Scientific Production of Silvio Franz from 2007

Production in the period 2007-2012: 21 articles in peer review Journals (14 published + 5 accepted papers) 1 review article on a topical book on dynamical heterogeneities in glasses. I also co-edited a special issue of Journal of Statistical Physics [22] and a Focus Issue of JSTAT [25].

Main Scientific Results:

- Introduction and first studies of Spin Glasses on Dyson lattices.
- Theory of heterogeneities and fluctuations in the β regime of glassy relaxation. Mapping on a Random Field Ising Model problem.
- First principle derivation of replica field theory of fluctuations from liquid theory
- Introduction of Boltzmann pseudo-dynamics, coarse grained dynamical description of slow relaxation and aging.
- Introduction of a statistical model for the evolution of a pathogen in presence of an effective model of interaction with the host immune system.

Here follows a detailed description of my work.

Glassy Systems

Renormalization Group approaches to the Spin Glass transition

Keywords: Spin Glasses, Dyson hierarchical models, overlap interfaces, Random Energy Model. Papers [21, 14, 13]. Collaborations with E. Brézin, T. Castellana, A. Decelle, T. Jörg, M. Mézard, G. Parisi.

Despite years of intense research, the nature of the spin glass phase in finite dimension is far from being clarified. One of the theoretical difficulties to achieve this task, consists in the lack of a reliable Real Space Renormalization Group (RSRG) scheme for the Spin Glass problem, and more generally for glassy systems. The Migdal-Kadanoff approximation for example, neglects most of the frustration and does not have the correct infinite dimensional limit. A promising approach consists in considering spin glass models with power law interactions on Dyson hierarchical lattices. Dyson lattices have played an important role in the theory of critical phenomena, providing models that can be solved exactly through the RG transformation. In [21] spin glasses on Dyson lattices are introduced and studied through the replica method and numerical simulations. It is convincingly shown that there is a critical value of the interaction parameter above which the model is in the mean-field universality class and below which the spin glass transition has non trivial exponents. In both cases a careful analysis of finite size interface free-energy indicates a low temperature phase following the lines predicted by mean field theory.

A different approach is taken in paper [14], where it is defined a hierarchical version of the Random Energy Model (REM). The main features of the models are that 1) differently from the ordinary case where an exponential time is needed, the free-energy can be computed in a time which is polynomial in the volume. 2) the strength of the coupling decreases with distance the system qualifies as a non-mean field model. Through small coupling series expansion and a direct numerical solution of the model, the paper provides compelling evidence for a spin-glass condensation transition similar to the one occurring in the usual mean-field REM. Interestingly, at variance with the mean field, the high temperature branch of the free-energy is nonanalytic at the transition point. The implications of this nonanaliticity for possible glassy phases of more realistic models has to be investigated.

To the problem of interfaces in spin glasses is devoted the paper [13] which discusses the finite size cost to force equal or opposite configurations at the boundaries of a spin glass system at its critical temperature.

Theory of Heterogeneities and Fluctuations in Glasses

Keywords: Dynamical heterogeneities in glassy dynamics, growth of correlations, replica field theory, random field Ising model. Papers [1, 3, 7, 5, 2, 11, 16, 17]. Collaborations with H. Jacquin, G. Parisi, F. Ricci-Tersenghi, T. Rizzo, P. Urbani, F. Zamponi.

In the papers [1, 2, 5, 11] it is proposed a theory of dynamical heterogeneities and fluctuations during the β regime in glassy systems. The theory, starting from the hypothesis that the beta regime corresponds to almost complete equilibration in a single metastable state, eliminates time from the description expressing dynamical fluctuation in a parametric way as a function of the correlation function itself. In this way the problem of fluctuations can be studied through static equilibrium techniques and the replica method. The universal theory describing fluctuations is a cubic theory involving a $n \times n$ matrix in the limit $n \to 1$. Remarkably, the unfolding of the replica formalism, allows to disentangle different sources of fluctuations: thermal fluctuations along dynamical trajectories and heterogeneities in different initial conditions. It turns out that the leading singular behavior of the perturbative expansion of the theory can be mapped into a Random Field Ising Model with cubic coupling. Differences in the initial condition are mapped in a self-generated effective random field that acts on the variables. One of the most characteristic predictions of the theory is that the dynamical four point function χ_4 ordinarily used to characterize heterogeneities and the growth of dynamical correlations can be split in two different contributions having different scaling properties.

Sharp dynamical glass transitions as they are found in mean field theory, become dynamical cross-overs in finite range systems. One can therefore question the existence of a range of applicability of the above theory. One can notice however that there are systems where within mean field theory depending on the parameter, there can be dynamical transition lines terminating in a critical point. This critical point is also the terminating point of a distinct equilibrium glass transition line, and it is not necessarily wiped out in finite dimension and it is potentially observable. It is therefore highly interesting to investigate the universal fluctuation properties of this critical point. In [3] the problem has been addressed within the replica field theory, and through simulations of a glassy model on a diluted graph. It has been found that these critical point are in the universality class of the ordinary (ϕ^4) Random Field Ising Model.

The [1, 3] undertake the task of deriving the theory of a full microscopic replica field theory of the dynamical transition in glasses. This can be done in a liquid theory context at the mean field level, supposing the existence of a dynamical transition and studying the soft modes that appear at the dynamical temperature. One then obtains an effective theory for the critical fluctuations where the parameter entering into the field theory are in principle derived from the potential of interaction between the particles. This leads to several interesting results: it gives an expressions for the mean field critical exponents, the critical behavior of a set of four-points correlation functions and a corresponding diverging dynamical correlation length. Through a one loop computation a Ginzburg criterion that identifies the region of validity of the mean-field approximation is derived. The analysis can be specified to particular approximation schemes of liquid state theory, all these quantities are then obtained within the hypernetted chain approximation for the Gibbs free energy.

Paper [7] addresses the problem of dynamical fluctuations in kinetically constrained models on random graph. These models, despite the fact of having trivial Hamiltonian, present a dynamics glass transition following the Mode Coupling phenomenology. In the paper it is shown by numerical simulations that if one concentrates on the right dynamical quantity, the fluctuation follow the theory developed in [10]. This is a surprising result and more work will be needed to understand all its implications.

Glassy Dynamics

Keywords: Boltzmann pseudodynamics, aging, liquid theory, mode coupling theory. Papers [26, 4, 6]. Collaborations with G. Parisi, P. Urbani

In [4] it is addressed the problem of describing quasi-equilibrium exploration of phase space in glasses in formal terms. Quasi-equilibrium exploration is formally equivalent to a chain of identical glassy systems in a constrained equilibrium where each bond of the chain is forced to remain at a preassigned distance to the previous one. The construction defines a Markov chain pseudo-dynamical process that under specific conditions of slow glassy dynamics can formally describe real relaxational dynamics the long time. Pseudo-dynamics is exactly analyzed in mean field glassy systems, where it is possible to recover the equation of Langevin relaxational dynamics in the long time limit both on approaching the glass transition and during aging at low temperature. This formal identity is an evidence that in these situations the configuration space is explored in a quasi-equilibrium fashion. Our general formalism, that relates dynamics to equilibrium puts slow dynamics in a new perspective and opens the way to the computation of new dynamical quantities in glassy systems.

The paper contains a number of important results: 1) The equivalence between the mean field picture of slow glassy dynamics and quasi-equilibrium exploration of phase space. 2) The equivalence of the supersymmetric formalism of dynamics with a replica approach. This suggests to use Boltzmann pseudo-dynamics as a coarse graining of dynamics in which the fast part is considered as instantaneous. 3) The scheme allows to use typical approximations devised for static equilibrium to study long time dynamics of glassy systems.

In [6] the quasi-equilibrium formalism is used in the context of liquid state theory in order to obtain a set of coarse grained long time dynamical equations for the two point density correlation functions. The approach is specified to the Hypernetted Chain (HNC) approximation and to the closure scheme of Szamel to the Yvonne-Born-Green hierarchies. In both cases one gets dynamical equations that have the structure of the Mode-Coupling (MCT) equations in the long time region. The equilibrium HNC approximation, is in this way generalized to a fully consistent scheme where long-time dynamic quantities can also be computed. In the context of this approximation one gets an asymptotic description of both equilibrium glassy dynamics at high temperature and of aging dynamics at low temperature. The Szamel approximation on the other hand is shown to lead to the exact Mode Coupling equation of Goetze for equilibrium dynamics. This unveils that according to MCT configuration space is sampled according to a quasi-equilibrium principle.

Probing glasses by boundary conditions

Keywords: Mosaic picture of the glass phase, interface energy in glasses. Papers [27, 18, 24, 12] Collaboration with G. Parisi, F. Ricci-Tersenghi, G. Semerjian, E. Zarinelli

Papers [27, 12] represent the first analytic study of the mosaic and dynamical length in glassy in a microscopic model, adn followed some phenomenological developments by Biroli and Bouchaud. The computation is performed in the Kac limit of long interaction range. As assumed by phenomenological mosaic picture, as the glass transition is approached one finds a growing correlation length below which the system behaves as glassy and above which it behaves as a liquid. Remarkably, a second length can be defined, which is growing much faster, that defines the minimal size of a system where relaxation can occur without activation.

In [24] it is addressed the problem of finite interaction-range corrections to the mean-field mosaic picture of the glass transition, as it emerges from the study of the Kac limit of large interaction range [27] [12]. To this aim it is introduced a 1D Random Energy Model with variable interaction range in a finite size window. It is then studied the mosaic correlation length associated to the "point-to-set" function, as a function of the range of interaction. In the Kac limit of infinite range, there is a sharp first-order transition separating a strongly correlated phase from an uncorrelated one. Correspondingly, we find for finite range there is a crossover and different curves as a function of the window size cross roughly at the value of correlation length. Nonetheless, the convergence to the Kac limit is very slow. This could explain the difficulty of measuring the mosaic length in realistic models.

The paper [18] aims at defining an *energy* tension between different states in glassy systems. This is done choosing two equilibrium states α and β and imposing them as boundary conditions at given distance between each other in a sandwich like geometry. Within a model with long range (Kac) interactions it is found that the energy tension as a function of the distance drops rapidly to zero after the mosaic length in the liquid state, while it remains finite even for infinite distance in the glassy phase.

Mathematical Physics of Spin Glasses

Keywords: Spin glass models on random graphs, Ghirlanda-Guerra identities. Paper [20]. Collaboration with L. De Sanctis

Spin glass problems on random graphs have important applications in complexity theory, theoretical computer science, information theory. Theoretical physics has provided important insight and exact solutions. However mathematical proves of the physicist's solutions are still lacking.

Recent years have seen important progresses in the mathematical comprehension of spin-glasses in the fully connected limit. A prominent role in this developments has been played by the Ghirlanda-Guerra identities, that restrict the possible values the overlap between different copies of a spin glass system can take.

In [20] a generalization of the Ghirlanda-Guerra identities to multioverlap is proposed. The implications of these identities have to be figured out.

Interdisciplinary Topics

My research in this period also includes several interdisciplinary applications of statistical physics, in the context of biological and social sciences.

Evolution of pathogens

Keywords: Evolution modeling, Wright-Fisher model, message passage algorithms, inference. Papers [8, 9, 19]. Collaborations with: G. Bianconi, L. Ferretti, D. Fichera, L. Peliti, C. Borile, M. Labarre, C. Sola, and G. Refregier

The ecology of pathogenic viruses is shaped by the interaction with the host immune system. In particular, the virus of Influenza A acts as hypermutator, it uses a high mutation rate in specific region of the genome to escape the acquired immunity of the hosts. The evolutionary dynamics of the viral population is characterized as an evolving quasispecies: the viral population at each time is concentrated in a strain with well defined genetic and antigenic properties. This kind of behavior has been ascribed to the mutual effect of the acquired immunity and competition caused by variability in fitness of the mutants. In the paper [8] an effective model to study the dynamics of evolving quasispecied is introduced. The effect of acquired immunity is modeled as a reduction in fitness of the different strains proportional to the prevalence of these strains in the viral population. The model shows the emergence of a stationary dynamics, marked by epochs of punctuated equilibria, where the population remains concentrated, separated by fast substitutions of one dominating strain by another.

The paper [19] presents a non-neutral stochastic model for the dynamics taking place in a meta-community ecosystems in the presence of migration. The model provides a framework for describing the emergence of multiple ecological scenarios and behaves in two extreme limits either as the unified neutral theory of biodiversity or as the Bak-Sneppen model. Interestingly, the model shows a condensation phase transition where one species becomes the dominant one, the diversity in the ecosystems is strongly reduced and the ecosystem is non-stationary. This phase transition extends the principle of competitive exclusion to open ecosystems and might be relevant for the study of the impact of invasive species in native ecologies.

Paper [9] in collaboration of a team of biologists of the university of Paris XI, uses the tools of statistical physics to classify the different strains of Mycobacterium tuberculosis into different groups, based on genetic differences observed in markers such as CRISPR and spoligotypes, similar to the ones used for DNA fingerprinting in humans. The paper first proposes measures of distances (domain wall distances) that reflect the mode of evolution of the marker. It then uses that measures of distance to define clusters using the affinity propagation clustering algorithm. The resulting classification refines some proposed groupings of the TBC strains.

Social Systems, Homophily

Keywords: Social networks, community detection, Ising model with memory. Papers [15, 23]. Collaborations with M. Marsili, P. Pin, F. Caccioli

The paper [15] concerns the field of social networks. Societies are heterogeneous in many dimensions such as census, education, religion, ethnic and cultural composition. The links between individuals e.g. by friendship, marriage or collaboration are not evenly distributed, but rather tend to be concentrated within the same group. This phenomenon, called inbreeding homophily, has been related to either (social) preference for links with owntype individuals (choicebased homophily) or to the prevalence of individuals of her same type in the choice set of an individual (opportunitybased homophily). Choices determine the network of relations we observe whereas opportunities pertain to the composition of the (unobservable) social network individuals are embedded in and out of which their network of relations is drawn. The paper proposes a method that, in the presence of multiple data, allows one to distinguish between opportunity and choice based homophily. The main intuition is that, with unbiased opportunities, the effect of choicebased homophily gets weaker and weaker as the size of the minority shrinks, because individuals of the minority rarely meet and have the chance to establish links together. The occurrence of homophily in the limit of very small minorities is therefore an indicator of opportunity bias. This idea is tested across the dimensions of race and education on data on US marriages, and across race on friendships in US schools.

The Ising model can be considered as a rudimentary model of opinion dynamics where individuals decide between two alternatives under the effect of social influence and possibly with some personal bias. In paper [23] it is introduced a learning dynamics in which individuals the more they have stayed in an opinion the more become reluctant to change it. The resulting system is a kinetic Ising model with a memory field. The probability of a spin-flip depends on the persistence time of the spin in a state. The more a spin has been in a given state, the less the spin-flip probability is. The growth and persistence properties of such a system are studied numerically on a two-dimensional square lattice. The memory introduces energy barriers which freeze the system at zero temperature. At finite temperature one observes an apparent arrest of coarsening for low temperature and long memory length. However, since the energy barriers introduced by memory are due to local effects, there exists a timescale on which coarsening takes place as for the Ising model. Moreover the two-point correlation functions of the Ising model with and without memory are the same, indicating that they belong to the same universality class.

References

[1] S. Franz, H. Jacquin, G. Parisi, P. Urbani, and F. Zamponi. Quantitative field theory of the glass transition. *Proceedings of the National Academy of Sciences (PNAS)*, 109, 2012.

[2] S. Franz, H. Jacquin, G. Parisi, P. Urbani, and F. Zamponi. Static replica approach to critical correlations in glassy systems. *arXiv preprint arXiv:1210.7304, J. Chem. Phys. accepted*, 2012.

[3] S. Franz and F. R.-T. Parisi, G. Glassy critical points and random field ising model. *arXiv preprint arXiv:1203.4849, JSTAT accepted*, 2012.

[4] S. Franz and G. Parisi. Quasi-equilibrium in glassy dynamics: an algebraic view. *arXiv preprint arXiv:1206.4067, JSTAT accepted*, 2012.

[5] S. Franz, G. Parisi, F. Ricci-Tersenghi, T. Rizzo, and P. Urbani. A note on weakly discontinuous dynamical transitions. *arXiv preprint arXiv:1211.1616, J. Chem. Phys. accepted*, 2012.

[6] S. Franz, G. Parisi, and P. Urbani. Quasi-equilibrium in glassy dynamics: a liquid theory approach. *arXiv preprint arXiv:1212.4291*, 2012.

[7] S. Franz and M. Sellitto. Finite-size critical fluctuations in microscopic models of mode-coupling theory. *arXiv preprint arXiv:1210.5952, JSTAT accepted*, 2012.

[8] G. Bianconi, D. Fichera, S. Franz, and L. Peliti. Modeling microevolution in a changing environment: the evolving quasispecies and the diluted champion process. *Journal of Statistical Mechanics: Theory and Experiment*, 2011(08):P08022, 2011.

[9] C. Borile, M. Labarre, S. Franz, C. Sola, and G. Refrégier. Using affinity propagation for identifying subspecies among clonal organisms: lessons from m. tuberculosis. *BMC bioinformatics*, 12(1):224, 2011.

[10] S. Franz, G. Parisi, F. Ricci-Tersenghi, and T. Rizzo. Field theory of fluctuations in glasses. *The European Physical Journal E: Soft Matter and Biological Physics*, 34(9):1–17, 2011.

[11] S. Franz, G. Parisi, F. Ricci-Tersenghi, and T. Rizzo. Replica field theory of the dynamical transition in glassy systems. *arXiv preprint arXiv:1105.5230*, 2011.

[12] S. Franz and G. Semerjian. *Analytical approaches to time and length scales in models of glasses in L. Berthier, G. Biroli, J.-P. Bouchaud, L. Cipelletti, W. van Saarloos (eds.). Dynamical heterogeneities in glasses, colloids and granular materials.* Oxford University Press, 2011.

[13] E. Brézin, S. Franz, and G. Parisi. Critical interface: Twisting spin glasses at t_ {c}. *Physical Review B*, 82(14):144427, 2010.

[14] M. Castellana, A. Decelle, S. Franz, M. Mézard, and G. Parisi. Hierarchical random energy model of a spin glass. *Physical review letters*, 104(12):127206, 2010.

[15] S. Franz, M. Marsili, and P. Pin. Observed choices and underlying opportunities. *Science and Culture*, 76:471, 2010.

[16] S. Franz, G. Parisi, F. Ricci-Tersenghi, and T. Rizzo. Critical behaviour of large scale dynamical heterogeneities in glasses: a complete theory. *arXiv preprint arXiv:1008.0996*, 2010.

[17] S. Franz, G. Parisi, F. Ricci-Tersenghi, and T. Rizzo. Properties of the perturbative expansion around the mode-coupling dynamical transition in glasses. *arXiv preprint arXiv:1001.1746*, 2010.

[18] E. Zarinelli and S. Franz. Surface tension in kac glass models. *Journal of Statistical Mechanics: Theory and Experiment*, 2010(04):P04008, 2010.

[19] G. Bianconi, L. Ferretti, and S. Franz. Non-neutral theory of biodiversity. *EPL (Europhysics Letters)*, 87(2):28001, 2009.

[20] L. De Sanctis and S. Franz. Self-averaging identities for random spin systems. *Progress in Probability*, 62:123–142, 2009.

[21] S. Franz, T. Jörg, and G. Parisi. Overlap interfaces in hierarchical spin-glass models. *Journal of Statistical Mechanics: Theory and Experiment*, 2009(02):P02002, 2009.

[22] S. Franz, J. Lebowitz, E. Marinari, M. Mezard, and N. Sourlas. *Special Issue of Journal of Statistical Physics dedicated to E. Brézin and G. Parisi, J. Stat. Phys. 135.* 2009.

[23] F. Caccioli, S. Franz, and M. Marsili. Ising model with memory: coarsening and persistence properties. *Journal of Statistical Mechanics: Theory and Experiment*, 2008(07):P07006, 2008.

[24] S. Franz, G. Parisi, and F. Ricci-Tersenghi. Mosaic length and finite interaction-range effects in a one-dimensional random energy model. *Journal of Physics A: Mathematical and Theoretical*, 41(32):324011, 2008.

[25] M. Evans, S. Franz, C. Godreche, and D. Mukamel. *Focus issue of JSTAT Dynamics of Non-Equilibrium Systems http://www.iop.org/EJ/journal/-page=extra.focus4/jstat.* 2007.

[26] S. Franz. On the dynamics of kac p-spin glasses. *Journal of Statistical Physics*, 126(4):765–780, 2007.

[27] S. Franz and A. Montanari. Analytic determination of dynamical and mosaic length scales in a kac glass model. *Journal of Physics A: Mathematical and Theoretical*, 40(11):F251, 2007.