PhD project proposal form

Thesis subject title :
Algebraic area enumeration of two-dimensional closed random lattice walks

- **Laboratory name :** **LPTMS**
- **Laboratory web site :** **http://lptms.u-psud.fr/fr**

- **PhD supervisor (contact person)**
  - **Name :** **OUVRY Stéphane**
  - **Position :** **DR1 CNRS**
  - **Email :** stephane.ouvry@u-psud.fr
  - **Phone number :** + 33 (0)1 69 15 36 30

- **Thesis proposal :**
  - The quantum Hofstadter problem [1] (a charge particle hopping on a square lattice coupled to a perpendicular magnetic field) is fascinating for several reasons : its spectrum is a rare example of a fractal emerging from quantum mechanics, its transport properties can shed an interesting light on the quantum Hall effect, …
  - The Hofstadter model happens to be related to closed random walks on a square lattice, more precisely there is a mapping between the n-th moment of the Hofstadter Hamiltonian and the generating function for the enumeration of close lattice walks of length n enclosing a given algebraic area.
  - Recently [2] a formula for the algebraic area enumeration has been obtained starting from the so called Kreft coefficients [3] which encode the Schrodinger equation for the Hofstadter model.
  - Several key observations made in [2] and on which the enumeration relies happen to be incompletely understood and not yet seated on solid mathematical grounds.
  - Also the final enumeration formula has a complexity which increases exponentially with n making it difficult to be used for walks with a large number of steps.
  - The thesis will focus on a better understanding and improving of [2] namely :

    i) derive some or all of the observations in [2] which lead to the enumeration formula
    ii) interpret in terms of 1d random walks characteristics (if possible) the building blocks of the enumeration formula and use this interpretation to push further its understanding
    iii) also if possible simplify and reduce the formula to make it more tractable for the algebraic aera enumeration of walks with a large number of steps

Last but not the least, coming back to the Hofstadter model via the mapping discussed above, can the enumeration formula gives new insights on the Hofstadter spectrum in the irrational limit where the flux per plaquette becomes an irrational number?

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- [1] D.R. Hofstadter, Phys. Rev. B 14 (1976) 2239.
- [2] S. Ouvry and S. Wu, arXiv: 1810.04098 « The algebraic area of closed lattice random walks »
- [3] C. Kreft, « Explicit Computation of the Discriminant for the Harper Equation with Rational Flux », SFB 288 Preprint No. 89 (1993)