M2 internship and/or Ph.D. proposal

Laboratoire de Physique Théorique et Modèles Statistiques (LPTMS) – UMR 8626
CNRS & Université Paris-Sud

Advisor : Raoul Santachiara

raoul.santachiara@gmail.com

06 07 84 92 22

http://lptms.u-psud.fr/raoul_santachiara/

Location : Orsay (3 minutes walk from the Orsay-ville RER)

Conformal bootstrap and stochastic geometry

(theoretical thesis/internship)

The random processes that generate fractal sets are of paramount importance in science. An extremely interesting subclass of two-dimensional fractal sets is given by the conformally invariant ones. This is the situation when invariance under rescaling and rotations is enhanced to invariance under any conformal (i.e. analytic and invertible) mapping. Two-dimensional Brownian motion, critical percolation cluster (see Fig 1) or clusters of like spin in critical Ising-type model (see Fig.2), to mention some of the most important and rich-of-applications random processes, belong to this subclass. The idea behind the conformal field theory (CFT) approach is that the observables associated to these fractal sets are given by the correlation functions of a local quantum field.

Very recently the **conformal bootstrap** has been shown to be a very useful method to attack long standing problems such as the 3D critical Ising model or the definition of the Liouville theory for all values of the central charge (which is a parameter defining the conformal current algebra). The conformal bootstrap try tosolve directly the infinite set of equations imposed by the conformal invariance. This perspective generally requires the help of well adapted numerical codes. Using conformal bootstrap techniques, this projects aims in finding **new CFT solutions** which capture the **geometry of stochastic fractal sets.** The intern/Ph.D. student will use recent theoretical and numerical (Python codes) tools to attack the problem. The answer to the problems suggested by the study of these fractal sets would provide further important progress in our understanding of conformal symmetry with a potential cascade of results with application in high-energy and condensed matter physics. It is important to stress that these approaches find their roots in different areas of theoretical physics and pure mathematics like **statistical field theory, integrable models, the theory of probability or algebra representation theory**.

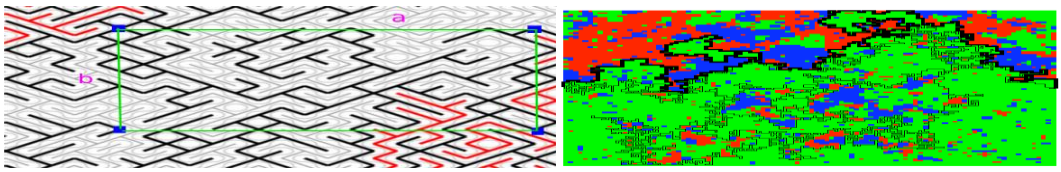

Fig 1 (left): Typical configuration of critical bond percolation. Open question: What is the probability of four points belonging to the same connected cluster? ***Fig2 (right):*** A typical configuration of a three-color spin model at the critical temperature. The clusters of like spin, the domain wall (black thick line) and the boundary of the FK clusters are examples of conformally invariant stochastic sets.