PhD thesis proposal in Theoretical Physics:
Separation of Variables and Correlation Functions of Quantum Integrable Systems

Quantum integrable systems are physical models, essentially in 1+1 dimensions (quantum 1D lattice models, quantum 1D field theories, 2D exactly solvable models of statistical physics...), for which it is possible to calculate exactly some quantities of physical interest such as the spectrum of commuting quantum integrals of motion, or correlation functions. More precisely, this terminology is commonly used for models associated to an R-matrix satisfying the Yang-Baxter relation, enabling one to construct an algebra (the Yang-Baxter algebra) containing an abelian sub-algebra generated by the so-called ``transfer matrix", which is a generating functional of the conserved quantities (including the Hamiltonian) of the system. The Yang-Baxter structure is then used to construct the eigenstates and compute the eigenvalues of the Hamiltonian, for instance by means of the *algebraic Bethe ansatz* (ABA) [1] or by the *quantum separation of variables* (SOV) [2] methods. The range of applications of the study of such integrable systems covers various domains of physics (from condensed matter and statistical physics to field and string theories) and of mathematics (quantum groups, knot theories, combinatorics...).

In recent years, important progresses have been made concerning the computation of form factors, correlation functions and structure factors, in particular for simple models solvable by ABA such as the XXZ Heisenberg spin-1/2 chain or the Lieb-Liniger model. The ABA approach has notably led to exact representations for the form factors in finite volume [3] which could be used both for the numerical of for the analytical (determination of the large-distance asymptotic behavior of the correlation functions at the thermodynamic limit [4]) study of the correlation functions and of the structure factors. These important new results led to new applications, in condensed matter physics (the numerical analysis permitted to establish connections with experimental measurements for spin chain materials) as well as in high energy physics (with notable applications in the framework of the AdS/CFT correspondence).

It happens that several integrable models, very interesting from the point of view of their applications, are not directly solvable by Bethe ansatz, whereas they are solvable by SOV, a method which therefore appears to be more general. Moreover, the SOV approach presents several interesting features with respect to ABA, one of them being the fact that it provides by construction a complete construction of the spectrum and eigenstates; this is in contrast to the ABA approach where the proof of completeness is in general a non trivial task. However, despite its promising features, the SOV approach has not yet been fully developed for the effective computation of form factors and correlation functions.

In this context, the purpose of this PhD is to understand how one can develop the SOV method towards the exact computation of form factors and correlation functions. The idea is to start with the study of the simple XXZ Heisenberg spin ½ chain, for which the results from ABA should in principle be recovered in the thermodynamic limit, but which already exhibits some interesting features which makes the application of SOV not completely trivial and probably very instructive. Then, one could try to apply the SOV approach to several models of various physical interest, not or hardly solvable by ABA. For instance one could considers spin chains with general integrable boundary conditions (interesting for the consideration of out-of-equilibrium problems), or higher rank spin chains (with possible applications in the context of the AdS/CFT correspondence).

Keywords: Quantum integrable models, Yang-Baxter algebra, Separation of variables, form factors, correlation functions

References:

[1] L.D. Faddeev, *How Algebraic Bethe Ansatz works for integrable model,* arXiv:hep-th/9605187.

[2] E.K. Sklyanin, *Quantum Inverse Scattering Method. Selected Topics,* arXiv:hep-th/9211111.

[3] N. Kitanine, J.M. Maillet and V. Terras, Nucl. Phys. B 554 (1999) 647, arXiv:math-ph/9807020.

[4] see for instance:
N. Kitanine, K. Kozlowski, J.M. Maillet, N. A. Slavnov and V. Terras, J. Stat. Mech. (2011) P12010, arXiv:1110.0803.
N. Kitanine, K. Kozlowski, J.M. Maillet, N. A. Slavnov and V. Terras, J. Stat. Mech. (2012) P09001, arXiv:1206.2630.
N. Kitanine, K. K. Kozlowski, J. M. Maillet and V.Terras, J. Stat. Mech. (2014) P05011, arXiv:1312.5089.

Contact: Véronique Terras (DR2 CNRS)
LPTMS - Université Paris Sud - Bâtiment 100
15 rue Georges Clémenceau
91405 Orsay CEDEX, FRANCE
e-mail: veronique.terras@lptms.u-psud.fr