

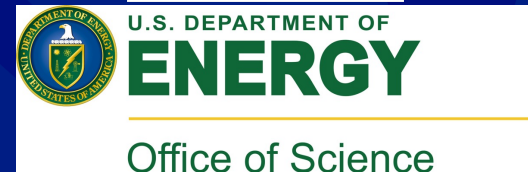
Correlated Chern insulator in two-dimensional materials

Shizeng Lin

Theoretical Division, Los Alamos National Laboratory

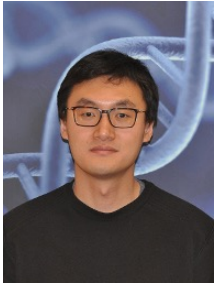
8/17/2022

ECRYS 2022@Cargese, Corsica, France



Outline

- Introduction
 - Integer quantum Hall and fractional quantum Hall effect
 - Chern insulator
- Moiré superlattice in two dimensional materials
- **A new route to Chern insulator: massive Dirac fermion in periodic potential**
- Fractional Chern insulator
- Summary



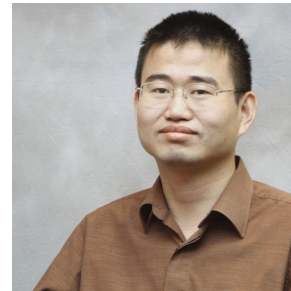
Ying Su (LANL=> UT Dallas)



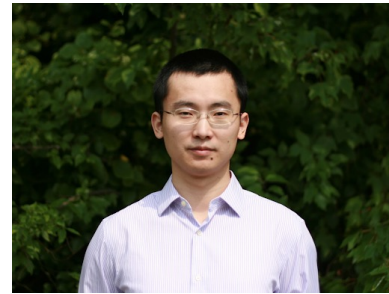
Umesh Kumar
(LANL=> Rutgers Univ.)



LANL student
Heqiu Li (U. Toronto)



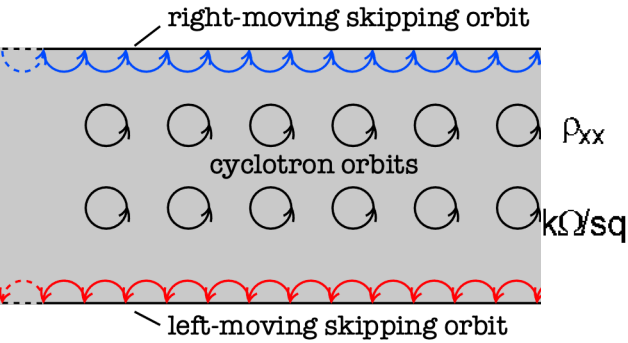
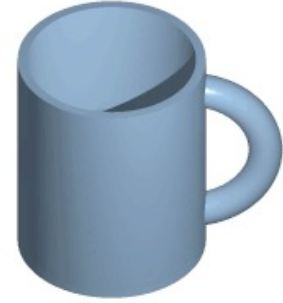
Chunwei Zhang
(UT Dallas)



Kai Sun (U. Michigan)

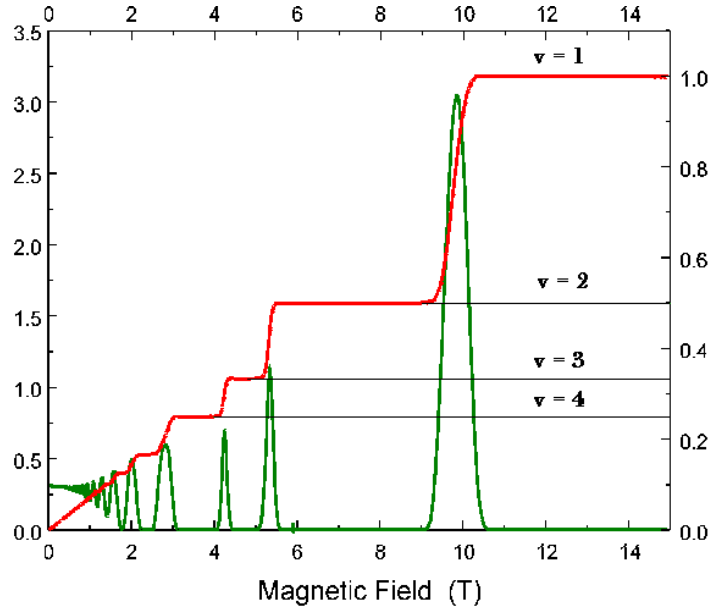
Topology

- From abstract mathematical concepts to materials
- Many topologically distinct classes of topological materials



Klaus von Klitzing

Integer quantum Hall effect



1 part in a billion precision!



GATE VOLTAGE

9/31/22

3

or



2am@ Cargese



Fractional quantum Hall effect

The Nobel Prize in Physics 1998

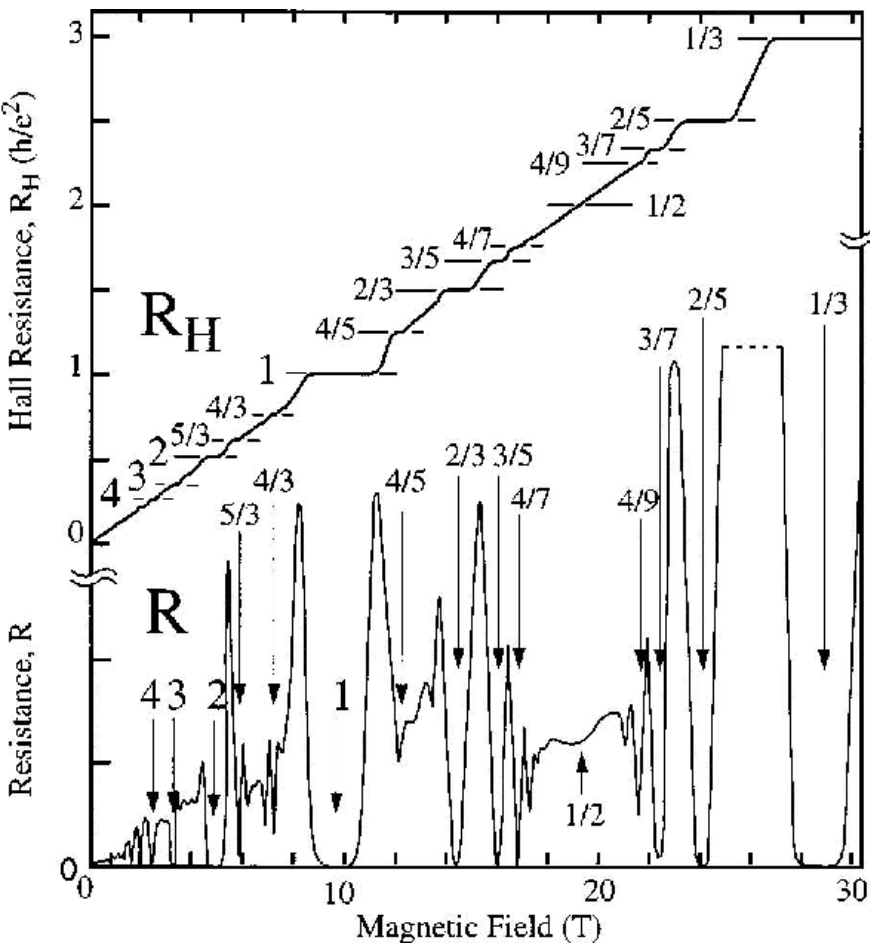


Photo from the Nobel Foundation archive.
Robert B. Laughlin
Prize share: 1/3



Photo from the Nobel Foundation archive.
Horst L. Störmer
Prize share: 1/3



Photo from the Nobel Foundation archive.
Daniel C. Tsui
Prize share: 1/3

VOLUME 48, NUMBER 22

PHYSICAL REVIEW LETTERS

31 MAY 1982

Two-Dimensional Magnetotransport in the Extreme Quantum Limit

D. C. Tsui,^{(a), (b)} H. L. Stormer,^(a) and A. C. Gossard
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 5 March 1982)

A quantized Hall plateau of $\rho_{xy} = 3h/e^2$, accompanied by a minimum in ρ_{xx} , was observed at $T < 5$ K in magnetotransport of high-mobility, two-dimensional electrons, when the lowest-energy, spin-polarized Landau level is $\frac{1}{3}$ filled. The formation of a Wigner solid or charge-density-wave state with triangular symmetry is suggested as a possible explanation.

Consequence of topology and strong correlation

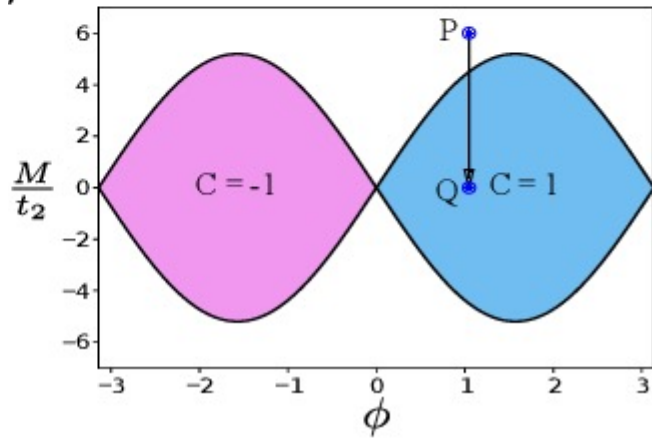
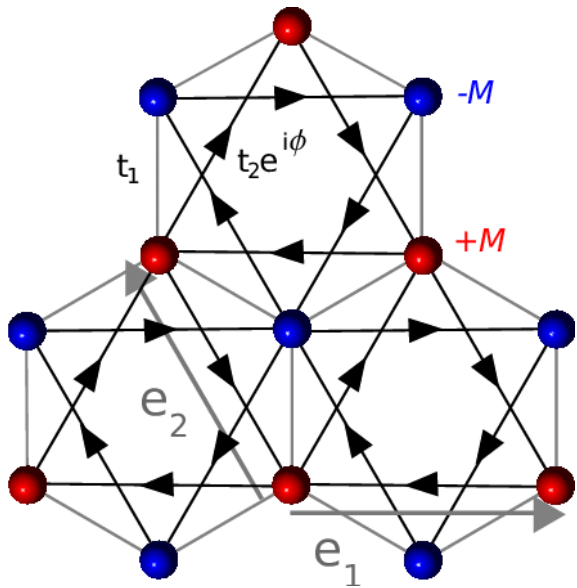
Lattice realization

- Landau levels are not essential, but need to break time reversal symmetry
Duncan Haldane, PRL 61, 2015 (1988)
- Integer quantum Hall effect → Chern insulator (quantum anomalous Hall insulator)
- Fractional quantum Hall effect → fractional Chern insulator

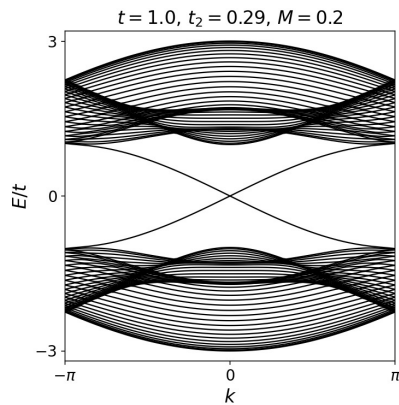
Haldane model



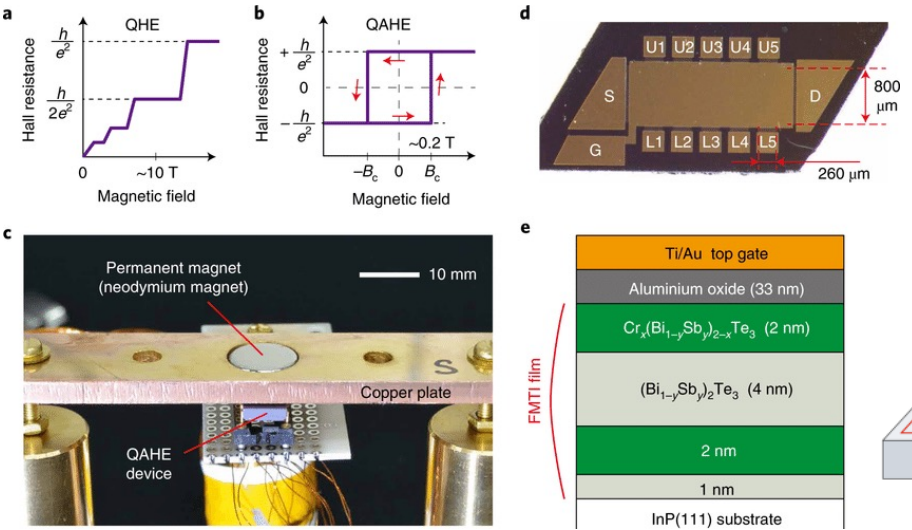
Duncan Haldane, PRL 61, 2015 (1988)
Nobel Prize 2016



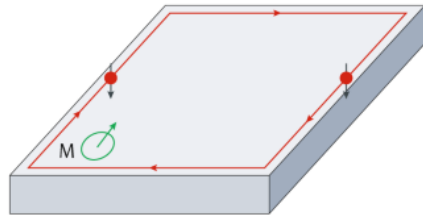
$$\mathcal{H}_H = t_1 \sum_{\langle ij \rangle} c_i^\dagger c_j + t_2 e^{i\phi} \sum_{\langle\langle ij \rangle\rangle} c_i^\dagger c_j + M \sum_i (-1)^{n_i} c_i^\dagger c_i$$



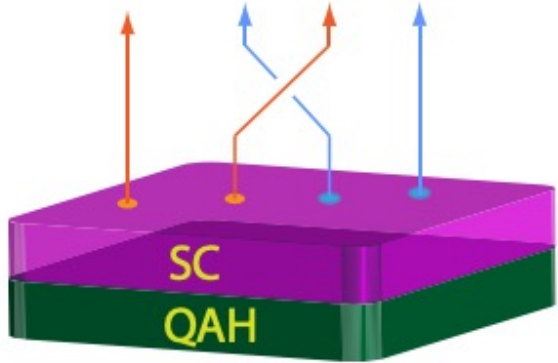
Why Chern insulator



Resistance standard without magnetic field
 10 parts per billion precision
 Okazaki et al., Nat. Phys. (2022)



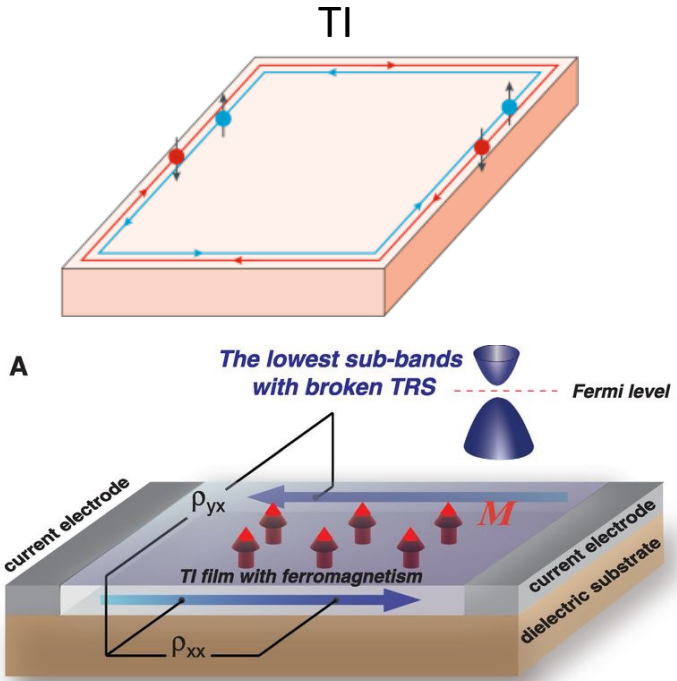
Dissipation free electronic devices



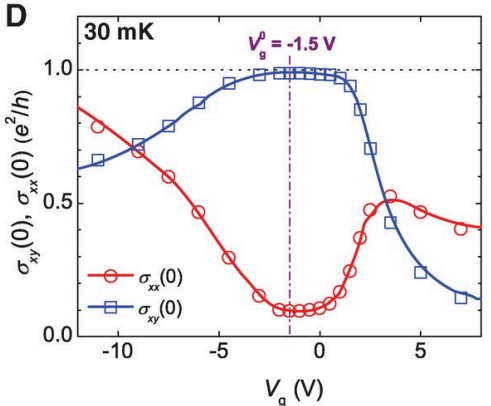
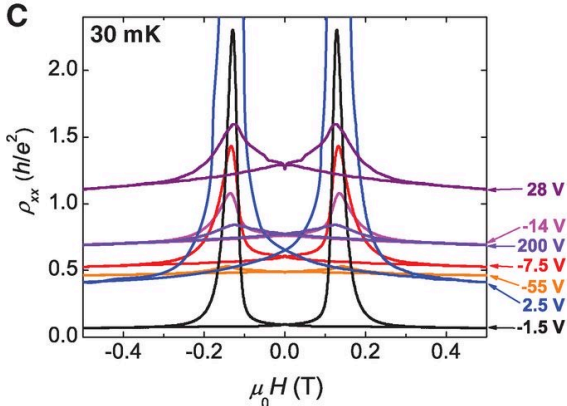
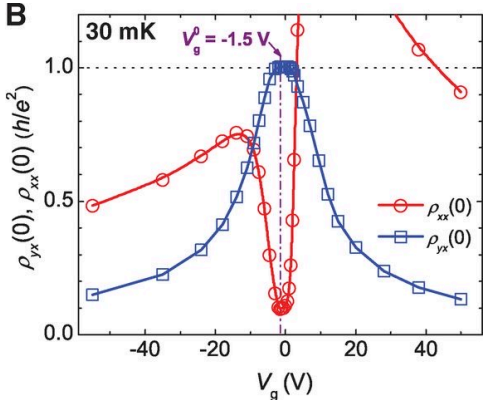
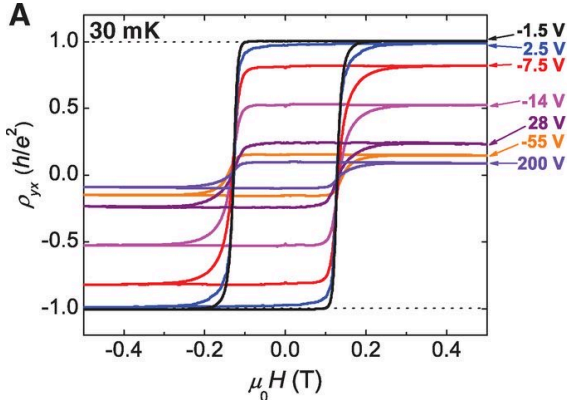
Platform for topological quantum computation

S. C. Zhang et al.

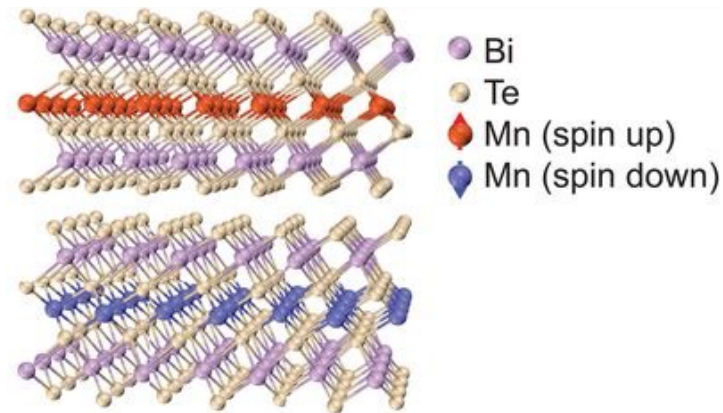
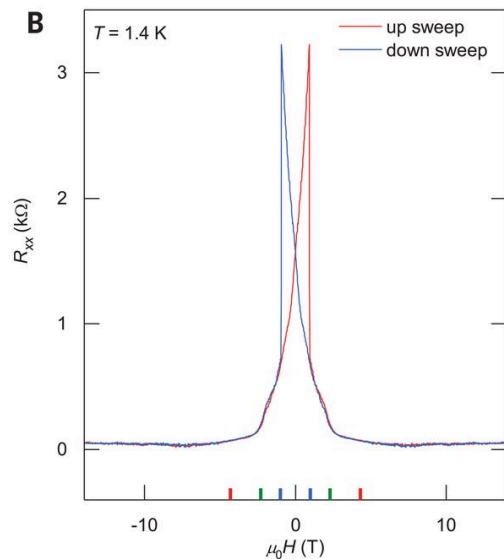
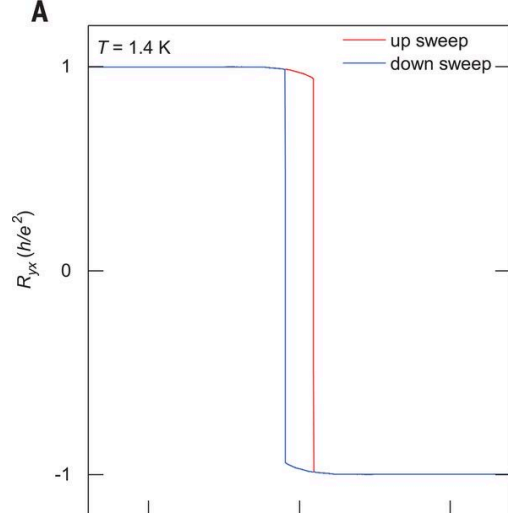
First experimental realization: doping topological insulator



chromium-doped $(\text{Bi,Sb})_2\text{Te}_3$



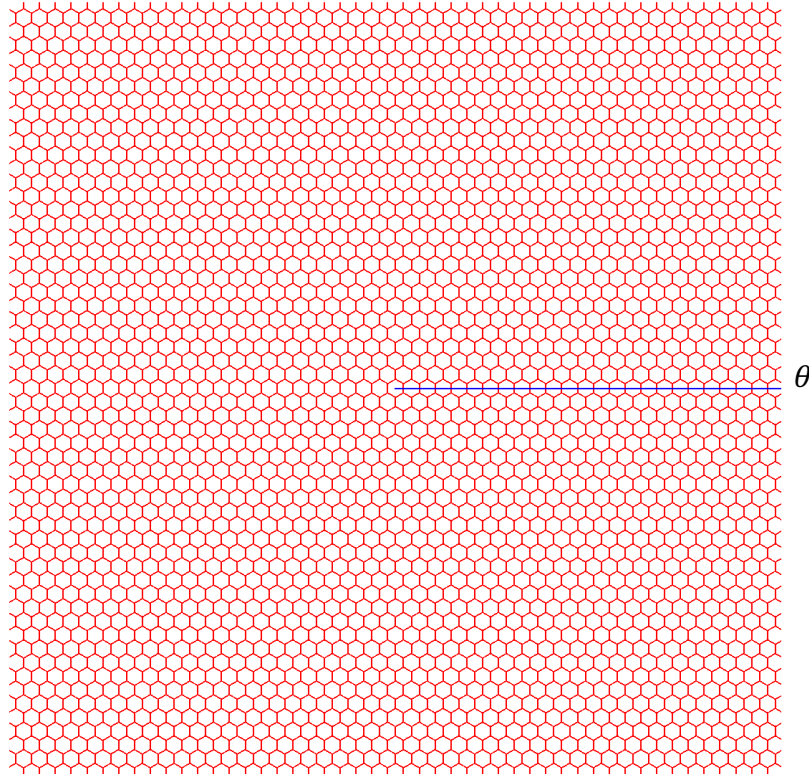
MnBi₂Te₄



Deng et al., Science (2020)

Chern insulator in Moiré materials

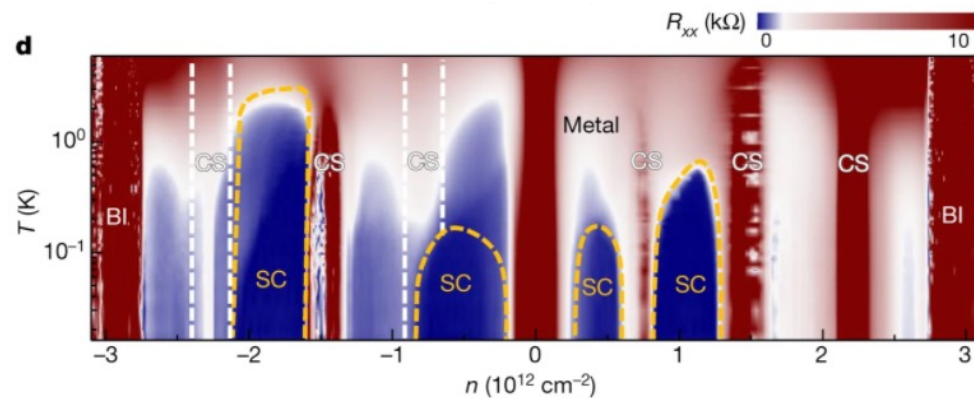
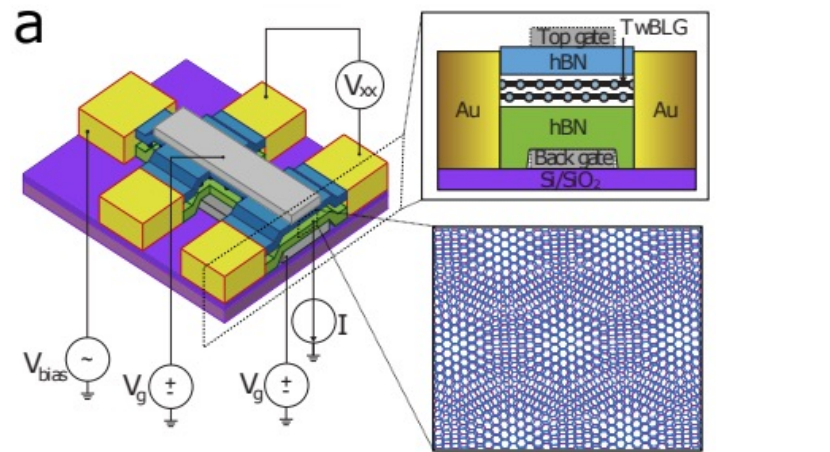
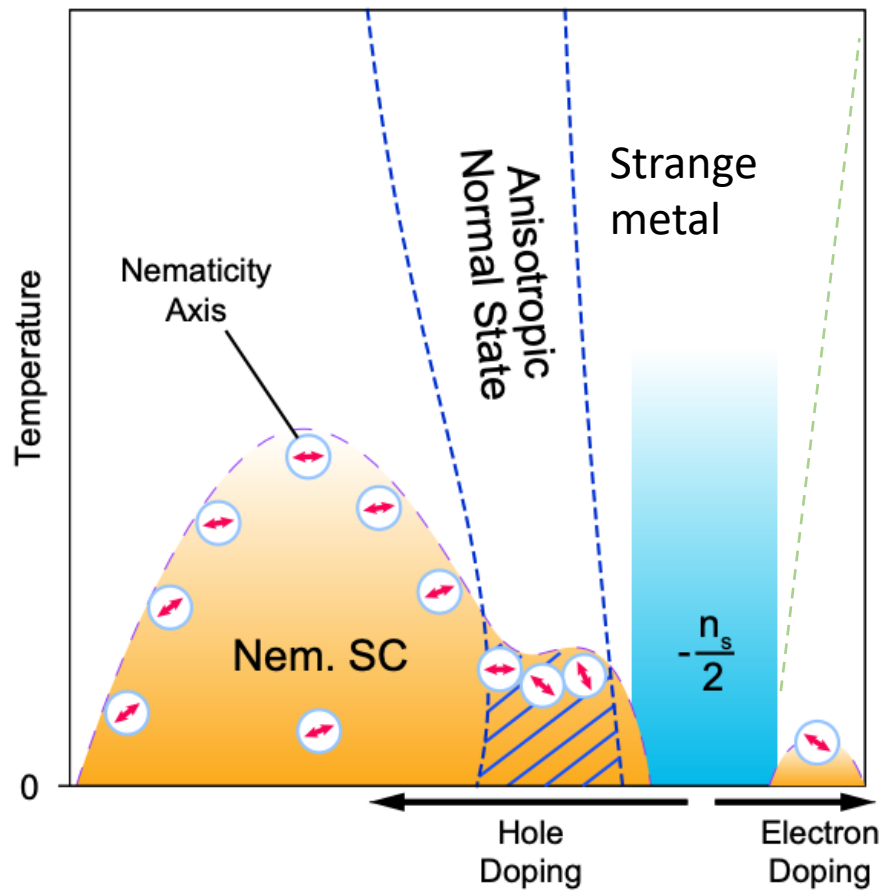
Twisted bilayer graphene



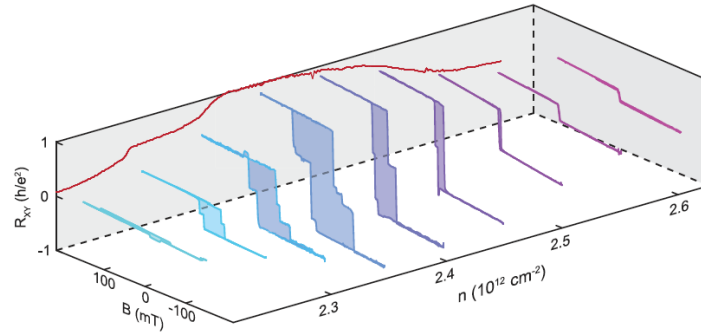
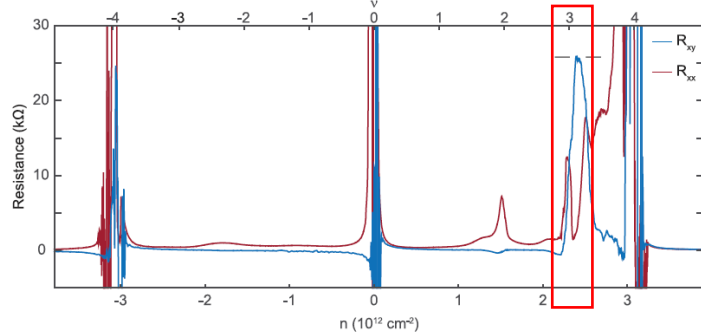
$$a_M \propto a/\theta$$

The rise of Moiré materials

R. Bistritzer, A.H. MacDonald, PNAS (2011)
 Y. Cao et al, Nature (2018)

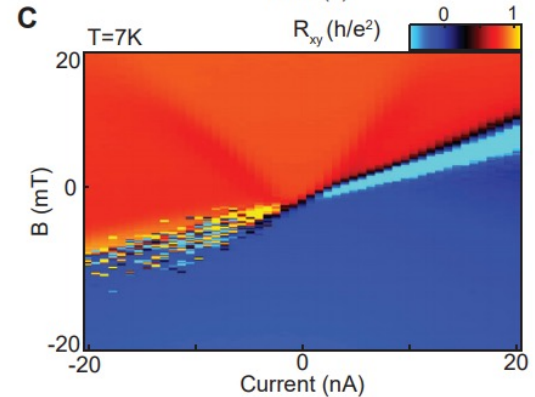
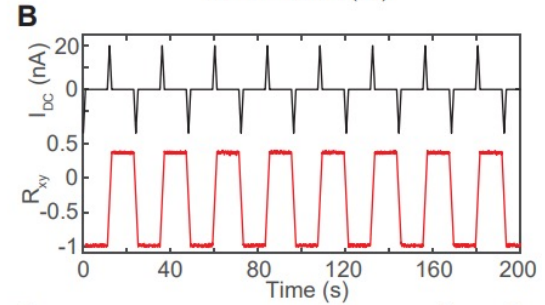
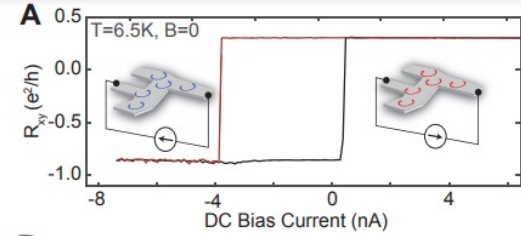


Chern insulator in twisted bilayer graphene



Inversion symmetry is broken by placing graphene on hBN

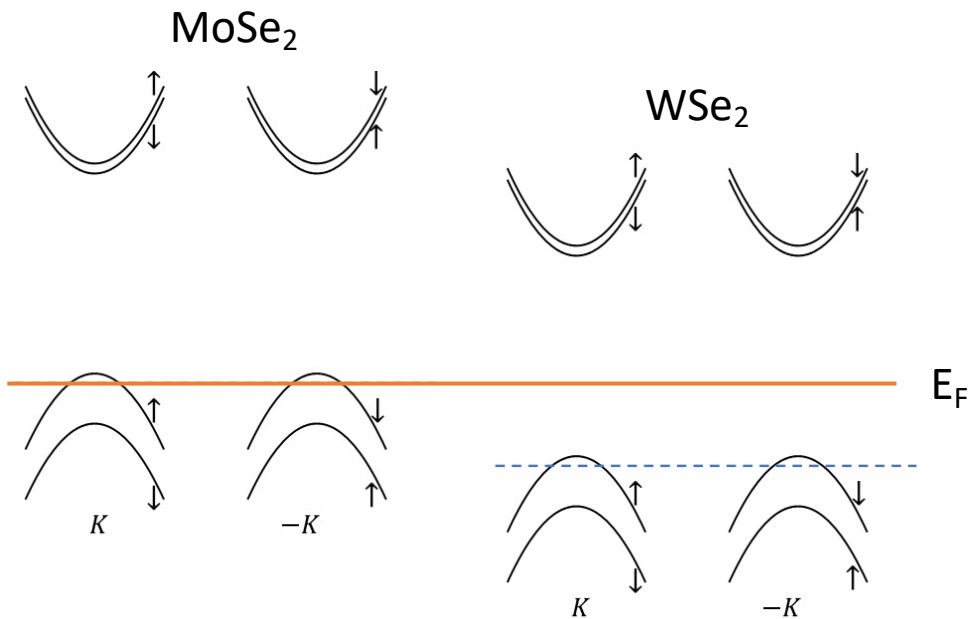
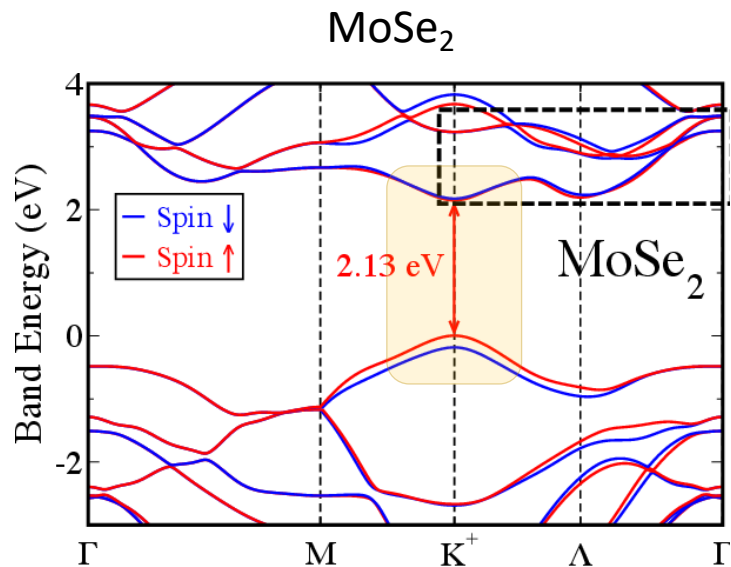
A. Sharpe et al, Science (2019)
M. Serlin et al, Science (2020)



Current induced switching of valley polarization and topology

Theory: Ying and Lin, PRL (2020)

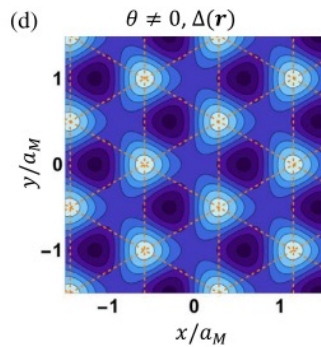
Transition metal dichalcogenide moiré superlattice



Effective Hubbard model

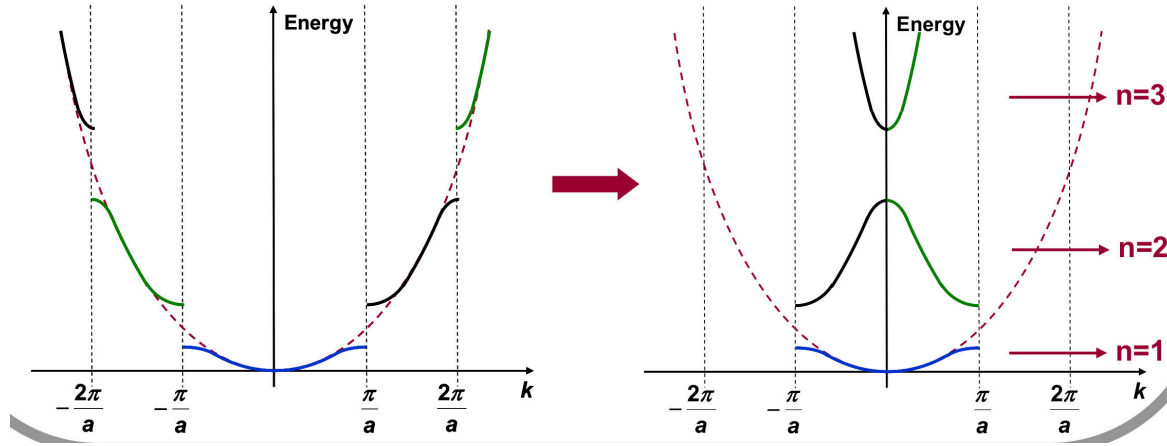
$$\mathcal{H} = -\frac{\hbar^2 \mathbf{Q}^2}{2m^*} + \Delta(\mathbf{r}), + \text{Coulomb}$$

$$\Delta(\mathbf{r}) = \sum_b V(\mathbf{b}) \exp[i\mathbf{b} \cdot \mathbf{r}],$$



Wu et al., PRL 121,
026402 (2018)

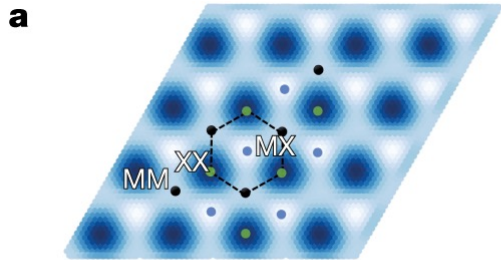
Band folding due to moiré superlattice



Coulomb $U \sim 1/a_M$, kinetic energy $K \sim 1/a_M^2$ for parabolic dispersion

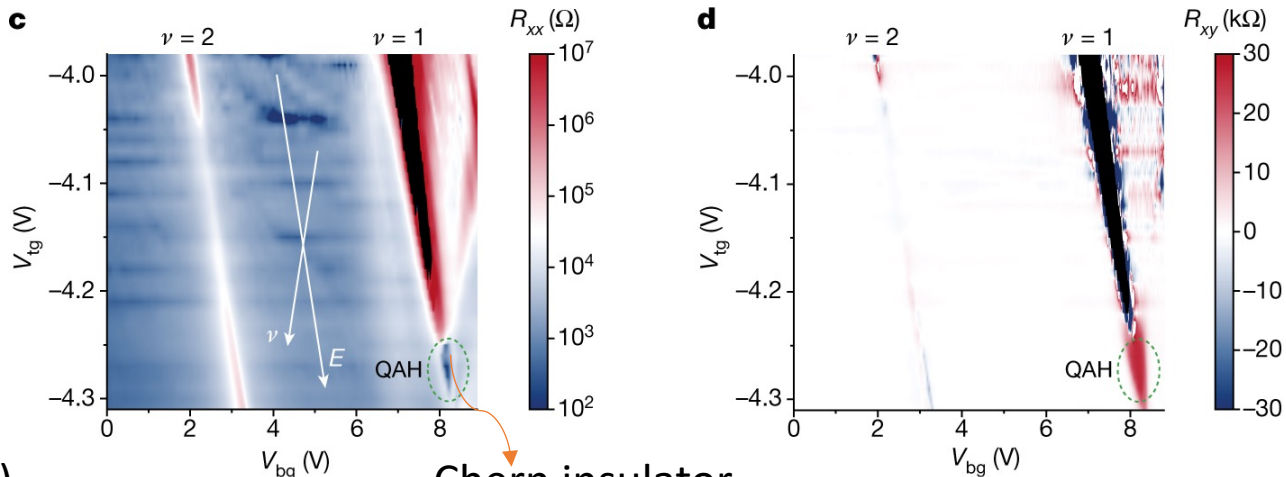
Coulomb dominates for a large a_M

Surprise: Chern insulator in TMD moiré superlattice

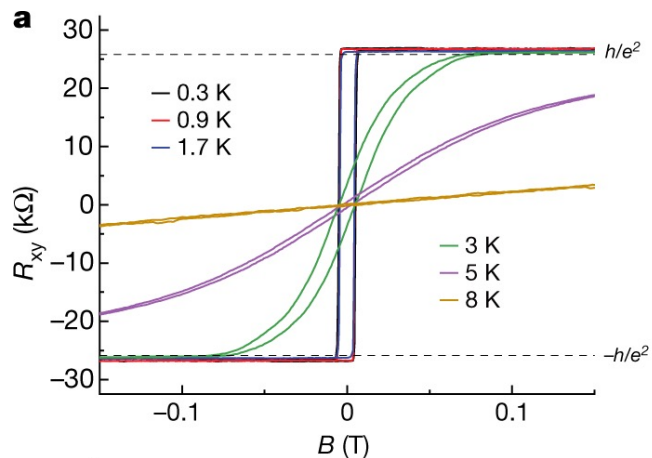


AB-stacked MoTe₂/WSe₂ heterobilayer

Li et al., Nature (2021)



Chern insulator



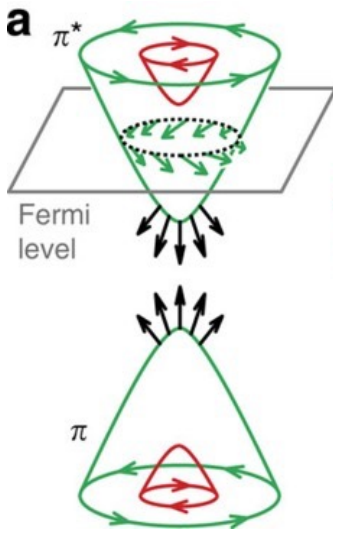
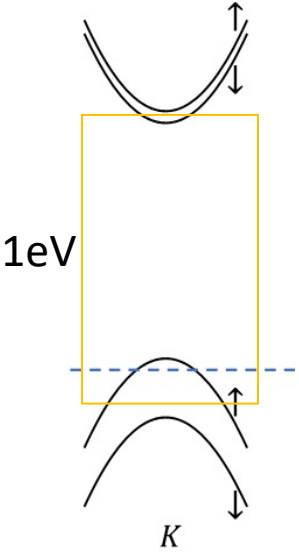
Massive Dirac fermion in a periodic potential

Su, Li, Zhang, Sun and SZL, arXiv:2110.02537

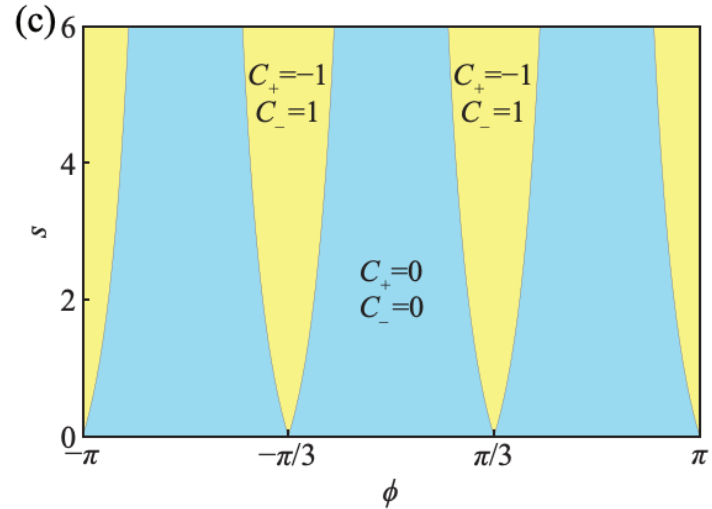
model

$$H_{\tau} = h_{\mathbf{k},\tau} + V(\mathbf{r}), \quad h_{\mathbf{k},\tau} = v_F (\tau k_x \sigma_x + k_y \sigma_y) + m \sigma_z,$$

$$V(\mathbf{r}) = 2V_0 \sum_{j=1}^3 \cos(\mathbf{G}_j \cdot \mathbf{r} + \phi)$$



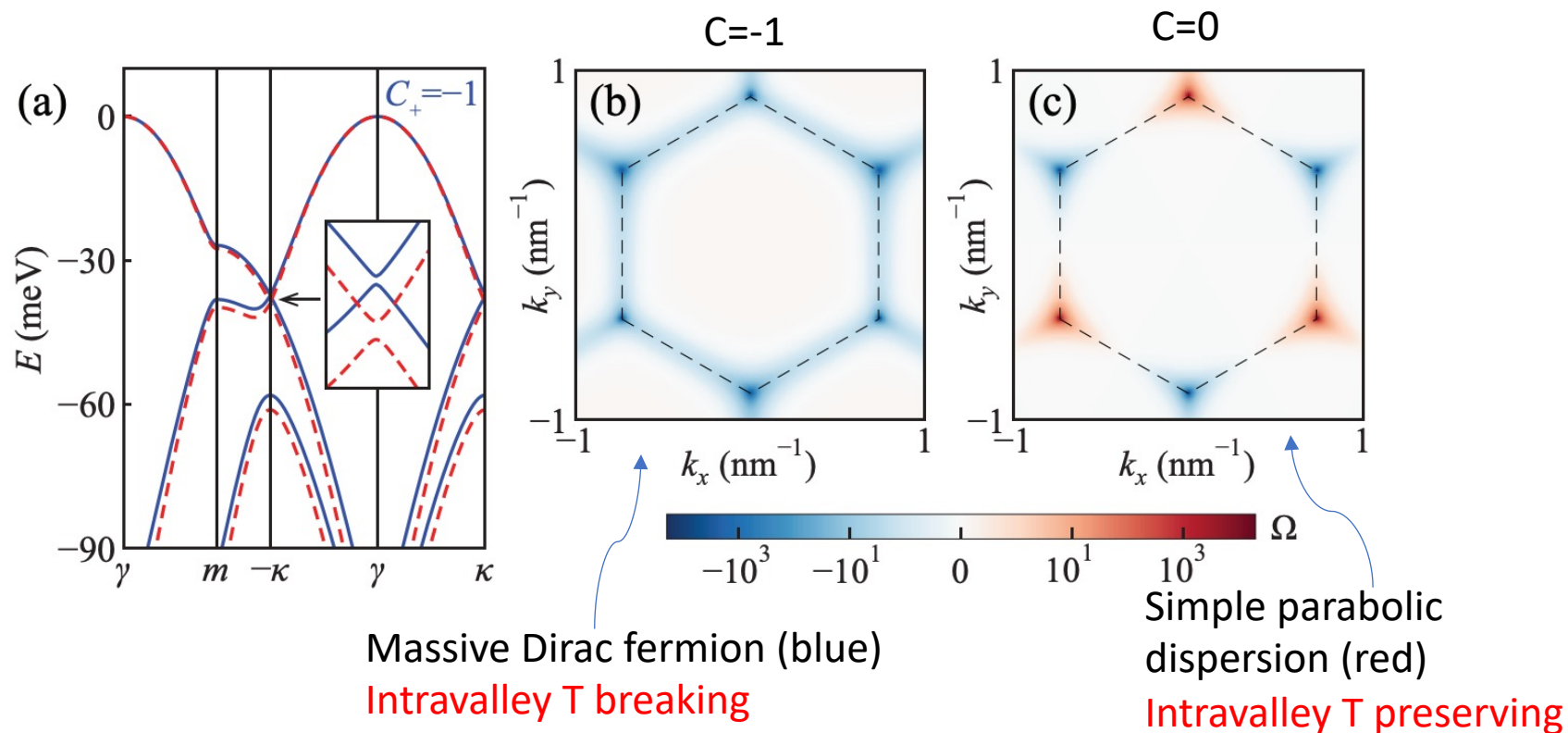
Meron texture



Parameter $s = 64\pi^2 v_F^2 / 9\Delta^2 a_M^2$ measures the initial Berry curvature

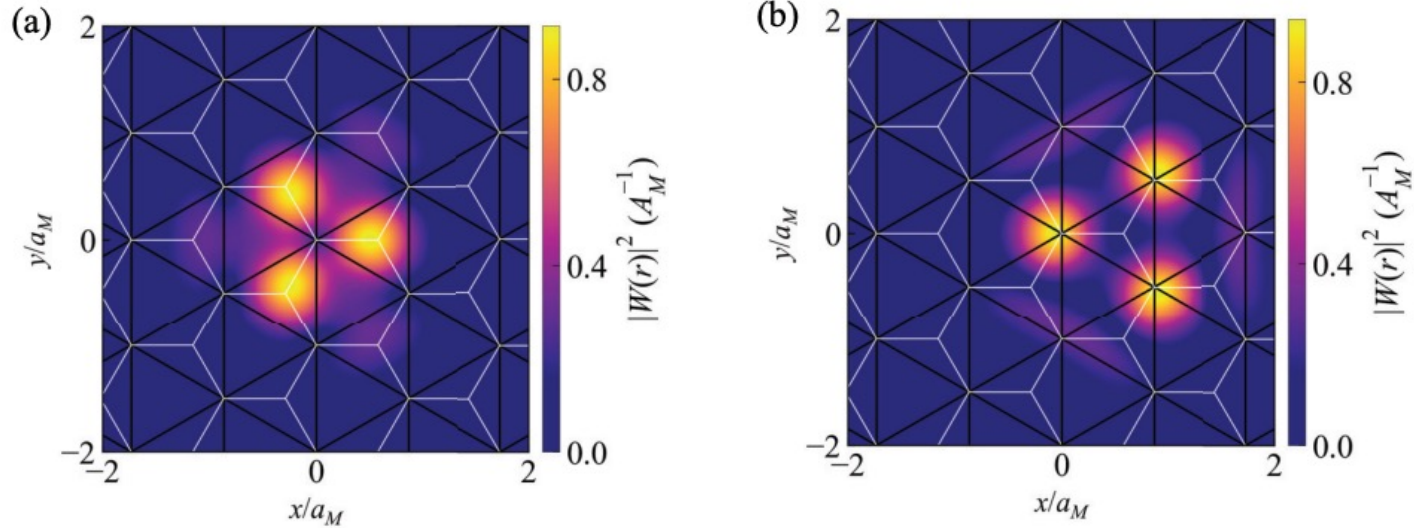
Small Berry curvature + Band folding → Chern band

Comparison between the two models



Band structure agrees well with two models, but the band topology is completely different!

Emergence of the Haldane physics



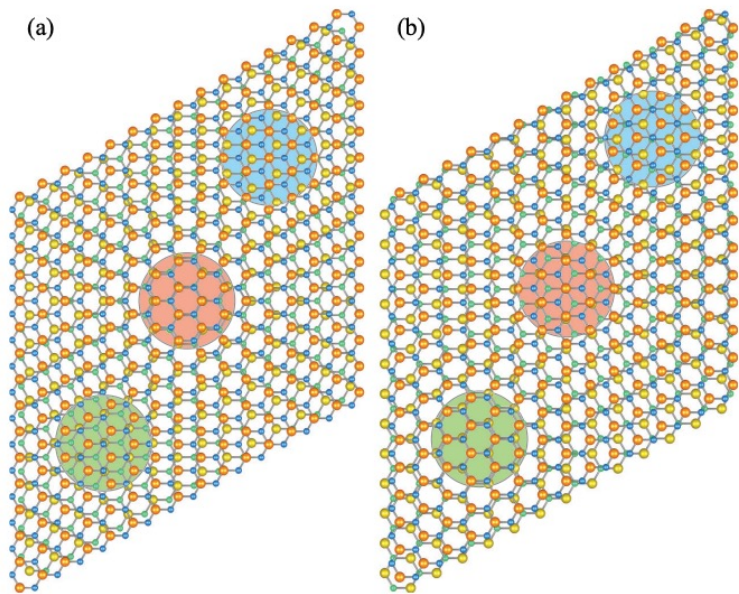
Wannier functions

$$W_{1,\tau}(\mathbf{r}-\mathbf{R}) = \frac{1}{\sqrt{2N}} \sum_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{R}} [\psi_{1,\mathbf{k},\tau}(\mathbf{r}) + \psi_{2,\mathbf{k},\tau}(\mathbf{r})],$$

$$W_{2,\tau}(\mathbf{r}-\mathbf{R}) = \frac{1}{\sqrt{2N}} \sum_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{R}} e^{i\theta_{1,\mathbf{k},\tau}} [\psi_{1,\mathbf{k},\tau}(\mathbf{r}) - \psi_{2,\mathbf{k},\tau}(\mathbf{r})]$$

- Wannier functions for two valence bands for a give valley form honeycomb lattice.
- ϕ serves as inversion symmetry breaking parameter

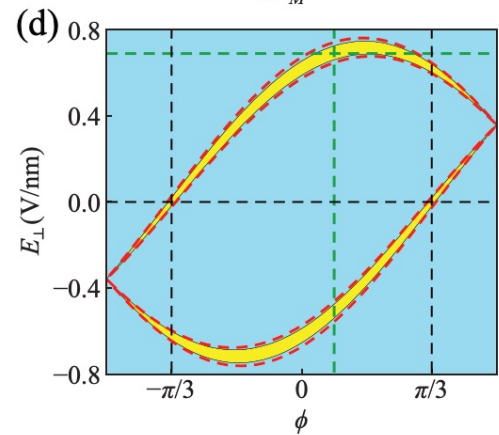
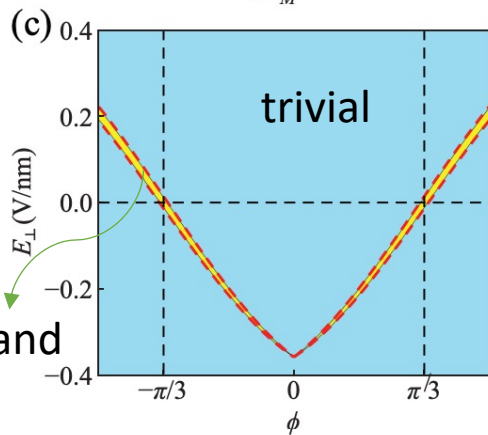
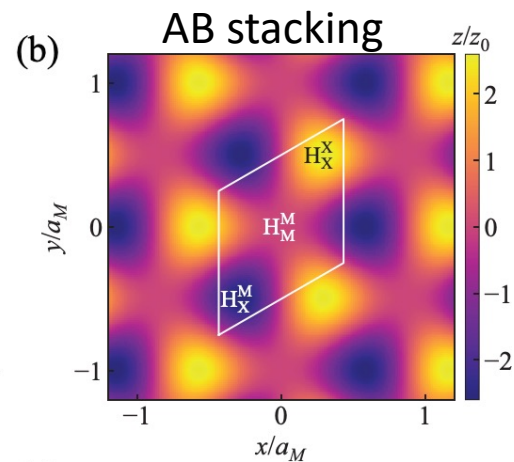
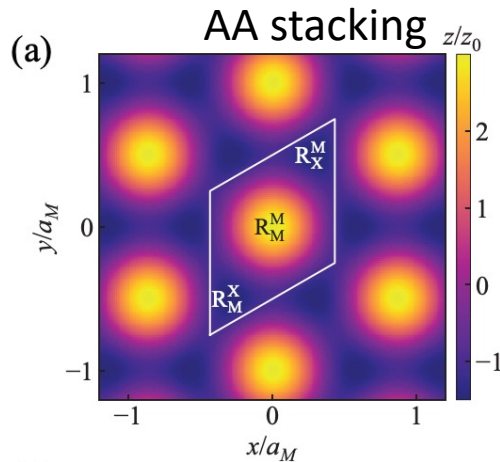
Electric field tuning of topology



$$H'_\tau = \hbar \mathbf{k} \cdot \boldsymbol{\tau} + V(\mathbf{r}) + eE_\perp z(\mathbf{r})$$

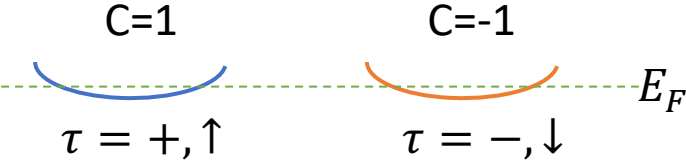
$$= \hbar \mathbf{k} \cdot \boldsymbol{\tau} + 2V'_0 \sum_{j=1}^3 \cos \left(\mathbf{G}_j \cdot \mathbf{r} + \frac{\phi + \phi'}{2} + \beta \right),$$

Chern band

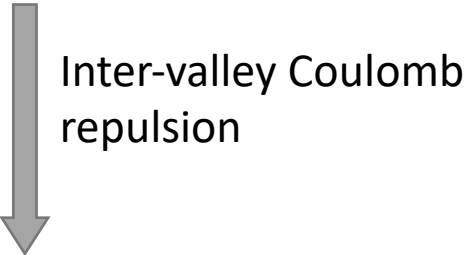


Spontaneous valley polarization

Time-reversal symmetric
Quantum valley/spin Hall insulator

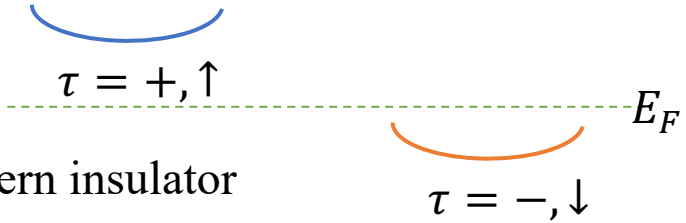


To realize Chern insulator, T-symmetry needs to be broken, which can be achieved by Coulomb interaction



Broken time-reversal symmetry

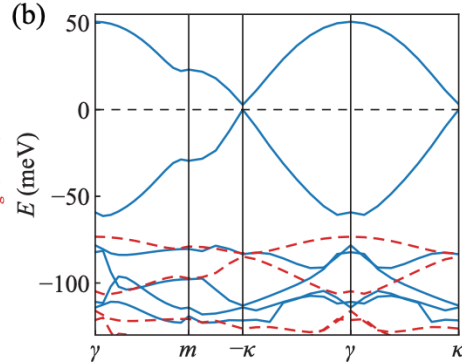
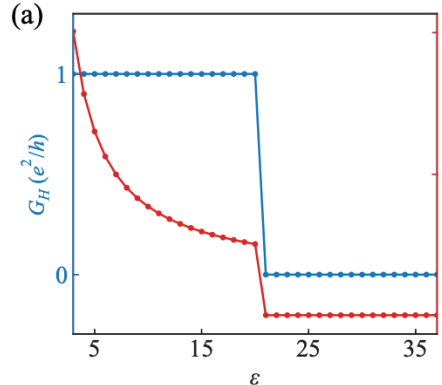
Chern insulator



Correlated Chern insulator

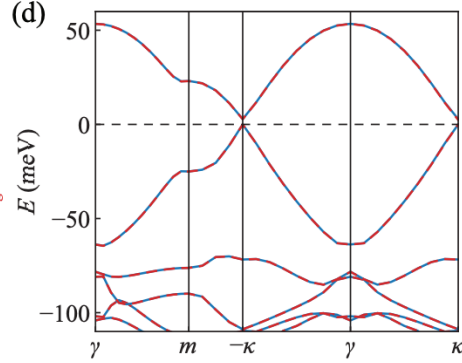
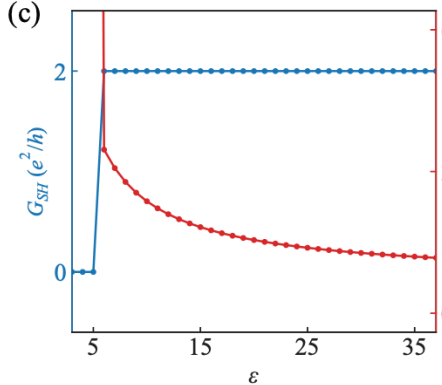
$$H = \sum_{n,\mathbf{k},\tau} (E_{n,\mathbf{k},\tau} - \mu) c_{n,\mathbf{k},\tau}^\dagger c_{n,\mathbf{k},\tau} + \frac{1}{2A} \sum_{\mathbf{q}} \rho(\mathbf{q}) V_{\mathbf{q}} \rho(-\mathbf{q}), \quad V_{\mathbf{q}} = e^2 \tanh(qd_{\perp}) / 2\epsilon_0 \epsilon q$$

$\nu = 1$ filling



Red: + valley
Blue: -valley

$\nu = 2$ filling



Fractional Chern insulator

Chern insulator in transition metal dichalcogenides Moiré materials

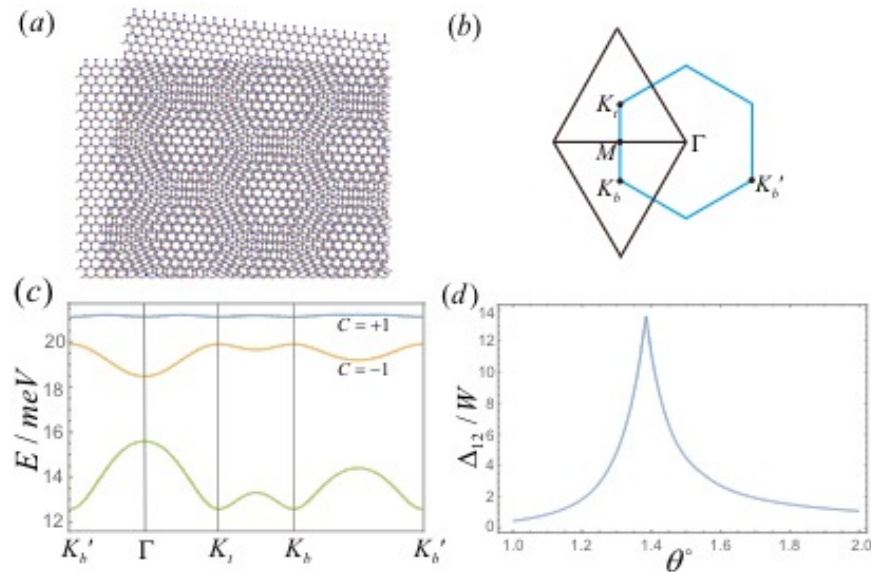
MoTe₂ homobilayer moiré superlattice

- Strongly correlated Chern band
- Interplay between correlation and topology
- Realization of Fractional Chern insulator

Tang et al., PRL (2011)

Sun et al., PRL (2011)

Neupert et al. PRL (2011)



$$H_+(\mathbf{k}, \mathbf{r}) = \begin{pmatrix} -\frac{\hbar^2(\mathbf{k}-\mathbf{K}_b)^2}{2m^*} + \Delta_b(\mathbf{r}) & \Delta_T(\mathbf{r}) \\ \Delta_T^\dagger(\mathbf{r}) & -\frac{\hbar^2(\mathbf{k}-\mathbf{K}_t)^2}{2m^*} + \Delta_t(\mathbf{r}) \end{pmatrix}$$

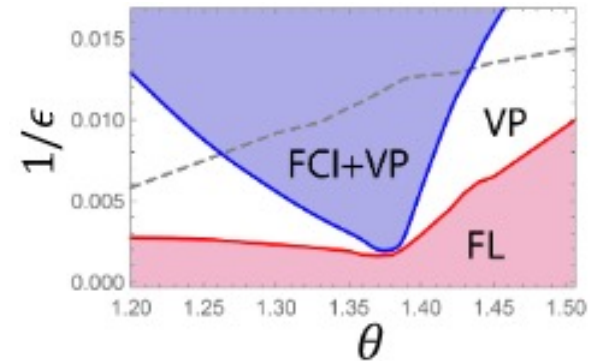
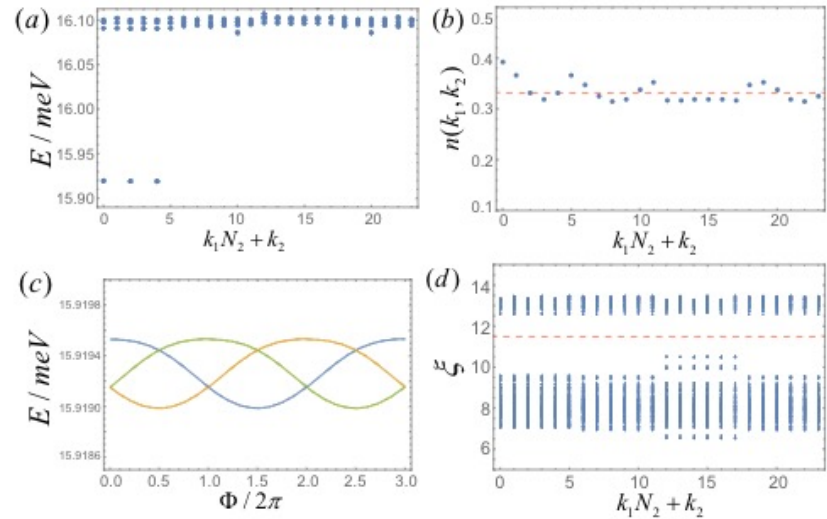
$$\Delta_T(\mathbf{r}) = w \left(1 + e^{-i\mathbf{G}_2 \cdot \mathbf{r}} + e^{-i\mathbf{G}_3 \cdot \mathbf{r}} \right)$$

$$\Delta_t(\mathbf{r}) = 2w_z \sum_{j=1,3,5} \cos(\mathbf{G}_j \cdot \mathbf{r} + l\psi),$$

Fractional Chern insulator

- Valley polarization by breaking T-reversal symmetry
- Fractional Chern insulating state at $\nu=1/3$ filling
- Quasi-hole excitation
- Symmetry breaking determines correlated topological phase and low energy excitations

H. Q. Li, U. Kumar, K. Sun and SZL, Phys. Rev. Research 3, 032070 (2021)



Charge neutral excitation

- Valley wave

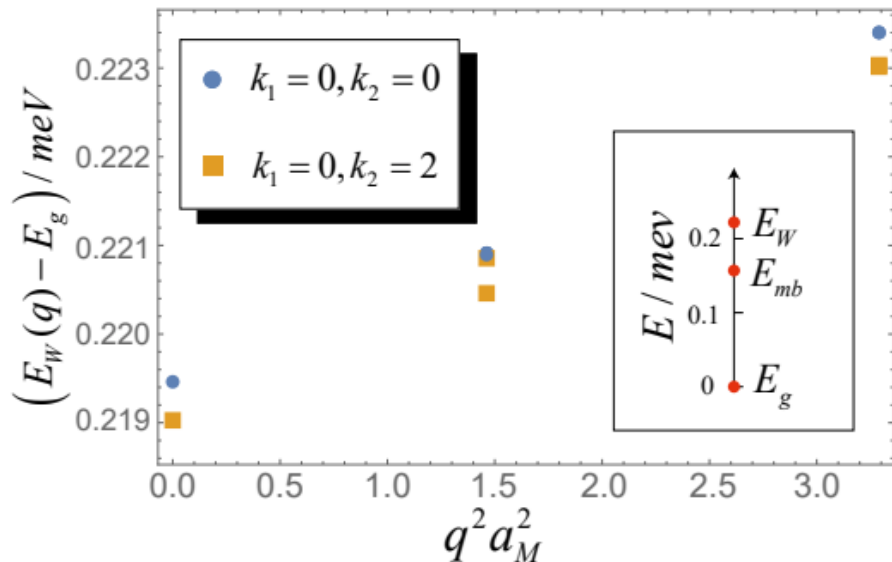
$$|\Psi_v(\mathbf{q})\rangle = \sum_{\mathbf{k}} z_{\mathbf{k}} C_+^\dagger(\mathbf{k} + \mathbf{q}) C_-(\mathbf{k}) |\Psi_-\rangle,$$

- Valley wave is gapped because the opposite valley has different Chern number: $SU(2) \implies U(1)$

- Hamiltonian for Valley pseudospin \mathbf{n}

$$H_n = \int dr^2 \left[\frac{J}{2} (\nabla \mathbf{n})^2 - \frac{A}{2} n_z^2 \right]$$

Individual valley skyrmion is not stable, but allow for skyrmion lattice stabilized by the Coulomb interaction



Summary

- A new mechanism to stabilize Chern insulator by placing massive Dirac fermion in a periodic potential

Su, Li, Zhang, Sun and SZL, arXiv:2110.02537

- Coulomb interaction spontaneously breaks time-reversal symmetry
- Our theory is a candidate to explain the experiments $\text{MoTe}_2/\text{WSe}_2$

Other theories: Zhang et al. arXiv:2107.02167

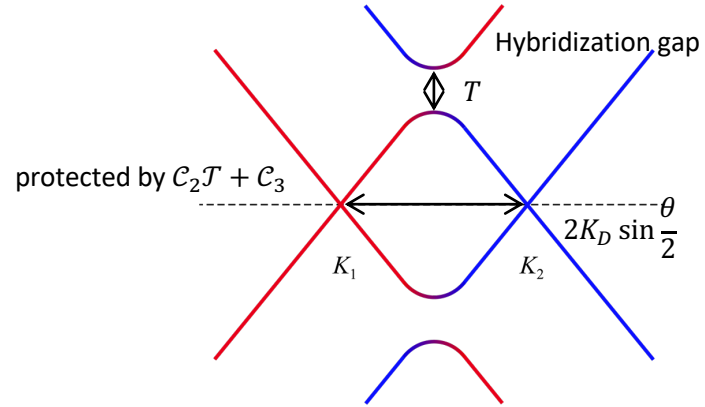
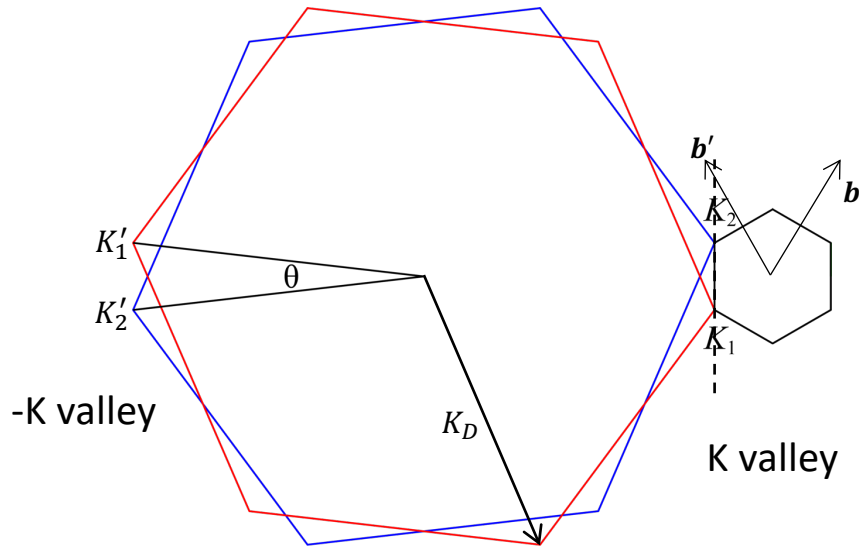
Xie et al., arXiv:2106.13991

Chang et al., arXiv:2203.10088

- Fractional Chern insulator can be realized when the interaction is strong

Li, Kumar, Sun and SZL, Phys. Rev. Research 3, 032070 (2021)

Narrow bands at magic angles



- When inversion symmetry is broken, we can pack two merons inside the same Moiré Brillouin zone
- Strong Coulomb interaction causes valley polarization → Chern insulator

Theory: Zhang, Mao, Senthil PRR (2019)