Quantum-quenched fluids of light in propagating geometries

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<td>Trento: I. Carusotto</td>
<td>Trento: S. Biasi, S. Manna, F. Ramiro-Manzano, F. Turri</td>
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<td>Trieste: A. Chiocchetta</td>
<td>Nice: M. Bellec, C. Michel</td>
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<td>Nice: M. Albert</td>
<td>Paris: A. Bramati, Q. Glorieux</td>
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<td>Edinburgh: D. Faccio</td>
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Quantum propagation of a paraxial beam of quasimonochromatic light in a nonabsorbing nonlinear optical medium

Nonabsorbing nonlinear optical medium:

\[ n(x, y, z, \omega) = n(\omega) + \Delta n(x, y, z, \omega) + n_2(\omega) \times (|\mathcal{E}|^2 \propto \text{Light intensity}) \]

Dispersion + Confinement, defects + Kerr nonlinearity

Paraxial beam of quasimonochromatic light:

\[ \hat{E}(x, y, z, t) = \frac{1}{2} \hat{E}_{\text{air}}(x, y, z, t) e^{i(kz - \omega t)} + \text{H.c.} \]

\[ \hat{E}(x, y, z, t) = \frac{1}{2} \hat{E}(x, y, z, t) e^{i(\beta_0 z - \omega t)} + \text{H.c.} \]

Input state \( \omega \)

\( k = \omega / c \)

Output state \( \omega \)

\( \beta_0 = nk \)
Quantum nonlinear Schrödinger formalism

- Equation of motion for the second-quantized electric field’s envelope $\hat{E}(x, y, t, z)$:

$$i \frac{\partial \hat{E}}{\partial z} = -\frac{1}{2 \beta_0} \left( \frac{\partial^2 \hat{E}}{\partial x^2} + \frac{\partial^2 \hat{E}}{\partial y^2} \right) + \frac{\beta_2}{2} \frac{\partial^2 \hat{E}}{\partial t^2} - \frac{\omega}{c} \Delta n(x, y, z) \hat{E} - \frac{\omega}{c} n_2 \hat{E}^\dagger \hat{E} \hat{E}$$

- Bose commutation relation at different $t$’s and equal $z$’s:

$$[\hat{E}(x, y, t, z), \hat{E}^\dagger(x', y', t', z)] = \frac{2}{n c} \frac{\hbar \omega}{\varepsilon_0} \delta(x - x') \delta(y - y') \delta(t - t')$$


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<th>Paraxial, quasimonochromatic light beam</th>
<th>Dilute atomic Bose gas</th>
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<td>Transverse coordinates $x$, $y$ and time parameter $t$</td>
<td>Spatial coordinates $x$, $y$, $\zeta$</td>
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<td>Propagation coordinate $z$</td>
<td>Time parameter $\tau$</td>
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<td>Optical field’s envelope $\hat{E}(x, y, t, z)$</td>
<td>Matter field $\hat{\Psi}(x, y, \zeta, \tau)$</td>
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<tr>
<td>Propagation constant $\beta_0$</td>
<td>Transverse mass $m_{x,y}$</td>
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<td>$-1/(\text{Group-velocity-dispersion parameter } \beta_2)$</td>
<td>Longitudinal mass $m_{\zeta} \neq m_{x,y}$</td>
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<td>$-(\omega/c) [\text{Refractive index’s spatial profile } \Delta n(x, y, z)]$</td>
<td>External potential $U(x, y, \tau)$</td>
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<tr>
<td>$-(\omega/c) (\text{Kerr-nonlinearity coefficient } n_2)$</td>
<td>Interaction constant $g$</td>
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• Entrance and exit faces treated with an antireflection coating not to spoil the $t \longleftrightarrow z$ mapping

• 3D evolution robust against modulational instabilities when:
  – $\beta_2 < 0$ (anomalous GVD) $\iff m_\zeta > 0$ (positive temporal mass)
  – $n_2 < 0$ (self-defocusing NL) $\iff g > 0$ (repulsive photon-photon interactions)

• Most of the already existing works:
  – $\hat{E}(x, y, A_t, z)$ (monochromatic limit) $\iff$ 2D evolution
  – $\iint \Phi^*(x, y) \hat{E}(x, y, t, z) \, dx \, dy$ (optical-fiber propagation) $\iff$ 1D evolution

• Normalization of the commutators from ab initio electrodynamics calculations

• Quantum features in (1) room-temperature (2) conservative systems of many interacting bosons:
  (1) $\neq$ Quantum fluids of matter
  (2) $\neq$ Quantum fluids of light in semiconductor-planar-microcavity architectures

• Quantum theory independent on the strength of the Kerr nonlinearity
  $\implies$ Strong quantum correlations appearing in the presence of a significant nonlinearity

• Low-dimensional quantum features
The propagating geometry as a platform to investigate the physics of quantum quenches in many-body Bose systems

- Sudden quenches of the system’s quantum Hamiltonian in the photon-photon interaction parameter at the entrance and the exit of the nonlinear medium
- Propagation distance $z < L$ across the medium $\iff$ Time elapsed after the first quantum quench

Light wave \( \omega \) through a Bulk nonlinear medium with the quantum quench in the nonlinearity. 

\[ \rho(x, y, t, z) = \hat{E}^\dagger(x, y, t, z) \hat{E}(x, y, t, z) - \langle \hat{\rho}(x, y, t, L) \rangle \langle \hat{\rho}(x', y', t', L) \rangle \]

**Density**: \( \hat{\rho}(x, y, t, z) = \hat{E}^\dagger(x, y, t, z) \hat{E}(x, y, t, z) \)

- **Dynamical Casimir emission** of pairs of correlated counterpropagating Bogoliubov phonons
- **Light-cone-like spreading** of the two-body quantum correlations at the Bogoliubov “speed” of sound:

\[ |x - x'|, |y - y'|, |t - t'| = 2 s_{x,y,t} z \]

\[ s_{x,y} = \sqrt{-\frac{\omega}{c} n_2 \beta_0 |E|^2}, \quad s_t = \sqrt{-\frac{\omega}{c} n_2 \beta_2 |E|^2} \]

Light wave $\omega$

1D nonlinear waveguide

Quantum quench in the nonlinearity

$z$ axis $\Longleftrightarrow$ Arrow of time

$g^{(1)}(|t - t'|) = \langle \hat{A}^\dagger(t, L) \hat{A}(t', L) \rangle$

- Optical confinement along the $z$ axis:

$$\hat{A}(t, z) = \int \int \Phi^*(x, y) \hat{E}(x, y, t, z) \, dx \, dy$$

$$i \frac{\partial \hat{A}}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 \hat{A}}{\partial t^2} - \frac{\omega}{c} \frac{n_2}{A_{\text{eff}}} \hat{A}^\dagger \hat{A} \hat{A}$$

- Prethermalization and loss of long-lived coherence at the Bogoliubov "speed" of sound:

$$\ln g^{(1)}(|t - t'| \leq 2 s_{1D} L) \left| \propto -T_{\text{eff}} |t - t'| \right| = \text{const}$$

$$s_{1D} = \sqrt{-\frac{\omega}{c} \frac{n_2}{A_{\text{eff}}} |A|^2}, \quad T_{\text{eff}} \propto \frac{\omega}{c} \frac{|n_2|}{A_{\text{eff}}} |A|^2$$

Conclusions

- **Quantum theory of the propagation of a paraxial beam of quasimonochromatic light in a nonabsorbing nonlinear optical medium:**
  - Quantum nonlinear Schrödinger formalism; space: \((x, y, t)\), time: \(z\)
  - Exact same-\(z\) commutation relations

- A propagating fluid of light experiences a pair of quantum quenches in the photon-photon interaction constant upon crossing the front and the back faces of a nonlinear medium
  \(\implies\) The propagating geometry as a platform to study the physics of quantum quenches in many-body Bose systems

- **Light superfluidity** revealed from the suppression of the optomechanical deformation of an illuminated solid dielectric immersed into a Kerr-type optical liquid — P.-É. L. and I. Carusotto, Phys. Rev. A 91, 053809 (2015)

- Work in progress:
  - “Phase characterization of photon absorption in one-dimensional nonlinear optical waveguides” — P.-É. L. and I. Carusotto, to appear on arXiv.org
  - Bogoliubov dispersion relation in a silicon-based waveguide (Trento exp.)
  - Superfluid light in a photorefractive crystal (Nice exp.)