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Parity violation in metals

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Résumé. — Dans cet article, nous étudions la violation de la parité induite dans les métaux par l'interaction électrofaible des électrons de conduction avec les noyaux du réseau. Les effets que nous discutons ici mettent en jeu le même paramètre électrofaible — la charge faible Q_W — que la violation de la parité dans les atomes et les variations avec le numéro atomique suivent également une loi en Z^3. Les mécanismes qui fournissent le renforcement nécessaire pour amener l'asymétrie gauche-droite au niveau de 10^{-6} à 10^{-5} sont cependant très différents de ceux qui opèrent dans les expériences de physique atomique. Ils peuvent être considérés comme des phénomènes coopératifs à longue portée associés à la délocalisation des électrons de conduction. Les effets de violation de la parité ne faisant pas intervenir des transitions entre bandes différentes n'apparaissent qu'en présence d'un champ magnétique inhomogène. Un exemple intéressant est la modification de l'interaction RKKY entre impuretés magnétiques. L'observation effective de cet effet, qui constitue un des rares cas de manifestation de la violation de la parité en régime statique, apparaît malheureusement comme une entreprise très difficile. Des conditions expérimentales plus favorables peuvent être créées en résonance magnétique électronique. La magnétisation oscillante transmise à travers une plaque métallique est assujettie à une petite rotation autour de la normale à la plaque qui est de l'ordre d'un milliradian par centimètre. Dans les circonstances où les niveaux de Landau sont bien séparés, la probabilité de transition à résonance présente une asymétrie gauche-droite de l'ordre de 2×10^{-6} cotg θ, où θ est l'angle entre champ magnétique statique et le champ oscillant. Bien que notre analyse soit de caractère général, les estimations numériques ont été faites dans le cas du césium métallique.

Abstract. — In this paper we study parity violation in metals, induced by the electroweak interaction of the conduction electrons with the nuclei of the lattice. The effects discussed here involve the same electroweak parameter — the weak charge Q_W — as in atomic parity violation and the variation with the atomic number also obeys a Z^3 law. The mechanisms which provide the enhancement necessary to bring the left-right asymmetry to the level of 10^{-5}-10^{-6} are however very different from the ones which are operating in atomic experiments. They can be viewed as long-range cooperative phenomena associated with the delocalization of the conduction electrons. The p.v. effects, not involving interband transitions, only appear when the metal is perturbed by an inhomogeneous magnetic field. An interesting example is the modification of the RKKY interaction between magnetic impurities. The actual observation of this effect, which is one of the very few cases of static manifestation of parity violation, appears unfortunately as a very difficult enterprise. More favourable experimental conditions could be achieved in magnetic electron spin resonance. The oscillating magnetization transmitted through a metallic slab is subjected to small rotation around the normal to the slab of about one milliradian per centimeter. Under these circumstances where the Landau levels are well separated, the spin resonance transition probability exhibits a left-right asymmetry of the order of 2×10^{-6} cotg θ, where θ is the angle between the a.c. and d.c. magnetic fields. Although our analysis is of general character, all the numerical estimates have been performed for the case of metallic caesium.

In this paper we give a theoretical analysis of parity violation phenomena in metals which stem from the weak interactions of the conduction electrons with the nuclei of the cristal lattice. Similar parity violation effects have been observed in heavy atoms [1] and particularly in atomic caesium [2] which remains, so far, the simplest atomic system where an electroweak interference has been clearly demonstrated. These experiments have led to a determination of a combination of electroweak coupling constants, the so called « weak charge » Q_W, not directly accessible to high-energy experiments ; they have also extended the explored range of momentum transfer towards the

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MeV region [3]. The effects discussed in this paper involve the same electroweak parameter Q_W and they also benefit from the enhancement known as the Z^3 law in atoms [4], which, by itself, is not sufficient to bring the left-right asymmetries to a level where a measurement looks possible. In atoms, the feasibility gap is bridged by looking at forbidden transitions where selection rules suppress the normal parity conserving amplitude, leaving the parity violating a better chance to be observed. In metals, we have searched for experimental situations where the enhancement of the p.v. effects could result from cooperative phenomena. Indeed, because of the delocalization of conduction electron wave functions, there can appear long range effective parity violating interactions, which, under favorable circumstances (very low temperature, high purity samples) lead to left-right asymmetries in the range 10^{-6} to 10^{-4}. Although the weak interaction physics involved in p.v. phenomena in metals and atoms is basically the same, the physical mechanisms which are behind the effects having a chance to be actually observed, are so different that we feel the rather detailed analysis given in this paper justified. This paper having an exploratory character, we have, to describe the conduction electron physics, used simple models which should nevertheless apply reasonably well to alkali metals. The numerical evaluations have been performed for metallic caesium which is clearly favoured by the Z^3 law.

In section 1 we give a description of the p.v. electron-nucleus interaction within the Bloch wave function formalism. We show, provided one neglects interband transitions, that the p.v. interaction can be replaced by an effective coupling having the very simple form : $a\boldsymbol{\sigma}.\mathbf{k}$, where $\boldsymbol{\sigma}$ is the spin of the electron and $\mathbf{k}$ its Bloch momentum. The coefficient a which crucially depends on the conduction electron wave function in the vicinity of the nucleus, has been computed in the case of caesium using the Wigner-Seitz « cellular » method.

The observable effects produced by the $a\boldsymbol{\sigma}.\mathbf{k}$ coupling appear in the presence of an inhomogeneous magnetic field $\mathbf{B}(\mathbf{r})$. In section 2 we show, using a SU(2) gauge transformation, that the static magnetic susceptibility tensor $\chi_{ij}(\mathbf{r}, \mathbf{r}')$ acquires an antisymmetric p.v. term :

$$\delta\chi_{ij}(\mathbf{r}, \mathbf{r}') = \varepsilon_{ijk} \frac{(\mathbf{r} - \mathbf{r}')_k}{l_{\text{p.v.}}} \chi_0(\mathbf{r}, \mathbf{r}')$$

where $\chi_0(\mathbf{r}, \mathbf{r}')$ is the normal scalar susceptibility and $l_{\text{p.v.}} = \dfrac{1}{2\,am^*}$ a basic length parameter which gives the typical size of the long range p.v. effects for conduction electrons (m^* is the effective mass of the valence electron). For caesium, $l_{\text{p.v.}}$ is found to be equal to 11 m. A direct consequence of the new term in $\chi_{ij}(\mathbf{r}, \mathbf{r}')$ is a modification of the indirect exchange interaction between magnetic impurities. The RKKY coupling $\mathbf{m}.\mathbf{m}'\, f_{\text{RKKY}}(\mathbf{r} - \mathbf{r}')$ gets a long range p.v. part :

$$(\mathbf{r} - \mathbf{r}').\mathbf{m} \wedge \mathbf{m}'\, f_{\text{RKKY}}(\mathbf{r} - \mathbf{r}')/l_{\text{p.v.}}\,.$$

The length $l_{\text{p.v.}}$ being very large compared to the oscillation length of the function $f_{\text{RKKY}}(\mathbf{r})$ the actual observation of this static manifestation of parity violation is going to be a very difficult enterprise.

A more favourable situation is encountered in magnetic spin resonance. In these circumstances where the diffusion motion of the electron is not affected by the static magnetic field $\mathbf{B}_0$, a formula similar to one written above holds for the oscillating magnetic susceptibility $\chi_{ij}(\mathbf{r}, \mathbf{r}', \omega)$. As a consequence, the transmitted magnetization $\mathbf{M}(L)$ through a metallic slab of size L is modified to

$$\mathbf{M}(L) + \frac{L}{l_{\text{p.v.}}} \mathbf{n} \wedge \mathbf{M}(L)$$

where $\mathbf{n}$ is a unit vector normal to the slab.

In sections 3 and 4 we study the effects of the $a\boldsymbol{\sigma}.\mathbf{k}$ coupling in a situation where the Landau levels are well defined, the electron collision relaxation time τ being large compared with the cyclotron period. We first treat the simple case where the skin depth is large compared with the cyclotron radius. (In practice these conditions could be fulfilled only in semiconductors where a reliable evaluation of $l_{\text{p.v.}}$ would require elaborate computational methods which go far beyond the scope of this paper.) The mixing of the Landau levels produced by the $a\boldsymbol{\sigma}.\mathbf{k}$ coupling leads to an electroweak interference between the spin resonance and the cyclotron resonance, which we compute using the dipolar approximation. In this rather ideal situation, the left-right asymmetry is found to be of the order of $\lambda/l_{\text{p.v.}}$ where λ is the wavelength of the resonant radio frequency field.

Section 4 is devoted to studying the influence of the Landau levels p.v. mixings on the conduction electron spin resonance; we properly take into account the complications due to the anomalous skin effect. With the help of a semi-classical method we are able to free ourselves from the dipolar approximation, here totally unjustified. The dominant p.v. mechanism can be viewed as an interference between magnetic multipole moments of different polarity, the rôle of electric amplitudes being negligible. The left-right asymmetry involves the pseudo scalar $\mathbf{n}.(\mathbf{B}_0 \wedge \mathbf{B})$ where $\mathbf{B}$ is r.f. magnetic field and its magnitude is given, up to a factor of order one, by the ratio $2\,\dfrac{r_c}{l_{\text{p.v.}}}$ where r_c is the cyclotron radius.

1. Parity violating effective potential for the conduction electrons of an alkali metal.

The weak interaction between a conduction electron and a nucleus of the crystal lattice is described, within the standard Weinberg-Salam model [5], by the exchange of a heavy neutral vector boson Z^0. It is well known [4] that the resulting dominant parity violating

interaction can be described in the non relativistic limit by the potential :

$$V^{eN}_{p.v.}(\mathbf{r}) = \frac{G_F Q_W}{4\sqrt{2}\, m_e} [\boldsymbol{\sigma}.\mathbf{p}\delta(\mathbf{r}) + \delta(\mathbf{r})\boldsymbol{\sigma}.\mathbf{p}]$$

$$Q_W = -[N + Z(4\sin^2\theta_W - 1)] \qquad (1)$$

N and Z are respectively the numbers of neutrons and protons inside the nucleus and θ_W is the Weinberg angle. We have kept only the term that is independent of the nuclear spin, since this term dominates for sufficiently heavy elements [4, 3].

In this section we would like to study the effect of $V^{eN}_{p.v.}$ on the properties of the conduction bands of a metal. The parity violating electron nucleus interaction

$$V_{p.v.}(\mathbf{r}) = \sum_{\mathbf{R}_i} V^{eN}_{p.v.}(\mathbf{r} - \mathbf{R}_i)$$

where the sum $\sum_{\mathbf{R}_i}$ runs over all the nuclei of the lattice, is invariant under the translation group of the lattice. The motion of the valence electron in the presence of this potential can still be described by a Bloch wave :

$$\psi^{p.v.}(\mathbf{r}) = e^{i\mathbf{k}.\mathbf{r}}\, u^{p.v.}(\mathbf{k}, \mathbf{r}). \qquad (2)$$

The wave vector $\mathbf{k}$ is taken inside the first Brillouin zone. The function $u(\mathbf{k}, \mathbf{r})$, for a given $\mathbf{k}$, is invariant under the translation group of the lattice. The energy bands in the presence of the parity violating interactions $\varepsilon_n^{p.v.}(\mathbf{r})$ are given by the following eigenvalue equation :

$$H(\mathbf{k})\, u_n(\mathbf{k}, \mathbf{r}) = \varepsilon_n^{p.v.}(\mathbf{k})\, u_n(\mathbf{k}, \mathbf{r})$$

$$H(\mathbf{k}) = \frac{1}{2\, m_e}(-i\nabla + \mathbf{k})^2 + U(\mathbf{r}) + \frac{G_F Q_W}{4\sqrt{2}\, m_e} [\{\boldsymbol{\sigma}.(-i\nabla), \delta(\mathbf{r})\}_+ + 2\,\boldsymbol{\sigma}.\mathbf{k}\delta(\mathbf{r})]$$

$U(\mathbf{r})$ is the periodic potential describing the interaction of the valence electron with the lattice ions. In the present paper we are interested in the motion of Bloch electrons within a band and we ignore the effect of $V^{eN}_{p.v.}$ on interband transitions. The quantity of interest will then be the modification of the band energy $\Delta\varepsilon_n^{p.v.}(\mathbf{k})$ by the parity violating potential. To first order in G_F, $\Delta\varepsilon_n^{p.v.}(\mathbf{k})$ is given by :

$$\Delta\varepsilon_n^{p.v.}(\mathbf{k}) = \frac{G_F Q_W}{2\sqrt{2}\, m_e} \times \left[\mathrm{Re}\left(u_n^*(\mathbf{k}, \mathbf{r})\,\boldsymbol{\sigma}\cdot\frac{\nabla}{i}\,u_n(\mathbf{k}, \mathbf{r})\right)_{\mathbf{r}=0} + u_n^*(\mathbf{k}, 0)\,\boldsymbol{\sigma}.\mathbf{k}\,u_n(\mathbf{k}, 0)\right] \qquad (3)$$

In the above expression, $u_n(\mathbf{k}, \mathbf{r})$ are the Bloch wave functions associated with the unperturbed Hamiltonian $H^0 = \frac{1}{2\, m_e}(-i\nabla + \mathbf{k})^2 + U(\mathbf{r})$. They are assumed to be normalized to the number of valence electrons inside one Wigner-Seitz primitive cell. The potential $U(\mathbf{r})$ is assumed to be invariant upon space reflexion. If, for a given $\mathbf{k}$, the energy level $\varepsilon_n(\mathbf{k})$ has no other degeneracies than the one associated with the spin angular momentum, we have the equalities :

$$\varepsilon_n(\mathbf{k}) = \varepsilon_n(-\mathbf{k})$$

and up to a phase factor

$$u_n(\mathbf{k}, \mathbf{r}) = u_n(-\mathbf{k}, -\mathbf{r}).$$

From the above relation it follows that $\Delta\varepsilon_n^{p.v.}(\mathbf{k})$ is an odd function of $\mathbf{k}$

$$\Delta\varepsilon_n^{p.v.}(\mathbf{k}) = -\Delta\varepsilon_n^{p.v.}(-\mathbf{k}). \qquad (4)$$

In order to evaluate $\Delta\varepsilon_n^{p.v.}$, in the case of an alkali metal, we shall use the cellular Wigner-Seitz method [6], together with an expansion of $u_n(\mathbf{k}, \mathbf{r})$ in power of the wave number $\mathbf{k}$:

$$u_n(\mathbf{k}, \mathbf{r}) = u_n^{(0)}(r) + i\mathbf{k}.\mathbf{r}\,u_n^{(1)}(r) + \cdots \qquad (5)$$

In the Wigner-Seitz cellular method, one proceeds by making the two following approximations :

i) the Wigner-Seitz primitive polyhedron is replaced by a sphere of radius r_s of the same volume;

ii) the potential inside the sphere is replaced by the atomic potential, which outside the ion core reduces to $V(r) = -e^2/r$.

For the band of lowest energy the functions $u^{(0)}(r)$ and $u^{(1)}(r)$ are spherically symmetric (we have dropped the band index n). The Wigner-Seitz boundary conditions reduce to :

$$\left.\frac{\partial u^{(0)}}{\partial r}\right|_{r_s} = 0 \quad u^{(1)}(r_s) = 0. \qquad (6)$$

The function $u^{(0)}(r)$ is the regular solution of the s-wave radial atomic Schrödinger equation, satisfying the above boundary condition. The corresponding eigenvalue E_0 is the minimum of the conduction band. Bardeen has shown [7] that the function $u^{(1)}(r)$ is obtained by adding to $-u^{(0)}(r)$ the regular solution of the p-wave radial wave equation corresponding to the energy E_0 :

$$u^{(0)} = \frac{1}{\sqrt{4\pi}} R_s(E_0, r)\,\chi_m$$

$$u^{(1)} = \frac{1}{\sqrt{4\pi}}\left(-R_s(E_0, r) + \frac{1}{r}R_p(E_0, r)\right)\chi_m \qquad (7)$$

χ_m ($m = \pm 1/2$) is a two-component Pauli spinor. $u^{(0)}(r)$ is normalized to unity in the Wigner-Seitz sphere. In order to satisfy the condition $u^{(1)}(r_s) = 0$

the p-wave radial wave equation is normalized in such a way that :

$$R_p(E_0, r_s) = r_s R_s(E_0, r_s) .$$

Using the above expressions for $u^{(0)}(r)$ and $u^{(1)}(r)$, we find the parity violating energy shift $\Delta\varepsilon^{p.v.}(\mathbf{k})$ to be :

$$\Delta\varepsilon^{p.v.}(\mathbf{k}) = a(\chi_m, \sigma\chi_m).\mathbf{k} \tag{8}$$

with

$$a = \frac{G_F Q_W}{\sqrt{2}\, 8\pi m_e} R_s(E_0, 0) \left(\frac{1}{r} R_p(E_0, r)\right)_{r=0} \times \times (1 + O(k^2)) \tag{9}$$

The quantity a has a structure very similar to the matrix element of $V^{eN}_{p.v.}$ between atomic $ns_{1/2}$ and $n' p_{1/2}$ states :

$$\langle ns_{1/2} \mid V^{eN}_{p.v.} \mid n' p_{1/2} \rangle = = \frac{3\, iG_F Q_W}{\sqrt{2}\, 16\pi m_e} R_{ns}(0) \left(\frac{1}{r} R_{n'p}(r)\right)_{r=0} .$$

We expect that $\Delta\varepsilon^{p.v.}$ will follow the same Z^3 law which has been established in reference [4].

As in parity violation phenomena in heavy atoms the relativistic corrections are of particular importance. The Wigner-Seitz cellular method can be easily extended to the relativistic case. It can be easily shown that the relativistic correcting factor K_r to be applied to the formula giving a is practically identical to the one involved in atomic physics calculation which is thorougly discussed in references [4] and [8].

We have performed an explicit computation of a in the case of caesium using the numerical values of $u^{(0)}(r)$ and $u^{(1)}(r)$ tabulated in reference [9]. In order to extrapolate the data to $r = 0$ we have used a representation of the regular radial wave function $R_l(r)$ given in reference [4] which is accurate in the region

$$0 \leqslant r \leqslant \frac{1}{Z}$$

$$rR_l(r) = c_l \left(\frac{8\varphi(r)}{\varphi'(r)}\right)^{1/2} J_{2l+1}(\sqrt{8\varphi(r)})$$

with

$$\varphi(r) = Z\left(1 - \zeta \frac{l(l+1)}{3}\right) r - \frac{\zeta}{3} Z^2 r^2 \tag{10}$$

ζ is a small parameter ($\zeta = 0.11$ for atomic caesium) which is proportional to the electrostatic potential of the core electrons near the nucleus.

We have obtained in this way :

$$R_s(E_0, 0) = 6.61\, a_0^{-3/2} .$$

$$\frac{1}{r} R_p(E_0, r)\,|_{r=0} = -\, 584.7\, a_0^{-3/2} . \tag{11}$$

It leads to the value of a :

$$a = \frac{G_F Q_W}{2\sqrt{2}\, m_e a_0^3} (-\, 307\, K_r) \tag{12}$$

(a_0 is the Bohr radius).

In the case of atomic caesium, the (nearly model independent) relativistic factor K_r is found to be

$$K_r \simeq 2.7 .$$

We arrive at the final expression of a :

$$a = \frac{G_F Q_W}{2\sqrt{2}\, m_e} (-\, 829) \frac{1}{a_0^3} . \tag{13}$$

It is of interest to compare this result with the one we would obtain using plane waves normalized to one in the Wigner-Seitz cell volume :

$$a_{p.w.} = \frac{G_F Q_W}{2\sqrt{2}\, m_e} \frac{1}{\Omega_s} = \frac{3\, Q_W G_F}{8\pi\sqrt{2}\, m_e r_s^3} .$$

From the value of r_s used in reference [9] $r_s = 5.735\, a_0$ we get

$$a_{p.w.} = \frac{G_F Q_W}{2\sqrt{2}\, m_e a_0^3} (1.27 \times 10^{-3}) .$$

As expected, there is first a considerable enhancement factor, of about 10^5, which comes from the steep increase of the electronic density near the nucleus. The second striking fact is the change of signs whose origin can be traced back to the Wigner-Seitz boundary condition and to the difference between the numbers of nodes of the s and p radial functions appearing in the Bloch wave functions. Using the asymptotic formulae for the radial wave functions given in reference [4], it is possible to obtain a closed formula for a in the limiting case where the two following conditions are simultaneously satisfied : $| E_0 r_s | \ll 1$ and $r_s \gg 1$ (E_0 and r_s are in atomic units) :

$$a = -\, \frac{G_F Q_W Z^2 (1 - \zeta) K_r}{m_e \sqrt{2}\, a_0^3 z_s \cos\pi(\mu_s(0) - \mu_p(0))} \tag{14}$$

where $z_s = \sqrt{8 r_s}$ and $\mu_p(E)$ and $\mu_s(E)$ are the interpolated quantum defects for p and s valence states respectively. This formula is unfortunately of no practical use for the caesium conduction band since its conditions of validity are clearly not fulfilled. We may note however that the Z^3 law is apparent and that the change of signs with respect to the free electron case occurs if the quantum defect of s and p-waves are sufficiently close so that $| \mu_s(0) - \mu_p(0) | < 0.5$, a condition which is satisfied in the case of caesium : $| \mu_s(0) - \mu_p(0) | \simeq 0.48$.

After computing the p.v. energy shift $\Delta\varepsilon^{p.v.}(\mathbf{k})$ given in formulae (8-9), in the following we shall use the

approximation that consists in ignoring the subtleties of the wave function near the nucleus, and describing the conduction electrons as an electron gas in a uniformly distributed positive background that ensures the electrical neutrality. The Hamiltonian describing this system, including the p.v. interaction, is then :

$$H = \sum_i \frac{\mathbf{k}_i^2}{2\,m^*} + \frac{1}{2}\sum_{ij} V(\mathbf{r}_i - \mathbf{r}_j) + \sum_i a\boldsymbol{\sigma}_i.\mathbf{k}_i \quad (15)$$

a is the constant given in (8-9), m^* is the effective mass of the electrons and $V(\mathbf{r}_i - \mathbf{r}_j)$ is the electron-electron interaction potential.

We study in the following, within this simple model, the new effects associated with the p.v. term $a\boldsymbol{\sigma}.\mathbf{k}$. Before proceeding to the more complicated cases discussed in the next sections, let us explain what happens in a uniform external magnetic field $\mathbf{B}_0 = B_0\,\mathbf{u}_z$. Due to the p.v. coupling, one expects a nonzero value of $\langle k_z \rangle$. Working to lowest order in a (which is certainly a very good approximation), one finds effectively that $\langle k_z \rangle = -am^* \langle \sigma_z \rangle$: there exists a longitudinal mean momentum proportional to the Pauli magnetization. However there is no convection current associated with this effect : the proper definition of the current operator $\mathbf{j}$ is :

$$\langle \psi \,|\, \mathbf{j} \,|\, \psi \rangle = \frac{e}{m^*} \operatorname{Im} \psi^*(\nabla\psi) - \frac{e^2}{m^*} \mathbf{A}\,|\,\psi\,|^2 + \\ + ea\psi^*\,\boldsymbol{\sigma}\psi$$

so that one finds : $\langle j_z \rangle = \frac{e}{m^*} \langle k_z + am^*\,\sigma_z \rangle = 0.$

This result confirms and explains a calculation of Vilenkin [10] in the relativistic case : there is no longitudinal current induced by the weak interactions in a uniform magnetic field. In the next section we discuss the case of a non uniform field.

2. Modification of the susceptibility in a non uniform magnetic field.

In this section we study the magnetization of the metal in the linear response to a small magnetic field. We first give the general formula which applies in the presence of parity violation, and then we specialize in two interesting cases :

— linear response to a small static field, and the subsequent modification of the interaction law of two magnetic impurities in a metal;

— in the presence of a strong static field $\mathbf{B}_0$, linear response to a radiofrequency field, i.e. the modification of conduction electron spin resonance, in a situation where the effect of $\mathbf{B}_0$ on the orbital motion can be neglected. This condition is fulfilled when the period of the cyclotron motion is much larger than the diffusion relaxation time τ of the conduction electrons. (The opposite situation, where the Landau levels are well defined, will be studied in the next two sections.)

a) *General formalism.*

Let us first study the general case where the system is in a magnetic field $\mathbf{B}$. In the effective mass approximation, the parity violating Hamiltonian is obtained from (15) by the gauge invariance prescription :

$$H = \sum_i \frac{(\mathbf{k}_i + |\,e\,|\,\mathbf{A}(\mathbf{r}_i))^2}{2\,m^*} + \frac{1}{2}\sum_{ij} V(\mathbf{r}_i - \mathbf{r}_j) + \sum_i \boldsymbol{\sigma}_i.[a(\mathbf{k}_i + |\,e\,|\,\mathbf{A}(\mathbf{r}_i)) + |\,\mu_B\,|\,\mathbf{B}(\mathbf{r}_i)] + H_{e-\varphi} + H_{e-i} \quad (16)$$

where $\mathbf{k}_i$ and $\mathbf{r}_i$ are conjugate variables. $H_{e-\varphi}$ and H_{e-i} stand for the Hamiltonians describing the electron-phonon and electron-impurity interactions. We shall not write them down explicitly but assume that they are spin-independent. Throughout this paper, we shall neglect terms of order a^2, which is certainly legitimate in view of the weakness of the p.v. effects. The Hamiltonian can then be rewritten in the form :

$$H = \frac{1}{2\,m^*}\sum_i (\mathbf{k}_i + |\,e\,|\,\mathbf{A}(\mathbf{r}_i) + am^*\,\boldsymbol{\sigma}_i)^2 + \frac{1}{2}\sum_{ij} V(\mathbf{r}_i - \mathbf{r}_j) + |\,\mu_B\,|\sum_i \boldsymbol{\sigma}_i.\mathbf{B}(\mathbf{r}_i) + H_{e-\varphi} + H_{e-i}\,.$$

This suggests to perform a canonical transformation on H : $H' = WHW^{-1}$ where the unitary operator W is given, up to corrections of order a^2, by :

$$W = 1 + iam^* \sum_i \boldsymbol{\sigma}_i.\mathbf{r}_i + O(a^2)\,.$$

One gets the following expression for H' :

$$H' = \sum_i \frac{(\mathbf{k}_i + |\,e\,|\,\mathbf{A}(\mathbf{r}_i))^2}{2\,m^*} + \frac{1}{2}\sum_{ij} V(\mathbf{r}_i - \mathbf{r}_j) + H_{e-\varphi} + H_{e-i} + \\ + |\,\mu_B\,|\sum_i \boldsymbol{\sigma}_i.[\mathbf{B}(\mathbf{r}_i) - 2\,am^*\,\mathbf{r}_i \wedge \mathbf{B}(\mathbf{r}_i)] + O(a^2). \quad (17)$$

This new form of the Hamiltonian makes the discussion particularly transparent : the only p.v. effect is in the spin coupling, and the magnitude of parity violation will be given conveniently by a characteristic length $l_{\text{p.v.}}$, defined by :

$$l_{\text{p.v.}} = \frac{1}{2\, am^*} \,. \tag{18}$$

(In the standard electroweak model $Q_W < 0$, so that, for caesium $a > 0$.)

From the expression of a given in (13), one can estimate $l_{\text{p.v.}}$ in the case of caesium : taking $\frac{m^*}{m} \simeq 0.73$, one gets :

$$l_{\text{p.v.}} \simeq 11.2 \text{ m} \,. \tag{19}$$

The relative order of magnitude of p.v. effects in metals will in general be given by the ratio of a typical interatomic length to $l_{\text{p.v.}}$. In order to get a feeling of the smallness of the effects we are looking for, we quote the value of the ratio of the Bohr radius a_0 to $l_{\text{p.v.}}$, in caesium again :

$$\frac{a_0}{l_{\text{p.v.}}} \simeq 0.47 \times 10^{-11} \,. \tag{20}$$

Let us study the modification of the magnetic susceptibility. In general, the average magnetization in the zero temperature limit is given by :

$$\langle M_i(\mathbf{r}, t) \rangle = \int_{-\infty}^{\infty} \mathrm{d}t' \, \mathrm{d}^3 r' \, \chi^{\text{R}}_{ij}(\mathbf{r}, t\,; \mathbf{r}', t') \, B_j(\mathbf{r}', t') \tag{21}$$

where the non-local susceptibility χ^{R}_{ij} is

$$\chi^{\text{R}}_{ij} = i\theta(t-t') \langle \Psi_0 | \, [\hat{M}_i(\mathbf{r}, t), \hat{M}_j(\mathbf{r}', t')] \, | \, \Psi_0 \rangle \tag{22}$$

with

$$\hat{M}_i(\mathbf{r}, t) = \mathrm{e}^{iHt} \left(\sum_n \mu_{\text{B}} \, \boldsymbol{\sigma}_n \, \delta(\mathbf{r} - \mathbf{r}_n) \right) \mathrm{e}^{-iHt} \tag{23}$$

(Ψ_0 is the ground state of H). The Hamiltonian H is split into two terms : $H = H_0 + H_{\text{p.v.}}$, where the p.c. Hamiltonian H_0 includes the electron-electron, electron-phonon and electron-impurity interactions together with the magnetic coupling with a static uniform magnetic field $\mathbf{B}_0$. In H_0 the only explicitly spin dependent interaction is assumed to be the coupling of the spin magnetic moment with $\mathbf{B}_0$.

Performing the unitary transformation W we get :

$$H' = W(H_0 + H_{\text{p.v.}}) \, W^{-1} = H_0 + \Delta H_{\text{p.v.}}$$

In this section we shall ignore the effects associated with

$$\Delta H_{\text{p.v.}} = - \mu_{\text{B}} \sum_n \frac{1}{l_{\text{p.v.}}} (\mathbf{r}_n \wedge \mathbf{B}_0) . \boldsymbol{\sigma}_n \,.$$

When there are no well defined cyclotron orbits, $\Delta H_{\text{p.v.}}$ plays the rôle of a very small random magnetic field.

Performing the unitary transformation W on the expression (22) giving χ^{R}_{ij} and setting $\Delta H_{\text{p.v.}} = 0$, we get the following expression for the parity violating magnetization :

$$\langle M_i(\mathbf{r}, t) \rangle_{\text{p.v.}} = \varepsilon_{ijk} \frac{x_j}{l_{\text{p.v.}}} \langle M_k(\mathbf{r}, t) \rangle - \\ - \varepsilon_{jkl} \frac{1}{l_{\text{p.v.}}} \left(\int \mathrm{d}t' \, \mathrm{d}^3 r' \, \chi^{\text{R}}_{ij}(\mathbf{r}, t\,; \mathbf{r}', t') \, x'_k \, B_l(\mathbf{r}', t') \right) \tag{24}$$

where we have set $\mathbf{r} = (x_1 \, \mathbf{u}_x + x_2 \, \mathbf{u}_y + x_3 \, \mathbf{u}_z)$ and ε_{ijk} stands for the completely antisymmetric third-order tensor.

b) *Modification of the* RKKY *interaction law.*

We first consider the case in which there is no uniform field $\mathbf{B}_0$, and we study the response to a small static field $\mathbf{B}(\mathbf{r})$. We keep to the approximation of neglecting the spin dependent interaction, so that χ^{R}_{ij} is diagonal in spin indices :

$$\chi^{\text{R}}_{ij}(\mathbf{r}, t\,; \mathbf{r}', t') = \mu_{\text{B}}^2 \, \delta_{ij} \, \pi(\mathbf{r}, t\,; \mathbf{r}', t')$$

where π is the correlation function of the electron density. In the static limit, we get from (24) :

$$\langle \mathbf{M}(\mathbf{r}) \rangle = \int \mathrm{d}^3 r' \, \chi_{\text{N}}(|\, \mathbf{r} - \mathbf{r}' \,|) \times \\ \times \left[\mathbf{B}(\mathbf{r}') + \frac{\mathbf{r} - \mathbf{r}'}{l_{\text{p.v.}}} \wedge \mathbf{B}(\mathbf{r}') \right] \tag{25}$$

where χ_{N} is the « normal » susceptibility in metals computed in the absence of p.v. terms. Let us quote the explicit form of $\chi_{\text{N}}(r)$, corresponding to the free electron gas approximation :

$$\chi_{\text{N}}(r) = \mu_{\text{B}}^2 \frac{m}{(2\pi)^3} \times \\ \times \left[\frac{\sin(2 k_{\text{F}} r) - 2 k_{\text{F}} r \cos(2 k_{\text{F}} r)}{r^4} \right] . \tag{26}$$

There is an alternative form for $\mathbf{M}$ which exhibits clearly the absence of effect in a uniform field : performing an integration by parts in (25), one gets :

$$\langle \mathbf{M}(\mathbf{r}) \rangle = \int \mathrm{d}^3 r' [\chi_{\text{N}}(|\, \mathbf{r} - \mathbf{r}' \,|) \, \mathbf{B}(\mathbf{r}') + \\ + \chi_{\text{p.v.}}(|\, \mathbf{r} - \mathbf{r}' \,|) \, \nabla \wedge \mathbf{B}(\mathbf{r}')] \tag{27}$$

where $\chi_{\text{p.v.}}$ is the p.v. susceptibility, related to χ_{N} by :

$$\chi_{\text{p.v.}} = - \frac{1}{l_{\text{p.v.}}} \int_r^{\infty} r' \, \mathrm{d}r' \, \chi_{\text{N}}(r') \,. \tag{28}$$

We have found that the weak interactions are responsible for a new p.v. term in the magnetic susceptibility. Furthermore, this modification is very small (see (20)), and will appear only in strongly

inhomogeneous magnetic fields. A situation where such fields are created is the case where there are magnetic impurities in the metal.

The indirect exchange coupling of nuclear magnetic moments by conduction electrons has been worked out by Ruderman and Kittel [11]. The p.v. term in the susceptibility (19) gives a new contribution to this coupling. Assuming that the contact part of the hyperfine interaction is dominant, we find that the interaction energy between an impurity of magnetic moment $\mathbf{m}$ located at $\mathbf{r} = 0$ and an impurity of magnetic moment $\mathbf{m}'$ located at $\mathbf{r}$ is :

$$\mathcal{H} = A\chi_N(\mathbf{r})\left[\mathbf{m}\cdot\mathbf{m}' + \frac{\mathbf{r}}{l_{p.v.}}\cdot \mathbf{m}\wedge\mathbf{m}'\right] \qquad (29)$$

A is a negative constant, proportional to the square of the hyperfine structure splitting.

Formula (29) is very simple and aesthetic. It gives an example of a rare situation : parity violation in a static (time independent) configuration. Furthermore, the p.v. interaction in (29) is long-ranged, due to the delocalization of the conduction electrons, although the weak interactions that create it are essentially of zero range on atomic scales.

An interesting problem in which the new term in (29) might be relevant is the indirect coupling between 4f electrons in rare earth metals. It has been shown by De Gennes [12] that these interactions are correctly described by a formula à la Ruderman-Kittel. In this case one has a periodic array of magnetic moments interacting *via* a Hamiltonian similar to (29) (but with a value for $l_{p.v.}$ different from that computed in (19) of course).

In the rare earths of the yttrium series, which have a hexagonal closed-packed structure, it is found experimentally that the stable spin configurations at low temperature are helicoïdal : all the ions lying in a plane orthogonal to the c-axis have parallel magnetic moments, lying in the plane also, and the angle between the magnetic moments in two neighbouring planes is φ. This can be understood from the oscillating nature of the RKKY interactions : forgetting for a while the p.v. term, the energy of a helicoidal configuration defined by an angle φ is

$$H_0(\varphi) = \sum_j F(2\,k_F\,r_{ij})\cos(\varphi_i - \varphi_j) \qquad (30)$$

(where $\varphi_i - \varphi_j = p\varphi$ when i and j are in planes at distance pd from each other). It has been found [12] that H_0 has a minimum at $\varphi = \pm\,\varphi^*$, where $\varphi^* \neq 0$.

It has been suggested [13] that the effect of the p.v. term is to lower the energy of one of these two helices (the left-handed one), the difference of energy-being of the order of 100 Hz. In order to get such a prediction one has to compute the value of $l_{p.v.}$ for each individual case (for instance for dysprosium). As we have seen before, the naive estimation cannot be expected to give a right result, and for instance the sign of $l_{p.v.}$ cannot be obtained from a crude approximation. (In fact, if we took the same sign as the one we found in caesium, we would find, using the arguments of [13], that the right-handed helix is more stable.)

Furthermore, there is a very simple argument which shows that the shift in energy between the two helices is in fact much smaller than expected : it is only a second-order term in $d/l_{p.v.}$. Taking into account the p.v. term in the sum (30), the energy of a helicoidal configuration becomes :

$$H(\varphi) = \sum_p \left[\cos p\varphi + \frac{pd}{l_{p.v.}}\sin p\varphi\right]\times \times \sum_{\mathbf{r}_T} F(2\,k_F\sqrt{p^2\,d^2 + \mathbf{r}_T^2}) \qquad (31)$$

where we have just separated the sum over all the spins in a given plane, then summing over all the planes. Hence one has the general formula :

$$H(\varphi) = H_0(\varphi) - \frac{d}{l_{p.v.}}\frac{\partial H_0}{\partial\varphi} \simeq H_0\left(\varphi - \frac{d}{l_{p.v.}}\right). \qquad (32)$$

Up to first order in $d/l_{p.v.}$, the only effect is that the stable configurations correspond to angles $\pm\,\varphi^* + d/l_{p.v.}$, but they have the same energy. Thus the result we get is that the effect of parity violation in these rare earth elements with helicoidal structure is to change slightly the pitch of the helix depending on whether the helix is right handed or left handed. This difference of the pitches is $2\,d/l_{p.v.} \sim 10^{-10}$ to 10^{-11}, its precise value and its sign must be computed in each case from the structure of the wave function of conduction electrons.

c) *Electron spin resonance : transmission resonance.* The general formula (24) can be applied to the problem of conduction electron spin resonance whenever the cyclotron orbits are not well defined (that is for $\omega_c\,\tau \ll 1$, where ω_c is the cyclotron pulsation and τ the electron relaxation time). In the theoretical model developed by Dyson [14] and extended by Lampe and Platzman [15], the phenomena of spin diffusion out of the skin into which the radiofrequency field penetrates, lead to a non-local susceptibility with a range δ_{mag} much larger than the skin depth δ. Consequently, in equation (24), the second term can be neglected, being of the order of δ/δ_{mag} relative to the first. Let us consider a metallic slab of thickness $L \sim \delta_{mag}$, and choose the y axis along the unit vector $\mathbf{n}$ normal to the surfaces. Equation (24) reduces to :

$$\langle \mathbf{M}(y,t)\rangle_{p.v.} = = \frac{y}{l_{p.v.}}\mathbf{n}_\wedge\,\langle \mathbf{M}(y,t)\rangle\,(1 + O(\delta/\delta_{mag})). \qquad (33)$$

Due to the phenomena of electron spin diffusion, the radiofrequency power is « transmitted » through the slab. For $y = L + O^+$ the r.f. magnetic field $\mathbf{B}_T$ can be shown to be equal to the magnetization for

$y = L + O^-$ times the surface impedance Z_0 [15]. The effect of parity violation is just to rotate $\mathbf{B}_T$ around the normal $\mathbf{n}$ of a small angle ψ given by :

$$\psi = \frac{L}{l_{p.v.}} \simeq \frac{\delta_{mag}}{l_{p.v.}} \simeq \frac{v_F (T\tau)^{1/2}}{l_{p.v.}} \qquad (34)$$

where v_F is the Fermi velocity, T the electron spin-lattice relaxation time, and τ the transport mean free time. Under « realistic conditions » this angle reaches values of the order of 10^{-4} to 10^{-5} radians.

3. Parity violating mixing of the Landau levels. Interference between the conduction electron spin resonance and the cyclotron resonance, in the dipolar approximation.

In the presence of a strong static uniform magnetic field, ($\mathbf{B}_0 = B_0 \mathbf{u}_z$), the dynamics of conduction electrons can reach a new regime, corresponding to the condition $\omega_c \tau \gg 1$, where $\omega_c/2\pi$ is the cyclotron frequency and τ is the collision time. Indeed, when this condition is fulfilled, the Landau levels are well defined and separated. (We shall also suppose in the following that $\omega_s \tau \gg 1$, so that the splitting of each Landau level into two spin sublevels is effective.) We shall work in the approximation of independent electrons, the energy levels of one electron being given by :

$$E_{n,k_z,\varepsilon} = \frac{1}{2m^*} k_z^2 + \left(n + \frac{1}{2}\right)\omega_c + \varepsilon \frac{\omega_s}{2}$$

with

$$\omega_c = |e| \frac{B_0}{m^*}, \omega_s = g_s \frac{|e| B_0}{2 m_e}, n \in \mathbb{N}, \varepsilon = \pm 1 .$$

In this section we study the effects of the p.v. potential in this simple picture. It turns out that, although the canonical transformation is helpful for the physical understanding of the problem, as seen in the preceding section, the calculations are somewhat simpler in the original formulation. So we start from the Hamiltonian :

$$H = \frac{(\mathbf{p} + |e| \mathbf{A}_0(\mathbf{r}))^2}{2m^*} + a\boldsymbol{\sigma}.(\mathbf{p} + |e| \mathbf{A}_0) + + |\mu_B| \boldsymbol{\sigma}.\mathbf{B}_0 \qquad (35)$$

and to be precise we shall use the Landau gauge :

$$A_{0x} = -yB_0 ; \quad A_{0y} = A_{0z} = 0 . \qquad (36)$$

The energy level $E_{n,k_z,\varepsilon}$ is degenerate with corresponding eigenfunctions :

$$\langle \mathbf{r} \mid n, k_x, k_z, \varepsilon \rangle = \frac{1}{2\pi} e^{i(k_x x + k_z z)} \times \times O_n\left(y - \frac{k_x}{|e| B_0}\right) \chi(\varepsilon) \qquad (37)$$

where $O_n(u)$ is the wave function of the nth level of the harmonic oscillator with frequency ω_c, and $\chi(\varepsilon)$ is a Pauli spinor.

The effect of $V_{p.v.}$ is twofold :

— it shifts each energy level $E_{n,k_z,\varepsilon}$ by a quantity $a\varepsilon k_z$;

— it mixes the neighbouring levels of opposite spins and parities, according to the formula :

$$\langle n', k'_x, k'_z, + \mid a\boldsymbol{\sigma}.(\mathbf{p} + |e| \mathbf{A}_0) \mid n, k_x, k_z, - \rangle = - a\sqrt{2 |e| B_0 n}\, \delta_{n',n-1}\, \delta(k'_x - k_x)\, \delta(k'_z - k_z) . \qquad (38)$$

One immediate consequence of this mixing is the possibility of an interference between cyclotron and spin resonances : a transverse radiofrequency electric field can induce a transition between the two perturbed spin sublevels, leading to a resonant absorption of r.f. energy at frequency ω_s. This transition amplitude will in general interference with the normal amplitude due to the transverse r.f. magnetic field (see Fig. 1).

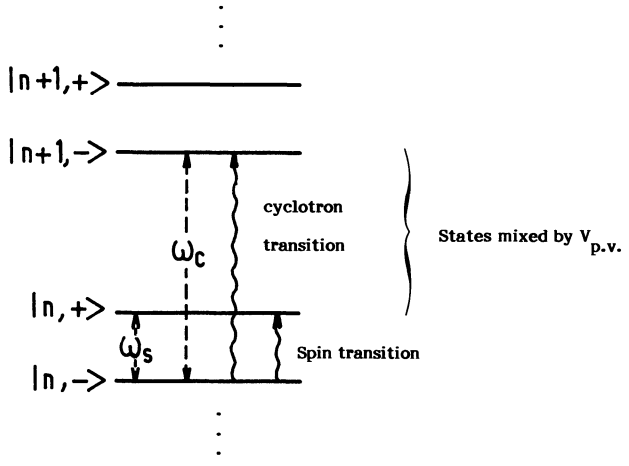

Fig. 1. — Coupling between the cyclotron and electron spin resonance induced by the effective $a\boldsymbol{\sigma}.\mathbf{k}$ parity violating interaction.

In the following we compute this interference and describe its signature. Since the frequencies involved are typically of order 10^{11} Hz, in an ordinary metal such as those we are considering in this paper, the (anomalous) skin effect and the resulting anisotropy play a major rôle as we shall see in the following. However, in order to give a clearer description of the phenomena, we first describe this interference in the simplest case of dipolar approximation, that is neglecting the spatial variation of the r.f. fields. This dipolar approximation would be valid in semiconductors, but there it is far from evident that the simple form (8) for the matrix elements of the p.v. potential holds, and one must carry through the whole analysis of section 1 with a detailed description of the wave functions of electrons and holes in the semiconductor one is studying. In this paper we shall carry on within the simplest scheme of a quasi free electron gas, with

the Hamiltonian given in (35), a model which should apply fairly well to alkali metals.

Within the dipolar approximation, we write the r.f. field as $\mathbf{E} = E\,e^{i\omega t}\,\mathbf{u}_x$, $\mathbf{B} = B\,e^{i\omega t}\,\mathbf{u}_y$; the interaction Hamiltonian linear in this field is :

$$H_{\text{r.f.}} = (H_1 + H_2 + H_3)\,e^{i\omega t} + \text{h.c.}$$

with

$$H_1 = \frac{|e|}{2m_e}\left(\frac{g_s}{2}\right)B\sigma_y \tag{39}$$

$$H_2 = \frac{-|e|}{i\omega m^*}E(p_x - y\,|e|\,B_0) \tag{40}$$

$$H_3 = \frac{-a|e|}{i\omega}E\sigma_x . \tag{41}$$

Let $|\,n, \widetilde{k_x}, k_z, \varepsilon\,\rangle$ be the eigenstates of the static Hamiltonian in the presence of the p.v. potential (treated in first-order perturbation theory). We need the transition amplitude $\mathcal{A}$ from $|\,n, \widetilde{k_x}, k_z, -\,\rangle$ to $|\,n, \widetilde{k_x}, k_z, +\,\rangle$. First of all we notice that the corresponding resonance frequency is shifted from its value without p.v. interaction : ω_s, to the value $\omega_s + 2ak_z$. But due to the symmetry $k_z \to -k_z$, this only leads to a broadening of the resonance signal of order a^2, and hence totally negligible. The resonance at frequency ω_s is associated with the amplitude :

$$\mathcal{A} = \langle\, n, k_x, k_z, +\,|\,H_1 + H_3\,|\,n, k_x, k_z, -\,\rangle + \frac{1}{\omega_c - \omega_s}\times$$
$$\times\{\langle\, n, k_x, k_z, +\,|\,H_2\,|\,n-1, k_x, k_z, +\,\rangle\langle\, n-1, k_x, k_z, +\,|\,V_{\text{p.v.}}\,|\,n, k_x, k_z, -\,\rangle -$$
$$-\langle\, n, k_x, k_z, +\,|\,V_{\text{p.v.}}\,|\,n+1, k_x, k_z, -\,\rangle\langle\, n+1, k_x, k_z, -\,|\,H_2\,|\,n, k_x, k_z, -\,\rangle\}. \tag{42}$$

Using (38) to (41), one easily finds :

$$\mathcal{A} = -i\left[\frac{|e|}{2m_e}\left(\frac{g_s}{2}\right)B + \frac{|e|\,aE}{\omega_c - \omega_s}\right]. \tag{43}$$

The ratio of the p.v. to the normal amplitude is

$$\rho = \left(\frac{2}{g_s}\right)\frac{2m_e a}{\omega_c - \omega_s}\frac{E}{B} = \frac{1}{2\pi}\frac{\omega_c}{\omega_c - \omega_s}\frac{\lambda}{l_{\text{p.v.}}}\frac{E}{cB} \tag{44}$$

where λ is the wavelength of the r.f. field. This result is quite impressive and shows why this situation is much more interesting for the detection of parity violation than those we studied in section 2 : the typical length to be compared with $l_{\text{p.v.}}$ here turns out to be λ (that is, a few centimeters), instead of the interatomic scale that appeared previously. This result can be understood in the following way : the parity mixing amplitude $\mathcal{F}$ is of order $\mathcal{F} \sim \frac{1}{2}\frac{v_c}{(\omega_c - \omega_s)\,l_{\text{p.v.}}}$ where v_c is the classical cyclotron velocity. To get ρ the parity mixing amplitude $\mathcal{F}$ has to be multiplied by the ratio $\frac{E_1}{M_1} \sim \frac{E}{cB}\frac{er_c}{\mu_B}$ where $r_c = \frac{v_c}{\omega_c}$ is the cyclotron radius which multiplied by e gives the typical size of the electric dipole in a cyclotron resonance. Finally there is an extra factor $\frac{1}{n}$ which accounts for the compensation between the two terms appearing in the right-hand side of (42).

What is the signature of parity violation in the interference we are studying here ? It is as usual associated with the existence of a pseudoscalar — time reversal invariant — quantity, which in this case turns out to be $\mathbf{B}_0.(\mathbf{E} \wedge \mathbf{B})$. As one sees immediately, when this quantity changes signs, the relative signs of the p.v. and the normal amplitudes also change, resulting in an asymmetry in the absorption rate, of relative magnitude

$$\mathcal{A}_{\text{R.L.}} = \frac{|\,\mathcal{A}_{\text{p.c.}} + \mathcal{A}_{\text{p.v.}}\,|^2 - |\,\mathcal{A}_{\text{p.c.}} - \mathcal{A}_{\text{p.v.}}\,|^2}{|\,\mathcal{A}_{\text{p.c.}}\,|^2} = 2\rho$$

$$\mathcal{A}_{\text{p.c.}} = \frac{ie}{2m_e}\left(\frac{g_s}{2}\right)B\,; \quad \mathcal{A}_{\text{p.v.}} = \frac{ieaE}{\omega_c - \omega_s}. \tag{45}$$

We won't try at this stage to give a precise estimate of the left-right asymmetry $A_{\text{R.L.}}$. As we have previously noticed, this calculation, based on the dipolar approximation, cannot be valid in a metal, since the scale of variation of the r.f. field is not given by the wavelength λ but rather by the penetration length δ of the anomalous skin effect, which, under realistic conditions, is much smaller than the cyclotron radius r_c. Furthermore, the ratio $|\,E/cB\,|$ for the radiofrequency field inside a metal is approximately given by $\delta/\lambda \ll 1$, so that the left-right asymmetry associated with a $E_1 \times M_1$ interference suffers from a considerable suppression :

$$A_{\text{R.L.}} \sim \left(\frac{\lambda}{l_{\text{p.v.}}}\right)\left(\frac{\delta}{\lambda}\right) = \frac{\delta}{l_{\text{p.v.}}} \sim 10^{-8}.$$

In the next section we shall study possible parity violation effects in electron magnetic-resonance experiments performed on high purity metallic samples placed in a large magnetic field in order to get well separated Landau levels. The dominant parity violation will turn out to be associated with $M_1 \times M_2$,

$M_2 \times M_3$,... interferences rather than with $E_1 \times M_1$ ones. The typical order of magnitude of the p.v. effects will be given by the ratio

$$\frac{r_c}{l_{p.v.}} \gg \frac{\delta}{l_{p.v.}} .$$

4. Parity violation in conduction electron spin resonance in metals.

We shall discuss the parity violation in a metal in the following configuration : the static magnetic field $\mathbf{B}_0$ is parallel to the surface of the metal, which is taken as the plane $y = 0$, the metallic matter is lying in the half space $y \geqslant 0$. (The z axis is taken along $\mathbf{B}_0$.) Inside the metal we shall keep only the dominant part of the radio-frequency field, namely the tangential component of the magnetic field $\mathbf{B}_T$. We shall also neglect the space variation of $\mathbf{B}_T$ in the x and z directions and write $\mathbf{B}_T$ as

$$\mathbf{B}_T(y) = B_1(y)\,(\mathbf{u}_z \cos\theta + \mathbf{u}_x \sin\theta) . \qquad (46)$$

Since we are dealing with an experimental situation where the cyclotron radius r_c is much larger than the skin depth δ, the effect of the metal surface on the motion of the valence electron cannot be ignored. In the usual treatment of conductivity, the effect of the surface is accounted for by replacing the half-space metallic medium by an infinite medium with an electro-magnetic field symmetric with respect to the plane $y = 0$. It turns out that this method is not convenient for our purpose, which is to look for a violation of space reflexion symmetry. So we shall work with a half-space medium and make simplifying assumptions concerning the metal surface. We shall assume that this surface coincides with a crystallographic plane and is relatively free of impurities. As a further assumption, the effect of the surface upon the motion of the valence electron will be described by a static potential $V(y)$ having a step function shape :

$$V(y) = \theta(-y)\, V_0 .$$

We shall neglect the distortion of the energy bands near the surface and use the effective mass approximation even in the vicinity of the surface. In absence of r.f. field, the Hamiltonian in Landau gauge (Eq. (36)) takes the following form :

$$H = \frac{(p_x - y\,|\,e\,|\,B_0)^2}{2\,m^*} + \frac{p_y^2 + p_z^2}{2\,m^*} + V(y) + \frac{1}{2}\,\sigma_z\,\omega_s \qquad (47)$$

where $\omega_s = \dfrac{|\,e\,|\,B_0}{2\,m_e}\,g_s$.

The eigenfunctions of H can be factorized as follows :

$$\langle \mathbf{r} \,|\, n, k_x, k_z, \varepsilon \rangle = \frac{1}{2\pi}\, e^{(ik_x x + ik_z z)}\, \Phi_n(y)\, \chi(\varepsilon).$$

($\chi(\varepsilon)$ is a Pauli spin function.)

The wave function $\Phi_n(y)$ obeys the one-dimensional Schrödinger equation :

$$\left(\frac{p_y^2}{2\,m^*} + \mathfrak{V}(y)\right) \Phi_n(y) = \mathcal{E}_n\, \Phi_n(y) . \qquad (48)$$

The effective potential $\mathfrak{V}(y)$ is given by :

$$\mathfrak{V}(y) = \frac{1}{2}\,m^*\,\omega_c^2 (y - y_0)^2 + V(y) \qquad (49)$$

where ω_c is the cyclotron frequency :

$$\omega_c = \frac{|\,e\,|\,B_0}{m^*} \qquad (50)$$

and y_0 is given by :

$$y_0 = \frac{k_x}{|\,e\,|\,B_0} . \qquad (51)$$

The reduced energies $\mathcal{E}_n$ are related to the total energies E_n by :

$$E_n = \frac{1}{2}\,\varepsilon\omega_s + \frac{k_z^2}{2\,m^*} + \mathcal{E}_n . \qquad (52)$$

Since the Fermi energy E_F is much larger than the cyclotron frequency, we are in the large quantum number limit and a semi-classical approximation is adequate. Classically $\mathcal{E}_n$ is nothing but the kinetic energy associated with cyclotron circular motion. It is convenient to introduce the classical cyclotron radius r_c as :

$$\mathcal{E}_n = \frac{1}{2}\,m^*\,\omega_c^2\,r_c^2 . \qquad (53)$$

Two cases have to be considered :

i) $y_0 > r_c$ (Fig. 2a) : the classical motion is purely harmonic, the electron does not « feel » the metal surface and the classical frequency ω_1 is just the cyclotron frequency ω_c ;

ii) $-r_c < y_0 < r_c$ (Fig. 2b) : the electron bounces upon the metal surface before it reaches the classical turning point of the harmonic motion. The motion classical frequency ω_1 is now larger than the cyclotron frequency and is given by

$$v = \frac{\omega_c}{\omega_1} = \frac{1}{\pi}\,\mathrm{Arcos}\left(-\frac{y_0}{r_c}\right). \qquad (54)$$

For $y_0 < -r_c$, no classical motion is possible.

It is convenient to rewrite the effective parity violating interaction in terms of the parameters of the one-dimensional harmonic motion [1] :

$$V_{p.v.} = \frac{1}{2\,m^*\,l_{p.v.}}\,(\sigma_x(p_x - |\,e\,|\,B_0\,y) + \sigma_y p_y)$$

$$= \frac{1}{2\,l_{p.v.}}\left(\sigma_y \frac{p_y}{m^*} - \sigma_x (y - y_0)\,\omega_c\right). \qquad (55)$$

([1]) The term $ak_z\,\sigma_z$ in $V_{p.v.}$ will be left out since it gives a level shift which averages to zero.

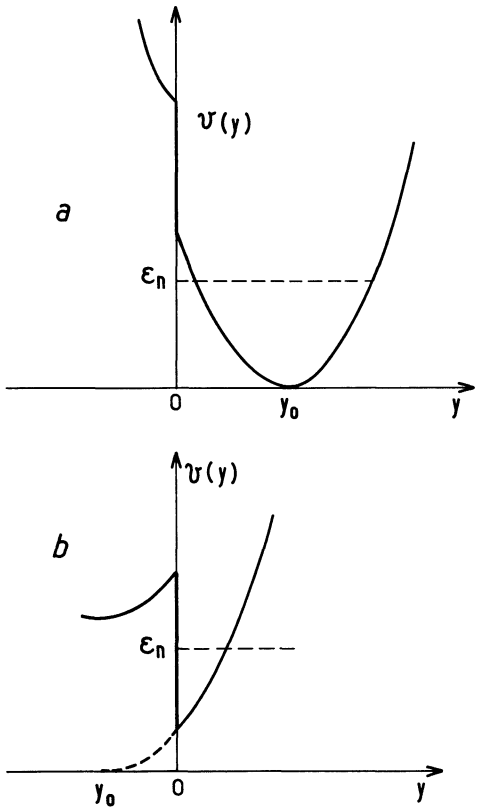

Fig. 2. — Effective potential for conduction electrons in a strong static field near the metal surface.

If one takes the classical limit, the expression for $V_{\text{p.v.}}$ can be written in terms of the velocity (v_x, v_y) of the circular motion :

$$V_{\text{p.v.}} = \frac{1}{2\, l_{\text{p.v.}}} (\sigma_x v_x + \sigma_y v_y) . \tag{56}$$

Before turning to a more detailed computation, let us briefly explain qualitatively the effect of $V_{\text{p.v.}}$ which, in this case, turns out to be equivalent to a small rotation of the static magnetic field $\mathbf{B}_0$ around the normal to the metal surface.

The electrons interact with the radiation field in a region of depth δ. Let us first consider the electronic states associated with the classical trajectories corresponding to case i) ($y_0 > r_c$). When one averages over the electrons which can interact with the r.f. field, one clearly sees in figure 3 that the average velocity is non-zero and positive. For trajectories corresponding to the case ii) ($| y_0 | < r_c$), the average value of v_x in the skin depth is also non-zero but could be negative when $y_0 < 0$ (see the trajectory labelled ii) of Fig. 3). So, at first sight, it looks as if the contributions to $\langle v_x \rangle$ coming from the two kinds of states could compensate. In fact, the compensation is strongly suppressed due to the presence in the thermal average of an important factor, namely, the level density of the states which is given in the semi-classical limit, by $\frac{1}{\omega_1} < \frac{1}{\omega_c}$. Consequently, the relative weight of state ii) associated with the trajectory drawn on figure 3 is just the ratio of the circular arc AB to the whole circumference. To conclude, when one performs the thermal average over the electrons which can participate in the transition induced by the radio-frequency field, the average velocity $\langle v_x \rangle$ is non-zero (and positive), thus we can write :

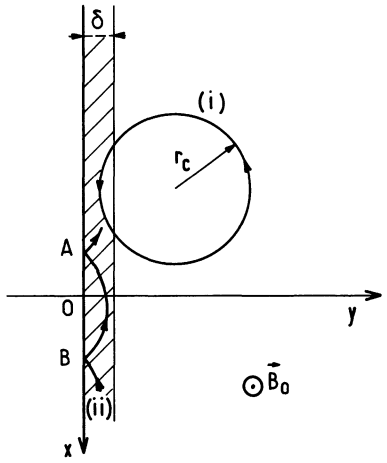

Fig. 3. — Classical cyclotron motion of conduction electrons near the metal surface.

$$\langle V_{\text{p.v.}} \rangle = \frac{1}{2\, l_{\text{p.v.}}} \sigma_x \langle v_x \rangle .$$

The effect of the parity violating interaction can be simulated by a small static field $\delta \mathbf{B}_0^{\text{p.v.}}$ along the x axis :

$$\delta \mathbf{B}_0^{\text{p.v.}} \sim \frac{r_c}{l_{\text{p.v.}}} \left(\frac{m}{m^*}\right) B_0\, \mathbf{u}_x . \tag{57}$$

Introducing the unit vector $\mathbf{n}$ normal to the metal surface ($\mathbf{n} = \mathbf{u}_y$), we rewrite $\delta \mathbf{B}_0^{\text{p.v.}}$ in such a way that its polar vector behaviour under space reflexion is manifest :

$$\delta \mathbf{B}_0^{\text{p.v.}} \sim \frac{r_c}{l_{\text{p.v.}}} \left(\frac{m}{m^*}\right) \mathbf{n} \wedge \mathbf{B}_0 . \tag{58}$$

The electronic spin flip probability in the presence of parity violation is proportional to the quantity :

$$| (\mathbf{B}_0 + \delta \mathbf{B}_0^{\text{p.v.}}) \wedge \mathbf{B} |^2 = \\ = | \mathbf{B}_0 \wedge \mathbf{B} |^2 - 2(\delta \mathbf{B}_0^{\text{p.v.}} . \mathbf{B})\, (\mathbf{B}_0 . \mathbf{B}) .$$

From the above expression we extract the following estimate of the left-right asymmetry :

$$A_{\text{R.L.}} \sim -2 \frac{r_c}{l_{\text{p.v.}}} \left(\frac{m}{m^*}\right) \frac{(\mathbf{B}_0 . \mathbf{B})\, [(\mathbf{n} \wedge \mathbf{B}_0) . \mathbf{B}]}{| \mathbf{B} \wedge \mathbf{B}_0 |^2} . \tag{59}$$

The more detailed treatment we are going to describe leads to a result which is, somewhat accidentally, very close to the above qualitative estimate.

Under the action of the effective parity violation interaction $V_{\text{p.v.}}$, the states $| n, k_x, k_z, \varepsilon \rangle$ get mixed to states with different cyclotron quantum numbers and opposite spins $-\varepsilon$:

$$| n, \widetilde{k_x, k_z}, \varepsilon \rangle = | n, k_x, k_z, \varepsilon \rangle - \frac{1}{2} \frac{\omega_c}{l_{\text{p.v.}}} \times \times \sum_n (\varepsilon\omega_s + \varepsilon_n - \varepsilon_{n'})^{-1} | n', k_x, k_z, -\varepsilon \rangle \times \left\langle \Phi_{n'} \left| \left(y - y_0 + i\varepsilon \frac{P_y}{m^* \omega_c} \right) \right| \Phi_n \right\rangle . \quad (60)$$

We shall compute the energy differences $\varepsilon_n - \varepsilon_{n'}$, and the matrix element of the one-electron operator $\hat{O}$, $\langle \Phi_{n'} | \hat{O} | \Phi_n \rangle$ using semi-classical formulae which are valid in the limit where $| n - n' | \ll (n + n')$

$$\varepsilon_n - \varepsilon_{n'} \simeq (n - n')\, \omega_1 = (n - n') \frac{2\pi}{T_1} \quad (61)$$

$$\langle \Phi_{n'} | \hat{O} | \Phi_n \rangle \simeq \frac{1}{T_1} \int_0^{T_1} dt\, e^{i(n'-n)\omega_1 t}\, O_c(t) \quad (62)$$

where $O_c(t)$ is the classical quantity at the time t associated with the operator $\hat{O}$ and $T_1 = 2\pi/\omega_1$ is the classical period.

Let us write the classical motion for the y coordinate in the case $-r_c \leqslant y_0 \leqslant r_c$:

$$y_c(t) = y_0 + \begin{cases} r_c \cos \omega_c t & \text{if } 0 \leqslant t \leqslant \frac{T_1}{2} \\ r_c \cos \omega_c (t - T_1) & \text{if } \frac{T_1}{2} \leqslant t \leqslant T_1 . \end{cases} \quad (63)$$

The matrix elements

$$\left\langle \Phi_{n'} \left| y - y_0 + i \frac{P_y \varepsilon}{m^* \dot{\omega}_c} \right| \Phi_n \right\rangle$$

needed for the evaluation of (60) are then given by :

$$\left\langle \Phi_{n'} \left| y - y_0 + \frac{i\varepsilon P_y}{m^* \omega_c} \right| \Phi_n \right\rangle = = \frac{1}{T_1} \int_0^{T_1} \left(y_c(t) - y_0 + \frac{i\varepsilon}{\omega_c} \dot{y}_c(t) \right) e^{i(n'-n)\omega_1 t} = \frac{2}{T_1} \int_0^{T_1/2} r_c\, e^{-i(\varepsilon\omega_c + (n-n')\omega_1)t}\, dt .$$

We immediately get the following expression for the Landau states perturbed by the p.v. interaction :

$$| n, \widetilde{k_x, k_z}, \varepsilon \rangle = | n, k_x, k_z, \varepsilon \rangle - \frac{r_c}{2 l_{\text{p.v.}}} \times \sum_{n'} \frac{\omega_c \sin \pi(\varepsilon\omega_c/\omega_1 + n - n')}{\pi(\omega_1(n-n') + \varepsilon\omega_s)\,(\varepsilon\omega_c/\omega_1 + n - n')} | n', k_x, k_z, -\varepsilon \rangle . \quad (64)$$

When $y_0 > r_c$ we have a pure cyclotron motion and the Φ_n are harmonic oscillator wave functions. The semi-classical result is identical to the exact one. The state $| n, \varepsilon \rangle$ is mixed only to the state $| n + \varepsilon, -\varepsilon \rangle$ and the mixing amplitude can be obtained by taking the limit $\omega_1 \to \omega_c$ in (64) :

$$| n, \widetilde{k_x, k_z}, \varepsilon \rangle = | n, k_x, k_z, \varepsilon \rangle - - \frac{r_c}{2 l_{\text{p.v.}}} \frac{\omega_c}{\varepsilon(\omega_s - \omega_c)} | n + \varepsilon, k_x, k_z, -\varepsilon \rangle . \quad (65)$$

In order to compute the transition amplitudes between the spin perturbed levels, we need the matrix elements of the r.f. magnetic field $B_T(y)$ between arbitrary quantized cyclotron states Φ_n :

$$B_{n',n} = \langle \Phi_{n'} | B_T(y) | \Phi_n \rangle \simeq \frac{1}{T_1} \int_0^{T_1} dt\, B_T(y_c(t))\, e^{i(n'-n)\omega_1 t} . \quad (66)$$

We shall assume that the y dependence of $B_T(y)$ is exponential. It is shown in the appendix that, in the case of anomalous skin effect with $\delta/r_c \ll 1$, the r.f. field $B_T(y)$ is still dominated by an exponential term :

$$B_T(y) = B_1 \exp(-e^{-i\varphi} Ky) . \quad (67)$$

After straightforward calculations and using changes of variables, we arrive at the following expressions for $B_{n',n}$:

$$B_{n+q,n} = B_1 \exp\left[-iKr_c \sin\varphi \cos\left(\frac{\omega_c}{\omega_1} \pi \right) \right] \times \times \left[R_q\left(\frac{\omega_c}{\omega_1}, Kr_c, \varphi \right) + iI_q\left(\frac{\omega_c}{\omega_1}, Kr_c, \varphi \right) \right] \quad (68)$$

where the integrals R_q and I_q are given by :

$$R_q(v, w, \varphi) = \frac{1}{\pi} \int_0^{\pi} du \exp\{ -w\cos\varphi[\cos(uv) - - \cos(v\pi)] \} \cos[w \sin\varphi \cos(vu)] \cos(qu) \quad (69)$$

$$I_q(v, w, \varphi) = \frac{1}{\pi} \int_0^{\pi} du \exp\{ -w\cos\varphi[\cos(uv) - - \cos(v\pi)] \} \sin[w \sin\varphi \cos(vu)] \cos(qu) . \quad (70)$$

These integrals have been computed numerically. (When $v \neq 0$ and $\neq 1$, asymptotic expansions valid when $w = Kr_c \geqslant 1$ can be easily derived.)

In the above computation, we have treated electronic states as stationary states. Even at very low temperature, the Landau states have finite lifetimes, due to the collisions with the metal impurities. We shall assume here that the collision relaxation time τ is much larger than the cyclotron period, i.e. : $\omega_c \tau \gg 1$. In other words, the width of the Landau level $\hbar/\tau$ is assumed to be small compared to the energy level

separation. Furthermore, we shall not attempt to compute the full resonance line shape in presence of p.v. but instead an asymmetry parameter $A_{\text{L.R.}}$ involving the transition probabilities for the electron spin transition $s_z = -1/2 \to s_z = 1/2$. To be precise, we are going to evaluate, for a resonant r.f. field ($\omega = \omega_s$), the transition probability $\mathcal{T}(s_z = -1/2 \to s_z = 1/2)$ after a finite time interval $\Delta t = t - t_0$, small compared both with the collision relaxation time τ and with the spin relaxation time T. The system will be assumed to be in thermal equilibrium at the initial time t_0. The transition probability averaged over the electronic states which participate in the transition is given, to lowest order in the radiation field, by :

$$\bar{\mathcal{T}}\left(-\frac{1}{2}\to\frac{1}{2}\right) = (\Delta t)^2\left(\int \mathrm{d}\nu\right)^{-1}\int \mathrm{d}\nu \times$$
$$\times\ |\langle n, \tilde{k_x}, \tilde{k_z}, \varepsilon=-1\,|\,\mu_B\,\boldsymbol{\sigma}.\mathbf{B}_T(y)\,|\,n, \tilde{k_x}, \tilde{k_z}, \varepsilon=1\rangle|^2 \tag{71}$$

where $\mathrm{d}\nu$ is the number of occupied levels which can be excited by the spin flip transition $s_z = -1/2 \to s_z = 1/2$.

In the zero-temperature limit, these levels are the ones around the Fermi level, and one has :

$$\mathrm{d}\nu = \omega_s\,\delta\left(E_F - \frac{k_z^2}{2m^*} - \mathcal{E}\right) \times \frac{L_x L_z\,\mathrm{d}k_x\,\mathrm{d}k_z}{(2\pi)^2}\,\frac{\mathrm{d}n}{\mathrm{d}\mathcal{E}}\,\mathrm{d}\mathcal{E}\,. \tag{72}$$

Depending on the value of k_z, $\mathcal{E}$ takes values between 0 and E_F, and hence r_c lies between 0 and

$$r_F = \sqrt{\frac{2E_F}{m^*\omega_c^2}}$$

(see (53)). We shall parametrize this variation of $\mathcal{E}$ (and r_c) through the variable

$$\rho = \frac{r_c}{r_F} \qquad 0 \leqslant \rho \leqslant 1 \tag{73}$$

k_x is related to the centre of the classical trajectory by $y_0 = k_x/|e|B_0$. As we have seen before, one must consider the two cases $|y_0| < r_c$ and $y_0 > r_c$ separately. We shall introduce besides ρ another dimensionless variable η :

$$\eta = \frac{y_0}{r_c}. \tag{74}$$

With these changes of variables, we get :

$$\mathrm{d}\nu \propto \frac{\rho^2\,\mathrm{d}\rho}{\sqrt{1-\rho^2}}\,\omega_c\,\frac{\mathrm{d}n}{\mathrm{d}\mathcal{E}}\,\mathrm{d}\eta\,. \tag{75}$$

(Notice that when $|y_0| < r_c$, $\eta < 1$, the value of the cyclotron energy $\mathcal{E}$, is most clearly characterized by the period of the classical motion, that is by the variable $v = \omega_c/\omega_1$ which has been introduced before. In this case we shall replace $\mathrm{d}\eta$ by $\pi \sin(\pi v)\,\mathrm{d}v$, and use the semi-classical formula $\omega_c\,\dfrac{\mathrm{d}n}{\mathrm{d}\mathcal{E}} = \dfrac{\omega_c}{\omega_1} = v$.)

Formulae (71) and (75), together with the expressions for the states $|n, \tilde{k_x}, \tilde{k_z}, \varepsilon\rangle$ given in (64) and the matrix elements of $B_T(y)$ given in (68-70), allow the computation of the transition probability $\bar{\mathcal{T}}(-1/2 \to 1/2)$. The simple considerations developed previously suggest the following general form for $\bar{\mathcal{T}}(-1/2 \to 1/2)$:

$$\bar{\mathcal{T}}\left(-\frac{1}{2}\to\frac{1}{2}\right) = \bar{\mathcal{T}}_0\left(-\frac{1}{2}\to\frac{1}{2}\right) \times$$
$$\times\left(1 + \frac{r_F}{l_{\text{p.v.}}}\,\mathcal{R}\,\frac{(\mathbf{B}_0.\mathbf{B})\,(\mathbf{n}.\mathbf{B}_0\wedge\mathbf{B})}{|\mathbf{B}_0\wedge\mathbf{B}|^2}\right) \tag{76}$$

where the dimensionless number $\mathcal{R}$ is expected to be negative and of order one. (Our previous simple estimate gives $\mathcal{R} \sim -2\dfrac{m}{m^*} = -2.9$).

It is convenient to write $\mathcal{R}$ as the ratio of an electroweak interference term $I_{\text{p.v.}}$ (where the electroweak scale parameter $r_F/l_{\text{p.v.}}$ has been factorized) by a reduced parity conserving probability $P_{\text{p.c.}}$:

$$\mathcal{R} = I_{\text{p.v.}}/P_{\text{p.c.}}\,. \tag{77}$$

The quantities $I_{\text{p.v.}}$ and $P_{\text{p.c.}}$ are given by a sum of two terms

$$I_{\text{p.v.}} = \sum_{s=1,2} I_{\text{p.v.}}^{(s)}\,; \qquad P_{\text{p.c.}} = \sum_{s=1,2} P_{\text{p.c.}}^{(s)}\,. \tag{78}$$

The values $s = 2$ and $s = 1$ are associated with cases ii) ($|y_0| < r_c$) and i) ($y_0 > r_c$) respectively. The explicit formulae giving $I_{\text{p.v.}}^{(s)}$ and $P_{\text{p.v.}}^{(s)}$ in terms of the integrals R_q and I_q read as follows :

$$I_{\text{p.v.}}^{(2)} = 2\int_0^1 \frac{\rho^3\,\mathrm{d}\rho}{\sqrt{1-\rho^2}}\sum_q \int_0^1 \mathrm{d}v\,\frac{(-1)^q\,(v\sin\pi v)^2}{(q - vm^*/m)\,(v-q)}\,[R_0(v,\rho K r_F,\varphi)\,R_q(v,\rho K r_F,\varphi) +$$
$$+\ I_0(v,\rho K r_F,\varphi)\,I_q(v,\rho K r_F,\varphi)] \tag{79}$$

$$P_{\text{p.c.}}^{(2)} = \int_0^1 \frac{\rho^2\,\mathrm{d}\rho}{\sqrt{1-\rho^2}}\int_0^1 \mathrm{d}v\,\pi v\sin\pi v\,[R_0^2(v,\rho K r_F,\varphi) + I_0^2(v,\rho K r_F,\varphi)]\,. \tag{80}$$

The formulae corresponding to case i) ($y_0 > r_c$) are much simpler; there is no summation upon q, $\omega_c \frac{dn}{d\mathcal{E}} = 1$ and the integration upon η can be done explicitly :

$$I^{(1)}_{p.v.} = 2 \int_0^1 \frac{\rho^2 \, d\rho}{\sqrt{1-\rho^2}} \frac{1}{(1 - m^*/m)\, 2 Kr_F \cos\varphi} [R_0(1, Kr_F \rho, \varphi) R_1(1, Kr_F \rho, \varphi) + \\ + I_0(1, Kr_F \rho, \varphi) I_1(1, Kr_F \rho, \varphi)] \quad (81)$$

$$P^{(1)}_{p.c.} = \int \frac{\rho \, d\rho}{\sqrt{1-\rho^2}} \frac{1}{2 Kr_F \cos\varphi} [R_0^2(1, Kr_F \rho, \varphi) + I_0^2(1, Kr_F \rho, \varphi)] . \quad (82)$$

Besides m/m^* and $l_{p.v.}$, the asymmetry depends only on the parameters K and φ, which characterize the penetration of the r.f. magnetic field inside the metal, and on $r_F = \sqrt{\frac{2 E_F}{m^* \omega_c^2}}$. Although the absolute values of $I_{p.v.}$ and $P_{p.c.}$ exhibit substantial variations with φ and the dimensionless parameter Kr_F, the ratio $\mathcal{R} = I_{p.v.}/P_{p.c.}$ stays around the value -2.3, with fluctuations of the order of a few percent, as is apparent in table I.

Let us give a numerical evaluation corresponding to a typical experimental situation : we shall consider the case of caesium, with $m^*/m \simeq 0.7$ and $l_{p.v.} \sim 11.2$ m, and assume that the collision time τ is sufficiently large : $\omega_c \tau \sim 10$ ([2]). It is shown in the appendix that in this situation K and φ are given by

$$K \simeq \frac{1.22}{\delta} \quad \text{and} \quad \varphi \simeq 1.37$$

where δ is the anomalous skin depth. δ and r_F depend on the wavelength λ of the radio-frequency field, and their values in caesium are :

$$r_F = 3.89 \times 10^{-4} \lambda \\ \delta \text{ (cm)} = 2.31 \times 10^{-5} (\lambda \text{ (cm)})^{1/3} . \quad (83)$$

Choosing $\lambda = 3$ cm as a typical value for the r.f. wavelength, we get :

$$Kr_F \cos\varphi = 8.67 \\ Kr_F \sin\varphi = 41.76 . \quad (84)$$

The ratio $\mathcal{R}$ computed with the above value is :

$$\mathcal{R} = I_{p.v.}/P_{p.c.} \simeq -2.28 . \quad (85)$$

The left-right angular asymmetry $A_{L.R.}(\theta)$ is given by :

$$A_{L.R.}(\theta) = \frac{\bar{\mathcal{F}}(\theta) - \bar{\mathcal{F}}(\pi - \theta)}{\bar{\mathcal{F}}(\theta) + \bar{\mathcal{F}}(\pi - \theta)} = \frac{r_F}{l_{p.v.}} \mathcal{R} \cot\theta . \quad (86)$$

Table I. — *Transition probabilities $P_{p.c.}$ and parity violating interference terms $I_{p.v.}$ for various values of anomalous skin effect parameters, $Kr_F \cos\varphi$ and $Kr_F \sin\varphi$. To get the left-right asymmetry the numbers in the last column have to be multiplied by $r_F/l_{p.v.} \cot\theta$.*

$Kr_F \cos\varphi$	$Kr_F \sin\varphi$	$P_{p.c.}$	$I_{p.v.}$	$I_{p.v.}/P_{p.c.}$
5	20	8.282×10^{-3}	-1.930×10^{-2}	-2.331
5	30	5.608×10^{-3}	-1.289×10^{-2}	-2.297
5	40	4.227×10^{-3}	-9.622×10^{-3}	-2.276
10	20	5.342×10^{-3}	-1.242×10^{-2}	-2.325
10	30	3.758×10^{-3}	-8.648×10^{-3}	-2.301
10	40	2.871×10^{-3}	-6.552×10^{-3}	-2.283
15	20	3.880×10^{-3}	-9.000×10^{-3}	-2.319
15	30	2.878×10^{-3}	-6.618×10^{-3}	-2.299
15	40	2.249×10^{-3}	-5.134×10^{-3}	-2.283
20	20	2.959×10^{-3}	-6.846×10^{-3}	-2.314
20	30	2.311×10^{-3}	-5.310×10^{-3}	-2.297
20	40	1.855×10^{-3}	-4.233×10^{-3}	-2.282
8.67	41.76	2.985×10^{-3}	-6.808×10^{-3}	-2.281

([2]) With $\lambda = 3$ cm the assumption $\omega_c \tau \sim 10$ corresponds to a relaxation time τ of the order of 1.6×10^{-10} s. Although values of τ 10 times larger have been achieved for instance with very pure copper samples, it is far from obvious that our assumed value of τ could be obtained with caesium samples.

Taking $\lambda = 3$ cm together with the value of $l_{\text{p.v.}} = 11$ m obtained in section 1, we obtain a typical value of the angular asymmetry :

$$A_{\text{L.R.}}(\theta) \simeq -2.38 \cot\theta \times 10^{-6} .$$

The order of magnitude of the left-right asymmetry is comparable to what has been observed in atomic caesium experiment, where an interference term between a parity violating electric dipole and an electric dipole amplitude induced by a static electric field $\mathbf{E}_0$ is measured. The left-right asymmetry is inversely proportional to $\mathbf{E}_0$. The value chosen for $\mathbf{E}_0$ results from a compromise between the signal to noise ratio and the magnitude of the left-right asymmetry which is the relevant parameter for the problem of systematic errors. A similar adjustment could be performed here by varying the angle θ.

Appendix.

When the condition $r_F/\delta \gg 1$ is satisfied, the Fourier transform of the conductivity tensor $\tilde{\sigma}_{ij}(q, \omega)$ is given by the following asymptotic formulae [16] :

$$\begin{aligned} \tilde{\sigma}_{xx} &= \frac{3\pi}{4a} \frac{\sigma_0}{1 - i\omega\tau} \coth \pi\gamma \left(1 + O\left(\frac{1}{a}\right)\right) \\ \tilde{\sigma}_{zz} &= \tilde{\sigma}_{xx} \left(1 + O\left(\frac{1}{a}\right)\right) \\ \tilde{\sigma}_{yy}/\tilde{\sigma}_{xx} &\sim \tilde{\sigma}_{xz}/\tilde{\sigma}_{xx} \sim O\left(\frac{1}{a}\right) \end{aligned} \tag{A.1}$$

where

$$\begin{aligned} a &= \frac{v_F \tau}{1 - i\omega\tau} |q| = \frac{\omega_c \tau}{1 - i\omega\tau} r_F |q| \\ \gamma &= \frac{1 - i\omega\tau}{\omega_c \tau} ; \quad \sigma_0 = n_e e^2 \tau/m^* . \end{aligned} \tag{A.2}$$

To leading order in $1/a$, the Maxwell equations in the metal admit a solution of the form :

$$\begin{aligned} \mathbf{B} &= B(y) (\mathbf{u}_x \sin\theta + \mathbf{u}_z \cos\theta) e^{-i\omega t} \\ \mathbf{E} &= E(y) (-\mathbf{u}_x \cos\theta + \mathbf{u}_z \sin\theta) e^{-i\omega t} \end{aligned} \tag{A.3}$$

with

$$B(y) = -i/\omega \frac{\partial E}{\partial y} .$$

The solution inside the metal ($y > 0$) can be continued to the whole space by the relation :

$$E(-y) = E(y) . \tag{A.4}$$

In order to have a non-trivial solution with the correct boundary condition for $y = 0$, the derivative $\frac{\partial E}{\partial y}$ must have a discontinuity for $y = 0$

$$\frac{1}{2} \left(\frac{\partial E}{\partial y}\bigg|_{y=0^+} - \frac{\partial E}{\partial y}\bigg|_{y=0^-} \right) = E'_T(0) . \tag{A.5}$$

Let us introduce the Fourier transforms of the fields **E**, **B**

$$\begin{aligned} E(y) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \tilde{E}(q) e^{iqy} dy \\ B(y) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \tilde{B}(q) e^{iqy} dy . \end{aligned} \tag{A.6}$$

The Maxwell equations, written in Fourier space (with a surface current added in order to account for the discontinuity of $\frac{\partial E}{\partial y}$) read as follows :

$$\left(-q^2 + \frac{\omega^2}{c^2}\right) \tilde{E}(q) = -i\mu_0 \tilde{\sigma}_{xx}(q) \tilde{E}(q) + \left(\frac{2}{\pi}\right)^{1/2} E'_T(0)$$

$$\tilde{B}(q) = \frac{q}{\omega} \tilde{E}(q) . \tag{A.7}$$

From the above equations, one immediately gets :

$$\tilde{B}(q) = \left(\frac{2}{\pi}\right)^{1/2} E'_T(0) \frac{q}{\omega} \frac{1}{q^2 - \frac{\omega^2}{c^2} - i\mu_0 \omega \tilde{\sigma}_{xx}(q)} . \tag{A.8}$$

In the above formula, one can drop the term in ω^2/c^2 since doing so one introduces a term of the order of δ^2/λ^2 where $\delta = \left(\frac{v_F \tau}{\sigma_0 \omega \mu_0}\right)^{1/3}$ is the anomalous skin depth. It is convenient to rewrite $\tilde{B}(q)$ as :

$$\tilde{B}(q) = -\left(\frac{2}{\pi}\right)^{1/2} E'_T(0) \frac{q}{\omega} \frac{1}{q^2 - \frac{1}{|q|} q_0^3} \tag{A.9}$$

where

$$q_0^3 = i \frac{3}{4} \pi \frac{1}{\delta^3} \coth \pi \left(\frac{1 - i\omega\tau}{\omega_c \tau} \right) . \tag{A.10}$$

The magnetic field $B(y)$ in coordinate space can be written as :

$$B(y) = -\left(\frac{1}{2\pi}\right)^{1/2} E'_T(0) \frac{1}{\omega} (I(y) - J(y)) \tag{A.11}$$

with

$$I(y) = \int_0^\infty \frac{q^2 dq e^{iqy}}{q^3 - q_0^3} \tag{A.12}$$

$$J(y) = \int_0^\infty \frac{q^2 dq e^{-iqy}}{q^3 - q_0^3} . \tag{A.13}$$

Let us consider first $I(y)$; the integrand has three poles which we write as :

$$q_0^{(n)} = \frac{1}{\delta} C e^{i\psi(n)} \quad (n = 0, 1, 2)$$

with $\psi^{(n)} = \psi^{(0)} + 2n\frac{\pi}{3}$ and C a real numerical

constant of order unity. If one takes $\omega_c \tau = 10$ and $\omega = \omega_s = \frac{m^* \omega_c}{m} = 0.7\, \omega_c$, one gets :

$$C = 1.2192$$
$$\psi^{(0)} = 0.2047$$
$$\psi^{(1)} = 2.299$$
$$\psi^{(2)} = 4.393$$

Performing a contour integral around the first quadrant of the complex plane, one obtains :

$$I(y) = \frac{2\pi i}{3} e^{iq_0^{(0)}y} + \int_0^\infty \frac{u^2\, du\, e^{-uy}}{u^3 - iq_0^3}.$$

We perform a similar operation for $J(y)$ but this time taking as contour the fourth quadrant. Since the integrand has no pole inside this contour, we get :

$$J(y) = \int_0^\infty \frac{u^2\, du\, e^{-uy}}{u^3 + iq_0^3}.$$

Performing the change of variables $u = q_0^{(0)} \xi$ and a suitable integration contour deformation, we get the final formula for $B(y)$:

$$B(y) = B_0 \left[e^{iq_0^{(0)}y} - \frac{3}{\pi} \int_0^\infty \frac{d\xi\, \xi^2}{1 + \xi^6} e^{-q_0^{(0)}\xi y} \right]. \quad (A.14)$$

Introducing the maximum cyclotron radius r_F corresponding to $\lambda = 3$ cm, we get the numerical formula :

$$B(y) = B_0 \left[\exp - \frac{y}{r_F} (8.67 - i\, 41.76) - \right.$$
$$\left. - \frac{3}{\pi} \int_0^\infty \frac{d\xi\, \xi^2}{1 + \xi^6} \exp - \frac{\xi y}{r_F} (41.76 + i\, 8.67) \right] \quad (A.15)$$

the function $\frac{\xi^2}{1 + \xi^6}$ is strongly peaked for the value $\xi_m = \left(\frac{1}{2}\right)^{1/6} \simeq 0.89$ so that the contribution from the exponential is expected to be dominant. We have verified that it is indeed the case, the contribution from the integral gives a correction of a few percents which has been neglected in the number quoted in section 4.

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