Skyrmion-electron bound states in a Néel antiferromagnet

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Topological textures:

- Domain walls
- Vortices
- Skyrmions
- …

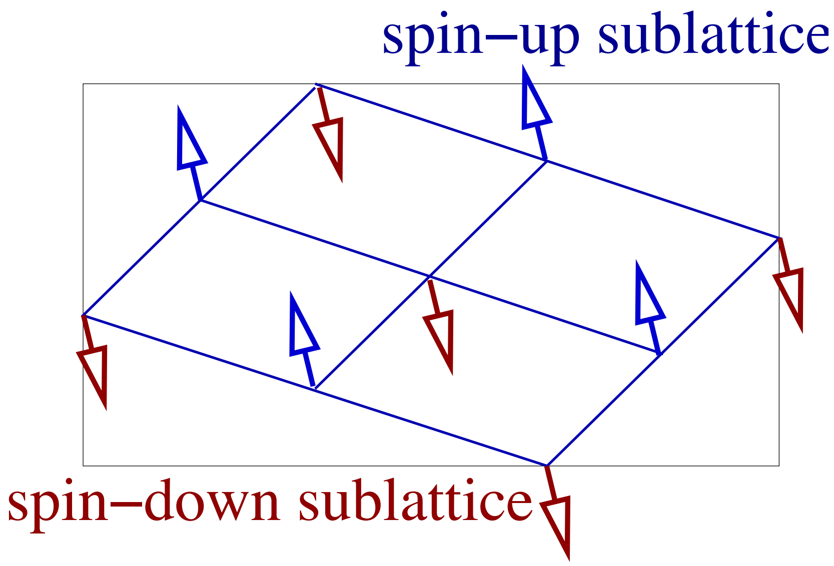
- Nuclear physics
- QCD
- Statistical Physics
- String theory
- Condensed matter

- Vortex lattice in a superconductor
- Skyrmion crystal in a magnet
- Memory devices using Skyrmions and domain walls

Outline:

- Néel antiferromagnet
- A texture
- Effective electron Hamiltonian
- A texture in the form of a Skyrmion
- Skyrmion-electron bound states
- Broader implications

Néel antiferromagnet :



$$\mathbf{M}(\mathbf{r}+\mathbf{a}) = -\mathbf{M}(\mathbf{r})$$

$$\mathbf{M}(\mathbf{r}) = \mathbf{M}_0 \exp[\, i\,(\mathbf{Q}\cdot\mathbf{r})\,]$$

$$\mathbf{Q} = \left(\frac{\pi}{a}\,,\ \frac{\pi}{a}\right)$$

$\mathbf{M}(\mathbf{r})$ couples to conduction electron spin σ :

$$J\,(\mathbf{M}(\mathbf{r})\cdot\sigma) \sim J\,(\mathbf{M}_0\cdot\sigma)\,e^{i\mathbf{Q}\cdot\mathbf{r}} = (\boldsymbol{\Delta}\cdot\sigma)e^{i\mathbf{Q}\cdot\mathbf{r}}$$

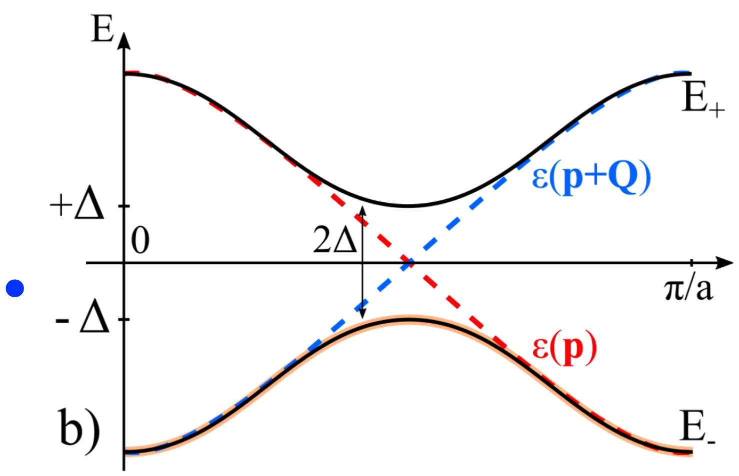
… that is, couples any momentum $\mathbf{p}$ to $\mathbf{p}+\mathbf{Q}$.

Bispinor $\Psi = (\psi_{\mathbf{p}}\,, \psi_{\mathbf{p}+\mathbf{Q}})$

The Hamiltonian : $\mathcal{H} = \begin{bmatrix} \varepsilon(\mathbf{p}) & (\boldsymbol{\Delta}\cdot\sigma) \\ (\boldsymbol{\Delta}\cdot\sigma) & \varepsilon(\mathbf{p}+\mathbf{Q}) \end{bmatrix}$

The spectrum $E_{\mathbf{p}} = \varepsilon_{+}(\mathbf{p}) \pm \sqrt{|\boldsymbol{\Delta}|^2 + \varepsilon_{-}^2(\mathbf{p})}$,

where $\varepsilon_{\pm}(\mathbf{p}) \equiv \frac{1}{2}\left[\varepsilon(\mathbf{p}) \pm \varepsilon(\mathbf{p}+\mathbf{Q})\right]$.



Generalisation for a non-uniform $\boldsymbol{\Delta}_{\mathbf{r}} = \hat{\mathbf{n}}_{\mathbf{r}}\Delta$?

Hamiltonian for a non-uniform $\mathbf{\Delta}_{\mathbf{r}} = \hat{\mathbf{n}}_{\mathbf{r}} \Delta$?

Effective-mass theory near band extrema:

$$\varepsilon(\mathbf{p}),\, \varepsilon(\mathbf{p}+\mathbf{Q}) \to \varepsilon_{\mathbf{p}_0}(-i\hbar\nabla)\,,\, \varepsilon_{\mathbf{p}_0+\mathbf{Q}}(-i\hbar\hat{\nabla})$$

Perform a spin rotation $U_{\mathbf{r}}$: $U_{\mathbf{r}}^{\dagger}(\hat{\mathbf{n}}_{\mathbf{r}} \cdot \sigma) U_{\mathbf{r}} = \sigma^z$

Off diagonal – uniform $\mathbf{\Delta}$: $\mathcal{H} = \begin{bmatrix} \varepsilon(\mathbf{p}) & (\mathbf{\Delta} \cdot \sigma) \\ (\mathbf{\Delta} \cdot \sigma) & \varepsilon(\mathbf{p}+\mathbf{Q}) \end{bmatrix}$

On the diagonal – a Peierls substitution:

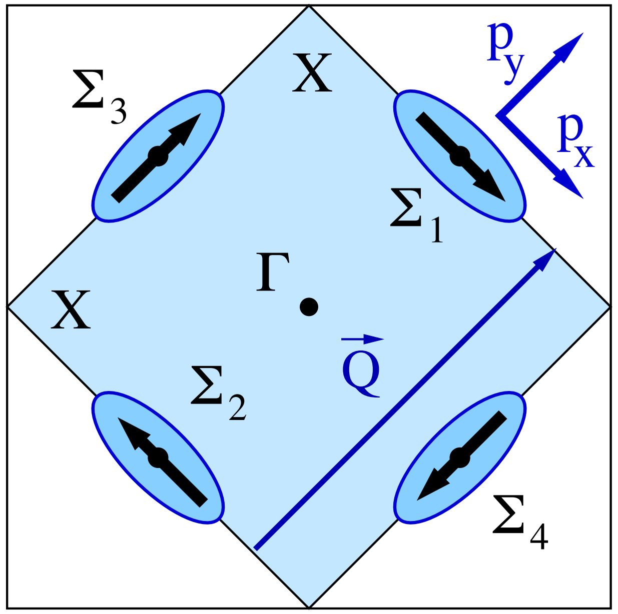
$\hat{p}_i \to \hat{p}_i + (\mathbf{A}_i \cdot \sigma)$ with $(\mathbf{A}_i \cdot \sigma) = A_i^{\alpha} \sigma^{\alpha} = -i\hbar U_{\mathbf{r}}^{\dagger} \partial_i U_{\mathbf{r}}$

Now, the effective Hamiltonian :

Stay close to the band edge ($\frac{E-\Delta}{\Delta} \ll 1$)

and go from "Dirac" to 2×2 "Schrödinger"

Where are our band extrema ?



What is the $\varepsilon_{\mathbf{p}_0}(\hat{\mathbf{p}})$?

$$\varepsilon_{\mathbf{p}_0}(\hat{\mathbf{p}}) = \mathbf{v} \cdot \hat{\mathbf{p}} + \frac{\hat{p}_i^2}{2m_i}$$ and ...

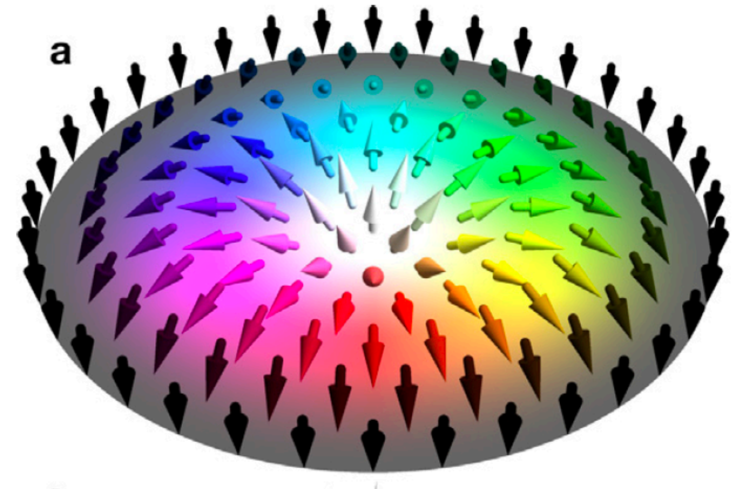
… and $$\mathcal{H}_\Sigma = \frac{\left(\hat{p}_i + A_i^z \sigma_z\right)^2}{2m_i^*} + \frac{\left(A_i^{\|}\right)^2}{2m_i} + v\left(\mathbf{A}_y^{\|} \cdot \sigma\right)$$

Skyrmion :

static input to the electron problem.

Now, on to our Skyrmion :

The 'Néel' profile :



$$\hat{\mathbf{n}}_{\mathbf{r}} = (\sin\theta_r \cos\phi, \sin\theta_r \sin\phi, \cos\theta_r), \; r = \sqrt{x^2+y^2}, \; \phi = \arctan\frac{y}{x}$$

Toy problem : isotropic antiferromagnet

Energy density : $J(\nabla \hat{\mathbf{n}}_{\mathbf{r}})^2$

… the Belavin-Polyakov Skyrmion (1975)

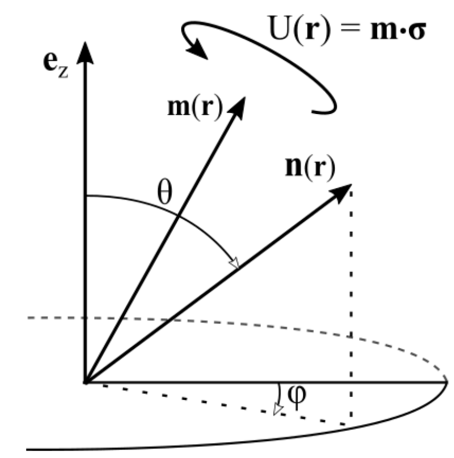
The Belavin-Polyakov Skyrmion:

Topological sectors, labelled by $\mathcal{Q} = 0, \pm 1, \pm 2 ...$

Energy $E = 4\pi J |\mathcal{Q}|$

A single length, radius R: $\sin\theta = \dfrac{2z}{1+z^2}, \; z = \dfrac{r}{R}$

Pick a gauge : $U_{\mathbf{r}} = (\mathbf{m}_{\mathbf{r}} \cdot \sigma)$



… and back to the Hamiltonian, term by term :

$$\mathcal{H}_\Sigma = \frac{\left(\hat{p}_i + A_i^z \sigma_z\right)^2}{2m_i^*} + \frac{\left(A_i^{\parallel}\right)^2}{2m_i} + v\left(\mathbf{A}_y^{\parallel} \cdot \sigma\right)$$

1: $\frac{\left(\hat{p}_i + A_i^z \sigma_z\right)^2}{2m_i^*}$, where $A_x^z = \frac{-\hbar y}{R^2 + r^2}$, $A_y^z = \frac{\hbar x}{R^2 + r^2}$

A_i^z produces geometric flux $\pm 2\pi\hbar$.

2: $\frac{\left(A_i^{\parallel}\right)^2}{2m_i} = \frac{\hbar^2}{2R^2}\left[\frac{1}{m_x} + \frac{1}{m_y}\right]\frac{1}{(1+z^2)^2}$ repulsive potential.

3: $v\left(\mathbf{A}_y^{\parallel} \cdot \sigma\right) = -\frac{\hbar v}{R}\frac{\sigma^x}{1+z^2}$

- Sk-induced SOC!
- Attraction!

Large Skyrmion ($R \gg \xi = \dfrac{\hbar v}{\Delta}$)

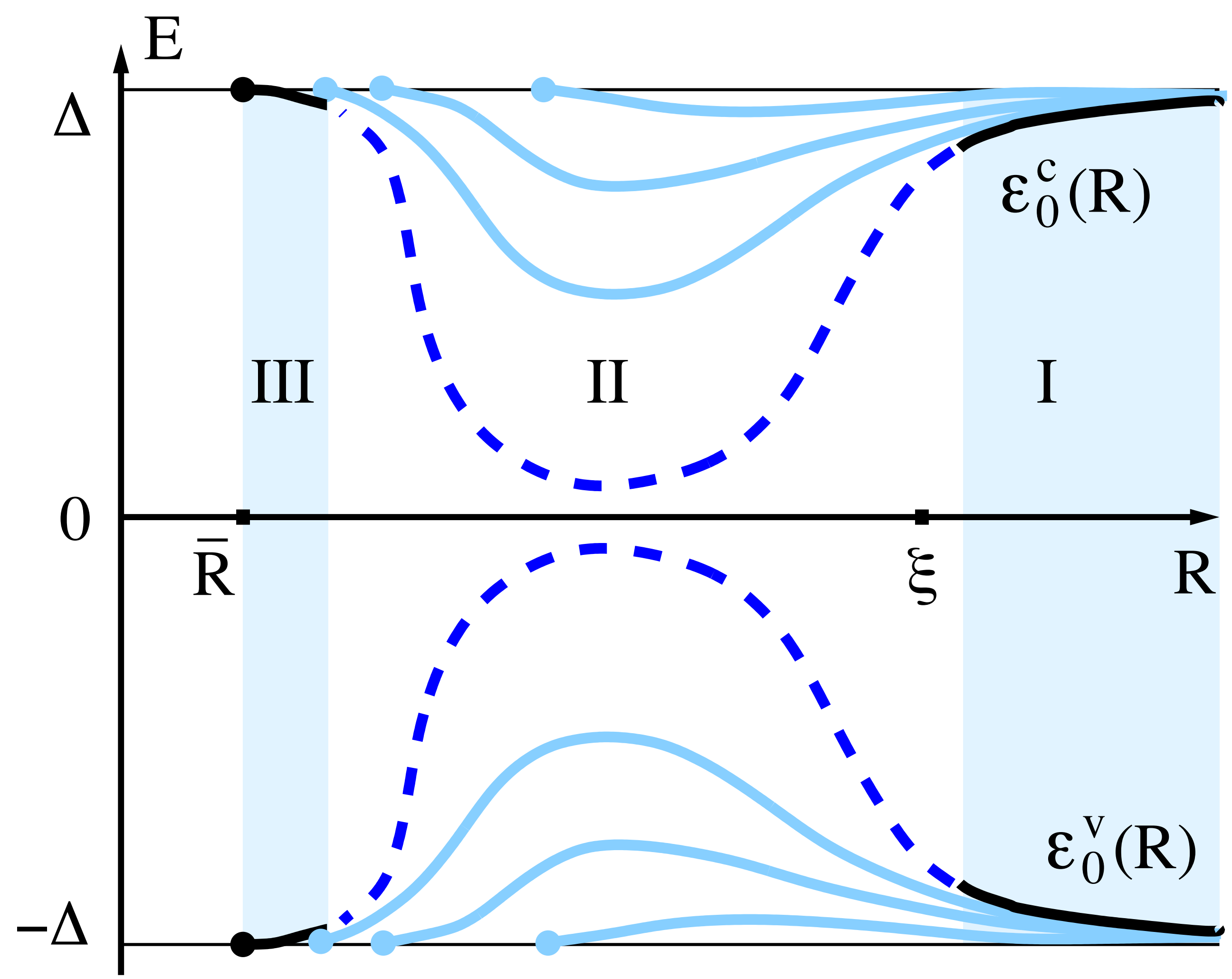
Attraction $-\Delta \dfrac{\xi}{R} \dfrac{\sigma^x}{1+z^2}$ dominates :

$$\epsilon_0^c(R) = -\Delta \frac{\xi}{R} \left[1 - \frac{1}{2^{1/2}} \sqrt{\frac{\xi}{R}} \right]$$

– For $R \gg \xi$, the bound state remains shallow.

– $R \sim \xi$: low-energy approximation breakdown.

Toy problem, full picture :



The scales : $\bar{R} \sim \sqrt{\frac{m_y}{m_x}}\sqrt{\xi a},\ a \ll \bar{R} \ll \xi\,;$

Summary :

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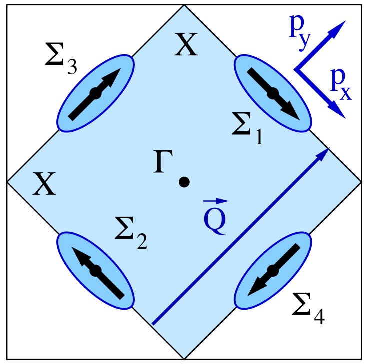
– the Skyrmion-electron bound states

– do ***not*** rely on the Belavin-Polyakov profile

– but do rely on real-space Sk-induced SOC $v\left(\mathbf{A}_y^{\parallel}\cdot\sigma\right)$

– and do hinge on lower symmetry of Σ points.



– The Skyrmion becomes charged

– and can be manipulated by electric field.

Thank you !