Acoustic black holes

The acoustic black hole is to sound what the gravitational black hole is to light.

\[ \delta_{\text{ac}}(q) = \frac{V h q}{(\text{Doppler shift})} \]

Exterior of the dumb hole
Horizon
Interior of the dumb hole

Dumb holes in quasi-1D Bose–Einstein condensates

Stationary Gross–Pitaevskii equation:

\[ \hat{H} = \frac{\hat{p}^2}{2m} + V(x) + g(x)\hat{n}[\hat{\Psi}] \]

- \( \hat{\Psi} \) (BEC order parameter,
- \( g(x) \) = contact-interaction constant,
- \( V(x) \) = external potential,
- \( \mu \) = chemical potential.

\( \delta \)-peak configuration: \( g(x) = C^2 \)

\[ U(x) = \Delta \delta(x) \]

\[ \delta \text{-peak configuration: } g(x) = C^2 \]

\[ \hat{\Psi}(x,t) = \hat{\Psi}(x) + \hat{\psi}(x,t) \text{ with } \hat{\psi} \ll \hat{\Psi} \]

- Bogoliubov approach:

\[ \hat{\psi}(x,t) = \hat{\Psi}(x) \hat{\Phi}(x,t) + \hat{\Phi}(x,t) \hat{\Psi}(x) \]

- Bogoliubov spectrum:

\[ \delta_{\text{bd}}(q) = \frac{V h q}{(\text{Doppler shift})} \]

\[ \delta_{\text{bd}}(q) = \frac{\text{c} h q}{\sqrt{q^2 + \omega^2}} \]

Quantum fluctuations: Bogoliubov approach

- Subsonic region

\[ \langle [S_{\omega}^{(\text{in})}(\omega)]^2 \rangle \text{ transmission or reflection coefficient for a } \omega \text{-going mode oscillating at pulsation } \omega \text{ scatters into a } \omega \text{-outgoing mode.} \]

- Supersonic region

\[ \langle [S_{\omega}^{(\text{out})}(\omega)]^2 \rangle \text{ absorption coefficient for a } \omega \text{-going mode oscillating at pulsation } \omega \text{ scatters into a } \omega \text{-outgoing mode.} \]

One-body Hawking signal

- Energy current associated to the emission of elementary excitations (deep outside the black hole):

\[ H_{\text{em}} = \left( \frac{\omega}{c} \right)^T x - \int_0^t \frac{d\omega}{h} [S_{\omega}(\omega)]^2 \]

- Radiation spectrum:

\[ |S_{\omega}(\omega)|^2 \approx \frac{\omega}{\omega_{\text{out}}^2} \]

- Low-\( \omega \) behaviour of \( S_{\omega}(\omega) \):

\[ S_{\omega}(\omega) = f_{\text{bd}} \left( \frac{\omega}{\omega_{\text{in}}} \right)^\alpha + h_{\text{bd}} \left( \frac{\omega}{\omega_{\text{in}}} \right)^\beta \]

\[ \gamma = -4 \text{Re}\left( f_{\text{bd}} f_{\text{bd}''} \right) \]

\[ T_H = 100 \text{K} \text{ } \mu = 100 \text{K} \]

Two-body Hawking signal in momentum space

- Analytical estimates of the gray-body factor and of the Hawking temperature:

\[ \Gamma = -4 \text{Re}\left( f_{\text{bd}} f_{\text{bd}''} \right) \]

\[ T_H = \frac{h \omega_{\text{in}}}{\pi c^2} \]

\[ \delta \text{ peak} \]

\[ \text{Waterfall} \]

\[ \xi = \left( 2 + \frac{m u}{\hbar} \right) \sqrt{\frac{\hbar}{\sqrt{\gamma} u + \hbar}} \]


Compressibility sum rule at zero temperature

- In the absence of black hole,

\[ \int dx' g_0^2(x,x') = -n(x) \]

- In the presence of black hole, the shape of the short-range antibunching is modified:

\[ \int dx' g_0^2(x,x') \approx (k(x) + \text{terms})_{\text{BH}} \]

Long-range correlations allow us to recover the sum rule:

\[ \int dx' g_0^2(x,x') \approx (k(x) + \text{terms})_{\text{BH}} \]

Because of the sum rule,

\[ (\text{Long-range correlations}) \]

\[ \xi = \frac{f_{\text{bd}}}{f_{\text{bd}''}} \sqrt{\frac{\hbar}{\sqrt{\gamma} u + \hbar}} \]

\[ f_{\text{bd}} f_{\text{bd}''} = 0 \]

\[ w_{\text{bd}} = \frac{V_{\text{bd}}}{c_{\text{bd}}} \]

Two-body Hawking signal

- Connected two-body density matrix:

\[ g^{(2)}(x,x') = \langle \hat{\Psi}(x,t) \hat{\Psi}^\dagger(x,t) \hat{\Psi}(x',t) \hat{\Psi}^\dagger(x',t) \rangle - n(x) n(x') \xi(x,x') g^{(1)}(x,x') \]

\[ \text{Correlated phonons} \]

\[ \hat{u}_{\text{out}} \rightarrow \hat{u}_{\text{bd}} \hat{d}_{\text{bd}'} \hat{d}_{\text{bd}''} \text{ Correlated} \]

\[ \text{Waterfall configuration} \]

- At time \( t \) after their emission, the phonons \( u_{\text{out}} \) and \( d_{\text{bd}''} \) are respectively located at:

\[ x = V_{\text{bd}}(t) \text{ and } x' = V_{\text{bd}}(t) \text{ along the line of slope } \]

\[ x' = \frac{V_{\text{bd}}(t)}{V_{\text{bd}}(t)} \text{ in the } \{ x, x' \} \text{ plane.} \]