Optical control of topological memory and induction of direct current

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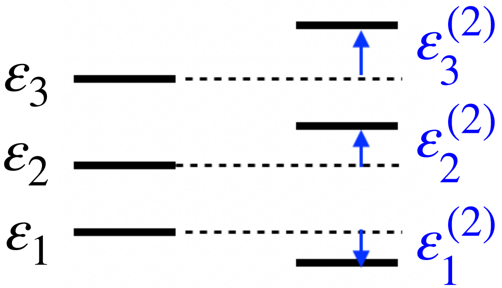
- Energy shift in atoms and solids in an oscillating electric field (the dynamical Stark shift)
- Optical control of the sign of magnetization in Chern insulators by circularly polarized light, PRB **105**, 064423 (2022)
- Direct current of bosonic atoms induced by circular stirring of an optical lattice, arXiv:2205.15981 (2022)

Dynamical Stark shift in atoms and solids

Energy levels of an atom



An oscillating electric field $\mathbf{E}(t) = \frac{1}{2}\left[\mathbf{E}(\omega)\, e^{-i\omega t} + \mathbf{E}^*(\omega)\, e^{i\omega t}\right]$,

$H' = -\, e\, \mathbf{r} \cdot \mathbf{E}(t)$, induces a dipole $d_n^{\alpha}(\omega) = \langle e r^{\alpha} \rangle_n = \chi_n^{\alpha\beta}(\omega)\, E_\beta(\omega)$

and the Stark shift of the n-th energy level to the 2nd order

$$\varepsilon_n^{(2)} = -\frac{1}{4} \chi_n^{\alpha\beta}(\omega)\, E_\alpha^*(\omega)\, E_\beta(\omega)$$

The electric polarizability tensor $\chi^{\alpha\beta}(\omega) = \chi_s^{\alpha\beta}(\omega) + \chi_a^{\alpha\beta}(\omega)$ (symmetric + antisymmetric)

In a crystal, Bloch wavefunctions $\psi_{n,\mathbf{k}} = e^{i\mathbf{kr}}\, u_{n,\mathbf{k}}(\mathbf{r})$ have energies $\varepsilon_n(\mathbf{k})$, denote $\varepsilon_{nm}(\mathbf{k}) = \varepsilon_n(\mathbf{k}) - \varepsilon_m(\mathbf{k})$

Shift of the energy bands in an oscillating electric field (Pershoguba-Yakovenko):

$$\varepsilon_n^{(2)}(\mathbf{k}) = \frac{e^2}{4\,(\hbar\omega)^2} \frac{\partial^2 \varepsilon_n(\mathbf{k})}{\partial k_\alpha \partial k_\beta} \operatorname{Re}\left[E_\alpha^*(\omega) E_\beta(\omega)\right] - \frac{e^2 \Omega_{n,\gamma}(\mathbf{k})}{4\,\hbar\omega} \epsilon^{\alpha\beta\gamma} \operatorname{Im}\left[E_\alpha^*(\omega) E_\beta(\omega)\right]$$

intraband symmetric — intraband antisymmetric

$$-\frac{e^2}{4} \operatorname{Re} \sum_{m \neq n} \frac{r_{nm}^{\alpha}(\mathbf{k}) r_{mn}^{\beta}(\mathbf{k})}{\varepsilon_{mn}(\mathbf{k}) - \hbar\omega} E_\alpha^*(\omega) E_\beta(\omega) - \frac{e^2}{4} \operatorname{Re} \sum_{m \neq n} \frac{\left[r_{nm}^{\alpha}(\mathbf{k}) r_{mn}^{\beta}(\mathbf{k})\right]^*}{\varepsilon_{mn}(\mathbf{k}) + \hbar\omega} E_\alpha^*(\omega) E_\beta(\omega)$$

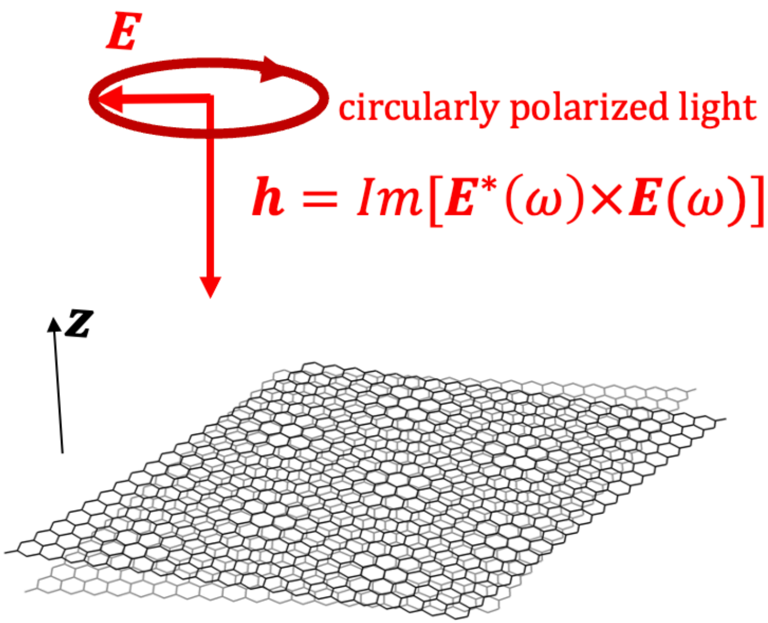
interband Stark shift — interband Bloch-Siegert shift

Berry connection $\mathbf{r}_{nm}(\mathbf{k}) = \langle u_{n,\mathbf{k}} \,|\, i \frac{\partial}{\partial \mathbf{k}} \,|\, u_{n,\mathbf{k}} \rangle$ Berry curvature $\boldsymbol{\Omega}_n(\mathbf{k}) = \frac{\partial}{\partial \mathbf{k}} \times \mathbf{r}_{nn}(\mathbf{k})$

Symmetric and antisymmetric, time-reversal even and odd terms

Equation	Permutation of indices	Time reversal
$\mathrm{Re}\left[E^*_\alpha(\omega)E_\beta(\omega)\right] = \mathrm{Re}\left[E^*_\beta(\omega)E_\alpha(\omega)\right]$	symmetric	even
$\mathrm{Im}\left[E^*_\alpha(\omega)E_\beta(\omega)\right] = \frac{1}{2}\epsilon_{\alpha\beta\gamma}h^\gamma$	antisymmetric	odd

The helicity $\mathbf{h}$ of the incident light represents its angular momentum



Renormalized energy:

$$\tilde{\varepsilon}_n(\mathbf{k}) = \varepsilon_n(\mathbf{k}) + \varepsilon_n^{(2)}(\mathbf{k})$$

bare — second-order correction

Energy shift splits in two terms

$$\varepsilon_n^{(2)}(\mathbf{k}) = \varepsilon_n^{(s)}(\mathbf{k}) + \varepsilon_n^{(a)}(\mathbf{k})$$

symmetric — antisymmetric

The symmetric energy shift

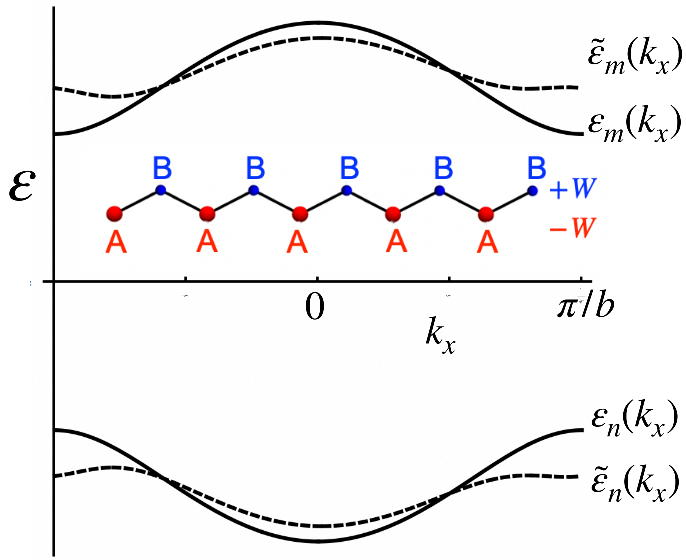
Newton's law for the electrons $\dfrac{d\tilde{\mathbf{k}}(t)}{dt} = \dfrac{e}{\hbar}\mathbf{E}(t)$

The momentum $\tilde{\mathbf{k}}(t) = \mathbf{k} + \delta\mathbf{k}(t)$ oscillates around its average value $\mathbf{k} = \langle \tilde{\mathbf{k}}(t) \rangle_t$

Expanding in small $\delta k \sim eE(\omega)/\omega$ and averaging over time

$$\langle \varepsilon_n[\mathbf{k} + \delta\mathbf{k}(t)] \rangle_t \approx \left\langle \varepsilon_n(\mathbf{k}) + \delta k_\alpha(t) \frac{\partial \varepsilon_n(\mathbf{k})}{\partial k_\alpha} + \frac{1}{2}\delta k_\alpha(t)\delta k_\beta(t) \frac{\partial^2 \varepsilon_n(\mathbf{k})}{\partial k_\alpha \partial k_\beta} \right\rangle_t = \varepsilon_n(\mathbf{k}) + e^2 \mathrm{Re}[E_\alpha^*(\omega) E_\beta(\omega)] \frac{1}{4(\hbar\omega)^2} \frac{\partial^2 \varepsilon_n(\mathbf{k})}{\partial k_\alpha \partial k_\beta}$$

The bare $\varepsilon(k_x)$ and renormalized $\tilde{\varepsilon}(k_x)$ energy dispersions for a two-band model



The full symmetric term for a two-band model

$$\varepsilon_n^{(s)}(\mathbf{k}) = e^2 \mathrm{Re}\left[E_\alpha^*(\omega) E_\beta(\omega)\right] \left\{ \frac{1}{4(\hbar\omega)^2} \frac{\partial^2 \varepsilon_n(\mathbf{k})}{\partial k_\alpha \partial k_\beta} - \frac{\varepsilon_{mn}(\mathbf{k})\,\mathrm{Re}\left[\mathbf{r}_{\mathbf{nm}}^\alpha(\mathbf{k}) \mathbf{r}_{\mathbf{mn}}^\beta(\mathbf{k})\right]}{2\left[\varepsilon_{mn}^2(\mathbf{k}) - (\hbar\omega)^2\right]} \right\}$$

intraband

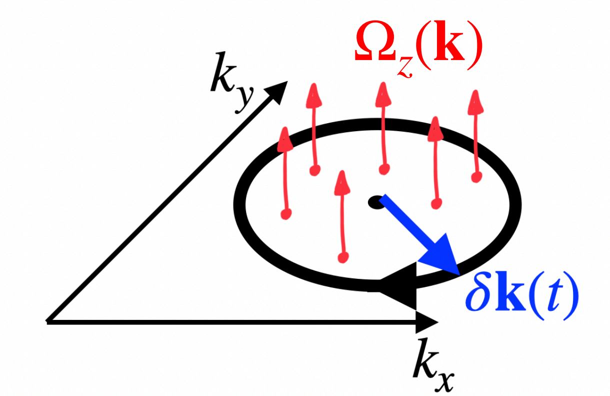
interband

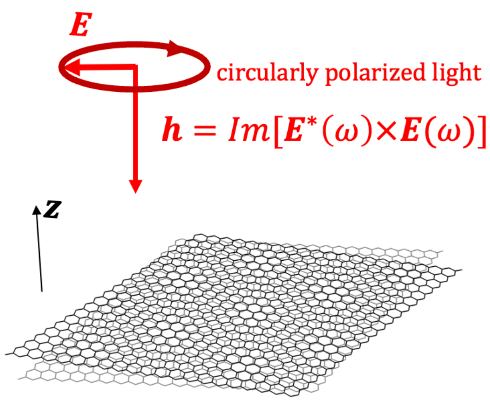
Oscillating electric field leads to flattening of energy spectrum.

Flatness controls emergence of strongly-correlated phases in Moire materials.

The antisymmetric energy shift in the presence of circularly polarized light

Circularly polarized light causes the momentum $\tilde{\mathbf{k}}(t) = \mathbf{k} + \delta\mathbf{k}(t)$ to move on a circular orbit with $|\delta\mathbf{k}| = eE/\hbar\omega$





The full antisymmetric energy shift for a two-band model

$$\varepsilon_n^{(a)}(\mathbf{k}) = -\frac{\varepsilon_{mn}^2(\mathbf{k})}{\varepsilon_{mn}^2(\mathbf{k}) - (\hbar\omega)^2}\frac{e^2\,\mathbf{h}\cdot\mathbf{\Omega}_n(\mathbf{k})}{4\,\hbar\omega}$$

The Berry phase accumulation over one loop per time period T

$$\phi_{loop} \approx \Omega_{n,z}(\mathbf{k})\,\pi\,(\delta k)^2 = \Omega_{n,z}(\mathbf{k})\,\pi\left(\frac{eE}{\hbar\omega}\right)^2$$

results in the energy shift

$$\varepsilon_n^{(a)}(\mathbf{k}) = \frac{\hbar\,\phi_{\rm loop}}{T} = \frac{(eE)^2\,\Omega_{n,z}(\mathbf{k})}{2\,\hbar\omega}$$

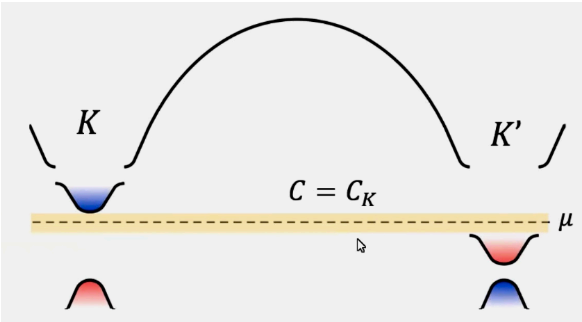
The total energy shift of the system of area $\mathscr{A}$ depends on the helicity ***h*** of light

$$U^{(a)} = -\frac{1}{4}\mathbf{h}\cdot\mathscr{M}(\omega), \quad \mathscr{M}_\gamma(\omega) = -\frac{1}{2}\epsilon_{\alpha\beta\gamma}\mathrm{Im}[\chi^{\alpha\beta}(\omega)]$$

$$\mathscr{M}(\omega) = \mathscr{A}e^2\int\frac{d^2k}{(2\pi)^2}\frac{\varepsilon_{mn}^2(\mathbf{k})}{\varepsilon_{mn}^2(\mathbf{k}) - (\hbar\omega)^2}\frac{\mathbf{\Omega}_n(\mathbf{k})}{\hbar\omega}$$

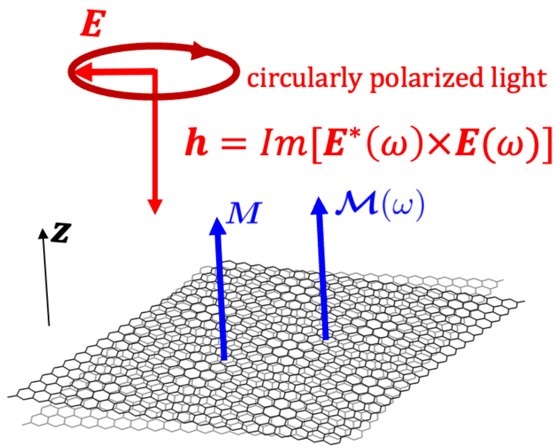
Optical control of orbital magnetism in Chern insulators

Spontaneous valley polarization induced by electron interactions

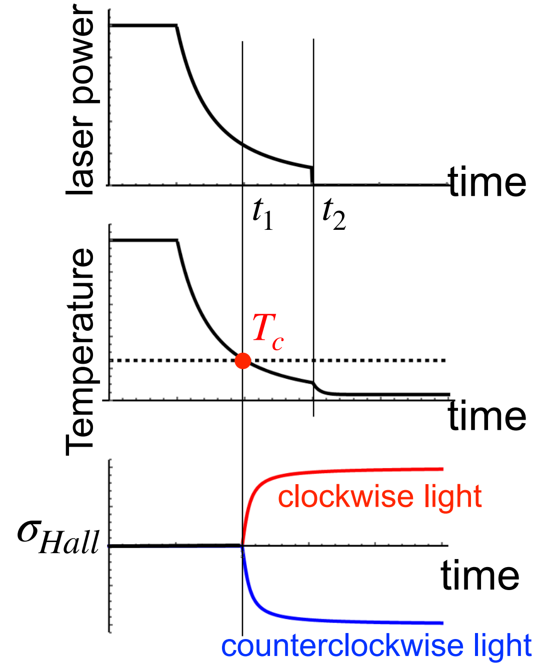


Graphene "Chernburgers", Song et al., PNAS **112**, 35 (2015)

Circularly polarized light incident on Moire superlattice



Cool through T_c in the presence of circular light



For a two-band model with occupied band (n) and empty band (m):

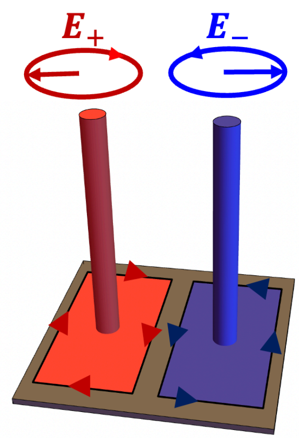
Quantum Hall conductivity of a Chern insulator $\sigma_H = -\frac{e^2}{\hbar}\int \frac{d^2k}{(2\pi)^2}\Omega_n(\mathbf{k})$

Orbital magnetization of a Chern insulator $\mathbf{M} = -\frac{e}{2\hbar}\int \frac{d^2k}{(2\pi)^2}[\varepsilon_m(\mathbf{k}) + \varepsilon_n(\mathbf{k}) - 2\varepsilon_F]\,\boldsymbol{\Omega}_n(\mathbf{k})$

The energy shift of a Chern insulator depends on the helicity ***h*** of light

$$U^{(a)} = -\frac{1}{4}\mathbf{h}\cdot\mathscr{M}(\omega), \quad \mathscr{M}(\omega) = \mathscr{A}e^2\int \frac{d^2k}{(2\pi)^2}\frac{\varepsilon_{mn}^2(\mathbf{k})}{\varepsilon_{mn}^2(\mathbf{k}) - (\hbar\omega)^2}\frac{\boldsymbol{\Omega}_n(\mathbf{k})}{\hbar\omega}$$

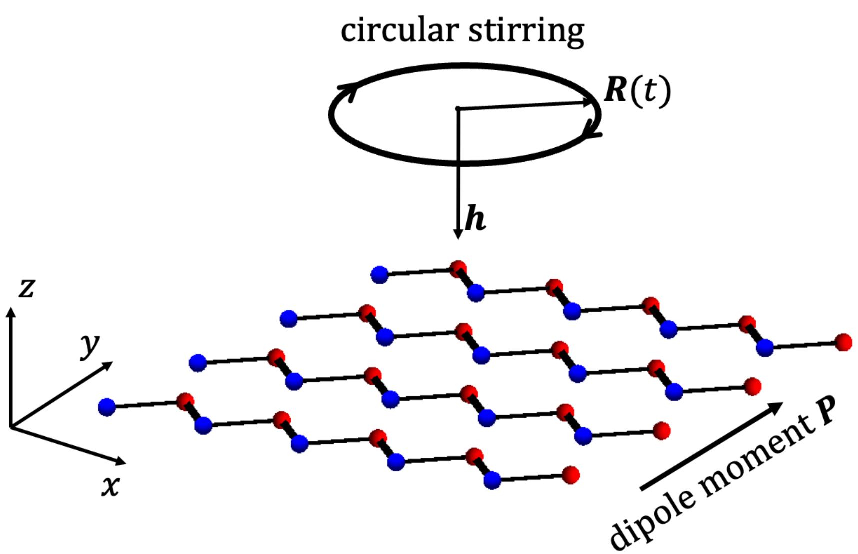
The coupling to the helicity *h* is linear in *M* and dominates over the Landau energy $U \sim (T - T_c)\,M^2 + M^4$ near T_c



Optical writing of topological domains

Circular stirring of an optical lattice

Stirred optical lattice realized experimentally by Jotzu, Esslinger et al., Nature **515**, 237 (2014)



The optical lattice potential $U[\mathbf{r}-\mathbf{R}(t)]$ is shifted by the displacement vector $\mathbf{R}(t)$

Schrodinger equation in the noninertial frame of the lattice

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t}=\left[\frac{\mathbf{p}^2}{2M}+U(\mathbf{r})-\mathbf{F}(t)\cdot\mathbf{r}\right]\psi(\mathbf{r},t)$$

Force of inertia plays the role of an electric field for neutral atoms

$$\mathbf{F}(t)=-M\frac{d^2\mathbf{R}}{dt^2}$$

For the periodical force

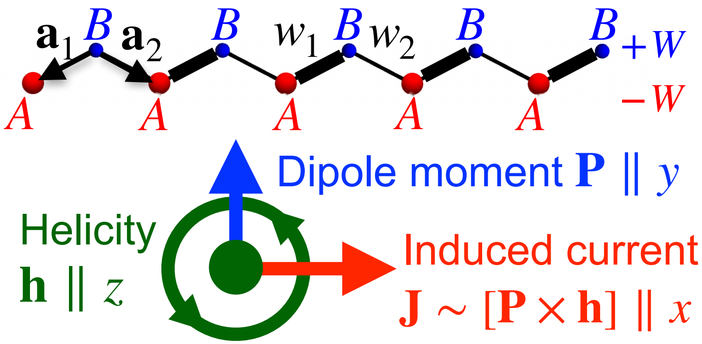
$$\mathbf{F}(t)=\frac{1}{2}[\mathbf{F}(\omega)\,e^{-i\omega t}+\mathbf{F}^*(\omega)\,e^{-i\omega t}]$$

the stirring helicity is

$$\mathbf{h}=\mathrm{Im}[\mathbf{F}^*(\omega)\times\mathbf{F}(\omega)]$$

Bosons in a 1D optical lattice

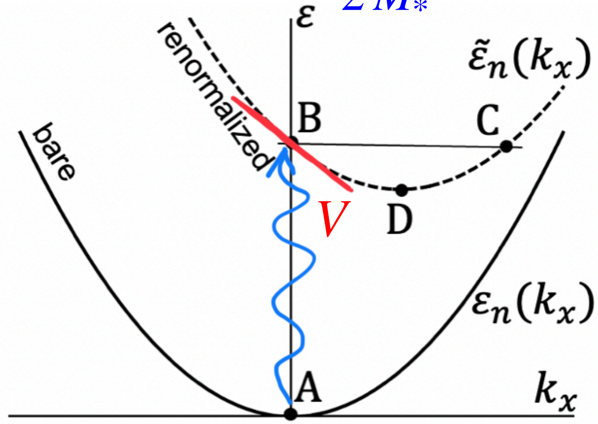
Dimerized 1D chain (Rice-Mele model)



Circular Photogalvanic Effect

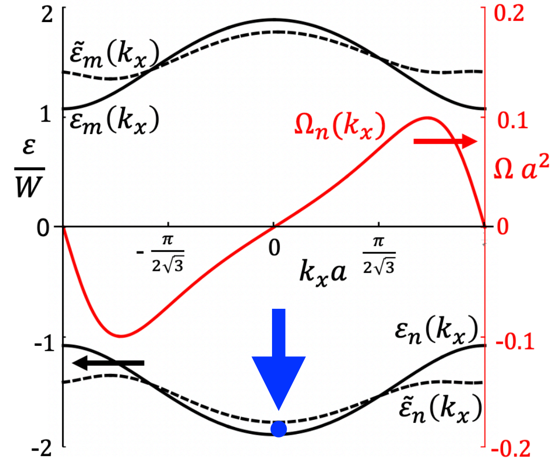
Energy dispersion near $\mathbf{k} = 0$

$$\tilde{\varepsilon}_n(k_x) \approx \tilde{\varepsilon}_n(0) + \frac{\hbar^2 k_x^2}{2M_*} + \hbar\, \mathbf{k} \cdot \mathbf{V}$$



Due to stirring, bosons are promoted from A to B.
In a finite system, they bounce between B and C.
Eventually, they relax to the lowest energy state D.

Bare and renormalized energy dispersion



upper band
$\varepsilon_m(\mathbf{k}) = |\mathbf{w}(\mathbf{k})|$

Berry curvature

lower band
$\varepsilon_n(\mathbf{k}) = -|\mathbf{w}(\mathbf{k})|$

Bosons occupy the lowest-energy state **k**=0

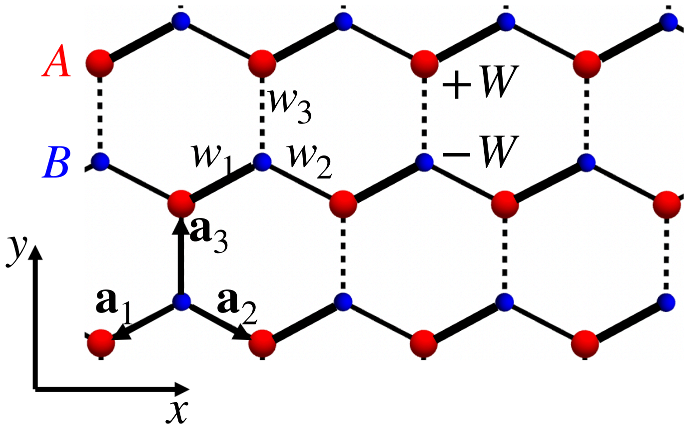
The renormalized energy dispersion has a nonzero group velocity at $\mathbf{k} = \mathbf{0}$:

$$\mathbf{V} = -\frac{\varepsilon_{mn}^2(0)}{\varepsilon_{mn}^2(0) - (\hbar\omega)^2} \frac{1}{4\hbar\omega} \frac{\partial[\mathbf{h} \cdot \mathbf{\Omega}_n(\mathbf{k})]}{\hbar\, \partial\mathbf{k}}\bigg|_{\mathbf{k}=0}$$

$$= \frac{w_1 w_2 W\, \mathbf{h} \cdot [\mathbf{a_1} \times \mathbf{a_2}]}{2\hbar^2\omega\, |\mathbf{w}(0)| \left[4|\mathbf{w}(0)|^2 - (\hbar\omega)^2\right]} (\mathbf{a_1} - \mathbf{a_2})$$

The induced current is transient and stops when bosons relax to the minimum of renormalized dispersion.

Bosons in anisotropic honeycomb lattice



The renormalized dispersion gives a nonzero group velocity at $\mathbf{k} = \mathbf{0}$

$$\mathbf{V} = C\,[w_1 w_2\,(\mathbf{a_1} - \mathbf{a_2}) + w_2 w_3\,(\mathbf{a_2} - \mathbf{a_3}) + w_1 w_3\,(\mathbf{a_3} - \mathbf{a_1})]$$

It is the sum of currents alongs three zigzag chains

$$C = \frac{W\,\mathbf{h}\cdot[\mathbf{a_1}\times\mathbf{a_2}]}{2\,\hbar^2\omega\,|\,\mathbf{w(0)}\,|\,\left[4\,|\,\mathbf{w}(0)\,|^2 - (\hbar\omega)^2\right]}$$

2×2 Hamiltonian $H(\mathbf{k}) = \sigma\cdot\mathbf{w}(\mathbf{k})$

$$\mathbf{w}(\mathbf{k}) = \left[\sum_{j=1}^{2} w_j \cos(\mathbf{k}\cdot\mathbf{a_j}),\ \sum_{j=1}^{2} w_j \sin(\mathbf{k}\cdot\mathbf{a_j}),\ W\right]$$

hoppings w_1 w_2 w_3 — site energy

- By tuning the hopping amplitudes, the current can be pointed in arbitrary direction.
- The current can be used to transport neutral atoms over a fixed distance by stirring for a fixed time.

Conclusions

PRB **105**, 064423 (2022) and arXiv:2205.15981 (2022)

- We derived the Stark energy shift for solids, where the wave functions are delocalized with the momentum $\mathbf{k}(t)$.
- The new intraband contributions result in (i) flattening of energy spectrum (ii) coupling to the helicity of light.
- Topological memory based on orbital magnetization in Chern insulators can be controlled by circularly polarized light.
- Stirring of an optical lattice filled with bosons induces a transient current, which can be used to transport neutral atoms.
- The effect is similar to circular photogalvanic effect in solids.
- The transient current was discussed by Belinicher et al. (1986) in the absence of resonant absorption, but different conclusions are offered in modern literature.
- An experiment in an optical lattice can help to resolve the dispute.