

# PCS Exam 2023 - Feedback

- $\beta$  being given, you had to solve for  $t$

$$\text{in } \beta = \frac{2 \sinh t}{1 + 2 \cosh t}$$

$$= \frac{e^t - e^{-t}}{1 + e^t + e^{-t}}$$

Changing variables to  $X = e^t$  yields a simple equation

- When a random variable  $X$  is such that " $X$  and  $-X$  are equally probable", it does not mean that  $X$  only takes two values.

- With  $\langle \dot{x} \rangle = \frac{1}{m\gamma} F \rightarrow \text{force}$ , there are velocity  $\leftarrow$  mass

about 20% error for the physical dimension of  $\gamma$  (an inverse time)  $\rightarrow$  not acceptable

- When the sum of IID random variables is such that the central limit theorem holds, this only yields the large deviation function  $\phi(\beta)$  for  $\beta \rightarrow 0$  (and it is then quadratic).

- When  $x_n = \sum_{i=1}^n \eta_i$  and the  $\eta$  are IID, then the variance  $V(x_n) = n V(\eta)$ .

This is a basic property of the variance, that can avoid messy calculations, going through  $\langle \sum_{i,j} \eta_i \eta_j \rangle \dots$

Too many errors on that  $\langle e^{tX} \rangle = e^{t\langle X \rangle + t^2 \sigma^2 / 2}$

- $\langle e^{tX} \rangle = e^{t\langle X \rangle + t^2 \sigma^2 / 2}$  only holds for a gaussian variable

- For the random walk in discrete time

$$x_n = \sum_{i=1}^n \eta_i$$

$\rightarrow$  if the  $\eta_i$  have a Pareto index  $\mu < 2$  then the walk is superdiffusive

$\rightarrow$  if the Pareto index is  $\mu > 2$ , the walk is not subdiffusive, but simply diffusive.

- When applying the Sanov theorem for computing a large deviation function, it is important to specify the alphabet over which the distribution is defined, before minimizing the Kullback-Leibler distance.

In (C), this alphabet was  $\{-1, 0, 1\}$