

Exercice 1:

$$n(\theta) = a - b \cos(n\theta) \quad a > b, \quad n \in \mathbb{N} \quad \dot{\theta} = \omega_0 = \text{cste}$$

$$\begin{aligned} 1.1 \quad \ddot{n} &= n \ddot{\mu}_n \quad \dot{\mu}_n = \dot{\theta} \ddot{\mu}_\theta; \quad \dot{\mu}_0 = -\dot{\theta} \ddot{\mu}_n; \quad \ddot{n} = \frac{d^2 n}{dt^2} = \frac{dn}{dt} \frac{d\theta}{dt} = \omega_0 \frac{dn}{d\theta} \\ \ddot{n} &= \dot{n} \ddot{\mu}_n + n \ddot{\mu}_0 \end{aligned}$$

$$= \omega_0 (-b(-n) \sin(n\theta)) \ddot{\mu}_n + n \omega_0 \ddot{\mu}_0$$

$$\vec{v} = n \omega_0 b \sin(n\theta) \ddot{\mu}_n + \omega_0 (a - b \cos(n\theta)) \ddot{\mu}_0$$

$$\begin{aligned} 1.2 \quad \ddot{\vec{r}} &= \ddot{\vec{v}} = n \omega_0 b n \dot{\theta} \cos(n\theta) \ddot{\mu}_n + n \omega_0 b \sin(n\theta) \dot{\theta} \ddot{\mu}_0 \\ &+ \omega_0^2 b n \sin(n\theta) \ddot{\mu}_0 + \omega_0 (a - b \cos(n\theta)) (-\omega_0 \ddot{\mu}_n) \end{aligned}$$

$$\boxed{\ddot{\vec{r}} = [a \omega_0^2 + b \omega_0^2 (\cos n\theta)(n^2 + 1)] \ddot{\mu}_n}$$

$$+ 2 b \omega_0^2 n \sin(n\theta) \ddot{\mu}_0$$

Exercice 2:

$$\boxed{2.1 \quad \vec{F} = -G \frac{M-m}{R^2} \ddot{\mu}_n}$$

$$\boxed{2.2. \quad \vec{F} \text{ dérive de } E_p = -\frac{GMm}{R}}$$

$$\boxed{E_m = \frac{1}{2} m v^2 - \frac{GMm}{R}}$$

2.3. d'après le cours, ...

$$\boxed{2.4. \quad E_m = 0 \Rightarrow \sigma_L = \sqrt{\frac{2GM}{R}}}$$

$$\boxed{2.5. \quad \ddot{\vec{r}} = (n - n \dot{\theta}^2) \ddot{\mu}_n + (2n\dot{\theta} + n\ddot{\theta}) \ddot{\mu}_\theta \text{ avec } n=R=a \Rightarrow \dot{n}=n=0}$$

$$\ddot{\vec{r}} = \dot{n} \ddot{\mu}_n + n \ddot{\mu}_\theta$$

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Exercice 3:

$$\left. \begin{aligned} \text{d'où} \quad \left\{ \begin{array}{l} \vec{r} = R \hat{e}_r \ddot{\theta} \\ n \ddot{\theta}^2 = \frac{v^2}{R} \end{array} \right. \quad \text{et} \quad \boxed{\ddot{\theta} = -\frac{v^2}{R^2} \ddot{r}} \end{aligned} \right\}$$

$$\boxed{2.6. \quad \text{PFD } m \ddot{\vec{r}} = -\frac{GMm}{R^2} \ddot{\mu}_n \Rightarrow R \ddot{\theta} = 0 \Rightarrow \dot{\theta} = \text{cste} \Rightarrow \ddot{\theta} = \text{cste}}$$

et

$$\Rightarrow -\frac{v^2}{R} = -\frac{GM}{R^2} \Rightarrow \boxed{v = \sqrt{\frac{GM}{R}}}$$

$$\boxed{2.7. \quad E_c = \frac{1}{2} m v^2}$$

$$\boxed{E_p = -m \frac{GM}{R} = -mv^2}$$

$$E_m = E_c + E_p$$

$v = \text{cste} \rightarrow \text{movement circulaire uniforme}$

Exercice 3:

3.1 Bilan: rég. galiléen, poids $\vec{P} = mg \hat{e}_y$, tension $\vec{T} = T \hat{e}_n$ avec $T \neq 0$

$$\text{base: } \ddot{\mu}_y = \cos \varphi \ddot{\mu}_y + \sin \varphi \ddot{\mu}_x$$

$$\ddot{\mu}_x = \cos \varphi \ddot{\mu}_x - \sin \varphi \ddot{\mu}_y$$

$$\boxed{3.2 \quad \ddot{\mu}_0(\vec{r}) = \vec{0} \wedge \vec{P} = l m g \ddot{\mu}_n \wedge (\cos \varphi \ddot{\mu}_y + \sin \varphi \ddot{\mu}_x) = m g l \cos \varphi \ddot{\mu}_z}$$

$$\boxed{3.3 \quad \ddot{\mu}_0(\vec{r}) = \vec{0} \wedge \vec{T} = \vec{0} \text{ car } \vec{T} \parallel \vec{r}}$$

$$\boxed{3.4. \quad \text{MC: } \frac{d\vec{r}}{dt} = \ddot{\mu}_0(\vec{r}) \rightarrow ml^2 \ddot{\varphi} = mg l \cos \varphi \text{ soit } \ddot{\varphi} = \frac{g}{l} \cos \varphi = \frac{g}{l} \cos \varphi = 0}$$

$$\boxed{3.5. \quad E_p(\vec{r}) = -mg l \cos \varphi \text{ et } \dot{\varphi} = l \dot{m} \ddot{\varphi}; \quad E_p(\vec{r}) = 0 \text{ car } \vec{r} \perp \vec{L} \text{ n'échange pas d'impulsion}}$$

$$\boxed{E_m = \frac{1}{2} ml^2 \dot{\varphi}^2 - mg l \cos \varphi \text{ en } \varphi = 0, \dot{\varphi} = 0 \text{ et } E_m \text{ est conservé donc}}$$

$$\boxed{3.6. \quad E_m = 0 \Rightarrow \frac{1}{2} ml^2 \dot{\varphi}^2 = mg l \cos \varphi \text{ et } \ddot{\varphi} = \frac{g}{l} \cos \varphi \text{ et } \ddot{\varphi} \in [0, \pi]}$$

$$\boxed{3.7. \text{ condition initiale: } x(0) = 0, y(0) = l, \dot{x}(0) = -\frac{g}{l} \cos \varphi, \dot{y}(0) = 0, \text{ Bilan: } P}$$

$$\boxed{\begin{cases} m \ddot{x} = 0 \\ m \ddot{y} = mg \end{cases} \Rightarrow \begin{cases} \ddot{x}(t) - \ddot{x}(0) = 0 \\ \ddot{y}(t) - \ddot{y}(0) = mg t \end{cases} \Rightarrow \begin{cases} \ddot{x}(t) = -\frac{g}{l} \cos \varphi t \\ \ddot{y}(t) = l + \frac{1}{2} g t^2 \end{cases}}$$

$$\boxed{3.8. \quad y(t) = 5l = l + \frac{1}{2} g t^2 \Rightarrow t = \frac{2\sqrt{4l}}{g} \text{ et } x_0 = x(t) \Rightarrow x_0 = -4l}$$

$$\boxed{3.9. \quad \ddot{x}(t) = -\frac{g}{l} \cos \varphi \text{ et } \ddot{y}(t) = g \ddot{t} = \frac{g}{l} \cos \varphi t \text{ soit } \ddot{r} = \frac{g}{l} \cos \varphi (2\ddot{y} - \ddot{x}_0)}$$