TRACK RECORD

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Undergraduate Studies (Sep 2000 - Sep 2005)

Università degli Studi di Parma (Italy)

1. Numerical and Analytical Studies on Sandpile Cellular Automata.

   Sandpile models have been introduced long ago as simple 2-d cellular automata with a deterministic updating rule [1]. In the absence of finely tuned external parameters, they are able to reach a self-organized critical state (SOC), such that local perturbations may affect the system at every scale (propagation of avalanches). The typical fingerprint of a SOC behavior is then the appearance of power-laws in the spectrum of physical observables (duration and size of the avalanches).

   The original model is defined on a square lattice with open boundaries, the dissipation of 'sand grains' through the borders being essential in driving the system towards the critical state.

   Many variants of the original version have since been proposed and analyzed, and together with my former Bachelor's thesis advisor Mario Casartelli, and with Luca Dall'Asta and Alessandro Vezzani, we gave analytical and numerical results on a 2d model with *closed* boundaries [2, 3], the so-called Fixed-Energy Sandpile (FES).

   The phenomenology of FES is completely different with respect to the dissipative case, as the SOC state is destroyed and the system enters a periodic orbit after a surprisingly short transient [4]. We explained quantitatively the appearance of short-period attractors in terms of conserved quantities or dynamical invariants, and extended the group-theoretical analysis of Abelian Sandpile Models [5] to conservative models.

2. Modifications of gravity for low accelerations.

   MOND (Modified Newtonian Dynamics) is a phenomenological theory of gravity proposed by M. Milgrom in 1983 [6] as an attractively simple alternative to the Dark Matter (DM) hypothesis.

   MOND is based on a quite elegant idea, aimed to shed light on one of the most fundamental and still unsolved cosmological puzzles. It is by now well established that stars in the outer rings of spiral galaxies rotate much faster than it is predicted by Newton's and Kepler's law if only the visible mass of the galaxy is taken into account. The commonly accepted solution for this problem postulates that a large amount of non-emitting and so far undetected kind of matter actually pervades the Universe. Although quite appealing as a solution for the missing mass problem, this new kind of dark matter has so far escaped a direct detection and also the quantitative details of the theory are often unsatisfactory and highly debated.

   In contrast with the rather *ad hoc* prescriptions of DM theories, MOND simply postulates that Newton's law breaks down for accelerations lower than a threshold $a_0 \sim 1.2 \times 10^{-10}$m s^{-2}, a scale which is of the same order of cH_0, where c is the speed of light and H_0 is the Hubble's constant.

   Newton's law $\vec{F} = m\vec{a}$ is thus replaced by $\vec{F} = m\mu(a/a_0)\vec{a}$, where $\mu(x)$ is a phenomenological function such that $\mu(x \ll 1) \sim x$ and $\mu(x \gg 1) \sim 1$. Unfortunately, the simple-minded MOND prescription is not free of problems itself: the functional form of $\mu(x)$ is not strictly constrained by the theory and a relativistic implementation is not straightforward. With my former advisor Enrico Onofri, I studied in my Master's thesis one of the relativistic formulation of MOND (TeVeS theory by J. Bekenstein [7]) in order to find further constraints on the function $\mu(x)$.

Postgraduate Studies (Sep 2005 - Sep 2008)

Brunel University - West London (United Kingdom)

I joined the Department of Mathematical Sciences at Brunel University (West London) in September 2005 after being awarded a Marie Curie Early Stage Training fellowship to start a Ph.D. within the NET-ACE (Network Theory and Applications to Computer Science and Engineering) project.

During my Ph.D. under the supervision of Gernot Akemann, my main research interests have been focused on the theory of non-Gaussian invariant random matrices with a statistical physics approach. I have been mainly working on four lines of research, namely:

1. Large deviations of the maximum eigenvalue in Wishart random matrices.
2. Connections between real β index and rotational invariance.
3. Superstatistical models of covariance matrices.
4. The quantum transport problem and the Jacobi ensemble.

Historically, random matrices with appropriate symmetries were employed first to model Hamiltonians of heavy nuclei, in order to extract some statistical information about the distribution of energy levels [8], but the technical tools and the general philosophy of RMT have found more recent applications in biological systems [9], finance and risk assessment [10], QCD [11], network theory [12], quantum chaos [13], number theory [14], wireless communication systems [15] and beyond.

The Gaussian ensemble of random matrices is probably the simplest and most studied case in RMT. It is composed by matrices $\mathbf{X}$ with real, complex or quaternion random entries following these two requirements:

1. the entries are independent;
2. the probability density $P[\mathbf{X}]$ for a matrix $\mathbf{X}$ to occur in the ensemble is invariant under a rotation in the matrix space.

Invariant ensembles are characterized by the so-called Dyson index $\beta = 1, 2, 4$ according to the number of real variables needed to specify a single entry. This index in turn identifies the symmetry group of the ensemble (orthogonal, unitary and symplectic respectively). As a general feature, the joint probability density of the eigenvalues $\{\lambda_i\}$ for invariant ensembles contains a Vandermonde factor $\prod_{j<k} |\lambda_j - \lambda_k|^\beta$ which is responsible for their peculiar strong correlations even when the entries are independent.

My research has been mainly focused on invariant ensembles with correlated entries, i.e. the condition 1) above does not hold. I give here a brief account of my achievements.

1. Large deviations of the maximum eigenvalue in Wishart random matrices.

   The Wishart matrices play a paradigmatic role among invariant non-Gaussian ensembles. Introduced for the first time by Wishart [16], a Wishart matrix $N \times N$ is a covariance matrix of a normally distributed data matrix $N \times T$. In the large N limit, with N/T fixed, the average density of eigenvalues follows the so-called Marčenko-Pastur distribution on a compact support $[x_-; x_+]$, whereas the largest eigenvalue (rescaled with $N^{1/3}$) has a Tracy-Widom distribution of order β around the upper edge x_+ [17]. This means that the largest eigenvalue fluctuates from one sample to another over a very narrow region of $\sim \mathcal{O}(N^{1/3})$ around the upper edge.

   However, from time to time the largest eigenvalue happens to be much *smaller* than expected, fluctuating over a much wider region of $\sim \mathcal{O}(N)$ to the left of the mean.

   What is the probability of occurrence of such anomalous events for large N? This question is of theoretical interest as the answer gives information about the efficiency of the PCA (Principal Component Analysis) technique for detecting patterns in multivariate data.

   In collaboration with S.N. Majumdar and O. Bohigas at LPTMS - Orsay we were able to answer this question analytically in [18], giving a large deviation expression for the sought probability.

2. Connections between real β index and rotational invariance.

   Ensembles of *non*-invariant random matrices having a general $\beta \in \mathbb{R}^+$ index and independent entries have been discovered by Dumitriu and Edelman [19]. Is it possible to generate an *invariant* version of these?

   At first sight, this seemed a hopeless task, as the index β for an *invariant* ensemble is strictly constrained to the values $1, 2$ or 4 as described above. On the contrary, together with S.N. Majumdar I proved explicitly that the two properties i) rotational invariance and ii) real $\beta > 0$ index, are in fact not incompatible [20], of course at the price of introducing correlations among the entries.

   Our explicit counterexample exhibits very curious and atypical features, and the technical tool we employed (representation of the Vandermonde-squared on the basis of power sums) may hopefully prove useful in future RMT studies.

3. Superstatistical models of covariance matrices.

   Superstatistics is a concept recently introduced by Beck and Cohen [21]. It refers to probability distributions of random variables which depend on one parameter, taken itself to be a random variable. The interplay between these two independent sources of randomness can be used to achieve a variety of interesting effects. Building on earlier works [22], together with my supervisor Gernot Akemann and Adel Abul-Magd, I have studied random covariance models of the Wishart class where the variance of the data is assumed to be a random variable (i.e. it changes from one matrix sample to another). We consider a χ^2 [23] or inverse χ^2 distribution [24] for the variances. Each case leads to analytical results for the spectral density (displaying non-compact supports) and for the level spacing distribution (where we generalized the well-known Wigner's surmise). Applications to the case of covariance matrices of financial data series were also put forward and furtherly developed in [25].

4. The quantum transport problem and the Jacobi ensemble.

   Electronic current fluctuations in mesoscopic devices have been subject to intense scrutiny in the past decade. For open cavities brought out of equilibrium by an applied external voltage, it is well established that current fluctuations (associated with the granularity of electron charge) persist down to zero temperature. The Random Scattering Matrix framework [26] has been very effective as a theoretical model: it is based on the assumption that the scattering matrix of the conductor may be well approximated by a statistical ensemble of matrices, with the overall constraint of unitarity. The transport properties of the cavity are encoded in the N transmission eigenvalues (if N denotes the number of electronic channels in the two leads connecting the cavity to the external world), a set of correlated random variables $\{T_i\}$ in the range $[0, 1]$ having the natural interpretation of probabilities that an electron gets transmitted through the i-th channel. Despite the remarkable simplicity of the joint probability distribution of the T_i's, the available analytical results for their statistics was quite limited and thus did not catch up with existing experimental capabilities. A part of my Ph.D. thesis was devoted to the derivation of the following:

   - A formula for the average of integer moments of the $T_i's$, $\langle\sum_{i=1}^{N} T_i^n\rangle$, valid for a *finite* number of open channels and previously unavailable [27]. This result is relevant for the statistics of the total amount of charge transmitted through the cavity. This work has been done in collaboration with E. Vivo (Parma).
   - A large deviation formula for the full probability distribution of experimental observables (previously unavailable) in the limit of a large number of open channels $N \gg 1$ [28]. This work has been done in collaboration with Satya N. Majumdar and O. Bohigas (LPTMS - Orsay).

My Ph.D. has been awarded on 13th September 2008 [External examiners: Yan V. Fyodorov (Nottingham) and Zdzisław Burda (Krakow)]. In July 2009, I was awarded the Brunel Vice-Chancellor's Prize for doctoral research 'in recognition of outstanding research achievements'.

Postdoctoral Experience (Sep 2008 onwards)

Abdus Salam ICTP - Trieste (Italy)

I joined the Abdus Salam International Centre for Theoretical Physics (ICTP) as a postdoctoral fellow at the end of September 2008 in the Condensed Matter and Statistical Physics Section, a position that I am currently holding. In the last year, I have broadened my network of collaborators including the following colleagues:

- Matteo Marsili and Antonello Scardicchio (ICTP).
- Fabio Caccioli and Raffaello Potestio (SISSA).
- Jean-Gabriel Luque (Rouen).

The lines of research I mainly pursued while at ICTP are as follows:

1. Models of financial markets.
2. Random Matrix approach to protein dynamics.
3. Selberg-like integrals, nonlinear statistics and applications to the quantum transport problems.
4. Distributions of the number of positive eigenvalues (the *index*) of Gaussian random matrices.
5. Statistical properties of entangled random pure states in bipartite systems.

1. **Models of financial markets.** Together with Matteo Marsili and Fabio Caccioli, I studied the effect of expanding the repertoire of derivative assets in a stylized model of interacting market [29]. The proliferation of financial instruments provides more tools for risk diversification, thus making the market more efficient and closer to the theoretical limit of *complete* market, where risk can be eliminated altogether. Nevertheless, a debate about the role of derivative contracts has surfaced in view of recent events [30].

   In order to get some insights about the role of derivative contracts, we set up a model of the market as an interacting system. In particular the impact of trading derivatives on underlying prices was considered. We showed that uncontrolled proliferation of financial instruments drives the system to a state which closely resembles the picture of the efficient arbitrage-free complete market described by Arbitrage Pricing Theory (APT), the mathematical basis of financial engineering [31].

   However, the same region of the phase space is also characterized by a phase transition between a supply-limited equilibrium and a demand-limited one. Close to the transition, small perturbations on the risk perception by credit institutions (banks) provoke dramatic changes in the volume of traded derivatives and large fluctuations are observed in response functions. This suggests that market completeness, often assumed in APT, may not be compatible with market stability.

   An intuition we gain from these studies is that derivative markets with volumes much larger than the underlying market are prone to be unstable. Indeed, when the market is complete, the production of more financial instruments introduces directions in the phase space where fluctuations can grow unbounded (Goldstone modes). This can be the ideal setting for the creation of bubbles, thus raising the challenging question of the relation between information efficiency and bubbles.

2. **Random Matrix approach to protein dynamics.** Proteins are biomolecules that are essential to life. The biological functionality of a protein often relies on its capability to undergo large-scale conformational changes. In order to study such motions, one may resort to molecular dynamics simulations, which are computationally very demanding. Alternatively, one may formulate elastic network models, where the protein structure is approximated by a network of anisotropic springs connecting aminoacids within a certain cutoff distance. The *covariance matrix* of aminoacid displacements contains essential information about the vibrational modes of the protein around its reference (equilibrium) structure. Together with my collaborators [32], I recently suggested that comparisons with randomly generated covariance matrices should help answering in a solid, quantitative way the following question: how many vibrational modes should we take into account to give a sufficiently reliable description of a protein's overall motion? Our recent proposal opens up a very promising direction of research, where the novel marriage between biophysical issues and random matrix techniques is expected to play a fruitful role.

3. **Selberg-like integrals, nonlinear statistics and applications to the quantum transport problems.** In collaboration with Jean-Gabriel Luque, I have studied a class of N-fold integrals of the Selberg type, arising when computing nonlinear averages in random matrix models [33]. Such quantities involve products of different eigenvalues and are notoriously harder to compute than the more standard sums (linear statistics). We based our analysis on the theory of hyperdeterminants (multidimensional generalizations of the conventional determinants) and symmetric functions. The resulting formulae are computationally very efficient and allow to perform a previously unattainable asymptotic analysis for $N \to \infty$, which is relevant for the statistics of experimental sample-to-sample fluctuations in ballistic chaotic cavities within the random scattering matrix approach.

4. **Distributions of the number of positive eigenvalues (the *index*) of Gaussian random matrices.** The Gaussian matrices are certainly the most studied and best understood among the classical ensembles. Still, after more than half a century, certain natural questions about eigenvalue distributions have not been answered yet, in spite of their relevance to a broad range of subjects.

   As a paradigmatic example, classical disordered systems offer the ideal environment where RMT ideas and tools may be applied. Physical systems such as liquids and spin glasses are known to exhibit a rich energy or free energy landscape characterized by many extrema (minima, maxima and saddles) and rather complex stability patterns [34] which play an important role both in statics and dynamics of such systems. The stability of a stationary point of an N-dimensional potential landscape $V(x_1, x_2, \ldots, x_N)$ is decided by the N real eigenvalues of the Hessian matrix $H_{ij} = [\partial^2 V/\partial x_i \partial x_j]$ which is evidently symmetric. The number of positive eigenvalues $0 \leq N_+ \leq N$, called the *index*, is a key object of interest as it determines the number of directions in which a stationary point is stable.

   In many situations, important insights about the system can be gained by simply assuming that the Hessian is a real symmetric random matrix drawn from a Gaussian ensemble, characterized by the Dyson index $\beta = 1$ (real elements). This *random Hessian model* (RHM) has been studied extensively in the context of disordered systems [35], landscape based string theory [36, 37] and quantum cosmology [38].

   The index distribution of Gaussian random matrices was so far addressed in the work by Cavagna *et al.* [35] by means of the replica trick and some additional approximations. In a recent work [39] with Satya N. Majumdar and Céline Nadal (LPTMS) and Antonello Scardicchio (ICTP), we revisited the paper [35]. Using a Coulomb gas analogy and our newly devised technique to tackle singular integral equations on disconnected supports, we have given a large deviation formula for the large N decay of the probability distribution of the index. Close to the peak, an unusual logarithmic singularity appears which is responsible for the slow (logarithmic) increase of the index variance with N, a feature that is confirmed by previous independent studies and numerical simulations.

5. **Statistical properties of entangled random pure states in bipartite systems.** Entanglement of random pure states in bipartite systems is currently a very active area of research, with possible applications to quantum information and quantum computation problems. The Schmidt decomposition of density matrices has been studied in the case where such states are distributed isotropically in certain manifolds. For the simplest case (the so called Hilbert-Schmidt measure), the Schmidt eigenvalues are distributed according to a fixed-trace Wishart-Laguerre measure. A delicate challenge is then to extract significant entanglement quantifiers from this intricate measure. In the last year at ICTP, I have succeeded in producing analytical results for ι) the density of Schmidt eigenvalues and average Rényi entropy for the most difficult orthogonal case, and $\iota\iota$) the density of the smallest eigenvalue for all symmetry classes, which gives information about the degree of entanglement of such states [40, 41].

Teaching. In the Academic Years 2008-09 and 2009-10 I have been teaching the Tutorials in Basic Physics for the ICTP Diploma courses.

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