Many-body quantum phenomena in fluids of nonlinear light

Tamara Bardon-Brun, Nicolas Cherroat, Dominique Delande, Pierre-Élie Larré, and Thibault Scoquart

From the “Complex Quantum Systems” group

Laboratoire Kastler–Brossel

UPMC—Sorbonne Universités, CNRS, ENS—PSL Research University, and Collège de France

Quantum fluid of light

- Propagation of a paraxial and quasi-monochromatic optical beam in a dispersive and nonlinear medium
  - Electric field: \( E(x,t) = \frac{i}{\hbar} \frac{\partial \hat{E}}{\partial t} = \left( \frac{\partial^2 \hat{E}}{\partial x^2} + \frac{\partial^2 \hat{E}}{\partial t^2} \right) + \frac{i}{c} \nabla \times E \)
  - Dispersion law: \( \frac{\partial^2 \hat{E}}{\partial t^2} + \frac{\partial^2 \hat{E}}{\partial x^2} = \frac{n_0}{c^2} |\hat{E}|^2 \hat{E} \)
  - Refractive index: \( n_0 + n_1(x,t) + n_2 |\hat{E}(x,t)|^2 \)

- Propagation equation of the quantized envelope \( E(x,y,t) \) of the complex electric field [1]:
  \[
  \frac{\partial \hat{E}}{\partial n} = -i \frac{\partial^2 \hat{E}}{\partial x^2} - \frac{i}{c} \nabla \times E \]

- Evolution equation of the quantized macroscopic wavefunction \( \psi(x,y,t) \) of a dilute atomic Bose–Einstein condensate:
  \[
  i \hbar \frac{\partial \psi}{\partial n} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V(x,y,t) \psi + \bar{g} \langle \psi^\dagger \rangle \bar{g} \psi \]

Optomechanical signature of a frictionless flow of superfluid light

- 2D Gross–Pitaevskii flow around a localized and stationary obstacle:
  \[
  E(x,y,z = 0) = E_{0e} e^{i\kappa x} + E_{0p} e^{-i\kappa x} \]

- The optomechanical deformation \( \xi(z) = F \) vanishes in the superfluid–flow regime [2]

Interferometry of the Bogoliubov dispersion relation of a fluid of light

- 1D Gross–Pitaevskii evolution with one- and two-photon losses:
  \[
  \hat{E}(t,z = 0) = \text{Pump} + \text{Probe} \]
  \[
  \hat{E}(t,z > 0) = \text{Pump} + \text{Signal} + \text{Idle} \]

- The probe’s dephasing is a nontrivial function of the Bogoliubov dispersion relation:
  \[
  k_{\text{Bog}}(\omega_{\text{Pump}} - \omega_{\text{Probe}}) = k_{\text{Signal}} = k_{\text{Pump}} \]

Postquench out-of-equilibrium dynamics of a quantum fluid of light

- Interaction quench [1]
  - Light detectors at the exit interface make it possible to probe the postquench out-of-equilibrium dynamics of the quantum fluid of light
  - Weak Kerr nonlinearity: Generation of correlated counterpropagating Bogoliubov excitations in the \( x, y, \), and \( t \) directions → Light-cone effect, prethermalization [1, 4]

Ongoing work

- Prethermalization [1]: 1D quantum fluid of scalar light, disorder, interaction quench:
  \[
  \frac{\partial \hat{E}}{\partial n} = -\frac{1}{2k_B} \left( \frac{\partial^2 \hat{E}}{\partial x^2} - \frac{\partial^2 \hat{E}}{\partial t^2} \right) + \frac{i}{c} \nabla \times \hat{E} \]

- Thermalization [2]: 2D classical fluid of vector light:
  \[
  \frac{\partial \hat{E}_\mu}{\partial n} = -\frac{1}{2k_B} \left( \frac{\partial^2 \hat{E}_\mu}{\partial x^\mu \partial x^\nu} - \frac{\partial^2 \hat{E}_\mu}{\partial x^\nu \partial x^\mu} \right) + \frac{i}{c} \nabla \times \hat{E}_\mu \]

Casimir effect [3]: 3D quantum fluid of vector light:
  \[
  \hat{E}_\mu = (\hat{E}_\mu + \hat{E}_\mu(x,y,z)) \]

References