Entanglement production in non-ideal cavities and optimal opacity

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Chapter 1

ENTANGLEMENT PRODUCTION IN A CHAOTIC QUANTUM DOT

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C.M. Marcus, A. Yacoby*
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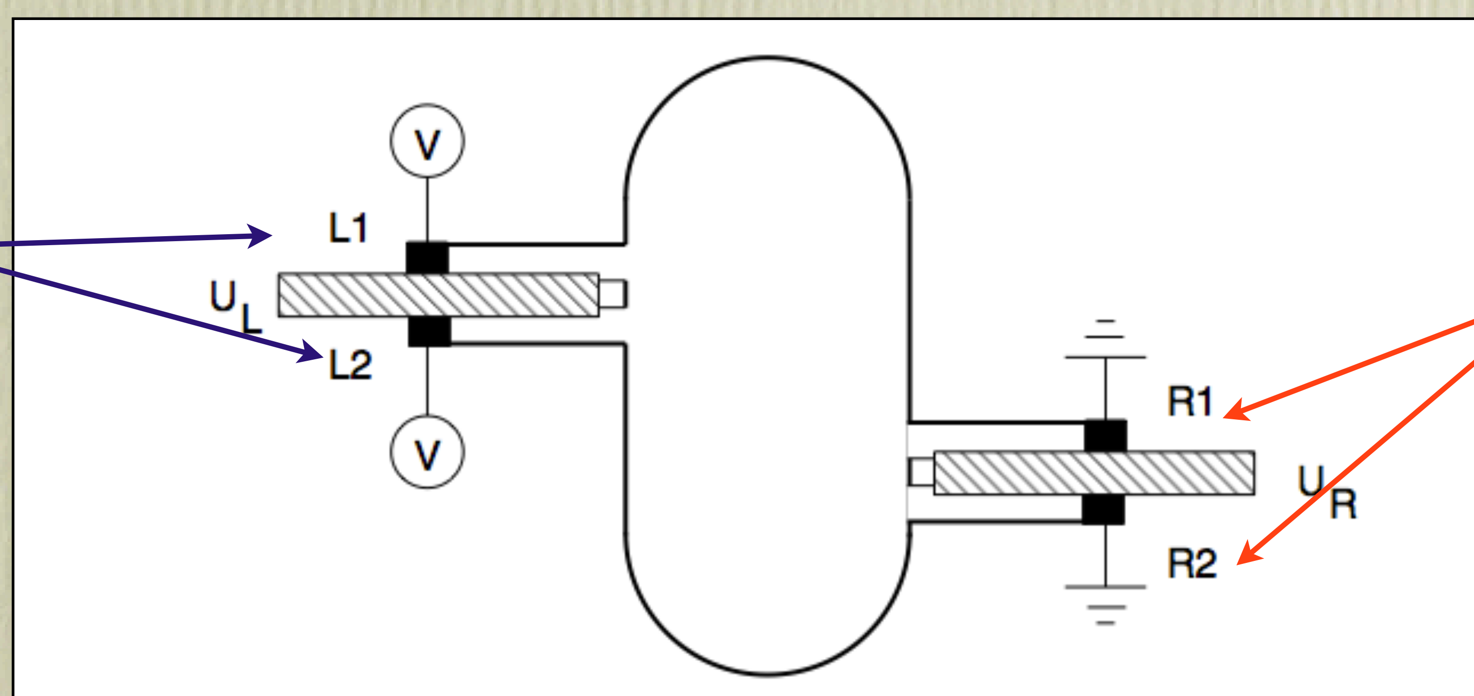
(2003)

Possibility to entangle electrons **without** interactions, using a chaotic quantum dot

2 channels to the left



2 channels to the right

2 electrons injected into the cavity from the left.
They get scattered into the cavity and then leave it.

$$|\psi_{\mathrm{out}}\rangle = |\psi_{\ell\ell}\rangle + |\psi_{rr}\rangle + |\psi_{\ell r}\rangle$$

Both electrons escape through the left lead

Both electrons escape through the right lead

An electron leaving the cavity can choose between channel 1 or 2.
Therefore, it is a two-level quantum system (qubit)

2 electrons injected into the cavity from the left. They get scattered into the cavity and then leave it.

$$|\psi_{\text{out}}\rangle = |\psi_{\ell\ell}\rangle + |\psi_{rr}\rangle + |\psi_{\ell r}\rangle$$

2-qubits (orbitally) entangled state

How to quantify the degree of entanglement?

4x4 Scattering Matrix

[unitary $(\beta = 2)$ or unitary and symmetric $(\beta = 1)$]

$$\mathcal{S} = \begin{bmatrix} \mathbf{r} & \mathbf{t}' \\ \mathbf{t} & \mathbf{r}' \end{bmatrix}$$

Hermitian matrix $\mathbf{t}\mathbf{t}^\dagger$

$(0 \leq \tau_i \leq 1)$

$$\mathcal{C} = \frac{2\sqrt{\tau_1(1-\tau_1)\tau_2(1-\tau_2)}}{\tau_1 + \tau_2 - 2\tau_1\tau_2}$$

Concurrence

[W. K. Wooters, *Phys. Rev. Lett.* **80**, 2245 (1998).]

$$\mathcal{N} = \tau_1 + \tau_2 - 2\tau_1\tau_2$$

Squared Norm

Random Scattering Matrix Approach

Europhys. Lett., **27** (4), pp. 255-260 (1994)

Universal Quantum Signatures of Chaos in Ballistic Transport.

R. A. Jalabert (*), J.-L. Pichard (**) and C. W. J. Beenakker (***)

Volume 73, Number 1 PHYSICAL REVIEW LETTERS 4 July 1994

Mesoscopic Transport through Chaotic Cavities: A Random *S*-Matrix Theory Approach

Harold U. Baranger[1] and Pier A. Mello[2]

Scattering matrix is drawn at random from the unitary group with measure given by Poisson kernel

$$P_\beta(\mathbb{S}) \propto \left[\det(\mathbb{1}_N - \bar{\mathbb{S}}\mathbb{S}^\dagger)\det(\mathbb{1}_N - \mathbb{S}\bar{\mathbb{S}}^\dagger)\right]^{\beta/2-1-\beta N/2}.$$

[P. W. Brouwer, *Phys. Rev. B* **51**, 16878 (1995)]

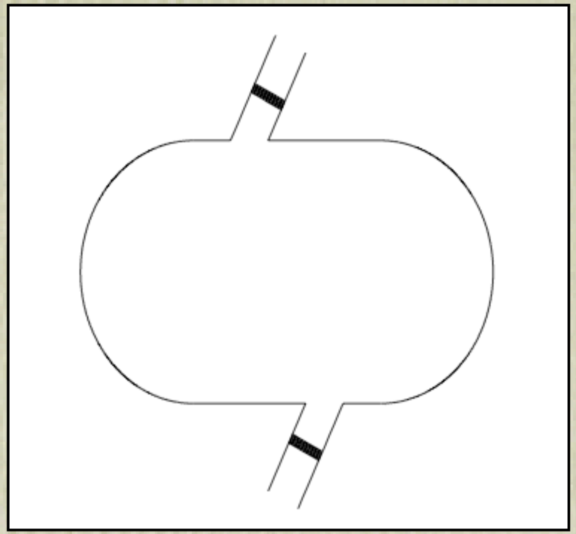
One cannot really argue with a mathematical theorem.
Stephen Hawking

$$P_\beta(\mathbb{S}) \propto \left[\det(\mathbb{1}_N - \bar{\mathbb{S}}\mathbb{S}^\dagger)\det(\mathbb{1}_N - \mathbb{S}\bar{\mathbb{S}}^\dagger)\right]^{\beta/2-1-\beta N/2}.$$

Ideal cavities

$$\bar{\mathbb{S}} = 0$$

Uniform distribution!



Non-Ideal cavities

$$\bar{\mathbb{S}} \neq 0$$

Coupling via tunnel barriers

Eigenvalues Γ_i of $\bar{\mathbb{S}}\bar{\mathbb{S}}^\dagger$ are tunneling probabilities, that can be tuned

[Gustavsson *et al.*, *Phys. Rev. Lett.* **96**, 076605 (2006)]

Random Scattering Matrix: statistical treatment of the entanglement production process

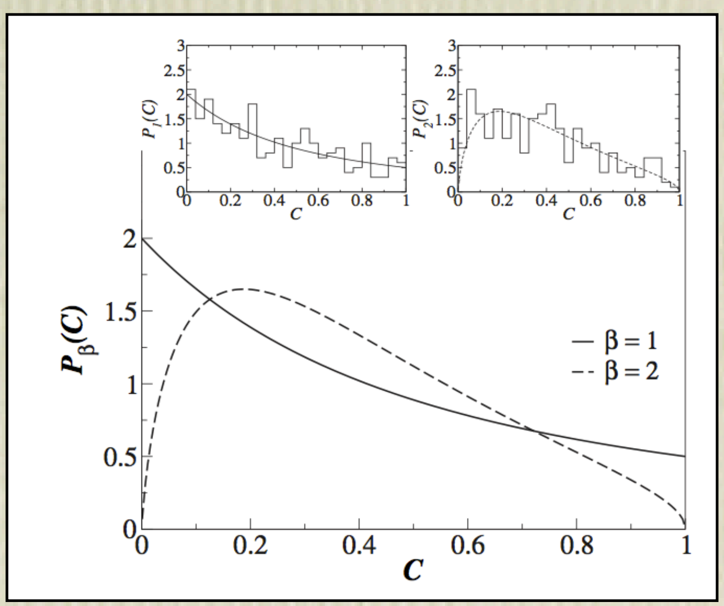
Ideal leads: summary

Uniformly distributed scattering matrix

$$p_\beta(\tau_1,\tau_2) = c_\beta|\tau_1-\tau_2|^\beta(\tau_1\tau_2)^{\beta/2-1}$$

- Average and variance of **Concurrence** [Beenakker et al. (2003)]
- Full distribution [Gopar and Frustaglia (2008)]

$$P^{(\beta)}(\mathfrak{C}) = \left\langle \delta\left(\mathfrak{C} - \frac{2\sqrt{\tau_1(1-\tau_1)\tau_2(1-\tau_2)}}{\tau_1+\tau_2-2\tau_1\tau_2}\right)\right\rangle$$



- Geometric constraints [Rodriguez-Perez and Novaes (2012)

$$\mathfrak{N}(1+\mathfrak{C}) < 1$$!

valid also in non-ideal case

What is known on entanglement production in non-ideal cavities?

PHYSICAL REVIEW B **82**, 115422 (2010)

Statistics of orbital entanglement production in a chaotic quantum dot with nonideal contacts

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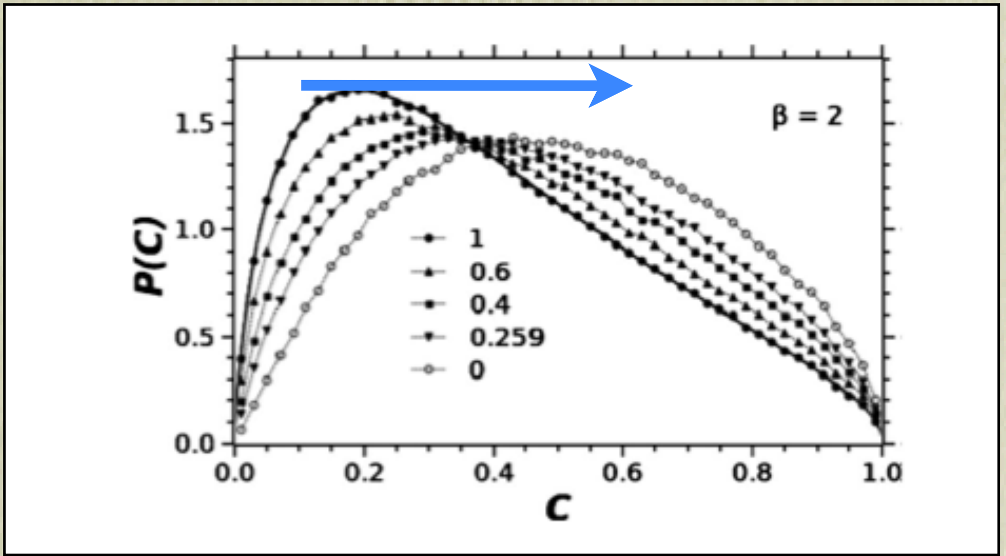
(Received 4 May 2010; revised manuscript received 12 August 2010; published 13 September 2010)

We investigate the statistics of orbital entanglement production between electrons in a chaotic quantum dot with two-channel leads. Through a random-matrix simulation, we obtain the probability density of two entanglement measures, concurrence and entanglement formation, by varying the transparency of the contacts in the presence and absence of time-reversal invariance of the electron dynamics inside the cavity. The results suggest that orbital entanglement production is optimized by increasing the asymmetry between the transparency of the contacts, especially when time-reversal invariance is broken.

One ideal and one non-ideal lead



Only numerical!

Analytical treatment of non-ideal cavities

PRL **108**, 206806 (2012) PHYSICAL REVIEW LETTERS week ending 18 MAY 2012

Statistics of Reflection Eigenvalues in Chaotic Cavities with Nonideal Leads

$(\beta = 2)$

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(Received 24 November 2011; published 16 May 2012)

Left Non-Ideal and Right Ideal lead

$$P_{(\hat{\gamma}_L|0)}(R_1,\ldots,R_{n_L}) = c_{n_L,n_R}\frac{\det^N(\mathbb{1}_{n_L}-\hat{\gamma}_L^2)}{\Delta_{n_L}(\hat{\gamma}_L^2)}\Delta_{n_L}(\boldsymbol{R})\det_{(j,k)\in(1,n_L)}[{}_2F_1(n_R+1,n_R+1;1;\gamma_j^2R_k)]\prod_{j=1}^{n_L}(1-R_j)^{\nu}.$$

Opacities

$$p_\beta(\tau_1,\tau_2) = c_\beta|\tau_1-\tau_2|^\beta(\tau_1\tau_2)^{\beta/2-1}$$

we recover Ideal leads

$$P_{(\hat{\boldsymbol{\gamma}}_L|0)}(R_1,\ldots,R_{n_L}) = c_{n_L,n_R}\frac{\det^N(\mathbb{1}_{n_L}-\hat{\boldsymbol{\gamma}}_L^2)}{\Delta_{n_L}(\hat{\boldsymbol{\gamma}}_L^2)}\Delta_{n_L}(\boldsymbol{R})\det_{(j,k)\in(1,n_L)}[{}_2F_1(n_R+1,n_R+1;1;\gamma_j^2R_k)]\prod_{j=1}^{n_L}(1-R_j)^{\nu}.$$

specialize to the case $n_L = n_R = 2$ $\gamma_1 = \gamma_2 = \gamma$

$$P_{\gamma,\gamma}(\tau_1,\tau_2) = \frac{(\gamma^2-1)^8\sum_{i,j=0}^4 A_{ij}(\gamma)(1-\tau_1)^i(1-\tau_2)^j}{\left(1-\gamma^2(1-\tau_1)\right)^6\left(1-\gamma^2(1-\tau_2)\right)^6}$$

Entanglement production in non-ideal cavities and optimal opacity

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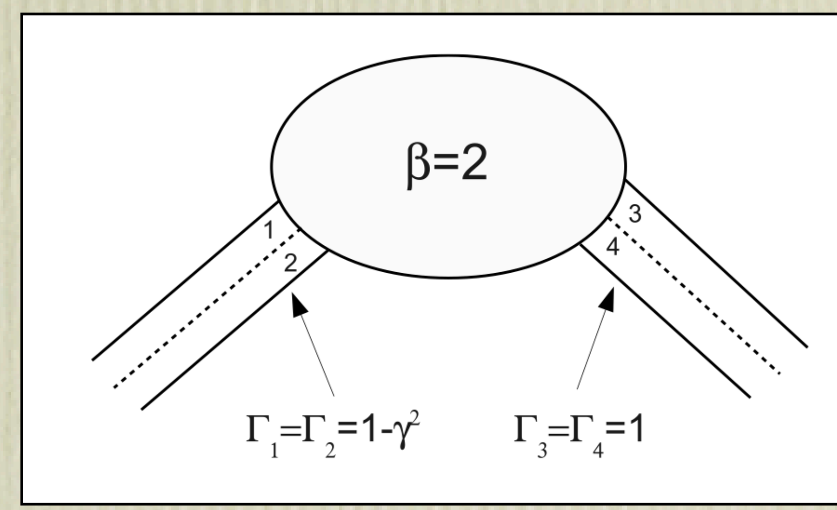
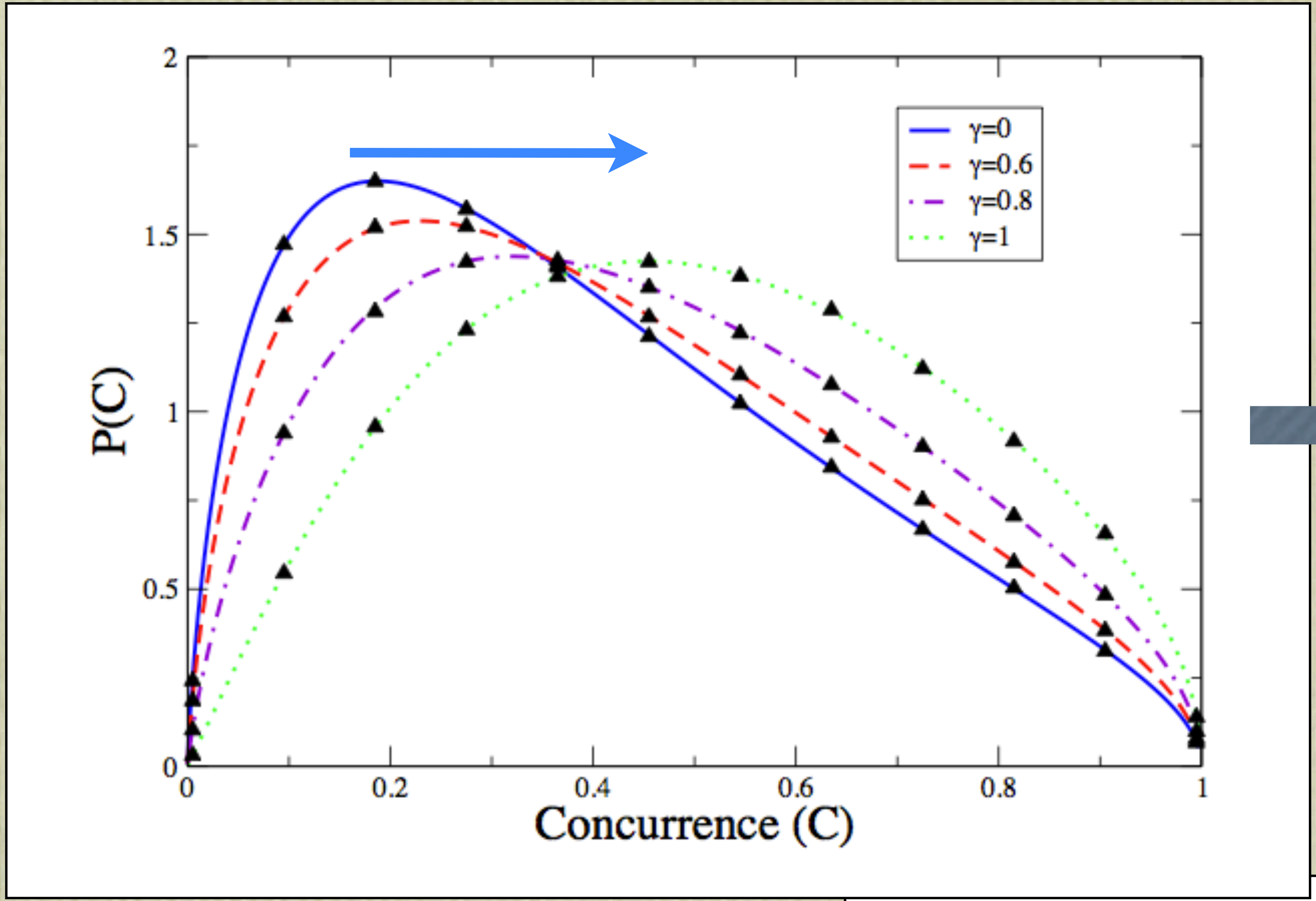
(Dated: February 21, 2013)

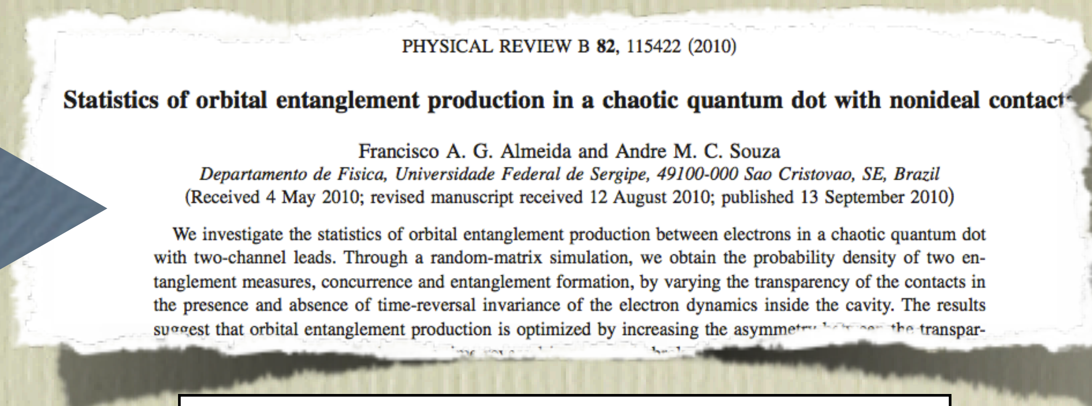


Joint distribution of concurrence and squared norm

$$P_\gamma(\mathcal{C},\mathcal{N}) = \left\langle \delta\left(\mathcal{C} - \frac{2\sqrt{\tau_1(1-\tau_1)\tau_2(1-\tau_2)}}{\tau_1+\tau_2-2\tau_1\tau_2}\right)\delta(\mathcal{N}-(\tau_1+\tau_2-2\tau_1\tau_2))\right\rangle$$



full agreement with numerics in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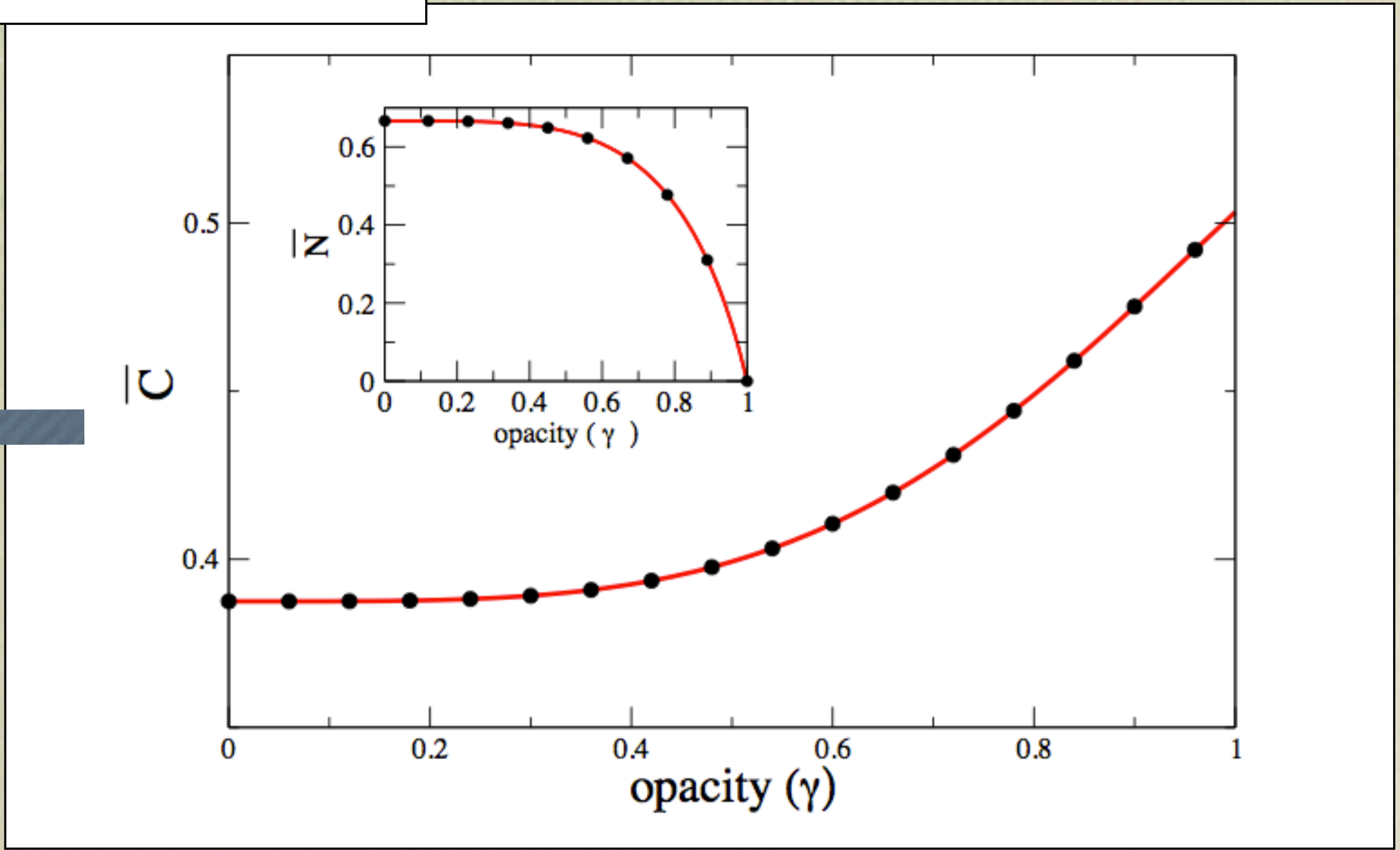
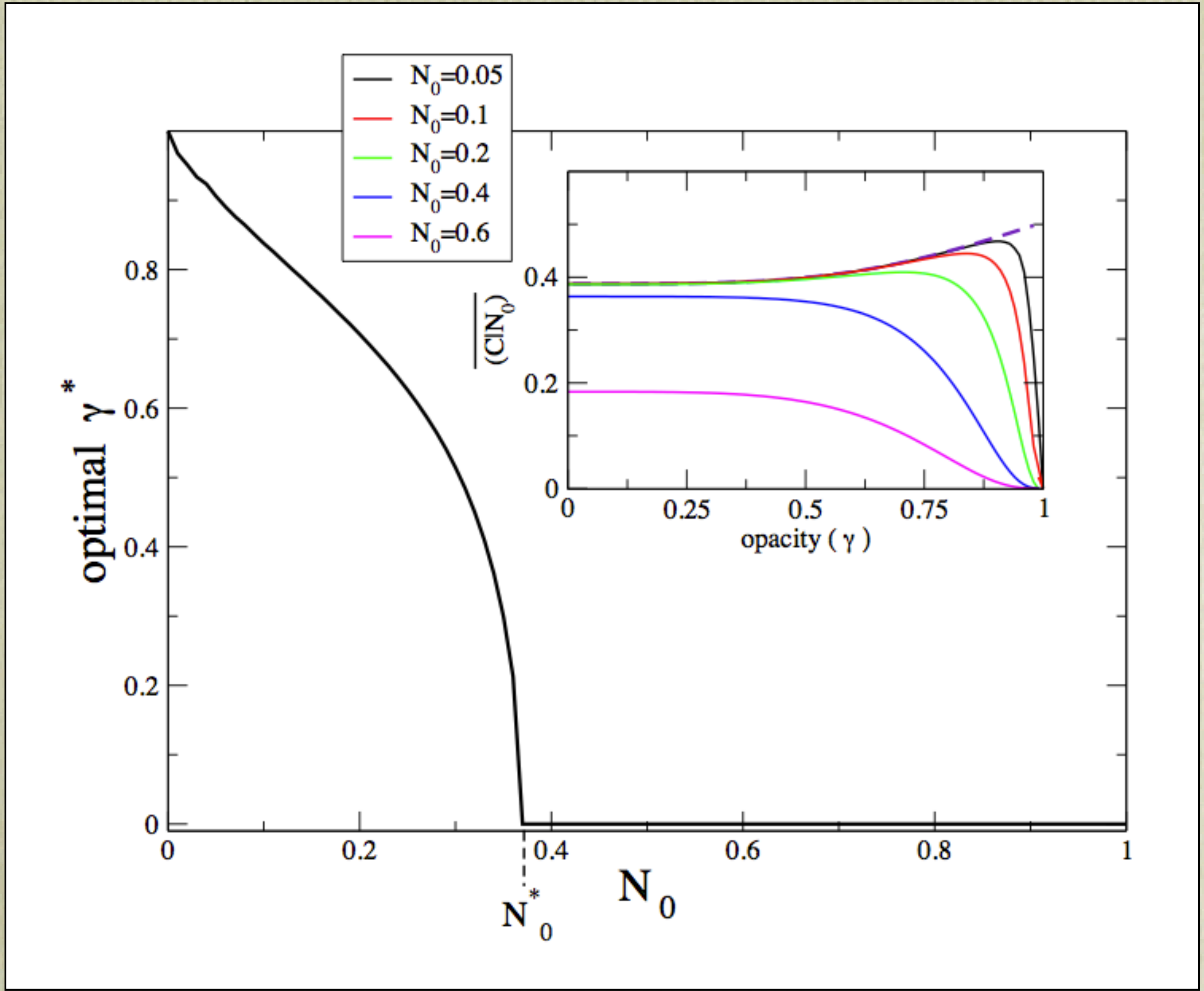
$$P_\gamma(\mathfrak{C}) = \sum_{k=0,4,6,8} c_k(\mathfrak{C})\gamma^k$$



is it possible to 'optimize' the entanglement production process?

$$\overline{(\mathfrak{C}|\mathfrak{N}_0)} = \int_0^1 d\mathfrak{C}\,\mathfrak{C}\int_{\mathfrak{N}_0}^1 d\mathfrak{N}\,P_\gamma(\mathfrak{C},\mathfrak{N})$$



Conclusions

- Electronic entanglement in a chaotic quantum dot with one ideal and one non-ideal lead

- Joint distribution of **concurrence** and **squared norm**

- Average **concurrence** is a increasing function (polynomial) of the opacity of the non-ideal lead, while the **squared norm** is a decreasing function

- The average **concurrence**, constrained to a minimal detectable **squared norm**, has an interesting behavior as a function of the opacity of the non-ideal lead