REVISION OF THE TIME DEPENDENT GINZBURG-LANDAU APPROACH TO EVOLUTION OF INHOMOGENEOUS STATES IN SLIDING CHARGE DENSITY WAVES.

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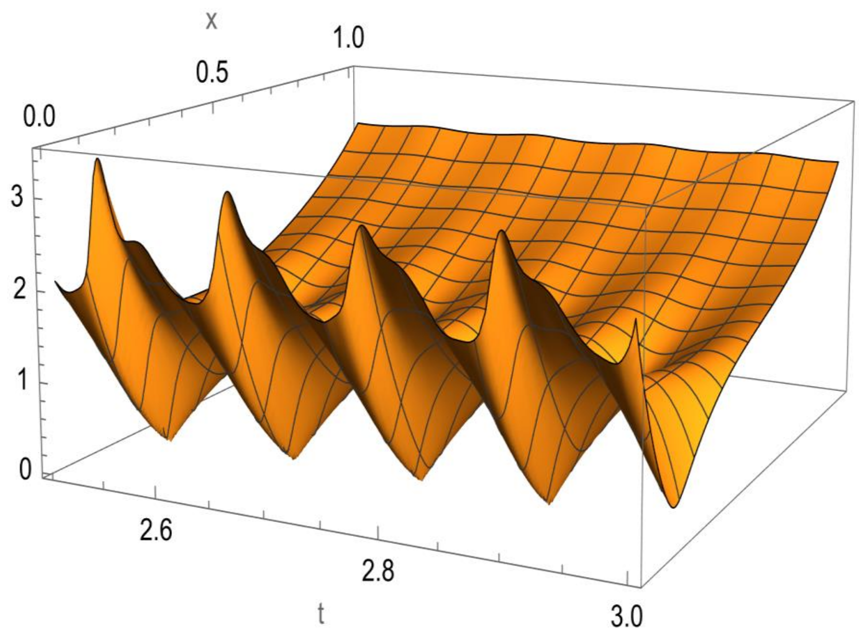
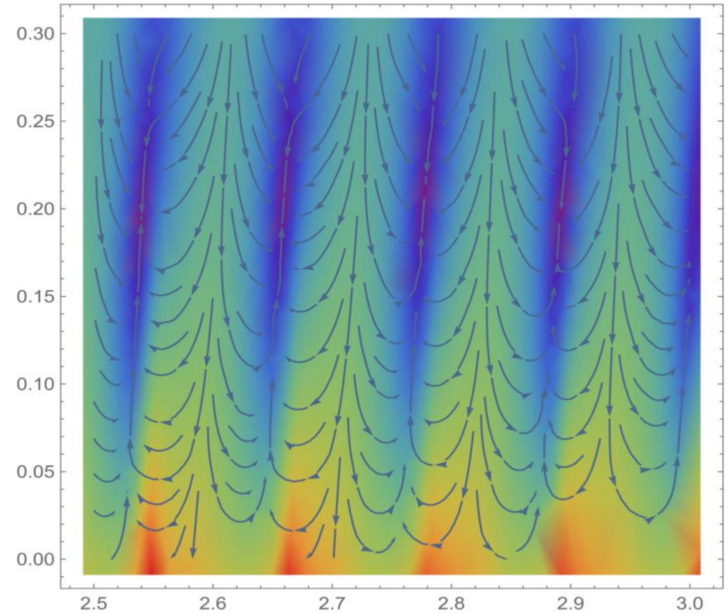




Symmetry breaking and multiple fluids

Thermodynamics of classical phase transitions $\boldsymbol{F(\eta)}$
Landau-Ginzburg free energy functional for the order parameter

Kinetics of phase transitions in quantum systems:
Bose condensates, Superconductors, CDW, etc
for the complex order parameter $\boldsymbol{\eta}$

$$\frac{\partial \boldsymbol{\eta}}{\partial \boldsymbol{t}} = -\frac{\delta \boldsymbol{F}}{\delta \boldsymbol{\eta}}$$

From microscopics (Green Functions, Gorkov and Keldysh technics, etc)

to Gross-Pitaevskii for bosons
or TDGL (time dependent Ginzburg Landau) for fermions
phenomenology for $\boldsymbol{\eta}$ alone – the follow-up carriers are integrated.

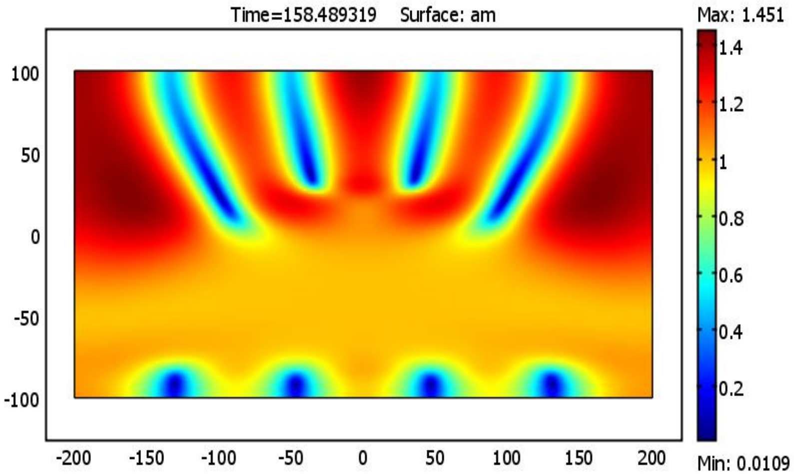
Collective effects related to the phase degeneracy

- Frohlich conduction by the collective sliding.
- Topological defects: solitons, dislocations (electronic vortices).
- Phase slips = instantons = spacio-temporary vortices
- Conversion among normal and condensed electrons.
  (to release the fracton fate from the vortex)

While the phase velocity or its deformation is the principle ingredient, no collective current can be set in without deformations of the CDW amplitude A(x,t), particularly with A passing though zero within the vortex core or the phase-slip event.

Rectangle.avi

Numerical modeling of nonstationary processes in CDWs within the TDGL phenomenology (+ electric field)
T. Yi, A. Rojo-Bravo, N. Kirova and SB.



traces of A=0

nucleations

Many vortices appear temporarily in the course of the evolution.
For that run, only one will be left.

The result is as spectacular as it is an ambiguity !
The TDGL approach is principally deficient here.

CDW=Acos(2K_Fx+ ϕ) → complex order parameters $\boldsymbol{\eta}$ =**Ψ= A exp[iφ]**

Degeneracy in the phase **φ** hence static vortices **=** dislocations, phase slips **=** instantons = **(x,t**) vortices

Spinons as amplitude solitons (kinks in **A**)

At the nominal amplitude **A=1**:

the collective density $\mathbf{n_c=\partial_x\varphi\,/\pi}$ - CDW dilatation

the collective current $\mathbf{j_c=-\,\partial_t\varphi\,/\pi}$

The charge conservation law is satisfied automatically !

$$\frac{\partial}{\partial x}\mathbf{n_c}+\frac{\partial}{\partial t}\mathbf{j_c}=\frac{\partial}{\partial x}\partial_t\varphi+\frac{\partial}{\partial t}(-\partial_x\varphi)\equiv 0$$

Ginzburg-Landau – like model

$$\psi = A\exp(i\varphi)$$

$$H_{CDW} = \int d^3r\left\{\left[\left|\frac{\partial\psi}{\partial x}\right|^2 + \alpha\left|\frac{\partial\psi}{\partial y}\right|^2\right] + |\psi|^2 \ln\frac{|\psi|^2}{e}\right\}$$

$$H_{\mathrm{int}} = \int d^3r\left[\Phi A^2\partial_x\varphi/\pi + \Phi n(\varsigma) + F(n) - |\nabla\Phi|^2\varepsilon/8\pi\right]$$

Φ - electric potential,

n=n

Only extrinsic (external to the CDW, from other bands) carriers *n_{ex}* are taken explicitly. In the GL spirit, the intrinsic carriers (in the gap region) are integrated out, their effect is hidden in the CDW amplitude A=|Ψ| and then parameters

Hidden problems with the TDGL model for CDW

Well established and works for stationary state and as a tool to reach it.
Takes explicitly the extrinsic carriers (not interacting with the CDW)

Restrictions:

The intrinsic carriers have been integrated out and come into the model only via the order parameter amplitude $\boldsymbol{A}$ and repated parameters.

Major problem:

Violation of the local charge conservation for the condensate .

automatically if A = const

$$n_c = \frac{A^2}{\pi}\frac{\partial\varphi}{\partial x} \qquad j_c = -\frac{A^2}{\pi}\frac{\partial\varphi}{\partial t} \quad \Longrightarrow \quad \frac{dn}{dt} = \frac{\partial n_c}{\partial t} + \frac{\partial j_c}{\partial x} = 0$$

In our case
$\boldsymbol{A(x,y,t)\neq \mathrm{cnst}}$

$$\pi\frac{dn}{dt} = \frac{\partial A^2}{\partial x}\frac{\partial\varphi}{\partial t} - \frac{\partial\varphi}{\partial x}\frac{\partial A^2}{\partial t} \neq 0$$

WRONG

Way of resolution: keep normal carriers in hand and decompose ***n,j***

More general scheme :

$\psi=(\psi_+,\psi_-)$ - electronic wave function components near **$\pm p_F$** decomposed in right and left moving fermions

$\Delta e^{i\varphi}$ – order parameter

Φ and ***A_x*** - scalar and vector potential,

v_F – Fermi velocity

$$H_{el} = \begin{pmatrix} i\dfrac{\partial}{\partial t} - i\hbar v_F \dfrac{\partial}{\partial x} + e\Phi + \dfrac{e}{c}A_x & \Delta e^{i\varphi} \\ \Delta e^{-i\varphi} & i\dfrac{\partial}{\partial t} + i\hbar v_F \dfrac{\partial}{\partial x} + e\Phi - \dfrac{e}{c}A_x \end{pmatrix}$$

Not convenient: the gap **Δ** is loaded with the essentially variable **x,t** dependent factor exp(±i **φ**)

Chiral transformation: $\psi_{\pm} \longrightarrow \psi_{\pm} e^{\pm i\phi/2}$

actually puts the electrons to the breathing frame of shifted Fermi momentum and Fermi energy: $\boldsymbol{\delta P_F = \partial_x \phi/2 \rightarrow \delta E_F = \hbar v_F/2\partial_x \phi}$

The gap **Δ** is unloaded from the phase factor,
we arrive at a semiconductor model,
but in expense of elongating the applied potentials:

$$\boxed{e\Phi \rightarrow e\Phi + \hbar v_F / 2\partial_x \phi} \qquad \boxed{e/cA_x \rightarrow e/cA_x + \hbar v_F / 2\partial_t \phi}$$

$$eE = -\partial_x \Phi + \partial_t A_x \rightarrow F = eE + (\partial_t^{\,2} - \partial_x^{\,2})\phi/2$$

F – the chiral invariant effective electric field is felt by electrons.
Resulting energy, collective charge and current, etc.
looks to be functions of entire ***F*** only.
E.g. the perturbed energy is expressed via the dielectric
permittivity $\boldsymbol{\varepsilon}$ of a semiconductor with the gap $\boldsymbol{\Delta}$:

$$W_e = -\frac{\varepsilon(k,\omega)F^2}{8\pi} = -\frac{\varepsilon(k,\omega)}{8\pi e^2}\left(-\frac{\partial e\Phi}{\partial x} - \frac{\hbar v_F}{2}\frac{\partial^2\varphi}{\partial x^2}\right)^2$$

As correctly derived as wrong in result:
misses or distorts all expected contributions from the CDW phase –
a semiconductor does not slide.
What was wrong?
Missed non-perturbative contribution known in field theory as the "chiral anomaly" (*rem. Krive for CDWs, Yakovenko for FISDWs*)

Resolution: the whole expression has been lost

Chiral anomaly

$$\delta W = \frac{\hbar v_F}{4\pi}\left(\frac{\partial\varphi}{\partial x}\right)^2 + \frac{e\Phi}{\pi}\frac{\partial\varphi}{\partial x}$$

On top of perturbational part

$$W_e^* = -\frac{\rho_n}{r_0^2}\frac{1}{8\pi}\left(e\Phi + \frac{\hbar v_F}{2}\frac{\partial\varphi}{\partial x}\right)^2$$

$$\zeta = \frac{\partial \mathrm{F}}{\partial \mathrm{n}}, \ \rho_{\mathrm{n}} = \mathrm{N}_{\mathrm{F}}^{-1}\frac{\partial \mathrm{n}}{\partial\zeta}; \ \rho_{\mathrm{c}} = 1 - \rho_{\mathrm{n}}$$

$$\varepsilon_e = \frac{1}{\lambda^2 k^2} = \frac{\rho_n}{r_0^2 k^2}$$

$$\boxed{W_e = W_e^* + \delta W = \rho_c\frac{\hbar v_F}{4\pi}\left(\frac{\partial\phi}{\partial x}\right)^2 + \frac{e}{\pi}\rho_c\Phi\frac{\partial\phi}{\partial x} - \frac{\rho_n(e\Phi)^2}{\pi\hbar v_F} - \frac{(\nabla\Phi)^2}{8\pi}}$$

The contribution of normal carriers erases from the T=0 anomalous action erasing it down to zero at $\mathbf{T_c}$ when A~$\boldsymbol{\Delta\to 0}$, $\boldsymbol{\rho_c \sim A^2}$

The chiral anomaly appears already above the CDW – in the normal metal

The most principle property of a conductor: expulsion of the electric field at the screening length $\mathbf{r_0}$

$$\frac{1}{r_0^2} = 4\pi e^2 \frac{dn(\zeta)}{d\zeta}$$

For a metal at T=0 from statistical mechanics:

$$\frac{dn(\zeta)}{d\zeta} = \frac{dn}{d\varepsilon_F} = N_F$$ - the DOS

From quantum mechanics - the Tomas-Fermi procedure:

The non-linearized Schroedinger eq. in WKB approximation

$$\psi(x) = \frac{C}{\sqrt[4]{E - e\Phi}} \exp(\pm i \int \sqrt{E - e\Phi(y)}\, dy)$$

$$n = \sum |\psi|^2 = \sum_E \frac{1}{\sqrt{E - e\Phi}} \sim \sqrt{\varepsilon_F - e\Phi}$$

δn~δΦ

When we first linearize the spectrum and decompose the wave function

$$\frac{p^2}{2m} - \frac{p_F^{\,2}}{2m} \cong \pm\hbar v_F k \qquad \psi = \psi_+ e^{ik_F x} + \psi_- e^{-ik_F x}$$

$$\mp i\partial_x \psi_\pm + e\Phi\,\psi_\pm = E\psi_\pm \qquad \psi_\pm = C\exp\left(\pm i\int dx(E - e\Phi(x))\right)$$

With **C=cnst** – the potential affects only the phase but no more the amplitude of the wave function

$$\boxed{n = \sum \left|\psi_\pm\right|^2 = cnst}$$

By the necessary but premature linearization we loose the possibility to change the density $|\psi|^2$ for any wave function

No density response to the potential already at one-particle level, hence no field screening

The chiral anomaly is invoked to restore the normality

Expressions for total density and current conserve number of particles

$$n=\frac{1}{\pi}\partial_x\varphi+n_{in}\;,\qquad j=-\frac{1}{\pi}\partial_t\varphi+j_{in}\Rightarrow\frac{dn}{dt}=0$$

But need a mechanism for n_{in} , n_{in} to compensate $\boldsymbol{\partial\varphi}$ at $\boldsymbol{A\to 0}$ to yield:

$$n_c=\frac{A^2}{\pi}\frac{\partial\varphi}{\partial x}\qquad j_c=-\frac{A^2}{\pi}\frac{\partial\varphi}{\partial t}$$

for ***A=cnst***

That will come implicitly from counter-charges,
counter- or backflow currents (rem. Littlewood, Artemenko)
which react to CDW bringing compensating contributions $-\rho_n\,\partial\varphi/\pi$

Local energy functional

$\psi = A\exp(i\varphi);$ $\quad$ $A=\Delta/\Delta_0$

$$W\{\phi,\Phi,n_{in},A\} = \frac{\hbar v_F}{4\pi}\left[\phi_x^2 + \alpha A^2\phi_y^2\right] + C\frac{\hbar v_F}{4\pi}\left[\left|\frac{\partial A}{\partial x}\right|^2 + \beta^2\left|\frac{\partial A}{\partial y}\right|^2\right] +$$

Expect A^2 – actually 1.

Non analytic in Ψ

terms come from the chiral anomaly.

$$\frac{\phi_x}{\pi}\Phi + \left(\Phi + \frac{\hbar v_F}{2}\phi_x\right)n_{in} - \frac{\varepsilon_h}{8\pi}(\nabla\Phi)^2 + F(A,n_{in})$$

$$\phi_i = \partial\phi/\partial x_i$$

$n_{in}=n=n_e-n_h$ "intrinsic" carriers – those which participate in CDW

$$F(n,A) = n^2/(2N_F) + (-\tau + (n/n_{cr})^2)(A\Delta_0)^2 N_F/2 + bA^4\Delta_0^2 N_F/4$$

Local electro-neutrality approximation $n=-\partial_x\varphi/\pi$

- the superconductivity form of the GL energy

True equations are not analytical in Ψ: phase gradients are not multiplied by A^2

$$\frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial x}+2\Phi+\pi\left(n_e-n_h\right)\right)+\alpha\frac{\partial}{\partial y}\left(A^2\frac{\partial\varphi}{\partial y}\right)=\gamma_\varphi A^2\frac{\partial\varphi}{\partial t}$$

$$\nabla^2 A+\alpha A\left(\frac{\partial\varphi}{\partial y}\right)^2+\frac{\partial F}{\partial A}=-\gamma_A\frac{\partial A}{\partial t}$$

$$-r_0^2\nabla^2\Phi=\frac{\partial\varphi}{\partial x}+\pi\left(n_e-n_h+n_{ex}\right)$$

$$\nabla\hat{\sigma}\nabla\mu=\partial_t n \qquad \mu=\zeta+\Phi+\partial_x\varphi/2$$

F=F(A,n) in principle

$$\zeta=\frac{\partial F}{\partial n},\ \rho_n=N_F^{-1}\frac{\partial n}{\partial\zeta};\ \rho_c=1-\rho_n$$

Former G-L like equations

$$\nabla A^2\nabla\varphi+\frac{\partial}{\partial x}A^2\Phi=\gamma_\varphi A^2\frac{\partial\varphi}{\partial t}$$

$$\nabla^2 A+A(\nabla\varphi)^2+\frac{\partial F}{\partial A}=-\gamma_A\frac{\partial A}{\partial t}$$

$$-r_0^2\nabla^2\Phi=A^2\frac{\partial\varphi}{\partial x}+n_{ex}$$

$$-\nabla\left[\sigma\nabla\left(\varsigma+\Phi\right)\right]+\frac{\partial n}{\partial t}=0$$

F=F(A) only

The limit of the local electro-neutrality $r_0 \to 0$ together with the infinite normal conductivity.

$$\partial_x \Phi + (\pi N_F)^{-1} \nabla_\perp A^2 \nabla_\perp \varphi - \gamma_\varphi \partial_t \varphi = 0 \; , \;\; \Phi = (n/N_F - \zeta)$$

$$\partial_x \varphi + \pi n = 0$$

Curiously, no commonly assumed longitudinal phase rigidity $\propto \partial_x^2 \varphi$

It is hidden in the term $\partial_x \Phi$ implicitly, via relations.

$$\frac{\rho_c}{\rho_n} \partial_x^2 \varphi + \kappa_\perp \nabla_\perp \left(A^2 \nabla_\perp \varphi \right) - \gamma_\varphi \partial_t \varphi = \pi N_F \frac{\partial \zeta}{\partial A} \partial_x A$$

Coulomb hardening (rem. Kirova talk) looks intuitive, but where is the driving force?

The drive comes only from the boundary conditions for Φ transferred to the phase via the local relations of Φ and $\partial\varphi$ mediated by n.

Electro-neutrality at a finite normal conductivity at D=1:

Static elastic force:
gradient of the normal chem. potential

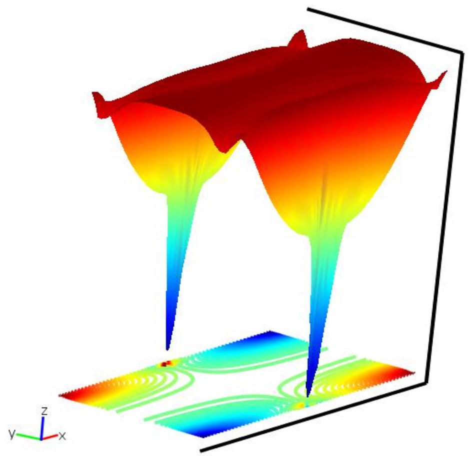
$$\frac{1}{\pi}\left(\frac{1}{\sigma_{CDW}}+\frac{1}{\sigma_n}\right)\partial_t\varphi+\partial_x\left(\frac{1}{\pi}\partial_x\varphi+\zeta\right)=\frac{-1}{\sigma_n}J(t)$$

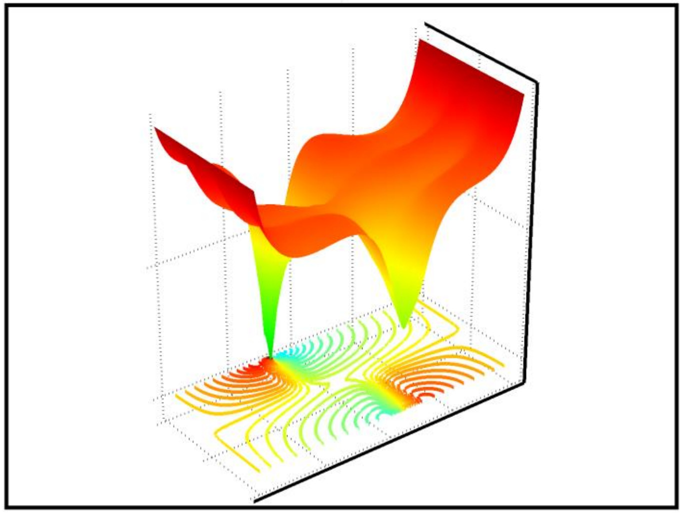
Serial CDW and normal resistivities

Current driving force:
(Total current)/(normal conduct.)

$$\frac{\rho_c}{\rho_n}(\partial_x\varphi)^2$$

Nonanalytic dependence on the amplitude requires new more complicated numerical studies.

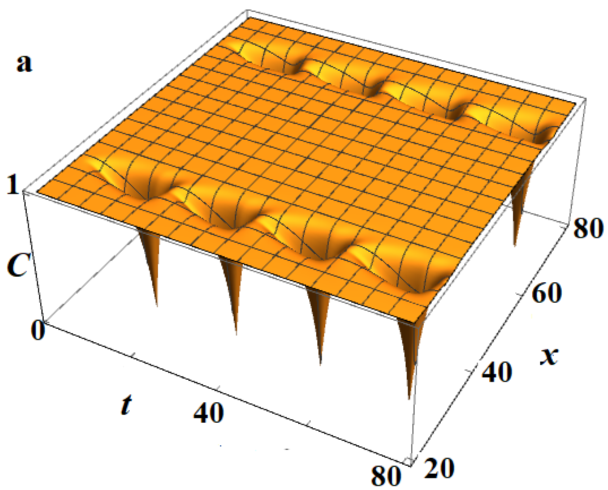




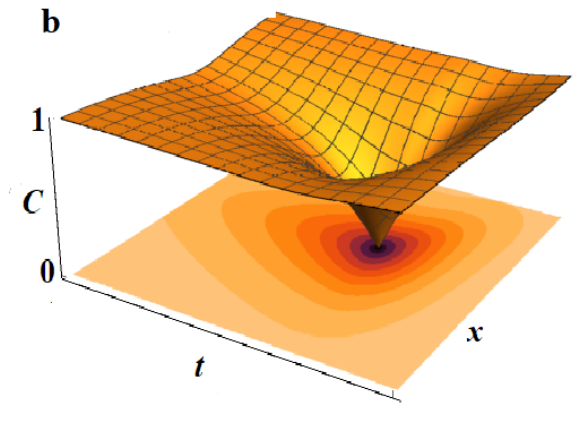
We still can run up to nucleation of vortices at a surface, rem. more in N. Kirova talk.
But we cannot trace proliferation of vortices as before.
A price for no explicit compensation of diverging **∂φ** by vanishing $\mathbf{A^2}$

Results for numerical solving of partial diff. eqs. for a substantial simplified free energy form.

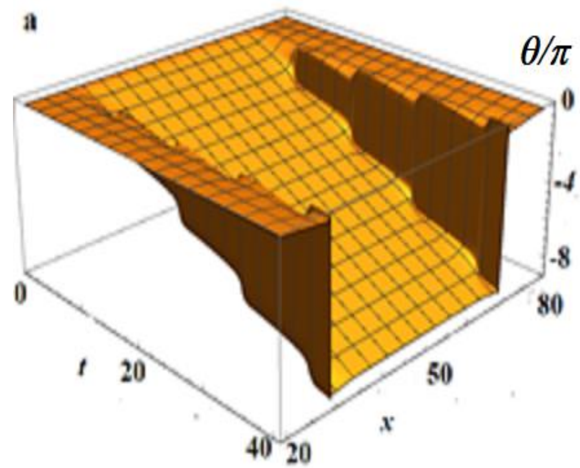
amplitude



Single x,t vortex



phase



Sequence of phase slips – spacio-temporary vortices – around the amplitude nodes.
It sets in near sample boundaries allowing for the mean phase velocity

Beyond quasi-1D and CDWs, e.g. for polaronic crystals

$\boldsymbol{u}$ – vector of unit cell displacements

$\boldsymbol{\nu}$ – units' filling factor

$$\boldsymbol{n_c} = -\nu\boldsymbol{\nabla u};\ \ \boldsymbol{j_c} = \nu\partial_t\boldsymbol{u}$$

$$\frac{dn_c}{dt} = \partial_t n_c + (\nabla\cdot\mathbf{j}_c) = (\nabla\nu\cdot\partial_t\mathbf{u}) - \partial_t\nu(\nabla\cdot\mathbf{u}) \neq 0$$

Need to bridge the elastic theory and inter-cell kinetics

Conclusion and perspective.

- Chiral transformations with account for chiral anomaly were applied to the sliding CDW model
- Two-fluid hydrodynamics was constructed for the order parameter and the normal liquid
- Topologically nontrivial dynamics appears under applied fields or charge injection
- The numeric procedure needs to be stabilized for the nonanalytic eqs.
- The problems of glide and climb should be considered