HYDRODYNAMICS OF PLASTIC FLOWS WITH CURRENT CONVERSION

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• Electronic crystals.
• Experiments on pastic flows and current conversion
• Two types of dislocation motion
• Kinematics at presence of dislocation lines/loops, invariant averaging.
• From kinematics to dynamics. Materials relations and kinetics.
• Where does the stress in density waves come from.
• 1D regime, electric field actions, correlation between stress and chemical potential.
• Conclusions
Main features of electronic crystals

- Periodicity follows a local concentration of electrons
- Crystal is sliding as a whole under the driving field above the threshold $E_t$.
- Collective sliding is a periodic highly coherent but strongly anharmonic process.
- Excess normal current is converted to the collective one via phase slip processes.
- Point defects, vacancies are favorable in compare to electrons as normal carriers.
- Perpendicular flow of dislocations is an ingredient of the sliding and of the current conversion.

**CDW/SDW specifics:** the energetics of dislocation lines/loops is determined by the Coulomb forces and by screening facilities of the free carriers.
Topological defects in electronic crystals: CDW, SDW

Solitons, phase slips and dislocation lines/loops

**Phase slips:**

*Microscopically* – selftrapping of electrons into solitons with their subsequent aggregation

*Macroscopically* – edge dislocation line proliferating/expanding across the sample.

![Diagram showing source, drain, and phase slips](image)
Adequate definition of **plastic deformations/flows**: distorted/sliding states of coherent media - crystals (superfluids) accompanied by local creation/annihilation of new periods.

**Realization:**

Electrons $\Rightarrow$ Amplitude solitons $\Rightarrow 2\pi$ phase solitons

**Originated by:**
- normal/collective current conversion
- surface strains
- impurities

History of dislocations in CDWs:
- Dumas, Feinberg, Friedel, Maki,
- Gorkov, Gill, Brazovskii, Matveenko,
- Kirova, Thorne, Monceau, ...
Our goal: theory of plastic flows of sliding superstructures

To describe:
• Current conversion: normal current ⇔ collective one
• Relaxation of injected carriers:
  normal electrons ⇔ crystal’s periods
• Collective motion in constrained geometry
• Plastic contribution to nonlinear I-V

\[ \phi = \omega t + kx^2/2 \]
\[ -\gamma \omega = CE \]
\[ -k^2 = (1-C)E \]
Partition \( C = ? \)

\[ F_{\text{frc}}(\phi) \]

\[ -\gamma \dot{\phi} + \nabla^2 \phi = E \]

\[ \Phi \; ; \; \frac{V_{ps}}{L} \; ; \; \frac{J}{\sigma} \]

? which \( \phi \)

\[ f(\nabla) \phi \]

normal carriers

Coulomb

? which \( E \)

? which \( \Phi \)

? which \( \frac{V_{ps}}{L} \)

? which \( J \)

? which \( \sigma \)
Current conversion

F. Nad’ et al
Plastic flows via space resolved diffraction

D. Rideau et al
Europhysics Letters 56 (2001) 289
Two types of dislocation motion: Glide and Climb

**Glide:** Force ≠ energy gradient

\[ \vec{F} \propto \int [\vec{T} \times d\vec{l}] \]

Supposed by analogy with magnetic forces:

\[ F_x \propto \oint \vec{T} \times d\vec{l} \iff -\frac{\partial}{\partial x} U = -\frac{\partial}{\partial x} \int \vec{T} d\vec{s} \]

Correct, because \( \vec{\nabla} \vec{T} = F_{frc}(\phi) \neq 0 \)

**Climb:** non dynamic but kinetic process – dislocation growth.

\[ \frac{dn_d}{dt} \neq 0, \text{ } n_d \text{ – density of defects} \]

\[ F_\perp \propto \int T_x dl \]

\( F_\perp \) promotes climb – forbidden expansion of loop – mass/charge conservation.

**Usual prescription:** project out, then we lose conversion

**Here:** consider climb only as a grows by adhesion of normal carriers
**Glide**: local versus integral currents. Nucleolus dislocation loop \(=2\pi\)-solit

\[
\begin{align*}
2e \\
\text{Soliton charge is transferred to the collective mode}
\end{align*}
\]

Does \(2\pi \equiv 2e\) soliton glide under applied electric field \(E\)?

**NO** (YES for pinned DW.)

Local charge \(2e\), local current \(2e\ v_x\).

\(n=0\) \(\delta\phi=2\pi\) \(n\neq0\) \(\delta\phi=0\)

Compensating charge in long range tails

\[
\varphi_{LR} \sim -\frac{xs}{(x^2 + r_s^2)^{3/2}} \Rightarrow \int \frac{d^2 r_s}{s} \varphi_{LR} = \varphi_{LR}(x) = -\pi \text{sgn}(x)
\]

**Long range tail charge** -2e, total charge = 0

no integral current

**Glide**: Force \(\neq\) energy gradient

\[
\bar{F} \propto \int [\bar{T} \times d\bar{l}]
\]

Supposed by analogy with magnetic forces:

\[
F_x \propto \int \bar{T}_\perp \times d\bar{l} \iff -\frac{\partial}{\partial x} U = -\frac{\partial}{\partial x} \int \bar{T}d\bar{s}
\]

**Correct**, by definition

Wrong, because

\[
\nabla \bar{T} = F_{jrc}(\phi) \neq 0
\]

**We need**: a careful separation of various contributions to the current.
Kinematics at presence of dislocation lines/loops (DL)

Geometric relations

Local deformations and velocities are not derivatives of the same phase

\[
\frac{\partial \varphi}{\partial x} \rightarrow \varphi_x; \quad \frac{\partial \varphi}{\partial y} \rightarrow \varphi_y; \quad \frac{\partial \varphi}{\partial z} \rightarrow \varphi_z \quad \frac{\partial \varphi}{\partial t} \rightarrow \varphi_t;
\]

Four variables: \(\varphi_x, \varphi_y, \varphi_z, \varphi_t\) instead of one: \(\varphi\)

\[
(\varphi_x, \varphi_y, \varphi_z) = (\omega_x, \omega_y, \omega_z) = \omega \quad \omega \neq \nabla \varphi \iff \nabla \times \omega \neq 0
\]

\[
\vec{\tau} = \frac{1}{2\pi} \left[ \nabla \times \vec{\omega} \right] \quad \text{- density of dislocation lines, space circulation of } \varphi
\]

\[
I = \frac{1}{2\pi} \left( \nabla \varphi_t - \frac{\partial \omega}{\partial t} \right) \quad \text{- flow of dislocation lines, space-time circulation}
\]

Compatibility relation

\[
\nabla \times \vec{I} = - \frac{\partial \vec{\tau}}{\partial t}
\]

Phase circulation

DL
\[ \vec{\nabla} \vec{\tau} = 0 \quad \Rightarrow \quad \vec{\tau} = -\frac{1}{2\pi} [\vec{\nabla} \times \vec{P}] \]

No terminations for dislocation lines.

\( P \) – density of discontinuities at arbitrary surfaces based on DL,

\( \omega = -P \) up to the gradient of one valued function, \textit{call it the phase} \( \phi \)

\[ \vec{\omega} + \vec{P} = \vec{\nabla} \phi \]

\[ \vec{I} = \frac{\partial \vec{P}}{\partial t} + \frac{1}{2\pi} (\vec{\nabla} \phi_t - \frac{\partial \vec{\omega}}{\partial t}) \]

\( \tau, I \) - physical singularity on DL,

\( P, \phi \) – non-physical singularities

\[ \vec{I} \propto \vec{v}_{DL} \times d\vec{l} \]
Fix the time dependent part of the gauge:

\[
\varphi_t = \frac{\partial \varphi}{\partial t} \quad \Rightarrow \quad \nabla \varphi_t = \frac{\partial}{\partial t} \left( \vec{\omega} + \vec{P} \right) \quad \Rightarrow \quad \vec{l} = \frac{\partial \vec{P}}{\partial t}
\]

\( \varphi_t, \omega, I \) are singular on DL only. Now the same holds for \( \vec{\varphi}_t \) and \( \vec{d} \). Discontinuity surface \( \vec{P} \) is arbitrary only at some initial \( t=0 \). Afterwards \( \vec{P} \) evolves only along the surface passed by DL, that is following the trace of physical singularities.

\[ j_d \neq 0, \text{ conservative motion }, \text{ allowed only along } \nu \]
Averaging over DL distribution

From non-unique \( \mathbf{P} \), \( \varphi \) to unique \( \mathbf{j}_d, \mathbf{n}_d \)

\( n_d \) – density of defects = DL area/volume

\( \mathbf{j}_d = (j_d, 0, 0) \) – current of defects

\( <P_x> = 2\pi n_d \) – projected DL area/volume

\( \langle \mathbf{r} \varphi \rangle = -\mathbf{v} \times \mathbf{\nabla} n_d \) - for plain loops

\( \mathbf{v} \times \langle \mathbf{r} \varphi \rangle = -\frac{\partial}{\partial t} \langle \mathbf{r} \varphi \rangle = \left[ \mathbf{v} \times \mathbf{v} \frac{\partial n_d}{\partial t} \right] \Rightarrow \langle \mathbf{r} \varphi \rangle = \mathbf{v} \frac{\partial n_d}{\partial t} + \mathbf{\nabla} f \)

Identify unknown \( f \equiv j_d \)

\( \langle I_x \rangle = \left( \frac{\partial P_x}{\partial t} \right) = 2\pi \frac{dn_d}{dt} = 2\pi \left( \frac{\partial n_d}{\partial t} + \frac{\partial j_d}{\partial x} \right) \)

\( \frac{1}{2\pi} \langle I \rangle = \mathbf{v} \frac{\partial n_d}{\partial t} + \mathbf{\nabla} j_d \)

**Climb:** \( \frac{\partial \langle P_x \rangle}{\partial t} - \frac{\partial \langle P_x \rangle}{\partial x} = 2\pi \frac{\partial j_d}{\partial x} \neq 0 \)

**Glide:** \( \frac{\partial \langle P_x \rangle}{\partial t} - \frac{\partial \langle P_x \rangle}{\partial x} = 0 \)
**Result:** There is a uniquely defined function $\chi$ such that

\[
\begin{align*}
\frac{\partial \chi}{\partial t} &= \langle \phi_t \rangle + 2\pi j_d \\
\frac{\partial \chi}{\partial x} &= \langle \phi_x \rangle - 2\pi n_d \\
\frac{\partial \chi}{\partial y} &= \langle \phi_y \rangle \\
\frac{\partial \chi}{\partial z} &= \langle \phi_z \rangle
\end{align*}
\]

Finding $\chi$ is equivalent to find the first integral to four equations for $\varphi_\alpha$, $\alpha = x, y, z, t$. 
A straightforward derivation

Forget the phases, follow stresses $T_i$, return to $\varphi_\alpha$ as self average values, average, find then separate Eqs. for $<\varphi_\alpha>$, try to find the first integral $\chi$.

**Procedure:** Introduce $P \rightarrow$ noninvariant integration of $\varphi_\alpha$ to $\partial_\alpha \varphi \rightarrow$ exclude unnecessary $\nabla_\perp \varphi \rightarrow$

**Equation for physical variables:** compression $\varphi_x$, field $E$

\[
\frac{1}{\pi} \left( \Delta - \gamma \frac{\partial}{\partial t} \right) \varphi_x + \frac{\partial E}{\partial x} - \frac{\partial^2 n_n}{\partial x^2} = \gamma \frac{\partial}{\partial t} (\vec{v} \vec{P}) + 2\alpha (\nabla \times \vec{v}) \tau
\]

R.H.S.$\Rightarrow$ $2\gamma \frac{dn_d}{dt} - 2\alpha \nabla_\perp n_d$

**Self averaging:** L.H.S. – by definition
R.H.S. – by construction
Final equations:

**Visco-elastic – plastic motion**

\[
\frac{1}{\pi} \left( \hat{\Delta} - \gamma \frac{\partial}{\partial t} \right) \chi = -\frac{\partial \Phi}{\partial x} - 2\gamma j_d - \frac{\partial}{\partial x} \left( n_i + 2n_d \right)
\]

+ 

**Poisson Eq.**

\[
r_0^2 \Delta \Phi + n_n + 2n_d + \frac{1}{\pi} \frac{\partial \chi}{\partial x} = 0
\]
From kinematics to dynamics.

Local material relations:
Velocity: \( v = \phi_t \leftarrow \partial \phi / \partial t \)
Strain: \( \omega = \omega(T) \leftarrow \nabla \phi \)
Stress: \( T = \delta W / \delta \omega \)
Equilibrium:
\( \nabla T = F_{\text{fric}}(v) \leftarrow m dv / dt + \gamma v \)

DW specific relations:
Poisson equation: \( \delta W / \delta \Phi = 0 \)
Normal carriers:
Chemical potential: \( \mu_n = \delta W / \delta n \)
Current: \( j_n = -\sigma_n \nabla \mu \)
Conservation:
\[
\frac{\partial n_n}{\partial t} + \nabla j_n = \frac{dn_n}{dt}
\]
Source – drain = conversion

\[
R (\mu_d - \mu_n) = \frac{dn_d}{dt} = -2 \frac{dn_n}{dt}
\]
normal electrons
\[
n_n = n_i + n_e
\]
R- phase slip rate
Supplements: material relations and kinetics

Diffusion Eqs. for carriers: \[ \frac{\partial n}{\partial t} = D \nabla^2 \mu = \frac{dn}{dt} \]

\[ n_n = n_i + n_e ; n_e = n_e(\Phi) ; n_i = n_i(V) ; n_d = n_d(U) \]
\[ d_{n_d}/dt = K(\mu_d-\mu_n) \]

\[ n_e, h : \mu_n = \mu_{loc}^n(n) + V ; V = e \Phi + \frac{v_F h}{2} \phi_x \]

\[ d : \mu_d \neq \mu_{loc}^n(n) + U \]
\[ \frac{\partial \mu_d}{\partial x} = \frac{\partial \mu_{loc}^d}{\partial x} + \alpha \Delta \chi \neq \frac{\partial U}{\partial x} \]

\[ U = \Phi + \frac{h v_F}{2} \phi_x + \frac{\pi h v_F}{2} n_i \]

DW stress contributions: electric elastic inequilibrium

2U – energy per chain paid to distort the CDW elastically by one period
What would be without Coulomb interactions

\[
\frac{1}{\pi} \left( \Delta - \gamma \frac{\partial}{\partial t} \right) T_x = \frac{\partial F}{\partial x} + \Delta_\perp (n_i + 2n_d) + \gamma \frac{\partial j_n}{\partial x} + \dot{n}_{\text{inj}} \quad T_x = U
\]

Where does the stress come from?
- \( \parallel \) gradient of external driving force \( F \)
- \( \perp \) profile of both normal carriers and defects
- \( \parallel \) gradient only of normal carriers
- charge injection

In terms of the invariant phase \( \chi \)

\[
\frac{1}{\pi} \left( \Delta - \gamma \frac{\partial}{\partial t} \right) \chi = -2\gamma j_d + \frac{\partial}{\partial x} \left( 2n_d + cn_i \right) + F
\]
CDW reality: Coulomb forces, low T

\[-\gamma \frac{\partial}{\partial t} + \hat{\Lambda} \Rightarrow \left[ -\gamma \frac{\partial}{\partial t} - \hat{\Lambda}^2 + \frac{r_0^{-2}}{\Delta} \frac{\partial^2}{\partial x^2} \right]\]

nonlocal elasticity
S.B & S. Matveenko

\[\frac{1}{\pi} \left[ r_0^2 \Delta \left( \Delta - \gamma \frac{\partial}{\partial t} \right) - \frac{\partial^2}{\partial x^2} \right] \chi = \frac{\partial}{\partial x} \left( n_n + 2n_d \right)\]

electroneutrality at \( r_0 = 0 \)

Driving force: only \( \parallel \) gradients of normal carriers and defects
1D regime: average crossection, $\langle r \rangle$

Charge
\[
\frac{1}{\pi} \frac{\partial \chi}{\partial x} + n_n + 2 n_d \equiv 0
\]

Current
\[
- \frac{1}{\pi} \frac{\partial \chi}{\partial t} + j_n + 2 j_d = J = \text{cnst}
\]
Electric field actions in 1D

\[-\frac{\partial \Phi}{\partial x} = E = -\gamma \frac{\partial \chi}{\partial t} + 2\gamma j_d + \frac{\partial}{\partial x}(n_e + n_0)\]

Also a resolution of the dilemma:

\[E = -\gamma \partial_t \phi + k \phi''\]

\(E \) drives the DW motion:

\[\partial_t \chi + j_d \leftarrow \text{the current of defects}\]

(takenether = total collective current)

\(E \) withstands the \(||\) gradient of normal carriers concentration

If \(\partial_t \chi = 0\), then \(V_{ps} = \Delta \Phi - EL = \hbar v_F n_e\)

gives the access to extrinsic carriers at the contacts
Correlation of the DW stress and the normal chemical potential

\[- \frac{\partial}{\partial x} \left[ T_x \sigma_c + \mu_n \sigma_n \right] = J_{tot} (t)\]

• DW is stressed only by normal carriers.
• Just the Kirchhoff law – parallel circuits of collective current driven by \(- \partial T_x / \partial x\) and \(- \partial \mu_n / \partial x\) correspondingly.
Modern experimental studies of CDW at constraint, meso & nano scales require for a transparent but complete description. The presented scheme is a minimal version of the multi-fluid hydrodynamics of plastic flows.
Main theory ingredients:

**Intrinsic carriers**: from DOS $N_F^i$, closed by the gap: $n_i, j_i, \rho_i, V_i$.

**Extrinsic carriers** in semimetallic DWs (NbSe$_3$): their DOS fraction $\beta_e$ is unaffected by the gap, $n_e, j_e, V_e$.

**Collective variables**: $\phi_x=q, n_c=\phi_x/\pi; j_c=-\phi_t; \rho_c=1-\rho_i=$ \begin{cases} 1 & T=0 \\ 0 & T=T_c \end{cases}

**Potentials**: $V_e=\Phi$, $V_i=\Phi+q/\pi N_F^i$

**DW stress contributions**: $U = \Phi + \frac{\phi_x}{\pi N_F^i} + \frac{n_i}{N_F^i}$

- Electric
- Elastic
- Nonequilibrium

2U – energy per chain paid to distort the CDW elastically by one period

Viscosity force $\gamma \phi_t$ and its generalization for the case of pinning

1D:

$$\gamma \frac{\partial \phi}{\partial t} \rightarrow \begin{cases} F_{pin} & |F| < E_t \\ F_{frc} \left( j_c \right) & |F| > E_t \end{cases}$$
Local energy functional

\[ W(\varphi, \Phi, n) = \frac{K}{2} \left[ \varphi_x^2 + \alpha_y \varphi_y^2 + \alpha_z \varphi_z^2 \right] + \frac{e}{8\pi} \varphi_x \Phi - \frac{S}{8\pi} (\nabla \Phi)^2 + \frac{S}{4\pi} (E D) + e n_e \Phi + \left( e \Phi + \frac{C}{2} \hbar v_F \varphi_x \right) n_i + W_n(n_i, n_e) \]

\[ \varphi_i = \frac{\partial \varphi}{\partial x_i}; \quad \frac{\varphi_x}{\pi} = \rho \quad \text{charge density} \]

\[ \Delta \varphi = 2\pi = \text{CDW period} = 2 \text{ electrons} \quad \text{CDW} = A \cos(Qx + \varphi), \quad \varphi' = q = \delta Q \]

\[ \Phi \quad \text{electric field potential,} \]
\[ W_n(n_i, n_e) \quad \text{free energy of normal carriers,} \]
\[ e \quad \text{free electron charge, } s \quad \text{area per one chain,} \]
\[ E \text{ and } D \quad \text{electric field and induction,} \]
\[ n_i, n_e \quad \text{concentrations of intrinsic and extrinsic free carriers,} \]
\[ C = 1 \quad \text{and } K_\parallel = \hbar v_F/2 \quad \text{links to chiral invariant and chiral anomaly} \]
What do we get when we measure:

\[
E = F_{frc}(j_c) - \beta_e \frac{\partial q}{\partial x} \frac{g}{\pi N_F^i} = \beta_i F_{frc} + \beta_e \frac{j_{tot} - j_c}{\sigma_n}
\]

\[
g = g(T) = \frac{\rho_c}{\rho_c} \rightarrow \begin{cases} 0 & T \rightarrow T_c \\ \infty & T \rightarrow 0 \end{cases}
\]

compressibility

Only in semiconducting DWs (\(\beta_e=0\)) electric field \(E\) gives access to friction. At \(\beta_e \neq 0\) strain contributes (NbSe\(_3\), some DWs)

\[
\frac{\partial}{\partial x} q \frac{g}{\pi N_F^i} = F_{frc}(j_c) - \frac{j_{tot} - j_c}{\sigma_n}
\]

\(A\)

E.g. to get \(j_c(x,t)\) from measured \(q(x,t)\)

\[
\frac{\partial q}{\partial t} + \frac{\partial j_c}{\partial x} = 2R(q, j_c) - v_{inj}
\]

\(B\)

Access to the phase slip rate \(R\) as a function of \(j_c\) and \(q-\eta\)

\[
\frac{gq}{\pi N_F^i} = \mu_n - U \equiv \eta
\]

\(A\) & \(B\) – full system of Eqs. to describe a space–time evolution