiv 2019

$: NSF-MRSEC, Simons Investigator

Physics Colloquium, Caltech, October 3, 2019

"White lies" about phases of condensed matter



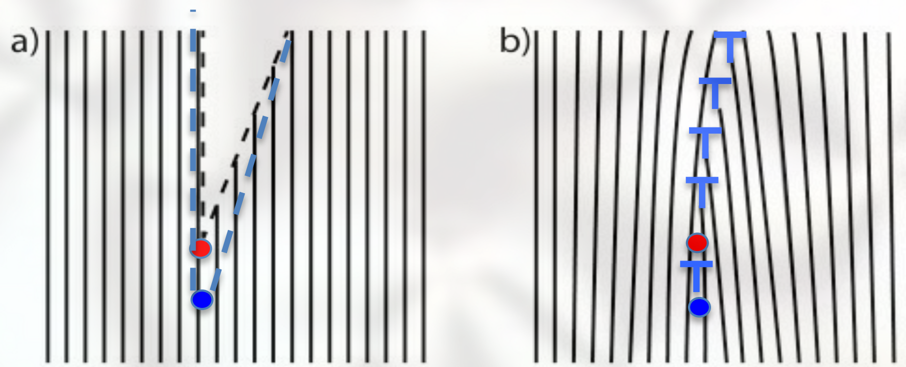


L. R., 2016
classical duality

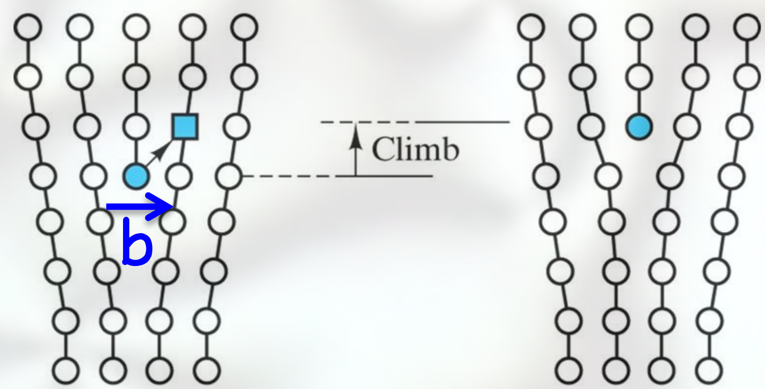
Topological defects in a crystal

$$\mathcal{H}_{el} = \frac{1}{2} B (u_{ij})^2 \longrightarrow \tilde{\mathcal{H}}_{el} = \frac{1}{2} B^{-1} (\nabla^2 \phi)^2 - i\phi(s + \hat{z} \cdot \nabla \times \mathbf{b})$$

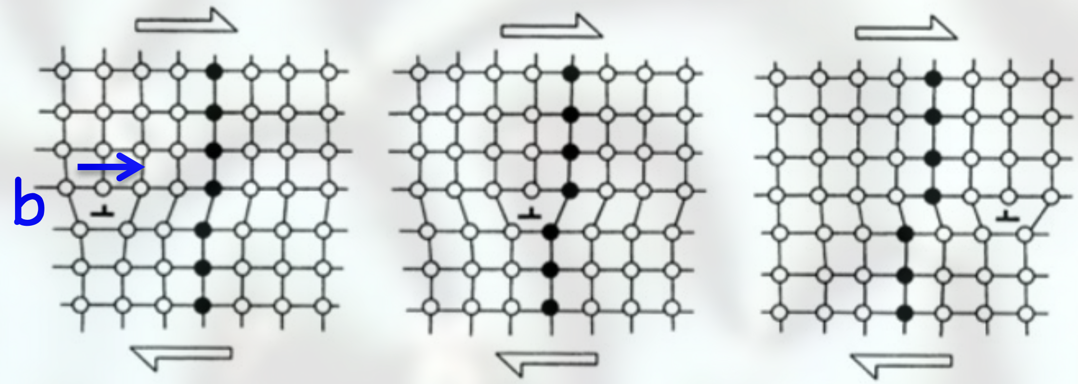
- *Disclination:*
  immobile



- *Dislocation* ***climb:***
  constrained by
  v/i diffusion



- *Dislocation* ***glide:***
  subdimension (d-1)
  motion



M. Pretko, L. R.,2017
PRLs 2018

Fracton-elasticity duality

- Elastic Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_t u^i)^2 - \frac{1}{2}C^{ijk\ell}u_{ij}u_{k\ell}$
- Disclinicity: $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s(\mathbf{x}) + \hat{\mathbf{z}}\cdot\nabla\times\mathbf{b}(\mathbf{x})$
- Momentum conservation (Newton) constraint: $\partial_t\pi^i - \partial_j\sigma^{ij} = 0$
- Electric, magnetic fields: $B^i = \epsilon^{ij}\pi_j$ $\quad E_\sigma^{ij} = \epsilon^{ik}\epsilon^{j\ell}\sigma_{k\ell}$

-> Faraday law: $\partial_t B^i + \epsilon_{jk}\partial^j E_\sigma^{ki} = 0$

-> Gauge fields: $B^i = \epsilon_{jk}\partial^j A^{ki}$ $\quad E_\sigma^{ij} = -\partial_t A^{ij} - \partial_i\partial_j\phi$

-> Gauge freedom: $A_{ij} \to A_{ij} + \partial_i\partial_j\alpha$ $\quad \phi \to \phi + \partial_t\alpha$

-> Peach-Koehler force: $F_i = E_{ij}\, p_j$

M. Pretko,
L. R.,2017

Fracton-elasticity duality

- Elastic Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_t u^i)^2 - \frac{1}{2}C^{ijk\ell}u_{ij}u_{k\ell}$
- Disclinations: $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s(\mathbf{x})$
- Electric, magnetic fields: $B^i = \epsilon^{ij}\pi_j$ $\quad E^{ij}_\sigma = \epsilon^{ik}\epsilon^{j\ell}\sigma_{k\ell}$

$B^i = \epsilon_{jk}\partial^j A^{ki}$ $\quad E^{ij}_\sigma = -\partial_t A^{ij} - \partial_i\partial_j\phi$

-> Fracton Hamiltonian: $[E^{ij}, A_{ij}] = i\delta^{(2)}(\mathbf{x})$

$$\mathcal{H} = \frac{1}{2}\tilde{C}^{ijk\ell}E_{ij}E_{k\ell} + \frac{1}{2}B^i B_i + \rho\phi + J^{ij}A_{ij}$$

-> Fracton charges, dipole currents:

$$\rho = s \qquad J^{ij} = \epsilon^{ik}\epsilon^{j\ell}(\partial_t\partial_k - \partial_k\partial_t)u_\ell = \epsilon^{(i\ell}v^{j)}b_\ell$$

-> Gauss' law, continuity: $\partial_i\partial_j E^{ij} = \rho$ $\quad \partial_t\rho + \partial_i\partial_j J^{ij} = 0$

-> Ampere's law: $\partial_t E^{ij} + \frac{1}{2}(\epsilon^{ik}\partial_k B^j + \epsilon^{jk}\partial_k B^i) = -J^{ij}$ $\quad \partial_t n_d + \partial_i J^i_d = -J^i_{\ i}$

Marchetti, L.R. 1998

M. Pretko,
L. R., 2017
PRLs 2018

Fracton dual "superconductor"

- trace over fractons and dipoles:

$$\mathcal{L} = \frac{1}{2}E_{ij}^2 - \frac{1}{2}B_i^2 - \cos(\partial_t\theta - A_0) + g\cos(\partial_i\partial_j\theta - A_{ij})$$

- Fractons in "normal" Coulomb phase (*crystal*)

- Higgs transition out of fracton phase (*liquid*)

L. R. 2016

Fractons via ***vector*** *gauge theory ?*

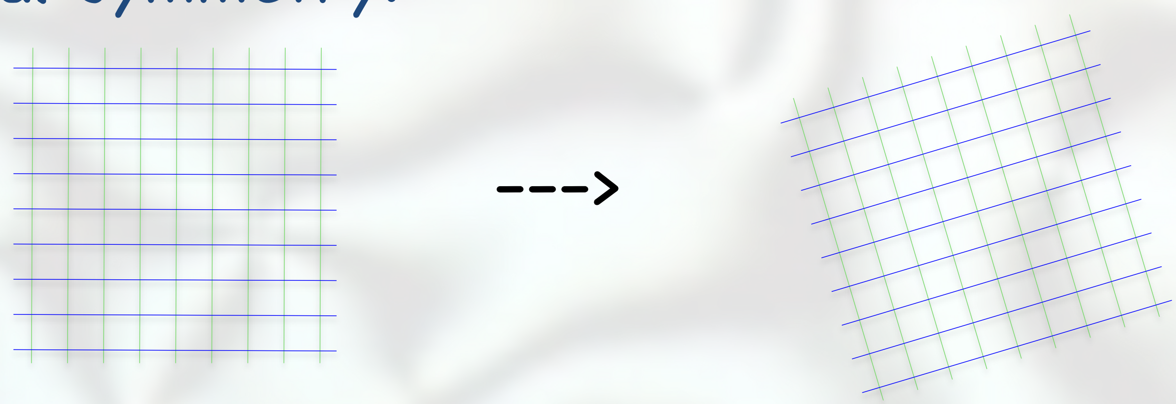
- *Flavored* xy model -> vector gauge duality *(no fractons)*

$$\mathcal{H} = \frac{1}{2}n_k^2 + \frac{1}{2}|\nabla\phi_k|^2 \longrightarrow \tilde{\mathcal{H}} = \frac{1}{2}(\nabla\times\mathbf{A}_k)^2 + \frac{1}{2}|\mathbf{E}_k|^2$$

- Reformulate elasticity into coupled xy models: $u_{ik} \longrightarrow \partial_i u_k$

$$\mathcal{H} = \frac{1}{2}\pi_k^2 + \frac{1}{2}Cu_{ik}^2 \longrightarrow \mathcal{H} = \frac{1}{2}\pi_k^2 + \frac{1}{2}C(\partial_i u_k - g\theta\epsilon_{ik})^2 + \frac{1}{2}L^2 + \frac{1}{2}K(\nabla\theta)^2$$

- Target space rotational symmetry:



-> $u_x = x(\cos\theta - 1) + y\sin\theta$

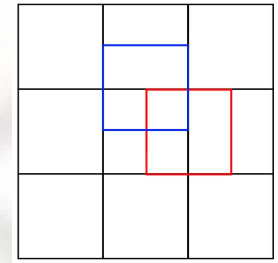
-> $u_y = -x\sin\theta + y(\cos\theta - 1)$

L. R. 2016

Fractons via **vector** gauge theory !

- Reformulate elasticity into flavored coupled xy models:

$$\mathcal{H} = \frac{1}{2}\pi_k^2 + \frac{1}{2}C(\partial_i u_k - g\theta\epsilon_{ik})^2 + \frac{1}{2}L^2 + \frac{1}{2}K(\nabla\theta)^2$$



- Dualize to a coupled vector gauge theory: $(A_a = \epsilon_{ik}A_{ik})$

$$\tilde{\mathcal{H}} = \frac{1}{2}C|\mathbf{E}_k|^2 + \frac{1}{2}(\nabla\times\mathbf{A}_k)^2 + \frac{1}{2}K|\mathbf{e}|^2 + \frac{1}{2}(\nabla\times\mathbf{a} + gA_a)^2 - \mathbf{A}_k\cdot\mathbf{J}_k^b - \mathbf{a}\cdot\mathbf{j}^s$$

- Gauss' law: $\nabla\cdot\mathbf{e} = s \qquad \nabla\cdot\mathbf{E}_k = \tilde{p}_k \qquad (\tilde{p}_k = p_k - e_k)$

- Gauge redundancy: $\mathbf{A}_k \to \mathbf{A}_k + \nabla\chi_k, \quad A_{0k} \to A_{0k} + \partial_t\chi_k$

  $a_k \to a_k + \partial_k\phi - \chi_k, \quad a_0 \to a_0 + \partial_t\phi$

- fractons:

  gauge invariance demands $\partial_t p_k + \nabla\cdot\mathbf{J}_k = j_k \longrightarrow \mathbf{j} = 0$

L. R.,
Hermele
2019

Fractons via **vector** gauge theory

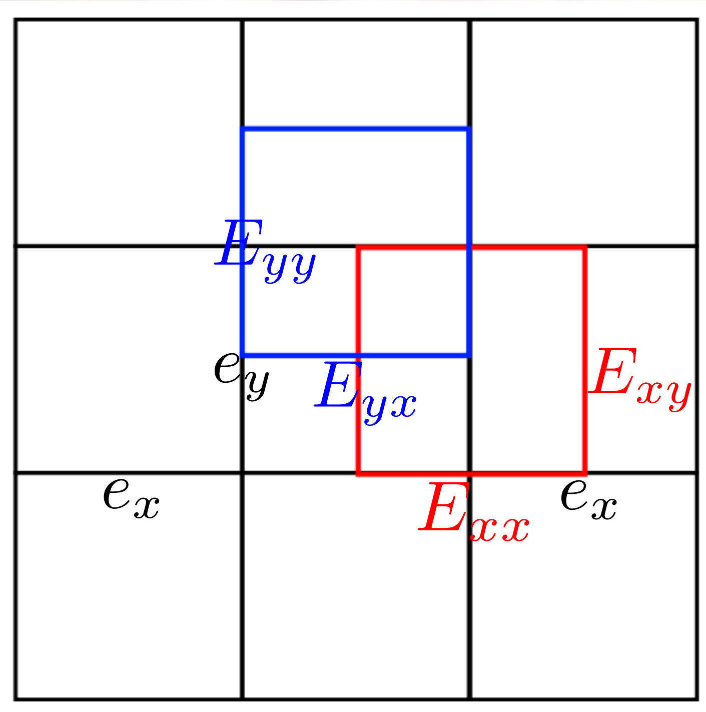
- Lattice fractonic vector gauge theory:

$$[\hat{A}_{ik}, \hat{E}_{jk'}] = -i\delta_{ij}\delta_{kk'}\delta^2(\mathbf{x}-\mathbf{x}'),$$

$$[\hat{a}_i, \hat{e}_j] = -i\delta_{ij}\delta^2(\mathbf{x}-\mathbf{x}')$$

  - Gauss' law:

$$\nabla\cdot\mathbf{e} = s \qquad \nabla\cdot\mathbf{E}_k = e_k$$



$$H = \frac{U_E}{2}\left[\sum_{\ell\in L_x} E_{x\ell}^2 + \sum_{\ell\in L_y} E_{y\ell}^2\right] + \frac{U_e}{2}\sum_{\ell\in L} e_\ell^2$$

$$+\frac{K_E}{2}\left[\sum_{\square_x}(\nabla\times\mathbf{A}_x)^2 + \sum_{\square_y}(\nabla\times\mathbf{A}_y)^2\right] - K_e\sum_{\square}\cos\left[(\nabla\times\mathbf{a})_\square + A_{xy} - A_{yx}\right]$$

L. R. 2019

Vector-tensor gauge theory equivalence

- Dualize to a coupled vector gauge theory: $(A_a = \epsilon_{ik} A_{ik})$

$$\tilde{\mathcal{H}} = \frac{1}{2} C |\mathbf{E}_k|^2 + \frac{1}{2} (\nabla \times \mathbf{A}_k)^2 + \frac{1}{2} K |\mathbf{e}|^2 + \frac{1}{2} (\nabla \times \mathbf{a} + g A_a)^2 - \mathbf{A}_k \cdot \mathbf{J}_k^b - \mathbf{a} \cdot \mathbf{j}^s$$

- Gauss' law: $\nabla \cdot \mathbf{e} = s \qquad \nabla \cdot \mathbf{E}_k = \tilde{p}_k \qquad (\tilde{p}_k = p_k - e_k)$

- Gauge redundancy:

$$\mathbf{A}_k \to \mathbf{A}_k + \nabla \chi_k, \quad A_{0k} \to A_{0k} + \partial_t \chi_k$$
$$a_k \to a_k + \partial_k \phi - \chi_k, \quad a_0 \to a_0 + \partial_t \phi$$

- *Vector* -> *tensor* gauge theory:

  * gauge away a_k: choose $\chi_k = a_k$ --> gaps out A_a --> symmetrizes A_{ik}

  * generalized Gauss' law: $\nabla \cdot \partial_k \mathbf{E}_k = \rho \qquad (\rho \equiv s + \nabla \cdot \mathbf{p})$

  * recover tensor gauge theory

M. Pretko, L. R., PRL2018

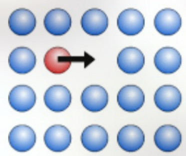
2D *Bosonic crystal*

Kumar Potter, 2018

- Coupled elasticity and bosonic vacancies/interstitials:

$$\hat{\mathcal{H}} = \frac{1}{2}\rho^{-1}\hat{\pi}^2 + \frac{1}{2}C^{ijk\ell}\hat{u}_{ij}\hat{u}_{k\ell} + \frac{1}{2}K(\nabla\hat{\phi})^2 + \frac{1}{2}\chi^{-1}\hat{n}^2 + g_1\nabla\hat{\phi}\cdot\hat{\pi} + g_2\hat{n}\hat{u}_{ii}$$

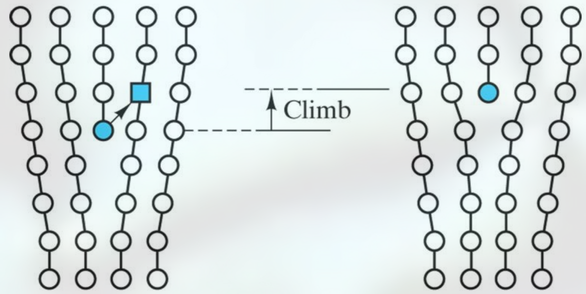
-> *Commensurate* (Mott-insulating) crystal



-> *Incommensurate* (supersolid) crystal

$$n_d = -\chi\partial_t\varphi = n + g_2\nabla\cdot\mathbf{u}, \qquad \mathbf{j}_d = K\nabla\varphi = \mathbf{j} - g_1\partial_t\mathbf{u},$$

$$\partial_t n_d + \nabla\cdot\mathbf{j}_d = g_2\partial_t\nabla\cdot\mathbf{u} - g_1\nabla\cdot\partial_t\mathbf{u} \equiv J_s,$$



$$J_s^{ij} = g\epsilon^{ik}\epsilon^{j\ell}(\partial_t\partial_k - \partial_k\partial_t)u_\ell = g\epsilon_{ik}v_k b_j$$

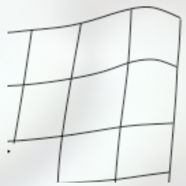
Marchetti, L.R. 1998

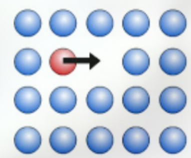
M. Pretko,
L. R., PRL2018

2D *Bosonic crystal*

- Coupled elasticity and bosonic vacancies/interstitials:

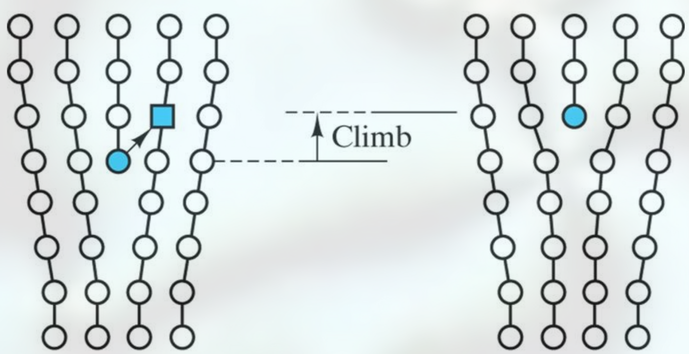
$$\hat{\mathcal{H}} = \underbrace{\frac{1}{2}\hat{\pi}^2 + \frac{1}{2}\hat{u}_{ij}^2}_{\text{elasticity}} + \underbrace{\frac{1}{2}(\nabla\hat{\phi})^2 + \frac{1}{2}\hat{n}^2}_{\text{vacancies/interstitials}} + \underbrace{\nabla\hat{\phi}\cdot\hat{\pi} + \hat{n}\hat{u}_{ii}}_{\text{coupling}}$$





-> *Commensurate* (Mott-insulating) crystal

-> *Incommensurate* (supersolid) crystal



$$\partial_t n_d + \partial_i J^i_d = -J^i_{\ i}$$

(→ Ampere's law)

Marchetti, L.R. 1998

M. Pretko,
L. R., PRL2018

2D *Symmetry-enriched fractons*

Kumar
Potter, 2018

- Hybrid U(1) vector-tensor gauge duality

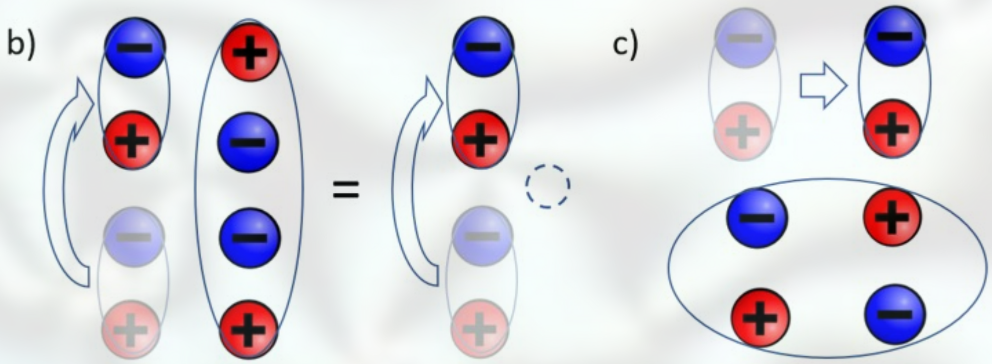
$$\mathcal{H} = \underbrace{\frac{1}{2}(E_{ij}^2 + B_i^2)}_{\text{elasticity}} + \underbrace{\frac{1}{2}(\mathbf{e}^2 + b^2)}_{\text{bosons}} + \underbrace{g(\mathbf{B}\cdot\mathbf{e} + E_{ii}b)}_{\text{“axion” coupling}} + \underbrace{J^{\mu\nu}A_{\mu\nu} + j^{\mu}a_{\mu}}_{\text{charges}}$$

-> supersolid -> “fracton superfluid” (mobile dipoles) F

-> normal crystal -> “fracton Mott insulator” (confined dipoles) $F_{U(1)}$

$\hat{a}_{\mathbf{r}}\hat{b}^{\dagger}_{x,\mathbf{r}+\hat{\mathbf{y}}}\hat{b}_{x,\mathbf{r}}e^{iA_{yy}}$

$E_{yy} \neq 0$



$\hat{b}^{\dagger}_{x,\mathbf{r}+\hat{\mathbf{x}}}\hat{b}_{x,\mathbf{r}}e^{iA_{xy}}$

$E_{xy} \neq 0$

- Vortex condensation: F --> $F_{U(1)}$
  - *-> fracton dipole dimensional confinement*
  - *-> superfluid to Mott-insulating fracton transition*

cf (0,1), “hollow GT”
Ma, et al; Bulmash, et al.
magnetic monopoles

M. Pretko, L. R.,2018

Fracton superfluid: supersolid

2D

- Hybrid U(1) vector-tensor gauge duality

$$\mathcal{H} = \frac{1}{2}\tilde{C}^{ijk\ell}E_{ij}E_{k\ell} + \frac{1}{2}B^iB_i + \frac{1}{2}K\mathbf{e}\cdot\mathbf{e} + \frac{1}{2}b^2$$
$$+g(\mathbf{B}\cdot\mathbf{e} + E_{ii}b) + J^{\mu\nu}A_{\mu\nu} + j^\mu a_\mu$$

with "mutual axion" electrodynamics

-> supersolid -> fracton superfluid (mobile dipoles)

-> normal crystal -> fracton Mott insulator (confined dipoles)

- Vortex condensation

-> fracton dipole dimensional confinement

-> superfluid to Mott-insulating fracton transition

- Fracton dipole condensation

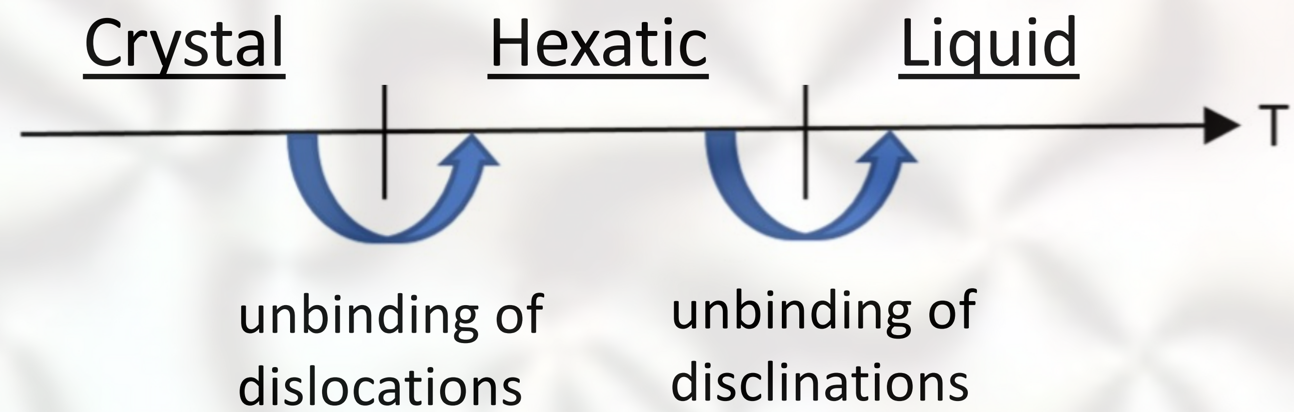
-> superhexatic

Berezinskii
Kosterlitz, Thouless
Halperin, Nelson
Young

Fracton condensation transition

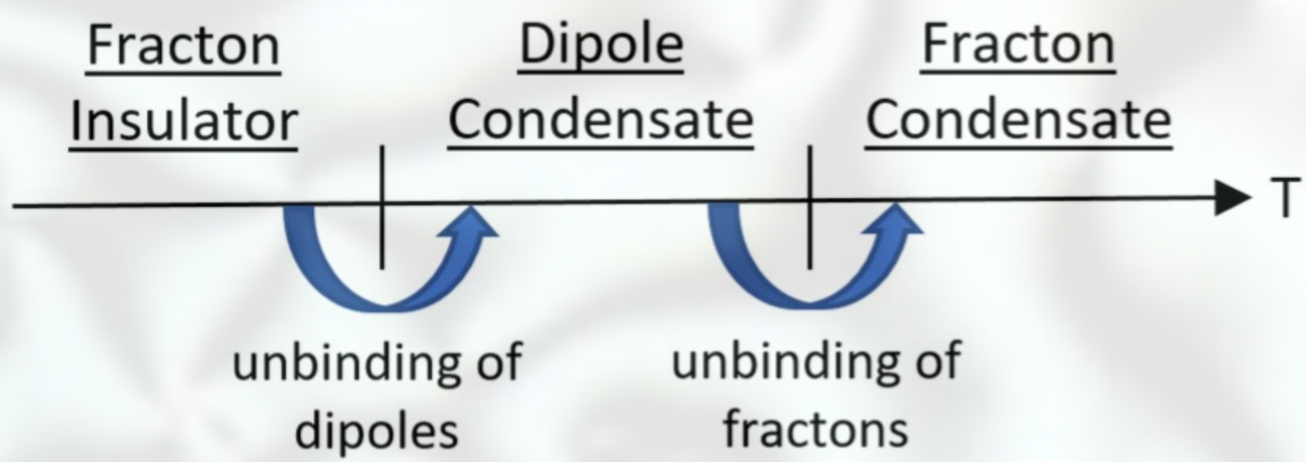
2D

2D crystal:



Z. Zhai, L. R., 2019

2D scalar fracton model:



$$\tilde{\mathcal{H}} = \frac{1}{2}B^{-1}(\nabla^2\phi)^2 - g_s \cos\left(\frac{2\pi}{6}\phi\right) - g_b \sum_{n=1,2,3} \cos\left(\mathbf{b}_n \cdot \hat{z} \times \nabla\phi\right)$$

L. R., 2016

L. R., 2018

Fracton "sliding phase"

Incompressible crystal -> "fracton Mott insulator"

$F_{U(1)}$

$$\hat{H} = \sum_{\mathbf{r}} \left[-t_x \hat{b}^\dagger_{x,\mathbf{r}+\hat{\mathbf{x}}} \hat{b}_{x,\mathbf{r}} e^{iA_{xy}} - t_y \hat{b}^\dagger_{y,\mathbf{r}+\hat{\mathbf{y}}} \hat{b}_{y,\mathbf{r}} e^{iA_{xy}} + \frac{1}{2} B_i^2 + \frac{1}{2} C_{ij} E_{ij}^2 \right]$$

- dispersionless along lines
- stability to interactions?
- $D_{climb} \sim e^{-\Delta/T} << D_{glide}$