

# Cours climat

## Constante solaire:

The sun shines light with a power distributed on a spherical surface. At distance  $D$  from the center of the sun this power per unit of surface is  $\mathcal{S}_s = \frac{P}{4\pi D^2}$  by mere geometrical effect.

$P$  from the blackbody law is given by  $P = 4\pi R_{\odot}^2 \sigma T_{\odot}^4$

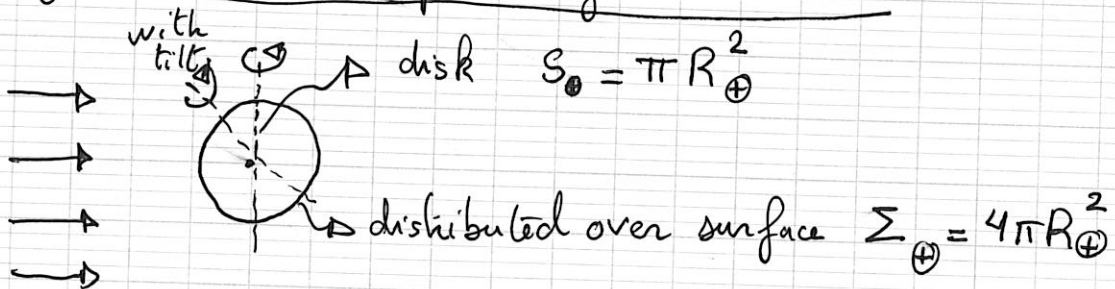
$$R_{\odot} \approx 7 \cdot 10^5 \text{ km}$$

$$T_{\odot} \approx 5780 \text{ K}$$

$$\sigma \approx 5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

surface temperature  $\downarrow$

## geometrical interception by the Earth



so the average power from sun by unit of surface is

$$\frac{S_{\oplus}}{\Sigma_{\oplus}} = \frac{P}{4\pi D^2} \frac{1}{4} = \boxed{\frac{\mathcal{S}_s}{4} \approx 342 \text{ W m}^{-2} = \mathcal{S}'_s}$$

Average albedo of the Earth:  $A_b \approx 0,3$

Flux on surface of Earth:  $\Phi_{\oplus} = \mathcal{S}'_s (1 - A_b) \approx 239 \text{ W m}^{-2}$

it is an average: the actual value fluctuates

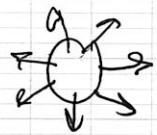
factor:  $1 \text{ W m}^{-2} = \overset{0,024}{\text{~~0,024~~ kWh/m}^2/\text{day}$

## Radiative balance: greenhouse effect

equivalent temperature: (zero couche)

$$\Phi_{\text{in}} = 4\pi R_{\oplus}^2 \Phi_0 \quad \text{received}$$

$$\Phi_{\text{out}} = 4\pi R_{\oplus}^2 \sigma T_{\text{eq}}^4 \quad \text{emits in all directions}$$



at equilibrium:  $\Phi_{\text{in}} = \Phi_{\text{out}}$

$$\Rightarrow \sigma T_{\text{eq}}^4 = \overset{\Phi_s'}{\text{~~0~~}} (1 - A_b)$$

or  $T_{\text{eq}} = \left( \frac{\Phi_s' (1 - A_b)}{4\sigma} \right)^{1/4}$  (plot table for planets)

A.N.: Earth:  $-18^\circ\text{C}$ ! (254 K)

## Greenhouse effect, model with one shell of atmosphere

[Question: why not re-emission of visible light?]

Balance: in / out

space:  $\overset{\text{out}}{\Phi_s'} = \overset{\text{in}}{\Phi_s' A_b} + (1 - \epsilon) \sigma T_a^4 + \epsilon \sigma T_a^4$

atmosphere:  $\Phi_s' + \sigma T_a^4 = \Phi_s' (1 - A_b) (1 - \alpha) + \overset{\text{in}}{\Phi_s' A_b} + (1 - \epsilon) \sigma T_s^4 + 2\epsilon \sigma T_a^4$

surface:  $\Phi_s' (1 - A_b) (1 - \alpha) + \epsilon \sigma T_a^4 = \sigma T_s^4$

key ingredient  $\Rightarrow$  feedback

Rk:  $T_s \neq T_a \Rightarrow$  radiative effect allows for setting a gradient of temperature between the high atmosphere and the surface.

we are looking for  $T_s$ , only two independent equations  
 second line using first

$$\left[ \begin{array}{l} \tilde{G}'_s + \sigma T_s^4 = \tilde{G}'_s (1-A_b)(1-d) + \tilde{G}'_s A_b + (1-\epsilon)\sigma T_s^4 + \epsilon\sigma T_a^4 \\ \dots \text{ gives back the third one} \end{array} \right]$$

$$\tilde{G}'_s (1-A_b) = \frac{(1-\epsilon)\sigma T_a^4 + \sigma T_s^4 - \tilde{G}'_s (1-A_b)(1-d)}{(2-\epsilon)\sigma T_s^4}$$

$$\tilde{G}'_s (1-A_b)(1+1-d) = (2-\epsilon)\sigma T_s^4 \Rightarrow \sigma T_s^4 = \frac{1-d/2}{1-\epsilon/2} \tilde{G}'_s (1-A_b)$$

$$\text{or } \boxed{T_s = \left( \frac{1-d/2}{1-\epsilon/2} \right)^{1/4} T_{eq}}$$

Rk: i) we could have put  $\epsilon_s \sigma T_s^4$  for the emission of the Earth (in practice  $\epsilon_s \approx 0,95$ )

ii) if  $\left| \begin{array}{l} \epsilon=1 \\ d=0 \end{array} \right.$ : fully absorbing atmosphere  $T_s = \sqrt[4]{2} T_{eq} > T_{eq}$

more effect of adding a piece of glass!

if  $d \uparrow$ ,  $T_s \downarrow$

if  $\epsilon \uparrow$ ,  $T_s \uparrow$

if  $A_b \uparrow$ ,  $T_s \downarrow$

(increases absorption in IR  $\Rightarrow$  greenhouse effect!)

### III Geostrophic wind formula:

$$f \mathbf{k} \wedge \vec{V}_g = -\frac{1}{\rho} \begin{pmatrix} \partial_x P \\ \partial_y P \end{pmatrix}$$

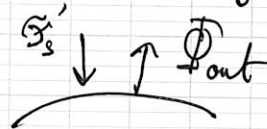
$$\begin{aligned} f(\mathbf{k} \wedge (\mathbf{k} \wedge \vec{V}_g)) &= f \left( (\mathbf{k} \cdot \vec{V}_g) \mathbf{k} - \mathbf{k}^2 \vec{V}_g \right) = -\frac{1}{\rho} \mathbf{k} \wedge \vec{\nabla} P \\ &= -\frac{1}{\rho} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \wedge \begin{pmatrix} \partial_x P \\ \partial_y P \\ 0 \end{pmatrix} = -\frac{1}{\rho} \begin{pmatrix} -\partial_y P \\ \partial_x P \\ 0 \end{pmatrix} \end{aligned}$$

$$\Rightarrow \boxed{\vec{V}_g = + \frac{1}{\rho f} \begin{pmatrix} -\partial_y P \\ \partial_x P \end{pmatrix}}$$

### III Radiative forcing and feedbacks

We look at the equation at the top of the troposphere:

$$N = \mathcal{F}'_s - \Phi_{out}$$



at equilibrium  $N=0$  (radiative balance)

Now  $N(\vec{E}, \vec{I}, T_s)$  is a function of external parameters,  $\vec{E}$  external parameters  $\vec{I}$  that are dependent on  $T_s$  and of  $T_s$ . An elementary variation of  $\Delta N$  writes

$$\Delta N = \underbrace{\sum_i \frac{\partial N}{\partial E_i} \Delta E_i}_{\Delta F_i} + \underbrace{\left( \frac{\partial N}{\partial T_s} + \sum_j \frac{\partial N}{\partial I_j} \frac{\partial I_j}{\partial T_s} \right) \Delta T_s}_{\text{variation of flux due to climate system response}}$$

$\hookrightarrow$  radiative forcing

variation of flux due to climate system response

total forcing is  $\Delta F = \sum_i \Delta F_i$   
 $\hookrightarrow$  independent contribution

in the end, at equilibrium  $\Delta N = 0$  and the climate system changes its surface temperature by  $\Delta T_s$  given by

$$\boxed{\Delta T_s = G \Delta F}$$

$\hookrightarrow$  in  $\text{Wm}^{-2}$

$G$  is the gain of the system in  $\text{KW}^{-1} \text{m}^2$   
 We observe that

$$G^{-1} = \underbrace{-\frac{\partial N}{\partial T_s}}_{G_0^{-1}} = \underbrace{\sum_j \frac{\partial N}{\partial I_j} \cdot \frac{\partial I_j}{\partial T_s}}_R$$

interpretation:

\*  $G_0$  corresponds to the bare gain of the system if all internal parameters are kept constant  $I_j$  in the presence of  $\Delta F$ , i.e. there is no feedback

$$\Delta T_s^{(0)} = G_0 \Delta F \quad \text{in this hypothetic situation}$$

Example: in the equivalent temperature model

$$N = \mathcal{G}_s' - \mathcal{G}_s' A_b - \sigma T_s^4$$

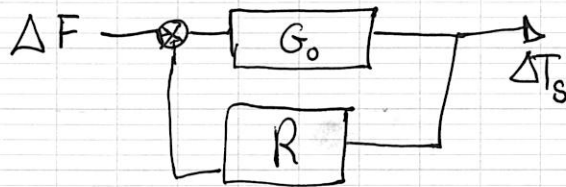
$$\frac{\partial N}{\partial T_s} = -4\sigma T_s^3 \quad \Rightarrow \quad \boxed{G_0 = (4\sigma T_s^3)^{-1}}$$

\*  $R$  is a feedback term showing that changing  $T_s$  to a new equilibrium value modify the  $I_j$  which in turn modifies  $\Delta N$ .

$$\boxed{G = \frac{1}{G_0^{-1} - R}} = \boxed{\frac{G_0}{1 - G_0 R}}$$

$\hookrightarrow$  amplification term

It has the following interpretation as in electronic circuits:



$$\Delta T_s = G_0 (\Delta F + \underbrace{R \Delta T_s}_{\text{feedback amplification term}})$$

$$\Leftrightarrow \Delta T_s = \frac{G_0}{1 - G_0 R} \Delta F$$

Example: same model as before

$$N = \mathcal{F}'_s (1 - A_b) - \sigma T_s^4$$

consider that  $\Delta T_s$  affect albedo  $A_b(T_s)$ , then

$$R = \frac{\partial N}{\partial A_b} \frac{\partial A_b}{\partial T_s} = - \mathcal{F}'_s \frac{\partial A_b}{\partial T_s}$$

if  $T_s \uparrow$  induces  $\frac{\partial A_b}{\partial T_s} < 0$  (as ice sea decreases albedo)

then  $R > 0 \Rightarrow$  positive retroaction (increase temperature)

$$\Delta T_s = \frac{G_0}{1 - G_0 R} \Delta F > G_0 \Delta F = \Delta T_s^{(0)}$$

$R > 0$ : positive feedback.

$R < 0$ : negative feedback.