Propagagation of a quantum fluid of light in a bulk nonlinear optical medium: General theory and response to a quantum quench

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Theory in collaboration with:

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Paraxial propagation of a quasimonochromatic electromagnetic wave

- \( \mathbf{x} = (x, y) \)
- Dispersive, inhomogeneous, nonlinear medium: \( n(\mathbf{x}, z, \omega) = n(\omega) + \Delta n(\mathbf{x}, z) + n_2 |\mathbf{E}|^2 \)
- Laser wave: \( E(\mathbf{x}, z, t) = \text{Re}\left[ E(\mathbf{x}, z, t) \, e^{i(\beta_0 z - \omega_0 t)} \right] \)

\[
\frac{i}{\beta_0} \frac{\partial E}{\partial z} = -\frac{1}{2 \beta_0} \frac{\partial^2 E}{\partial x^2} + \frac{D_0}{2} \frac{\partial^2 E}{\partial t^2} - \frac{i}{v_0} \frac{\partial E}{\partial t} - \frac{\omega_0}{c} \Delta n(\mathbf{x}, z) \, E - \frac{\omega_0}{c} n_2 |\mathbf{E}|^2 \, E
\]

Gross–Pitaevskii-like wave equation

New coordinates: \( \tau = z/v_0 \) and \( \zeta = v_0 \, t \) \( \Rightarrow \)

\[
\frac{i}{\frac{\beta_0}{v_0}} \frac{\partial E}{\partial \tau} = -\frac{1}{2 m_x} \frac{\partial^2 E}{\partial x^2} - \frac{1}{2 m_\zeta} \frac{\partial^2 E}{\partial \zeta^2} - \frac{i}{v_0} \frac{\partial E}{\partial \zeta} + U(\mathbf{x}, \tau) \, E + g \, |\mathbf{E}|^2 \, E
\]

- \( m_x = \beta_0 / v_0 \): Diffraction “mass”
- \( m_\zeta = -1/(v_0^3 \, D_0) \): Dispersion “mass”
- \( U(\mathbf{x}, \tau) = -(\omega_0 \, v_0 / c) \Delta n(\mathbf{x}, z) \): “External potential”
- \( g = -(\omega_0 \, v_0 / c) \, n_2 \): Photon-photon interaction constant
Classical field theory

\[ [\text{Action}] = \text{Cst} \int d\tau \int dx \, d\zeta \left[ \text{Im} \left( \frac{\partial \mathcal{E}^*}{\partial \tau} \mathcal{E} \right) - \frac{1}{2} \frac{\partial \mathcal{E}^*}{\partial x} \frac{\partial \mathcal{E}^*}{\partial x} + \frac{1}{2} \frac{\partial \mathcal{E}^*}{\partial \zeta} \frac{\partial \mathcal{E}^*}{\partial \zeta} \right] + v_0 \text{Im} \left( \frac{\partial \mathcal{E}^*}{\partial \zeta} \mathcal{E} \right) \]

Quantum field theory

\[ [\hat{\mathcal{E}}(x, \zeta, \tau), \hat{\mathcal{E}}^\dagger(x', \zeta', \tau)] = \frac{\hbar}{\text{Cst}} \delta(x - x') \delta(\zeta - \zeta') \quad \text{Equal} \quad \tau's \equiv \text{Equal} \quad z's \]

\[ \left[ \text{Many-body Hamiltonian} \right] = \text{Cst} \int dx \, d\zeta \left[ \frac{1}{2m_x} \frac{\partial \hat{\mathcal{E}}^\dagger}{\partial x} \cdot \frac{\partial \hat{\mathcal{E}}}{\partial x} + \frac{1}{2m_\zeta} \frac{\partial \hat{\mathcal{E}}^\dagger}{\partial \zeta} \frac{\partial \hat{\mathcal{E}}}{\partial \zeta} - \frac{v_0}{2i} \left( \frac{\partial \hat{\mathcal{E}}^\dagger}{\partial \zeta} \hat{\mathcal{E}} - \hat{\mathcal{E}}^\dagger \frac{\partial \hat{\mathcal{E}}}{\partial \zeta} \right) \right. \]

\[ \left. + U(x, \tau) \hat{\mathcal{E}}^\dagger \hat{\mathcal{E}} + \frac{g}{2} \hat{\mathcal{E}}^\dagger \hat{\mathcal{E}}^\dagger \hat{\mathcal{E}} \hat{\mathcal{E}} \right] \]

Bogoliubov theory of quantum fluctuations

\[ \text{Quasimonochromaticity} + \text{weak nonlinearity} \Rightarrow \hat{\mathcal{E}}(x, \zeta, \tau) = \mathcal{E}(x, \zeta, \tau) + \delta \hat{\mathcal{E}}(x, \zeta, \tau) \]

\[ \delta \hat{\mathcal{E}} \ll \mathcal{E} \]

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Application of the theory: Response to a quantum quench

- Response of a propagating quantum fluid of light to a pair of “temporal” quenches of the photon-photon interaction constant

\[
\left[ \text{Vacuum “out”} \right]_{\tau_{\text{out}} > L/(2v_0)} \neq \left[ \text{Vacuum “in”} \right]_{\tau_{\text{in}} < -L/(2v_0)}
\]

- Antireflection coating at the \( z = \mp L/2 \) surfaces of the dielectric slab

Propagation of a quantum fluid of light in a bulk nonlinear optical medium: General theory and response to a quantum quench
Near-field two-body correlations

Intensity: \( \hat{I}(x, \zeta, \tau) = \hat{E}^\dagger(x, \zeta, \tau) \hat{E}(x, \zeta, \tau) \)

\[
\frac{\langle :\hat{I}(x, \zeta, \tau_{\text{out}}) \hat{I}(x', \zeta', \tau_{\text{out}}) : \rangle_{\text{in}}}{\langle \hat{I}(x, \zeta, \tau_{\text{out}}) \rangle_{\text{in}} \langle \hat{I}(x', \zeta', \tau_{\text{out}}) \rangle_{\text{in}}} - 1
\]

Far-field two-body correlations

- \( q = (q_x, q_y) \)
- Momentum distribution: \( \hat{n}(q, q_\zeta, \tau) = \hat{a}^\dagger(q, q_\zeta, \tau) \hat{a}(q, q_\zeta, \tau) \)

\[
\frac{\langle \hat{n}(q, q_\zeta, \tau_{\text{out}}) \hat{n}(q', q'_\zeta, \tau_{\text{out}}) \rangle_{\text{in}}}{\langle \hat{n}(q, q_\zeta, \tau_{\text{out}}) \rangle_{\text{in}} \langle \hat{n}(q', q'_\zeta, \tau_{\text{out}}) \rangle_{\text{in}}} - 1
\]

\( \propto \delta(q - q') \delta(q_\zeta - q'_\zeta) + \delta(q + q') \delta(q_\zeta + q'_\zeta) \)

Dynamical Casimir emission (DCE)

\( (q, q_\zeta) \leftrightarrow (x, \zeta) \leftrightarrow (x', \zeta') \leftrightarrow (q', q'_\zeta) \) Correlated

\( \kappa_n^2(\Delta \tau = L/v_0)/2 \)

\[
= -1 + \sqrt{1 + \left( \frac{n \pi}{g |E|^2 \Delta \tau} \right)^2}
\]
Conclusion

- Effective theory of quantum fluctuations in cavityless bulk nonlinear optical media
  - Propagation coordinate $\leftrightarrow$ time
- DCE after a quench of the Kerr nonlinearity along the propagation axis
  - Experimental implementation by Daniele Faccio’s team, Edinburgh

Related works in progress

- Breakdown of laser coherence by local emergence of thermal-like correlations in a quenched one-dimensional quantum fluid of light (with José Lebreuilly)
- Reaching the Tonks–Girardeau regime in a photon fluid (with José Lebreuilly)
- Pump-probe measurement of the sound velocity in a photon fluid (with Stefano Biasi and Fernando R. Manzano, Trento)
- Creating an acoustic black-hole horizon in a photon fluid (with Daniele Faccio’s team, Edinburgh)
Laser propagation in optical fibers

- Adiabatic approximation:

\[
\mathcal{E}(x, z, t) = \Psi(z, t) \left( \Phi(x; |\Psi(z, t)|^2) \right)
\]

(Longitudinal profile) ("Freezed" transverse profile)

- If $|\Psi|^2 \ll 1/(\beta_0^2 |n_2|)$, the transverse profile is a Gaussian function of width $w = \sqrt{2/\beta_0 \kappa}$ and

\[
i \frac{\partial \Psi}{\partial z} = \frac{D_0}{2} \frac{\partial^2 \Psi}{\partial t^2} - \frac{i}{v_0} \frac{\partial \Psi}{\partial t} + \frac{\kappa}{2} \Psi + \alpha |\Psi|^2 \Psi,
\]

where

\[
\alpha = -\beta_{\text{vac}} n_2 / (\pi w^2).
\]

At the exit of the fiber

\[
\log |\Psi|^2 \quad [\text{a. u.}]
\]

Coherence

0

-1

-2

-3

|t - t'| [a. u.]

0

20

40

Sound "speed":

\[
s = \sqrt{\frac{\alpha |\Psi|^2}{-1/(v_0^2 D_0)}}
\]