TIME CRYSTALS AND TIME OPERATORS FROM CHARGE DENSITY WAVES

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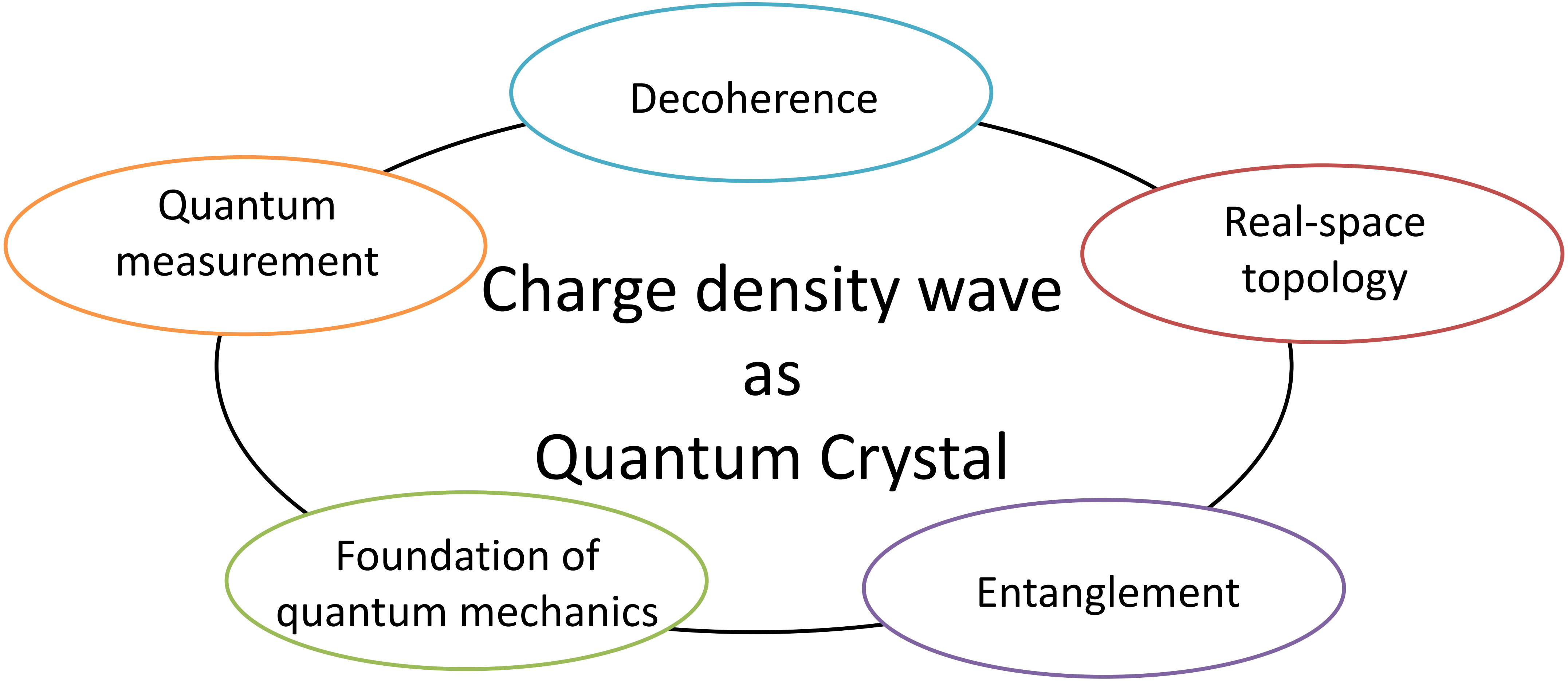
Aug. 11, 2022 ECRYS2022

My view of charge density wave

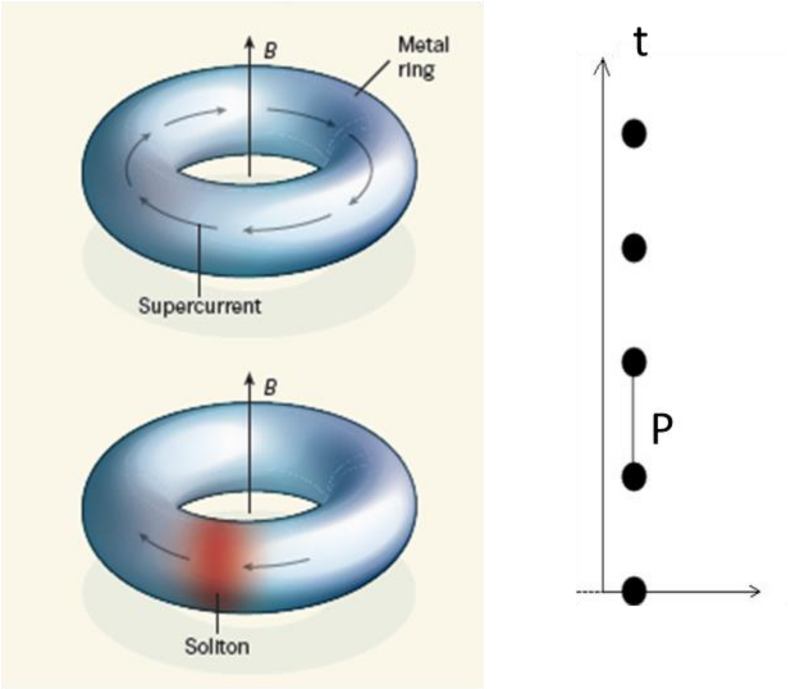


Time Crystal, Time operators!

The original proposal of time crystal

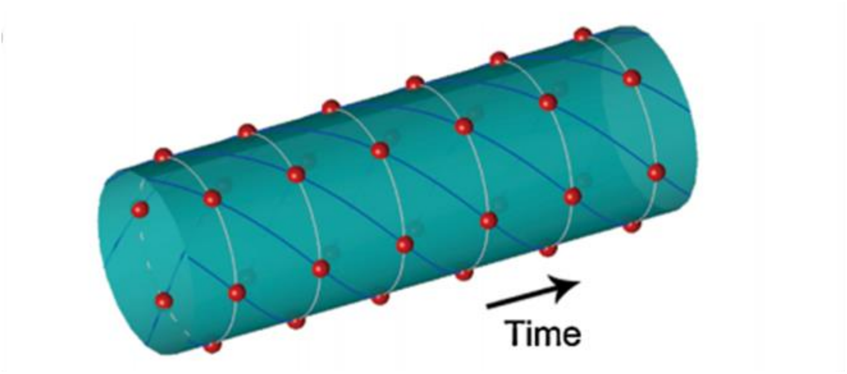
Solion Model

F. Wilczek, Phys. Rev. Lett. **109**, 160401 (2012)



Ion Trap Model

Li et al. Phys. Rev. Lett. 109, 163001 (2012)



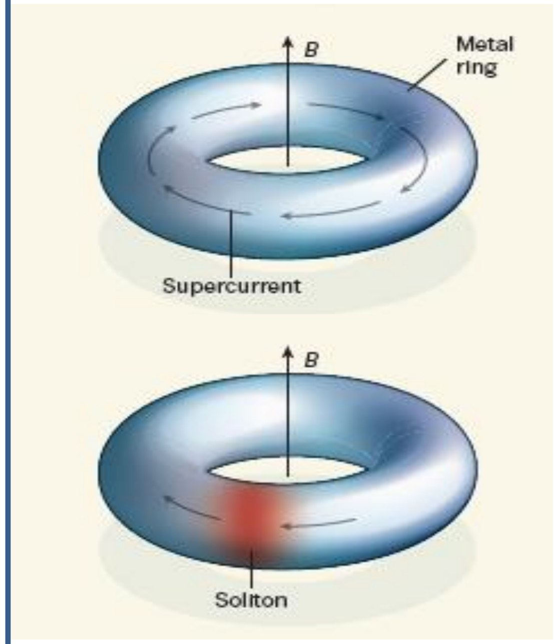
Proposal: Periodically oscillating expectation value at ground state.

Method: Spontaneous breaking of time translation symmetry.

New Types of Crystals?

Time Crystal

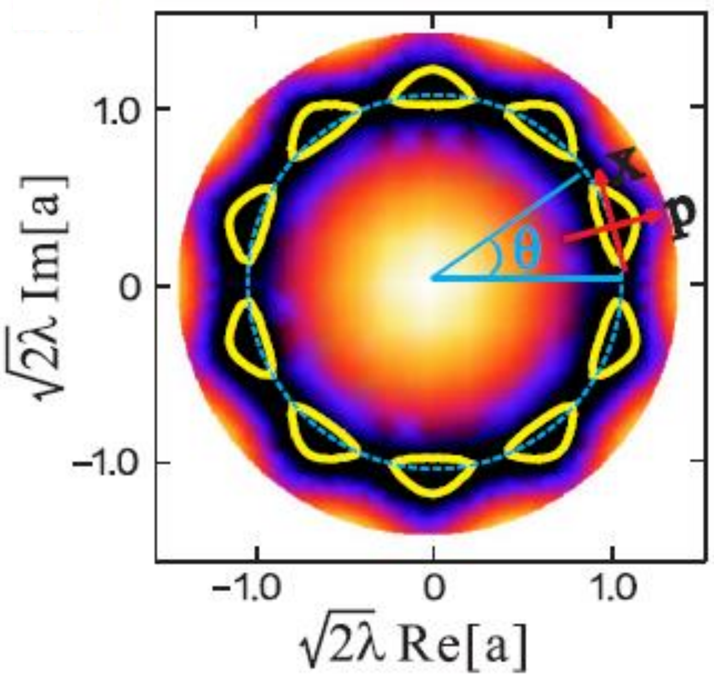
F. Wilczek, PRL **109** (2012)



T space?

Phase-space Crystal

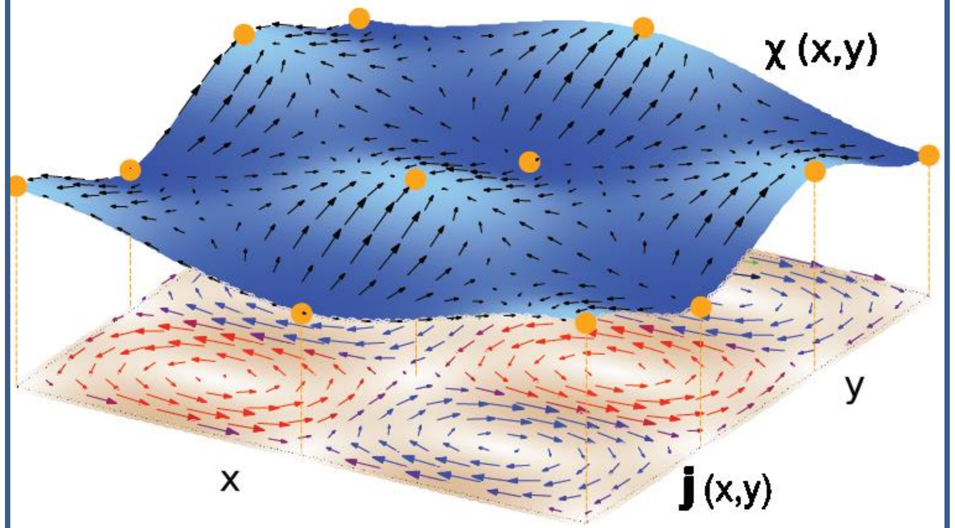
L. Guo et. al., PReL **111** (2013)



p space?

Phase Crystals

P. Holmvall et. al., arXiv:1906.04793



ϕ space?

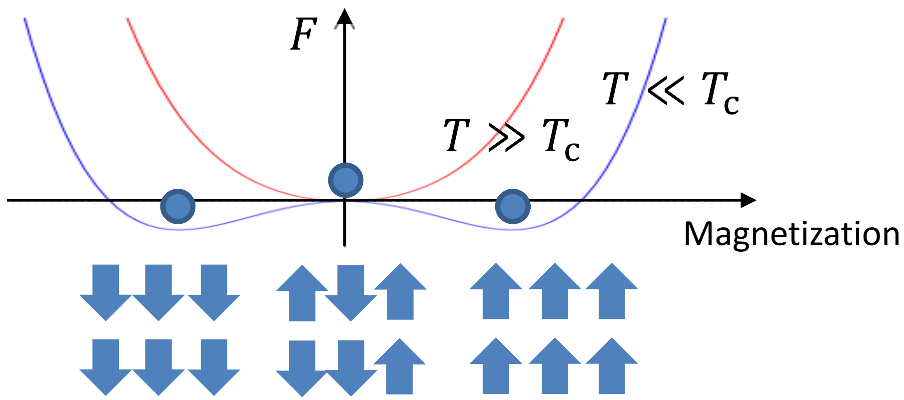
Can these new type of crystals really exist?

Spontaneous Symmetry Breaking

Spontaneous symmetry breaking: Ground state breaks symmetry of Hamiltonian

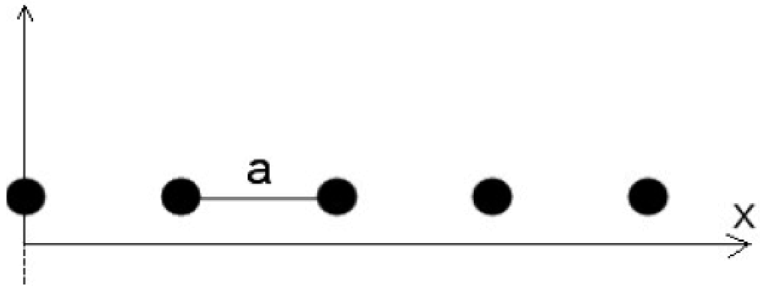
Ising Model: Inversion symmetry symmetry ($S_n \to -S_n$) is broken

$$H = -J\sum_n S_n S_{n+1}$$



Crystal : *Continuous* spatial translation symmetry ($x_i \to x_i + \epsilon \;, \epsilon \ll 1$) becomes *discrete* ($x_i \to x_i + a$)

$$H = \sum_i \frac{p_i^2}{2m_i} + \frac{1}{2}\sum_{i\neq j} U_{ij}(r_i - r_j)$$

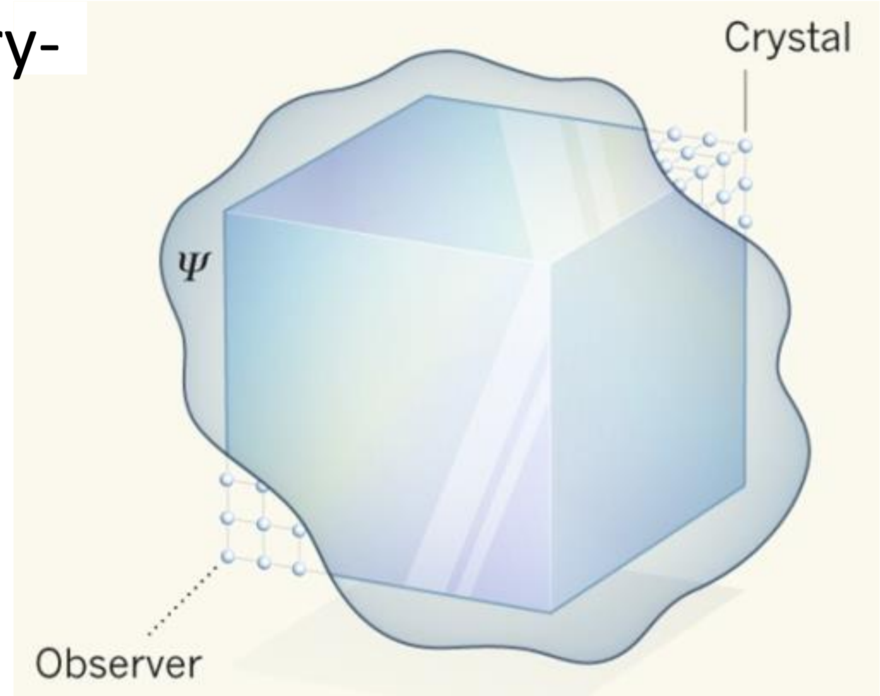


Symmetry Breaking in Quantum Systems?

Ground state of a crystal = Superposition of lattices

- Periodicity is not apparent from crystal's wave function
- Periodicity becomes apparent after a symmetry-breaking interaction (such as measurement).

F. Wilczek, Phys. Rev. Lett. **109**, 160401 (2012)
P. Coleman, *Nature* **493,** 166 (2013)



P. Coleman, *Nature* **493,** 166 (2013)

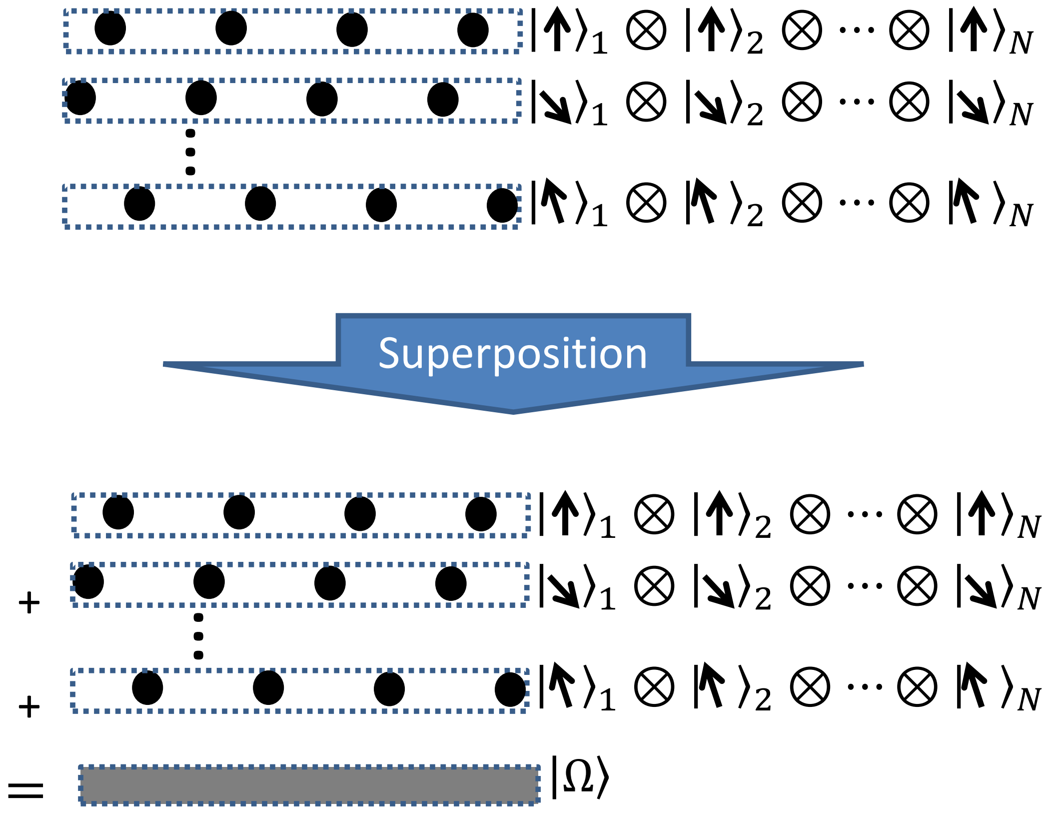
Loss of coherence plays a central role

Quantum Crystals

The symmetry broken state is **not** the true ground state

- Periodicity is not apparent from crystal's wave function
- Periodicity appears after measurement

F. Wilczek, Phys. Rev. Lett. **109**, 160401 (2012)
P. Coleman, *Nature* **493,** 166 (2013)

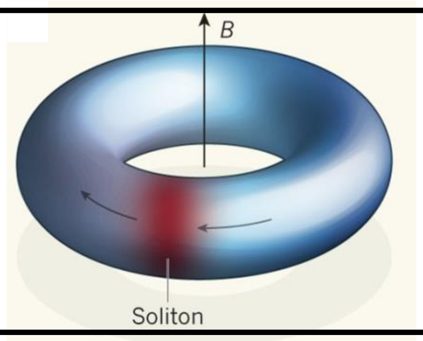


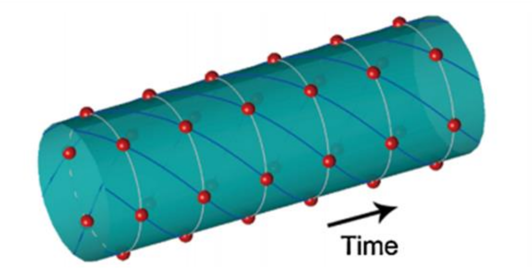
Entangled state

A Brief History of Time Crystals

Original Idea: Spontaneous breaking of time translation symmetry at ground state

F. Wilczek, Phys. Rev. Lett **109**, 160401 (2012)





No-go theorem for $V \to \infty$ limit

H. Watanabe and M. Oshikawa, *PRL* **114** (2015)

Floquet Time Crystal:
Spontaneous breaking of discrete time translation symmetry

K. Sacha, *PRA* **91**, 033617 (2015); etc

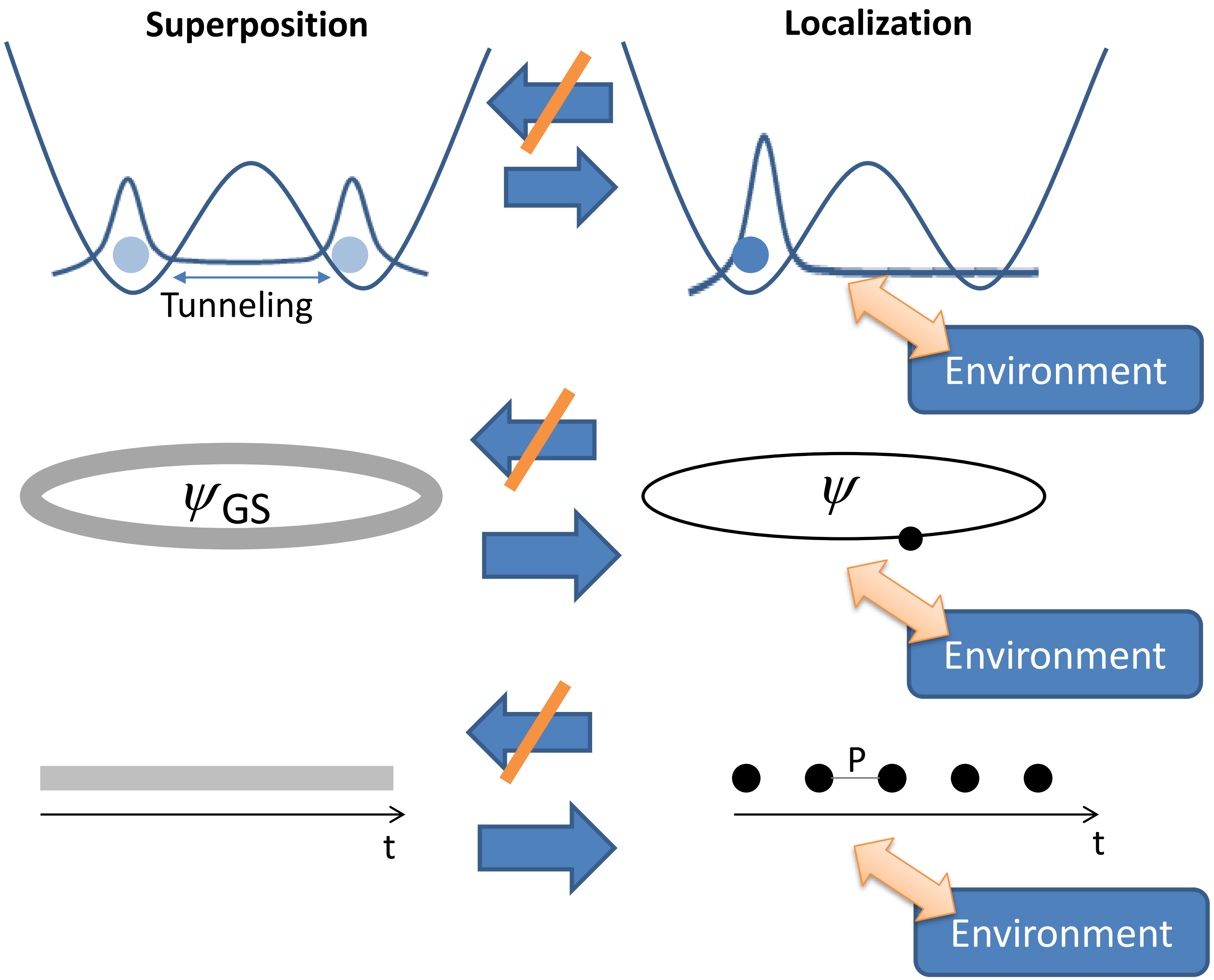
Metastable time crystal with finite lifetime

G. E. Volovik, JEPT Lett. **98**, 491 (2013)

Focus on relation between Topology and Quantization

Time crystal by *decoherence*

Symmetry Breaking by Decoherence



2-state system (Leggett *et al.*, Rev. Mod. Phys. 59, 1 (1987))

Coupling with environment will localize a particle on a ring

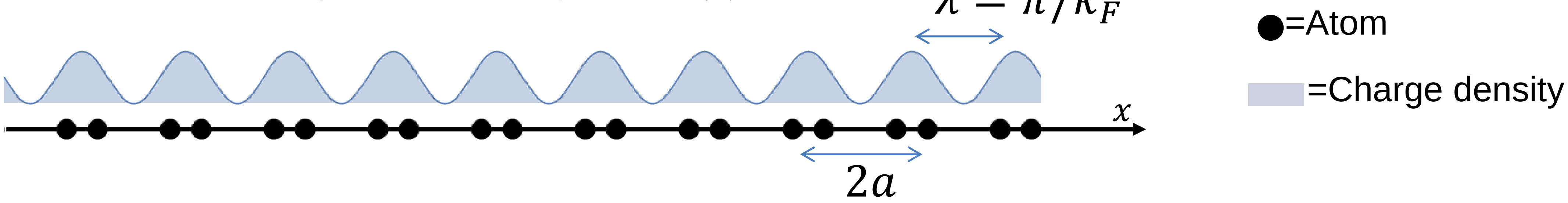
Idea: Breaking of time translation symmetry by decoherence?

Goal

Propose a model of quantum space crystal and quantum time crystal by decoherence using charge density wave.

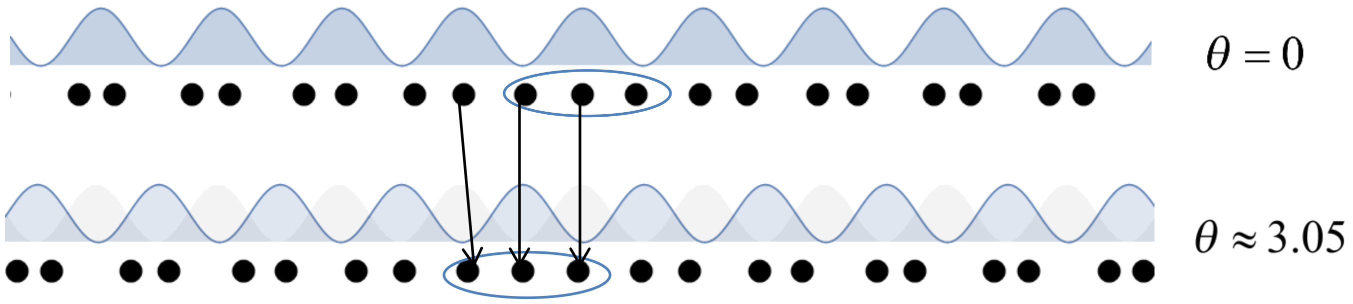
Incommensurate Charge Density Wave (ICDW)

$$n(x,t) = n_0 + n_1 \cos(x/\lambda + \theta(t))$$

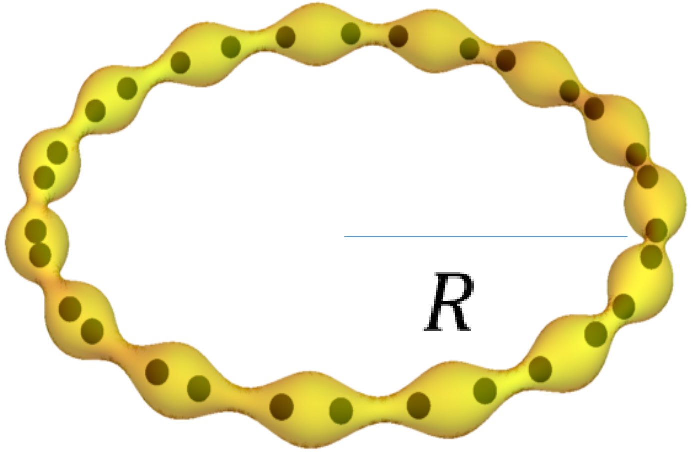


- ❑ Order Parameter: Macroscopic wave function
- ❑ Incommensurate CDW : λ/a =irrational number
  Energy independent of θ ⟹ Fröhlich superconductivity

$$\frac{\lambda}{a} = 2.11$$

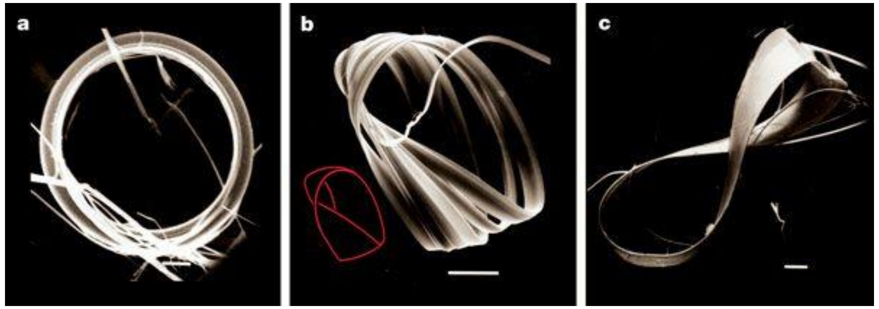


ICDW Ring



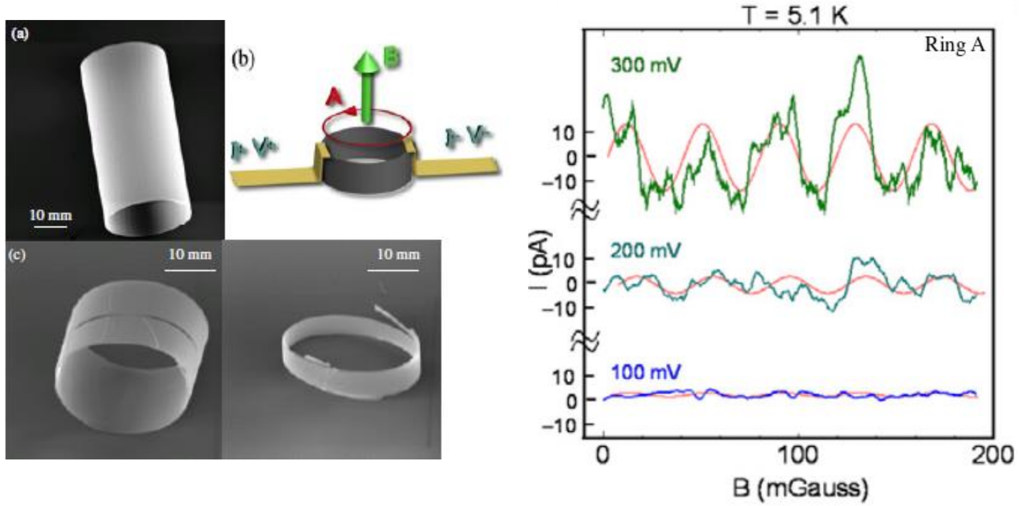
- ❑ A boson, so superposition is possible
- ❑ Macroscopic wave function
- ❑ A space crystal, direct measurement is possible

Ideal for quantum space crystal with coherence.



Topological crystals

S. Tanda et.al., *Nature* **417**, 139 (2002)



CDW Aharonov-Bohm effect

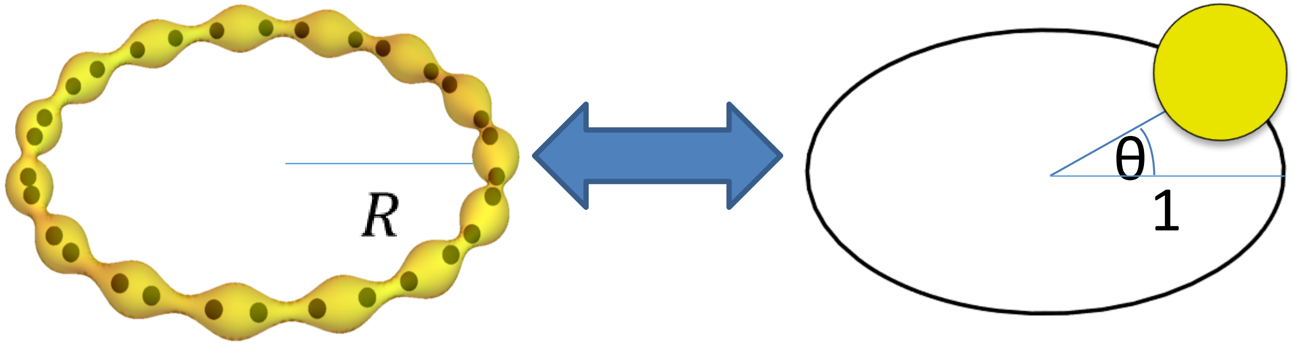
Tsubota et al, Europhys. Lett. **97**, 57011 (2012)

Classical Dynamics of ICDW ring

Assumption: $\theta' = \frac{\mathrm{d}\theta}{dx} = 0$

Consider the CDW phase Lagrangian

[c.f. H. Fukuyama, JPSJ **41**, 513 (1976)]



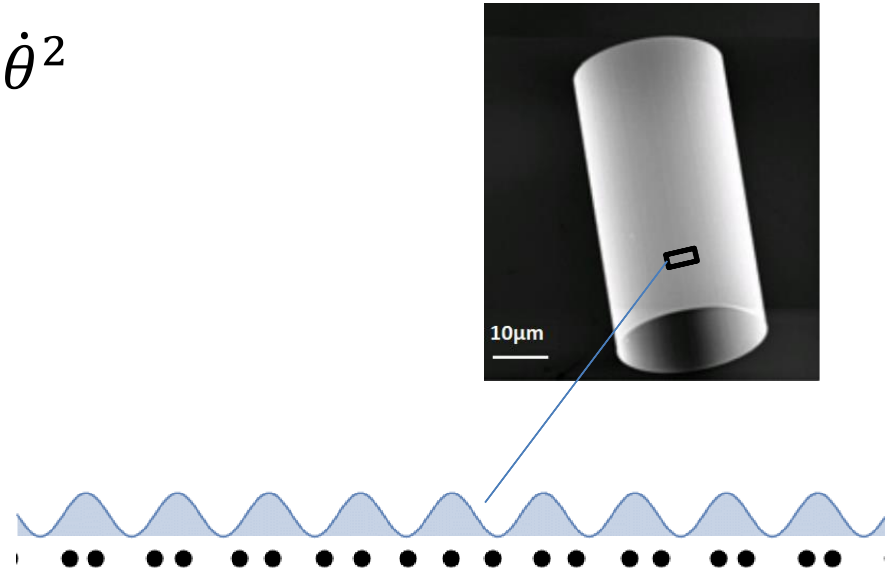
$$L(\dot{\theta}, \theta') = \frac{\hbar v_{\mathrm{F}}}{4\pi c_0^2} \int_0^{2\pi R} dx \left[\dot{\theta}^2 - c_0^2 \theta'^2\right] = \frac{I}{2} \dot{\theta}^2$$

Fermi Velocity : v_{F}

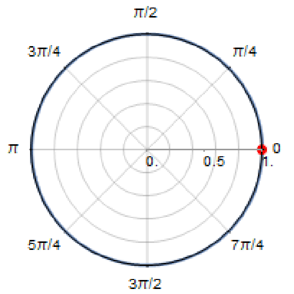
Phason velocity : c_0

Moment of Inertia : $I = R\hbar v_{\mathrm{F}} / c_0^2$

Collective coordinate : $\theta(t) = 2k_{\mathrm{F}} x(t)$



Phase



- ➢ Equivalent to a single free particle!

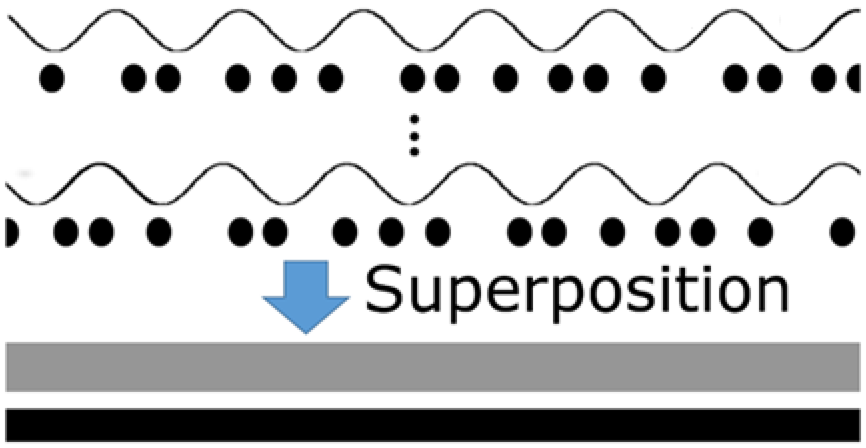
How can we quantize this system?

Ground state of ICDW

Charege Density Operator: $\hat{n} = n_0 + \frac{n_1}{2}\left(e^{2ik_{\mathrm{F}}x}\widehat{e^{i\theta}} + e^{-2ik_{\mathrm{F}}x}\widehat{e^{i\theta}}^{\dagger}\right)$

Expectation value: $\langle\hat{n}\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathrm{d}\theta[n_0 + n_1\cos(2k_{\mathrm{F}}x + \theta)] = n_0$

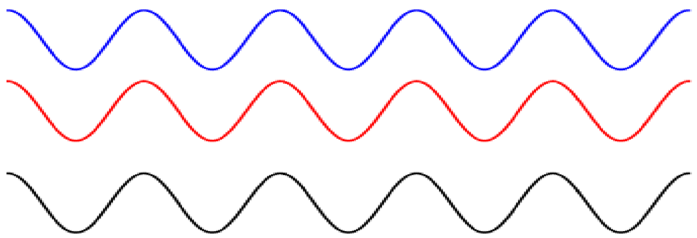
The ground state is a uniform superposition of CDW
(The phase if not determined)



Ground state of ICDW ring

Charege Density Operator : $\hat{n} = n_0 + \frac{n_1}{2}\left(e^{2ik_F x}\widehat{e^{i\theta}} + e^{-2ik_F x}\widehat{e^{i\theta}}^{\dagger}\right)$

Expectation value : $\langle \hat{n}(t) \rangle = n_0 + n_1 \int_{-\pi}^{\pi} \mathrm{d}\theta \, \frac{1}{2\pi} [n_L(\theta, t) + n_R(\theta, t)] = n_0$



$n_L(\theta, x, t)$

$n_R(\theta, x, t)$

$[n_L(\theta, x, t) + n_R(\theta, x, t)]/2$

Periodic oscillation

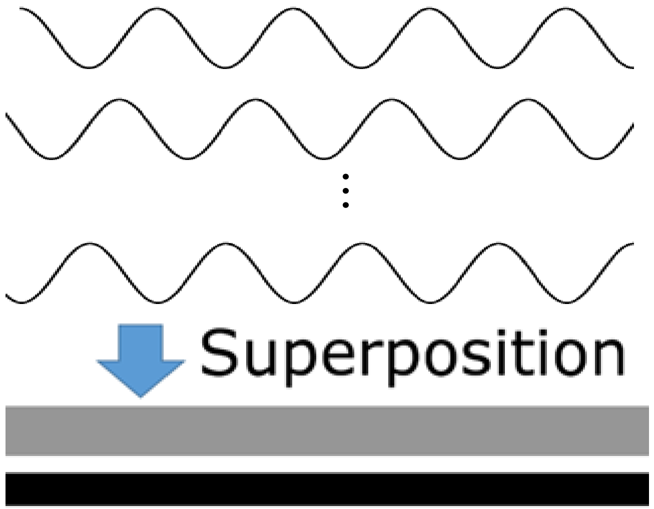
$P = 4\pi I/\hbar \propto R,$

$I =$ Moment of Inertia

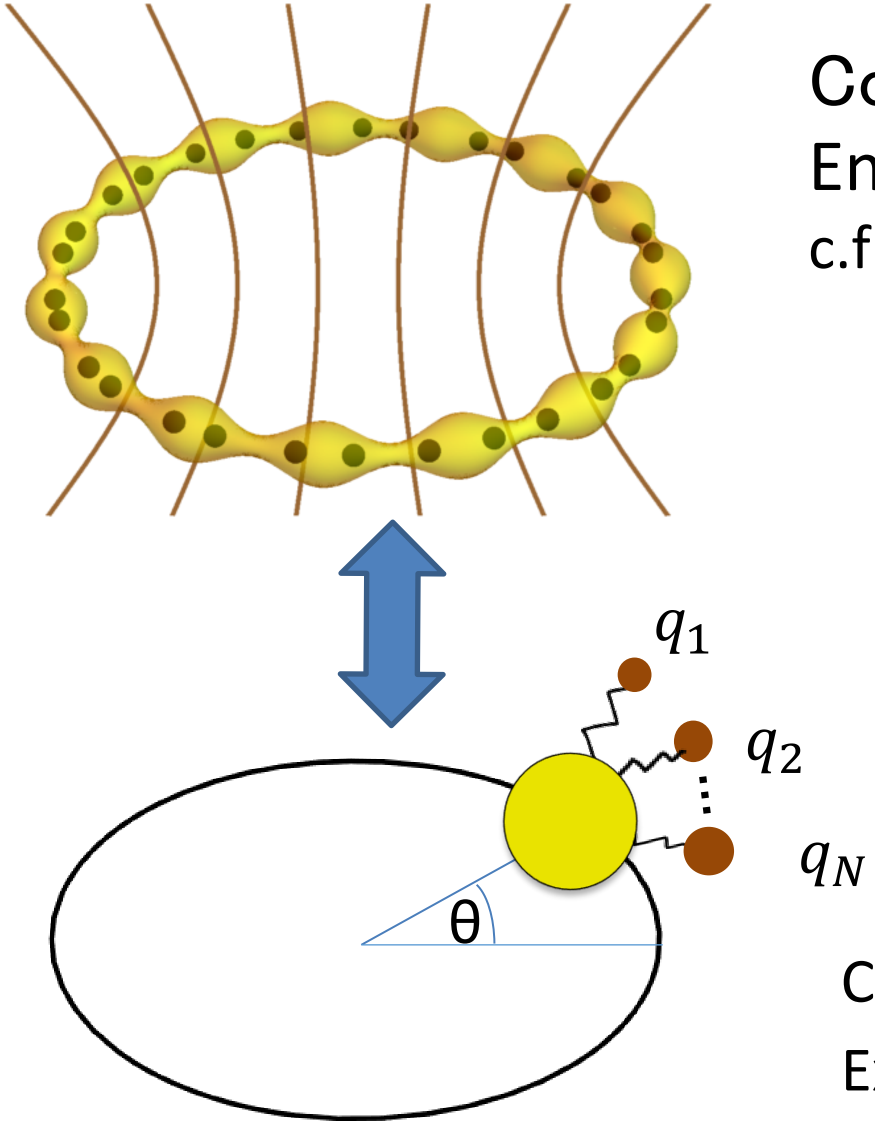
The ground state is a uniform superposition of CDW

(The phase if not determined)

∴We need decoherence to break superposition



Model: ICDW Ring With Environment



Consider fluctuating magnetic flux

Environment : Harmonic oscillators $\{q_i\}$

c.f. Caldeira and Leggett. Physica A **121**, 587 (1983)

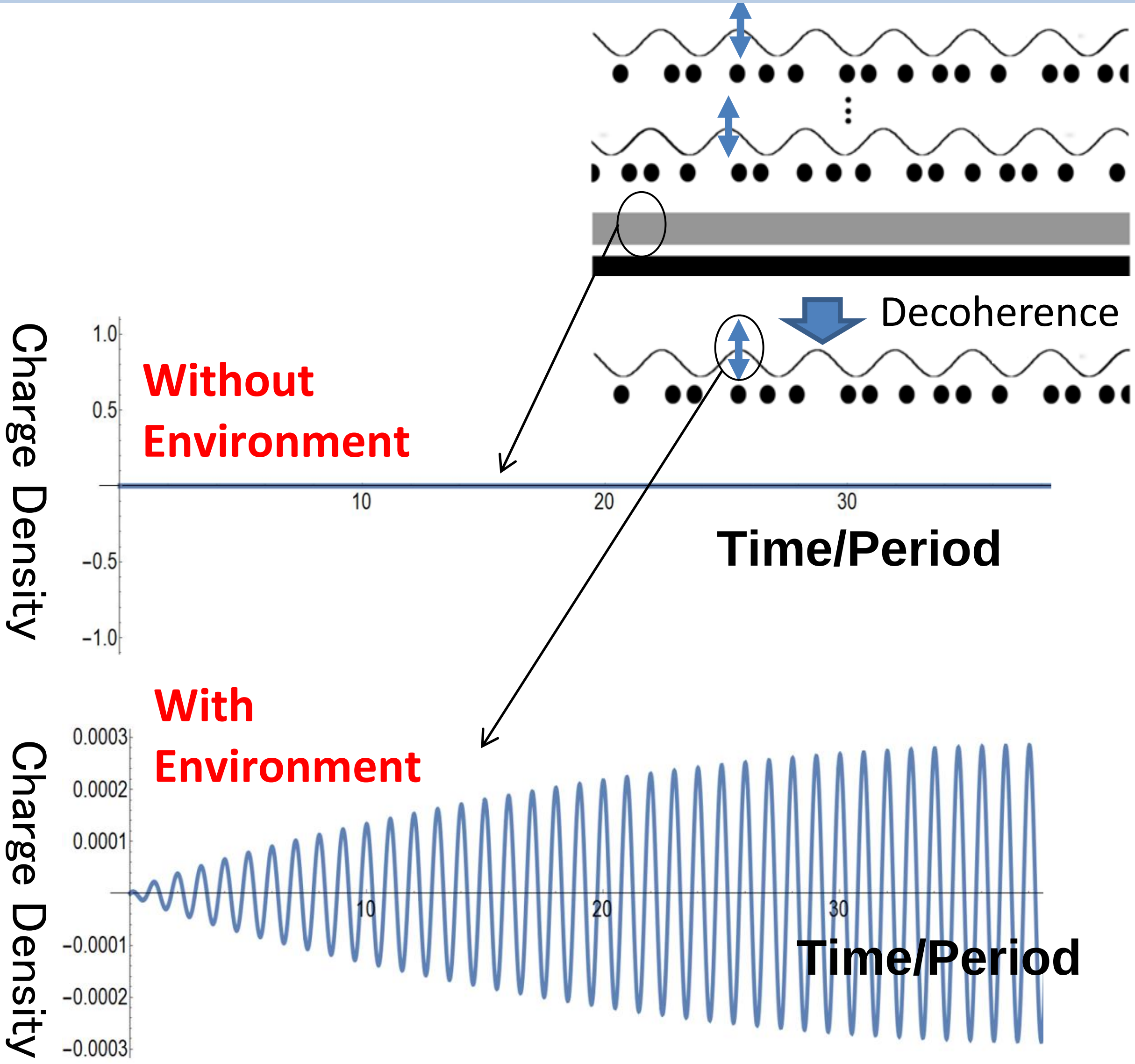
$$\hat{H} = \frac{1}{2I}\left(L - \sum_j c_j \hat{q}_j\right)^2 + \sum_j \left(\frac{1}{2m_j}\hat{p}_j^2 + \frac{m_j \omega_j^2}{2}\hat{q}_j^2\right)$$

Interaction with Environment   Environment Hamiltonian

Charge density operator : $\hat{n} = n_0 + n_1 \hat{C}$

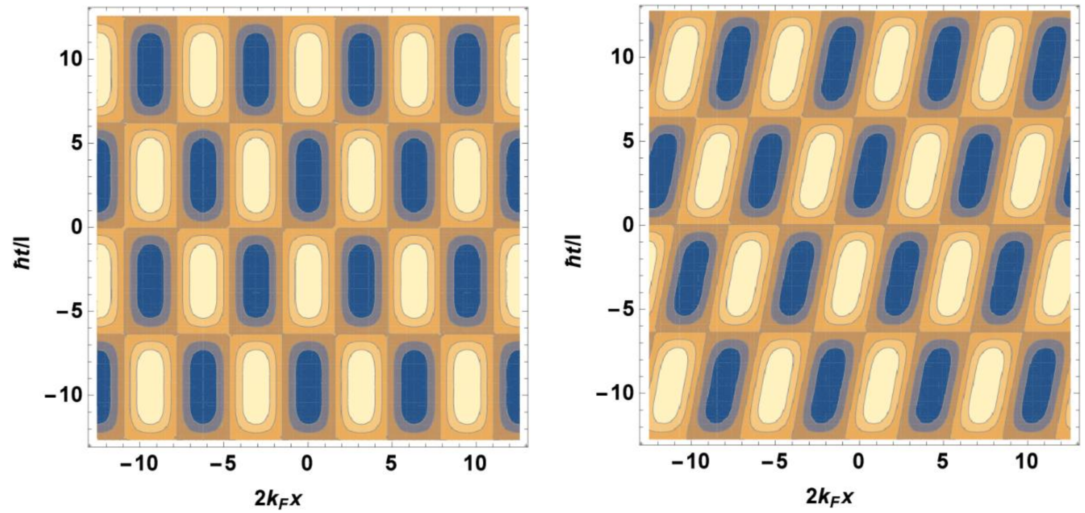
Expectation value : $n(t) = \langle \psi_{\mathrm{GS}}, e^{i\hat{H}t/\hbar}\hat{n}e^{-i\hat{H}t/\hbar}\psi_{\mathrm{GS}}\rangle$

Time Crystal by Decoherence

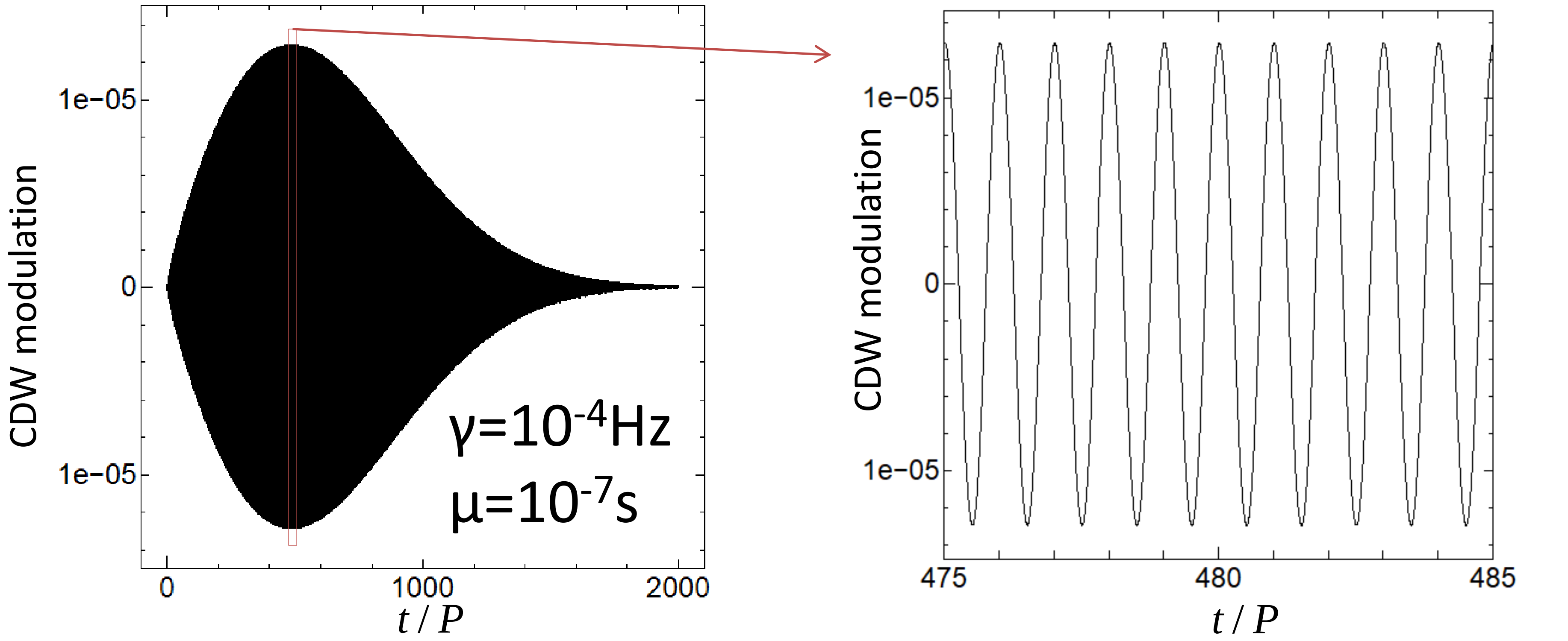


$R \sim 10^{-6}$m: radius,
I: moment of inertia,
$v_{\mathrm{F}} \sim 10^{6}$m/s: Fermi velocity,
$v' \sim 10^{-3} v_{\mathrm{F}}$: CDW phason velocity

Period $P = \dfrac{4\pi I}{\hbar} = \dfrac{4\pi R}{v'} \sim 1\mu\mathrm{s}$



Finite Lifetime due to Dissipation



Metastable time crystal

$$P \ll \tau \propto 1/\gamma$$

QTC period

"Life time"

(γ: Interaction strength with environment)

G. E. Volovik, JEPT Lett. 98, 491 (2013)

Origin of Oscillation: Uncertainty Principle

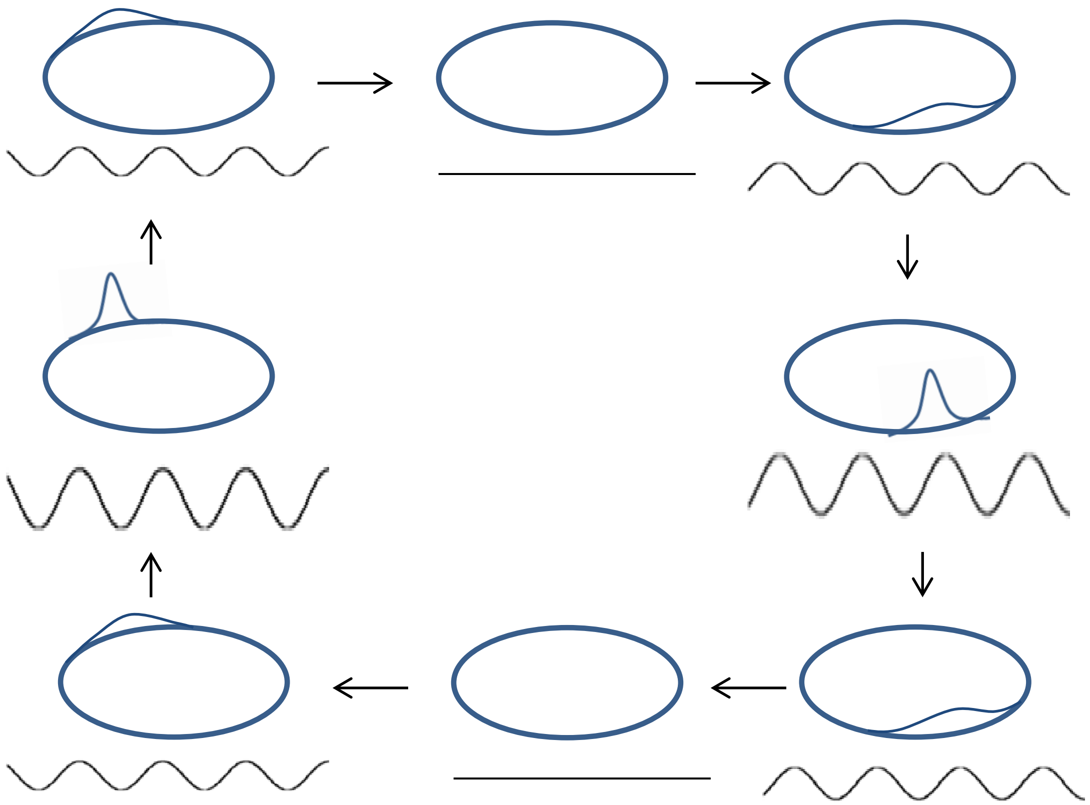
$\Delta x \Delta p \geq \hbar/2$

Momentum uncertainty: $\Delta p \sim \hbar/2R$

wave packet spreads with velocity

$$\mathrm{v} \sim \frac{\Delta p}{m} \sim \frac{\hbar}{2Rm}$$

Assemble on opposite side of the ring

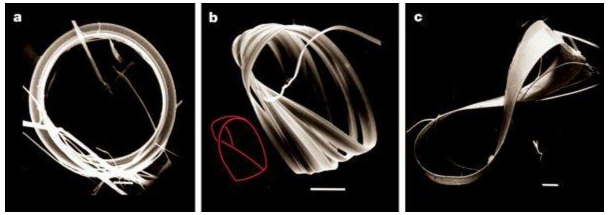


Period $P = \frac{2\pi R}{v} = \frac{4\pi m R^2}{\hbar} = \frac{4\pi I}{\hbar}$

Interference of wave function+uncertainty relation+ring system=Time crystal!

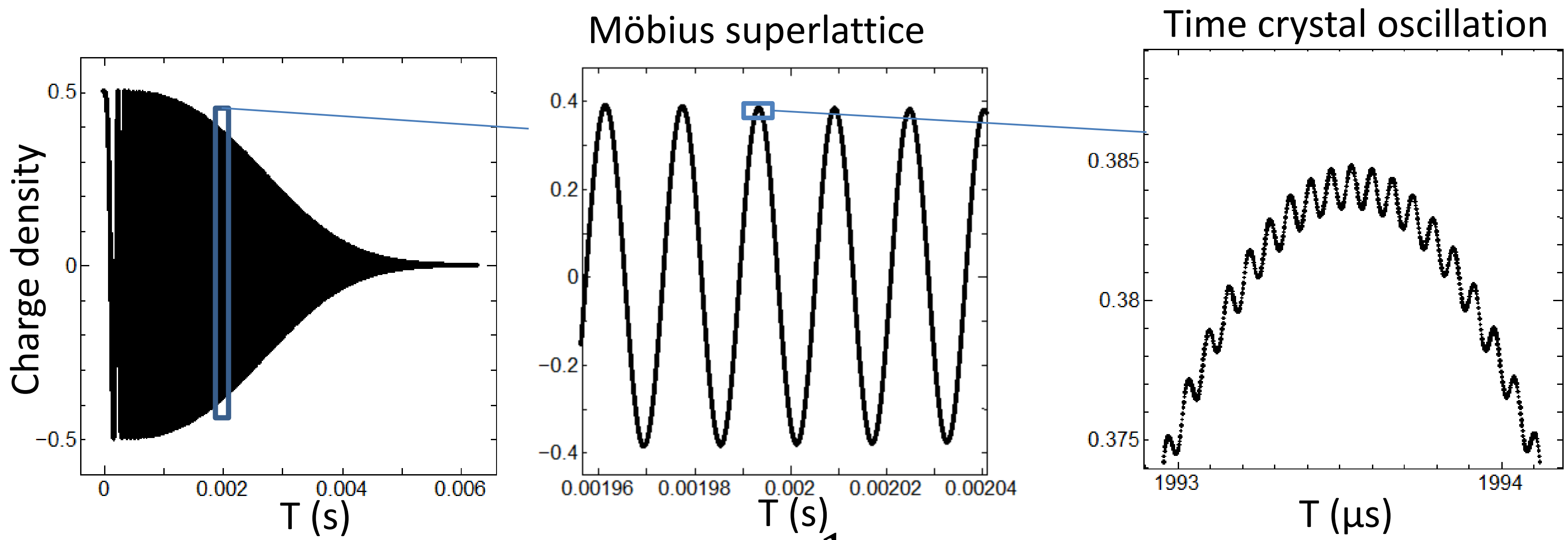
But, this is incomplete. We must consider “time operators” (later).

Time Crystal in Möbius ring



Antiperiodic boundary condition: $\psi(\theta + 2\pi) = -\psi(\theta)$

Ground state: $\psi_{\rm GS}(\theta) \propto \cos\frac{\pi\theta}{2}$

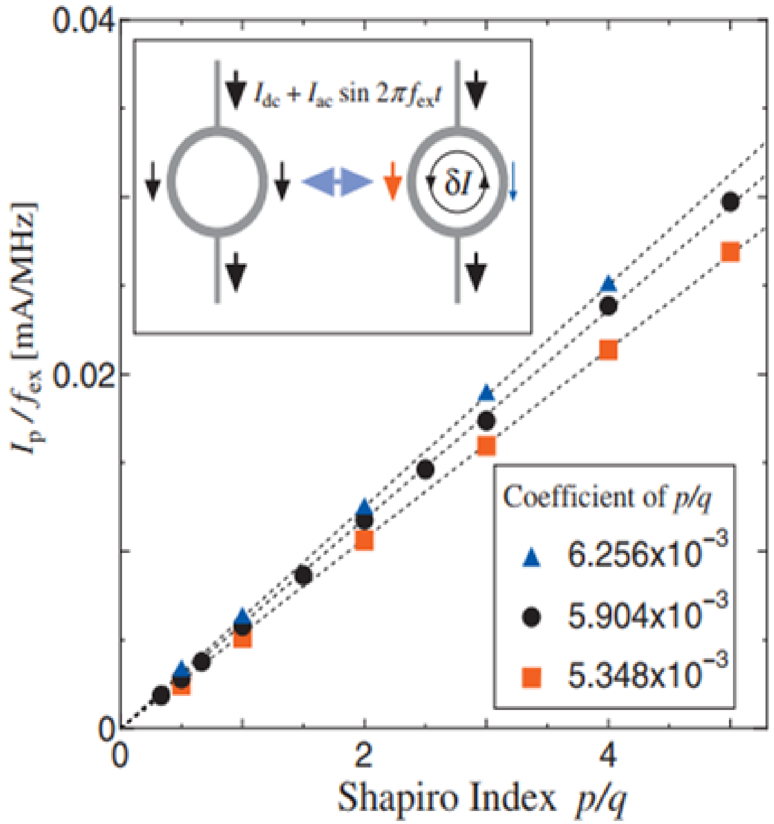


"Superlattice" with length γ^{-1}

Small oscillation=Periodicity of ring

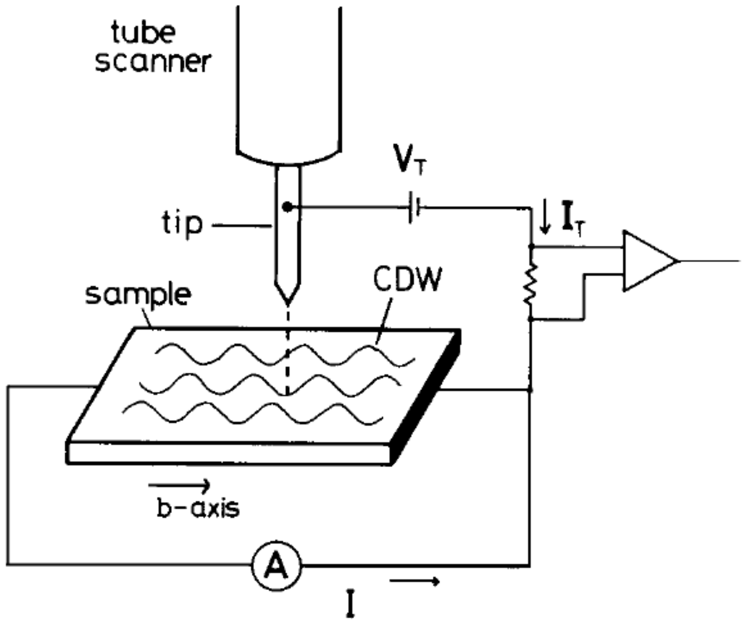
Strong oscillation from the beginning

Experiments?



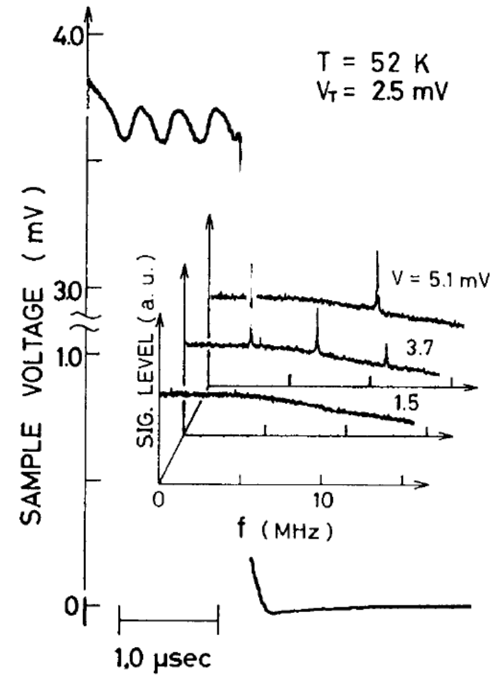
- Existence of circulating CDW current
- Elongation of current damping time.

T. Matsuura, K. Inagaki, & S. Tanda PRB 79, 014304 (2009)



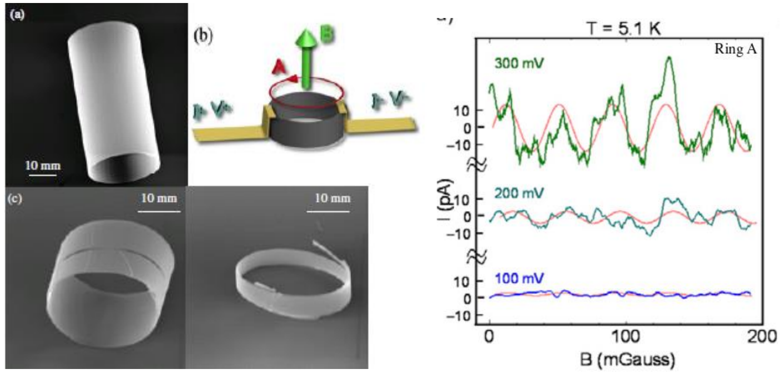
STM observation of sliding charge-density wave

K. Nomura, K. Ichimura, Journal of Vacuum Science & Technology A, **8** (1990)



Direct observation using narrow-band noise

M. Ido *et al.*, Physica B+C, **143** (1986)



ICDW ring Tsubota et al, EPL. **97**, 57011 (2012)

Summary of Results

I Proposed a model of metastable quantum time crystal by decoherence using ICDW ring

- ❑ Ground state without environment= superposition of CDW with various phases
- ❑ Decoherence breaks time translation symmetry
- ❑ Periodic Interference of ICDW wave function

Next:

What is the origin of quantum time crystal?

Consider **time operators.**

Quantum Time Crystal

Time Operator

Is Time a Physical Observable?

- In quantum mechanics, position and momentum are operators (=physical observable).
- From the point of view of relativity, it is desirable to define time operators

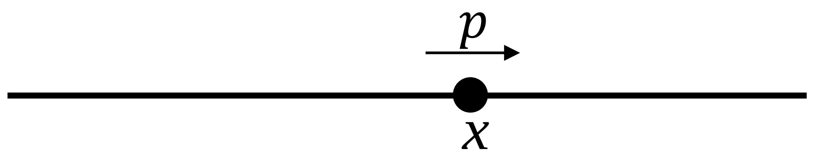
$$[\hat{x}, \hat{p}] = i\hbar \xrightarrow{\text{Analogy}} [\hat{H}, \hat{T}] = i\hbar \ ?$$

- However, how to define time operators is still an open problem
  →In quantum mechanics, time is treated as a parameter, not an observable (W. Pauli 1926)

- Time Crystal :Continuous time becomes discrete
  F. Wilczek Phys. Rev. Lett **109**, 160401, (2012);
  K. Nakatsugawa, T. Fujii and S. Tanda, Phys. Rev. B **96**, 094308 (2017);
- Can we solve the problem of time operators using time crystals?

Time-of-arrival operator

Free particle in one dimension



Y. Aharonov & D. Bohm, Phys. Rev. 122, 1649 (1961)

$$\hat{H} = \frac{\hat{p}^2}{2m}$$

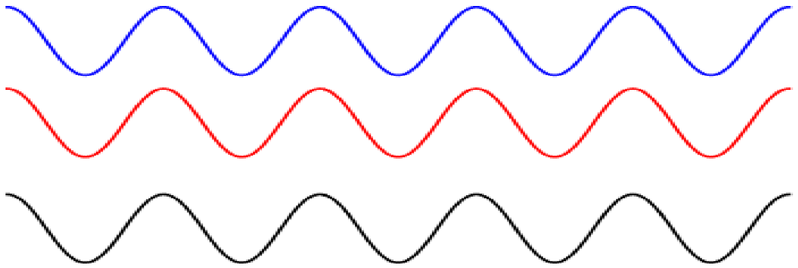
$$\hat{T}_{\mathbb{R}} = -\frac{m}{2}\left(\frac{1}{\hat{p}}\hat{x} + \hat{x}\frac{1}{\hat{p}}\right)$$

$$[\hat{x}, \hat{p}] = i\hbar \Rightarrow \left[\hat{H}, \hat{T}_{\mathbb{R}}\right] = i\hbar$$

Physical interpretation of Eigenvalues of $\hat{T}_{\mathbb{R}}$: Time required to travel between two points

Time operator from quantum time crystal

ICDW provide new time operators
(Mathematical origin of time crystal)



K. Nakatsugawa, T. Fujii, A. Saxena & S. Tanda, J. Phys. A **53**, 025301 (2020)

$$\hat{H} = \frac{\hat{p}^2}{2m}$$

$$\hat{T}_{S^1} = \frac{mR^2}{2i}\left[\frac{1}{\hat{\pi}_\theta + \hbar/2}\widehat{W}^\dagger - \widehat{W}\frac{1}{\hat{\pi}_\theta + \hbar/2}\right]$$

$$[\hat{S}, \hat{\pi}_\theta] = i\hbar\hat{C} \rightarrow [\hat{x}, \hat{p}] = i\hbar$$

$$[\widehat{H}, \hat{T}_{S^1}] = i\hbar\hat{C} \rightarrow [\widehat{H}, \hat{T}_{\mathbb{R}}] = i\hbar$$

$$\hat{S} = (\widehat{W} - \widehat{W}^\dagger)/2i \quad \hat{C} = (\widehat{W} + \widehat{W}^\dagger)/2$$

$$\widehat{W} = \widehat{e^{i\theta}}$$

Physical interpretation of Eigenvalues of $\hat{T}_{\mathbb{R}}$: Time required to move by one wavelength

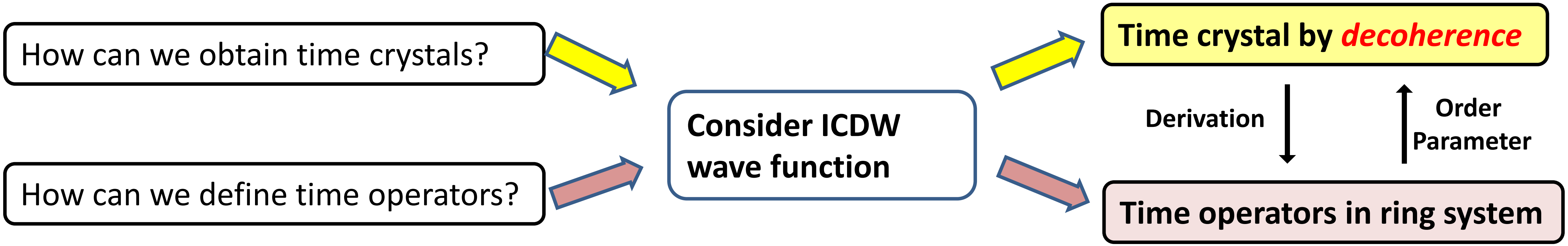
Summary

- **Quantum Time Crystal by Decoherence :**

  K. Nakatsugawa, T. Fujii & S. Tanda, Physical Review B **96**, 094308 (2017)

- **Time Operators from Time Crystals :**

  K. Nakatsugawa, T. Fujii, A. Saxena & S. Tanda, J. Phys. A **53**, 025301 (2020)



Quantization of CDW lead to new phenomena!