Recovering Mermin-Wagner ($d_{lower} = 2$) with a continuous symmetry

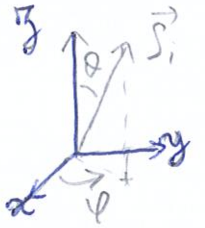
iCFP 13 bis

Start from Heisenberg $\mathcal{H} = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$ with $|\vec{S}_i| = 1$, $\forall i$

Degenerate ground-state, investigate the stability of state where all spins // z axis.

Fluctuations assumed small;

$$\vec{S}_i \left| \begin{matrix} \sin\theta_i \cos\varphi_i \\ \sin\theta_i \sin\varphi_i \\ \cos\theta_i \end{matrix} \right. \quad ; \quad \vec{S}_i \cdot \vec{S}_j = \sin\theta_i \sin\theta_j \left(\cos\varphi_i \cos\varphi_j + \sin\varphi_i \sin\varphi_j \right) + \cos\theta_i \cos\theta_j$$



spherical coordinates

θ is the co-latitude

θ_{ij} small

$$\simeq \theta_i \theta_j \cos(\varphi_i - \varphi_j) + 1 - \frac{\theta_i^2}{2} - \frac{\theta_j^2}{2}$$

1 to leading order in θ

$$\simeq 1 - \frac{1}{2}(\theta_i - \theta_j)^2$$

This yields the simplified $\mathcal{H} \simeq +\frac{J}{2} \sum_{\langle ij \rangle} (\theta_i - \theta_j)^2 \xrightarrow[\text{limit}]{\text{continuous}} \frac{J}{2} \int \frac{d\vec{r}}{a^d} a^2 (\vec{\nabla}\theta)^2$

Fluctuations are thus Gaussian; $\quad a \equiv$ lattice spacing

$$\langle S_z \rangle = \langle \cos\theta \rangle \simeq 1 - \frac{1}{2}\langle \theta^2 \rangle$$

Remember that for a Gaussian vector with probability density $\propto e^{-\frac{1}{2} x_i \Gamma_{ij} x_j}$,

$$\langle x_i x_j \rangle = (\Gamma^{-1})_{ij} \equiv G_{ij}$$

Here $\Gamma(\vec{r}, \vec{r}\,') = -\delta(\vec{r} - \vec{r}\,') \frac{\beta J}{a^{d-2}} \nabla^2_{\vec{r}}$; $G(\vec{r}) = \int \frac{d\vec{q}}{(2\pi)^d} \frac{e^{i\vec{q}\cdot\vec{r}}}{q^2} \frac{a^{d-2} kT}{J}$

$$\Rightarrow G(\vec{0}) = \langle \theta^2 \rangle = a^{d-2} \frac{kT}{J} \underbrace{\int \frac{d\vec{q}}{(2\pi)^d} \frac{1}{q^2}}_{\begin{cases} L^{2-d} & \text{for } d < 2, \text{ divergent} \\ a^{2-d} & \text{for } d > 2, \text{ convergent} \end{cases}} \quad \text{where } \int_{\frac{1}{L}}^{\frac{1}{a}} \text{; } a \text{ fixed, } L \to \infty\text{ ?}$$

For $L \to \infty$ and $d \leqslant 2$, $\langle \theta^2 \rangle \to \infty$, inconsistent. NO LONG RANGE ORDER

For $d > 2$, $\langle \theta^2 \rangle \simeq \frac{kT}{J}$, small for small T, order is possible.

$\to$ NO LONG RANGE ORDER for $d \leqslant 2$