Wave pattern generated by an obstacle moving in a one-dimensional polariton condensate

Pierre-Élie Larré

Laboratoire de Physique Théorique et Modèles Statistiques
Université Paris-Sud 11, Orsay

Séminaire du LPTMS (shared with Paul Soulé)
— 09/04/2013 —

Анатолий Камчатнов

Institute of Spectroscopy
Russian Academy of Sciences, Troitsk

Nicolas Pavloff
LPTMS
Université Paris-Sud 11, Orsay
Microcavity polaritons

- Reflectors
- Semiconductor microcavity
- Exciton
- QW
- $\sim 5\,\text{nm}$
- $\lambda/4$
- $\sim 5\,\text{nm}$
- Upper branch (UB)
- Lower branch (LB)
- Cavity photons
- Excitons

\[
\frac{1}{\sqrt{2}} \left( |\text{Photon}\rangle + |\text{Exciton}\rangle \right) = |\text{Polariton}\rangle
\]

- Photon, exciton: bosons $\implies$ Polariton: boson
- Polariton effective mass (LB): $m^*_P \lesssim 10^{-4} m_e$
- Polariton lifetime: $\tau_P = \tau_\gamma \lesssim 50\,\text{ps}$
Polariton condensation

- Interacting bosons
- Spontaneous appearance of temporal coherence and long-range spatial coherence
- Low $m_p^* \implies$ High $T_c \sim 10$ K
- Finite polariton-lifetime $\implies$ Direct experimental access to internal properties of the polariton fluid just by optical detection of the light emitted by the gas: no intrusive measurements


- Grenoble: Institut Néel
- Lausanne: EPFL
- Marcoussis: LPN
- Paris: LKB (Jussieu)

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Phase coherence in out-of-equilibrium systems

M. Richard et al., PRL (2005)

G. Roumpos et al., PNAS (2012)

\[ \ell_T \equiv \frac{\hbar c_s}{T} \]

\[ \rho_s : \text{superfluid density} \]

\[ \lambda_T \equiv \sqrt{2\pi\hbar^2/(mT)} : \text{thermal wavelength} \]

\[ \rho_s \lambda^2_{T_{\text{BKT}}} = 4: \text{vortex/antivortex pairs unbind (Berezinskii–Kosterlitz–Thouless critical point)} \]

\[ \rho_s \lambda^2_T > 4: \text{vortex proliferation in a phase with finite-range correlations (condensed phase)} \]

\[ \rho^{(1)}_{2D}(r \equiv |x - y|) \equiv \langle \hat{\Psi}^\dagger(x) \hat{\Psi}(y) \rangle \]

\[ \frac{r}{\lambda_T} \to 0 \quad \frac{r}{\ell_T} \to \infty \]

\[ \propto e^{-\pi r^2/\lambda_T^2} \quad \propto (\ell_T/r)^{1/(\rho_s \lambda^2_T)} \]

(thermal) (condensed)

\[ \rho_s \lambda^2_T \big|_{\text{exp}} \simeq (0.8 - 1.1) < 4 \]

Out-of-equilibrium processes: the pumping noise excites phase fluctuations not triggered by vortex proliferation.
Superfluidity in polariton condensates

Landau criterion

★ Weakly perturbing obstacle moving at constant velocity $V$ in a conservative quantum fluid at zero temperature

★ $\implies$ There can exist a critical velocity $V_{\text{crit}}$ such that:

(1) when $V < V_{\text{crit}}$, no excitation is emitted away from the obstacle and there is no drag force: $F_d = 0$ (superfluid regime);

(2) when $V > V_{\text{crit}}$, a Cherenkov radiation of linear waves occurs and the obstacle is subject to a finite drag-force: $F_d \neq 0$ (dissipative regime).

$U_{\text{ext}}(x, t) = \kappa \delta(x + Vt)$ in a quasi-1D BEC

$\omega_{\text{Bog}}(q)$

$\omega(q_*)$ $Vq$ $c_s q$

$q_*$ $q$

$F_d$

$\frac{2m\rho_0 \kappa^2}{\hbar^2}$

$V_{\text{crit}} = c_s$

$V$

A. Amo et al., Nat. Phys. (2009)
Nonresonantly-pumped polariton condensates at zero temperature: a simple one-dimensional model

Phenomenological modification of the Gross–Pitaevskii equation

\[ i \partial_t \psi = -\frac{1}{2} \partial_{xx} \psi + U_{\text{ext}}(x, t) \psi + \rho \psi + i \eta (1 - \rho) \psi \]

★ \( \psi(x, t) \): condensate wavefunction (scalar because \( \sigma = \pm 1 \))
★ \( \rho(x, t) = |\psi(x, t)|^2 \): longitudinal density
★ \( U_{\text{ext}}(x, t) \): potential of an external obstacle

\[ \partial_t \psi = \eta \psi \]
\[ \partial_t \psi = -\eta |\psi|^2 \psi \]
\[ \Rightarrow \partial_t |\psi|^2 = \eta (1 - |\psi|^2) \psi \]

\( \eta \equiv (\text{Gains due to pumping}) - (\text{Losses } \propto 1/\tau_p) > 0 \)
Gain saturation
Dynamical equilibrium between gains and losses
\[ \Rightarrow \text{Steady-state configuration with } |\psi_0|^2 = 1 < \infty \]

Finite-size obstacle moving at constant velocity \(-M \hat{x} \) \( (M \equiv V/c_s > 0) \):

\[ U_{\text{ext}} = U_{\text{ext}}(X \equiv x + Mt) \xrightarrow{|X| \to \infty} 0 \]

Uniform and stationary solution in the absence of external obstacle:

\[ \psi_0(x, t) = e^{-it}, \quad \rho_0(x, t) = |\psi_0(x, t)|^2 = 1 \]
Flow past a weakly perturbing impurity

Linear-response theory

\[ \psi(x, t) = [1 + \delta \psi(x, t)] e^{-it}, \quad |\delta \psi(x, t)| \ll 1 \]

\[ \delta \rho(X) = \int_{\mathbb{R}} \frac{dq}{2\pi} \chi(q, -Mq) U_{\text{ext}}(q) e^{iqX} \]

Critical velocity \( M_{\text{crit}} \equiv V_{\text{crit}}/c_s \)

\[ M_{\text{crit}}^2(\eta) = 1 - \frac{3}{2} \eta^2 \left( \frac{3}{\sqrt{\sqrt{1 + \eta^2} + 1}} \right. \]

\[ \left. - \frac{3}{\sqrt{\sqrt{1 + \eta^2} - 1}} \right) \]

\[ U_{\text{ext}}(X) = \kappa \delta(X) \]

\[ U_{\text{ext}}(X) = \frac{\kappa}{\sigma \sqrt{\pi}} \exp \left( -\frac{X^2}{\sigma^2} \right) \]

\[ M > M_{\text{crit}}: \text{emission of a damped wake ahead of the obstacle} \]
Perturbative drag-force

\[ F_d \equiv \int_{\mathbb{R}} dx \left| \psi(x, t) \right|^2 \frac{\partial U_{\text{ext}}}{\partial x}(x, t) = \left( \text{drag force experienced by the obstacle} \right) = -\int_{\mathbb{R}} dX \frac{d\delta \rho}{dX}(X) \ U_{\text{ext}}(X) \]

\[ U_{\text{ext}}(X) = \kappa \delta(X) \]

\[ F_d / \kappa^2 \]

\[ \eta = 0.1 \]
\[ \eta = 0.5 \]
\[ \eta = 1 \]

\[ \frac{2 |U_{\text{ext}}(qM)|^2}{\kappa^2}, \quad qM \equiv 2 \sqrt{M^2 - 1} \]

\[ \star F_d |_{\delta M \to 0} \approx \eta M \kappa^2 \propto M: \text{“viscous” drag of Stokes type (} \eta \sim \text{viscosity)} \]

\[ \star F_d (M_{\text{crit}}) |_{\delta} = \frac{2}{9} \kappa^2 = \text{fct}^\circ(\eta): \text{onset of (damped) Cherenkov radiations (wave resistance)} \]

\[ \star F_d |_{\delta M \to \infty} \approx 2 \kappa^2 = \begin{cases} \text{fct}^\circ(\eta): \text{pure wave-drag} \\ \text{fct}^\circ(M): \delta\text{-peak artifact} \end{cases} \]

\[ \star F_d |_{\delta \eta \to 0} \approx 2 \kappa^2 \Theta[M -(M_{\text{crit}} \equiv 1)]: \text{discontinuous behavior in the absence of “viscosity” (well-known result in the atomic-condensation context)} \]

\[ \star F_d |_{\text{Gaussian}} \overset{\sigma \to 0}{\to} F_d |_{\delta} \]
Counterintuitive effect

\[ F_d (M > 1) \downarrow \text{ when } \eta \uparrow \]

\[ 2|\mathcal{W}_{\text{ext}}(q_M)|^2/\chi^2 \]
Superfluidity?

★ Within the framework of our model at zero temperature, a small object moving in a nonresonantly-pumped polariton condensate experiences a finite drag-force at any velocity, revealing a nonsuperfluid behavior of the quantum fluid according to the Landau criterion.

★ Similar behavior observed in other related works:
    [resonantly pumped polaritons]

★ However, the drag-force profile presents a (smooth) crossover between a low-velocity regime and a large-velocity one, recalling the case of nondissipative BECs at $T = 0$,
  (i) so maybe superfluidity is compatible with “viscous” drag,
  (ii) and then maybe $F_d(V)$ is not the best-suited observable to probe superfluidity in dissipative systems.

★ After all, $\rho_n \equiv \rho_{\text{tot}} - \rho_s \neq 0$, $\forall T \in ]0, T_\lambda]$ in (superfluid) helium II...
Nonlinear theory for a narrow obstacle

\[
\left( \frac{M^2}{2} + 1 \right) \psi = \left[ -\frac{1}{2} \partial_{xx} + \kappa \delta(x) + |\psi|^2 \right] \psi + i\eta (1 - |\psi|^2) \psi, \quad M > 1 \quad \eta \ll 1
\]

\[
\psi(x) = \sqrt{\rho(x)} e^{i\theta(x)} \quad \text{with} \quad \theta(x) = \int x \, dx' \, u(x')
\]

\[
\begin{align*}
\partial_x(\rho \, u) &= 2\eta \rho (1 - \rho) \\
\frac{u^2}{2} + \rho + \frac{(\partial_x \rho)^2}{8\rho^2} - \frac{\partial_{xx} \rho}{4\rho} &= \frac{M^2}{2} + 1
\end{align*}
\]

Whitham modulation theory \((x < 0)\)

\* \(\{\lambda_i(x)\}_{i=1,2,3,4}\): Riemann invariants

\[
\rho(x) = \frac{1}{4} (\lambda_1 - \lambda_2 - \lambda_3 + \lambda_4)^2 + (\lambda_1 - \lambda_2)(\lambda_3 - \lambda_4) \times \text{sn}^2 \left[ \sqrt{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_4)} \, x, \frac{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_4)}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_4)} \right]
\]

\* \(V\varphi \equiv 0, j, a, L = \text{function}\left(\{\lambda_i(x)\}_{i}\right)\)

\* \(\eta \ll 1\): parameters of the dispersive shock-wave vary weakly over one wavelength \(\implies\) Perturbed Whitham equations:

\[
\frac{d\lambda_i}{dx} = \frac{2}{L} \frac{G_1(\{\lambda_j\}_{j}) \lambda_i + G_2(\{\lambda_j\}_{j})}{\prod_{j \neq i} (\lambda_i - \lambda_j)}
\]

Hydraulic approx\(^\circ\) \((x > 0)\)

\* \(\partial_x \rho = O(\eta \ll 1)\): one neglects derivatives of \(\rho\) in Eq. (B)

\* Eqs. (A) and (B) \(\implies\)

\[
\partial_x \left[ \rho \sqrt{M^2 + 2(1 - \rho)} \right] = 2\eta \rho (1 - \rho)
\]
The two types of steady flows identified in the weak-perturbation limit are separated by a time-dependent regime for strong-enough external potentials, as typically observed in ultracold atomic vapors.
Nonresonantly-pumped spinor polariton condensates

Phenomenological model in 1D

\[ i\hbar \partial_t \psi_\sigma = -\frac{\hbar^2}{2m} \partial_{xx} \psi_\sigma \]
\[ + U_{\text{ext}}(x + Vt) \psi_\sigma - \sigma \hbar \Omega \psi_\sigma \]
\[ + (g_1 |\psi_\sigma|^2 + g_2 |\psi_{-\sigma}|^2) \psi_\sigma \]
\[ + i(\gamma - \Gamma \rho) \psi_\sigma \]

\( \sigma = \pm 1: \) spin projections onto the z axis

\((\psi_+ \psi_-)^T: \) condensate wavefunction

\(\rho(x, t) = |\psi_+|^2 + |\psi_-|^2: \) density

\(\Omega \propto B_z: \) Zeeman splitting between the two polarized states \(\psi_+\) and \(\psi_-\)

\(g_1, g_2: \) interactions between polaritons with parallel \((g_1)\) and antiparallel \((g_2)\) spins; repulsion dominates: typically,

\[-g_1/10 \sim g_2 < 0 < g_1\]

Linearized theory

\(\star \) Two critical velocities: \(V_{\text{crit}}^{(d)} < V_{\text{crit}}^{(p)}\)

\(\star \) \(V > V_{\text{crit}}^{(d)}: \) Cherenkov radiation of damped density-waves

\(\star \) \(V > V_{\text{crit}}^{(p)}: \) Cherenkov radiation of weakly damped polarization-waves

P.-É. L., N. Pavloff, A. M. Kamchatnov

In preparation
**Dumb holes in spinor condensates**

### Acoustic horizon for the polarization modes

\[
i\hbar \partial_t \hat{\Psi}_\sigma = -\frac{\hbar^2}{2m} \partial_{xx} \hat{\Psi}_\sigma + (g_1 \hat{n}_\sigma + g_2 \hat{n}_{-\sigma}) \hat{\Psi}_\sigma \quad \hat{n}_\sigma(x, t) = \hat{\Psi}_\sigma^\dagger \hat{\Psi}_\sigma \quad 0 < g_2 < g_1
\]

Lab-frame dispersion relation of elementary excitations in the long-wavelength limit:

\[
\hbar \omega_{\text{lab}}(q) \simeq V_0 \hbar q \quad \text{(Doppler shift)} \quad \pm \left( \frac{c(p)}{c(d)} \right) \hbar q \quad \text{with} \quad \frac{c(p)}{c(d)} = \sqrt{\frac{g_1 - g_2}{g_1 + g_2}} < 1
\]

### Correlations

\[
\hat{\sigma}_z(x, t) = \sum_{\sigma = \pm 1} \sigma \hat{n}_\sigma(x, t)
\]

\[
\langle :\hat{\sigma}_z(x, t) \hat{\sigma}_z(x', t) : \rangle
\]

correlates with

I. Carusotto, S. Finazzi, P.-É. L., N. Pavloff, A. Recati

*In preparation*
Wave patterns in polariton condensates: conclusion

★ Analyzis of the flow of a one-dimensional scalar polariton condensate in motion with respect to a localized obstacle in a situation of nonresonant pumping at zero temperature

★ **Weak-perturbation limit**: smooth crossover from a viscous flow to a regime where the drag is mainly dominated by wave resistance

★ Onset of (damped) Cherenkov radiations at a velocity $V_{\text{crit}}(\eta) \leq c_s$ only depending on the parameter $\eta$ (≈ pumping and losses processes in the system)

★ Absence of long-range wake ≠ absence of dissipation — Drag force ≠ best-suited observable to probe superfluidity in dissipative condensates?

★ Whitham modulation theory and hydraulic approximation in the case of a supersonic fluid flowing past a δ-peak impurity of arbitrary amplitude

★ The two types of steady flows identified in the weak-perturbation limit are separated by a time-dependent regime for strong-enough external potentials

★ Ejection of a weakly damped polarization-wave: does it make it possible to probe Hawking-like radiation in spinor polariton condensates?

- P.-É. L., N. Pavloff, A. M. Kamchatnov, *In preparation*