

# Direct measurement of the phase coherence length in a GaAs/GaAlAs square network

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(Dated: December 10, 2004)

The low temperature magnetoconductance of a large array of quantum coherent loops exhibits Altshuler-Aronov-Spivak oscillations which periodicity corresponds to  $1/2$  flux quantum per loop. We show that the measurement of the harmonics content provides an accurate way to determine the electron phase coherence length  $L_\varphi$  in units of the lattice length with no adjustable parameters. We use this method to determine  $L_\varphi$  in a square network realised from a 2D electron gas (2DEG) in a GaAs/GaAlAs heterojunction, with only a few conducting channels. The temperature dependence follows a power law  $T^{-1/3}$  from 1.3 K to 25 mK with no saturation, as expected for 1D diffusive electronic motion and electron-electron scattering as the main decoherence mechanism.

PACS numbers: 73.23.-b ; 73.20.Fz

The characteristic scale on which quantum interference can occur in a conductor, the phase coherence length  $L_\varphi$ , is the key parameter of quantum transport. In particular, the dependence of  $L_\varphi$  on temperature can discriminate between the various scattering mechanisms which limit phase coherence: electron-electron (e-e), electron-phonon or electron-magnetic impurity interactions. Interference on the scale of  $L_\varphi$  gives rise to two different types of contributions to the conductance in a transport experiment. Some are sample specific and depend on the particular disorder configuration. These are conductance fluctuations (magnetofingerprints) and the  $\phi_0$  periodic Aharonov-Bohm (AB) oscillations ( $\phi_0 = h/e$  is the flux quantum). Their amplitudes are governed both by  $L_\varphi$  and the thermal length  $L_T$ , in general smaller than  $L_\varphi$ . This makes an accurate determination of  $L_\varphi$  difficult [1, 2, 3]. The second type of contribution, called the weak localisation (WL) correction, is obtained after ensemble averaging of quantum interferences on many configurations of disorder. It originates from interferences between time reversed electronic trajectories, which are the only ones surviving the disorder average. It is also observed in samples of size  $L \gg (L_\varphi, L_T)$  and only depends on  $L_\varphi$  since it involves trajectories at the same energy. Manifestations of WL are the magnetoconductance (MC) of large connex samples [1, 2, 3] and the Altshuler-Aronov-Spivak (AAS)  $\phi_0/2$  periodic oscillations resulting from the ensemble average of AB oscillations in a long cylinder or large arrays of connected phase coherent rings [4, 5, 6]. The WL provides thus in general a much more direct measurement of  $L_\varphi$  than sample specific corrections.

The analysis of the MC in 1D diffusive metallic wires (with transverse dimensions smaller than  $L_\varphi$ ) has led to accurate determinations of  $L_\varphi$ . It was found that the dominant phase breaking mechanism at very low temperature, in the absence of magnetic impurities, is due to e-e scattering and is well described by the Altshuler-Aronov-Khmelnitskii (AAK) theory [1, 7] yielding  $L_\varphi \propto T^{-1/3}$

with no saturation down to 40 mK [2, 3]. Such a remarkable agreement between theory and experiment has not been established for semiconducting wires, where most WL experiments have been performed only above 0.2 K or with insufficient ensemble averaging [8, 9, 10, 11]. It is however essential to check the validity of the AAK theory for these systems which correspond to radically different physical parameters: fewer conducting channels and larger screening lengths. In this Letter we present MC data down to 25 mK of networks fabricated from a GaAs/GaAlAs 2DEG, which contain  $10^6$  square loops in the diffusive transport regime, and determine  $L_\varphi$  *without adjustable parameters* from the analysis of the AAS oscillations (Fig. 1). Following [12, 13], we explain how to calculate the harmonics content of these oscillations and show that it depends only on  $L/L_\varphi$  where  $L$  is the circumference of the elementary loop. It is then possible to determine  $L_\varphi$  and its temperature dependence exclusively from geometrical parameters of the network. This new method of determining  $L_\varphi$  is especially interesting in these 2DEG wires for which basic transport parameters such as the electron density and wire width ( $W$ ) are not straightforwardly determined, unlike metals.

Moreover once  $L_\varphi$  is determined, we deduce from the analysis of the high field positive MC the elastic mean free path ( $l_e$ ),  $W$ , and make a detailed comparison with theoretical predictions of the AAK theory on dephasing by e-e interactions. We find a very good quantitative agreement in the regime, never explored before, of very few conducting channels.

In the weakly disordered diffusive regime ( $k_F l_e \gg 1$ ), the WL correction is directly related to the Cooperon, which can be computed from the time integrated return probability  $P_c(\vec{r}, \vec{r})$  for a diffusive electron. In a cylinder or an array of connected loops the contribution to the Cooperon of trajectories enlacing at least one loop oscillates with a flux periodicity of  $\phi_0/2$  giving rise to the AAS oscillations. A systematic way of calculating

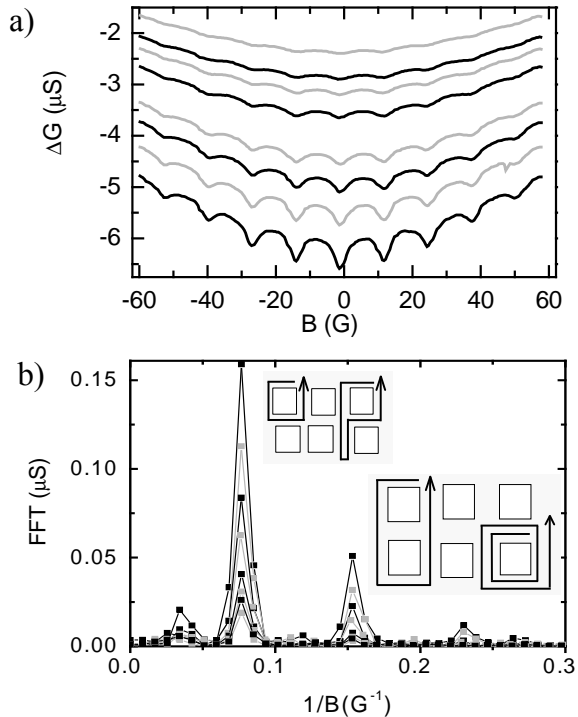


FIG. 1: a) Conductance versus magnetic field between 25 mK to 220 mK. b) Fourier transform of the MC (after subtraction of the envelope) for different temperatures. Left (right) inset: some orbits contributing to the first (second) harmonic.

WL in a mesoscopic network of diffusive wires was derived in [12]. More recently [13], a relation was found between the WL correction and the spectral determinant  $S(\gamma) = \det(\gamma - \Delta)$  of the Laplace operator  $\Delta$  defined on the network. If we write  $\Delta\tilde{\sigma} = \langle \Delta\sigma \rangle h/e^2$ , then

$$\Delta\tilde{\sigma} = -4 \int \frac{d\vec{r}}{\text{Vol}} P_c(\vec{r}, \vec{r}) = \frac{-4}{\text{Vol}} \frac{\partial}{\partial \gamma} \ln S(\gamma), \quad (1)$$

where  $\gamma = 1/L_\varphi^2$ . Eq.(1) assumes an exponential relaxation of phase coherence. This approach, which is meaningful only for regular networks, is particularly efficient because  $S(\gamma)$  can be computed systematically for any given network in terms of the determinant of a finite size matrix encoding the network's characteristics (topology, length of the wires, magnetic flux). It can also be shown that the WL can be expressed, in the small  $L_\varphi$  limit, as a trace expansion over periodic orbits, denoted  $\mathcal{C}$ ,

$$\frac{\partial \ln S(\gamma)}{\partial \gamma} = \frac{1}{2\sqrt{\gamma}} \left[ \mathcal{L} + \frac{V-B}{\sqrt{\gamma}} + \sum_{\tilde{\mathcal{C}}} l(\tilde{\mathcal{C}}) \alpha(\mathcal{C}) e^{-\sqrt{\gamma} l(\mathcal{C}) + i\theta(\mathcal{C})} \right]$$

where  $V$  ( $B$ ), is the total number of vertices (bonds).  $\tilde{\mathcal{C}}$  is the primitive orbit related to  $\mathcal{C}$ .  $\text{Vol} = \mathcal{L}$  is the total length. We explain briefly this formula, demonstrated in [14] and discussed in detail in [15]. Each orbit contributes to the MC with a phase factor which depends

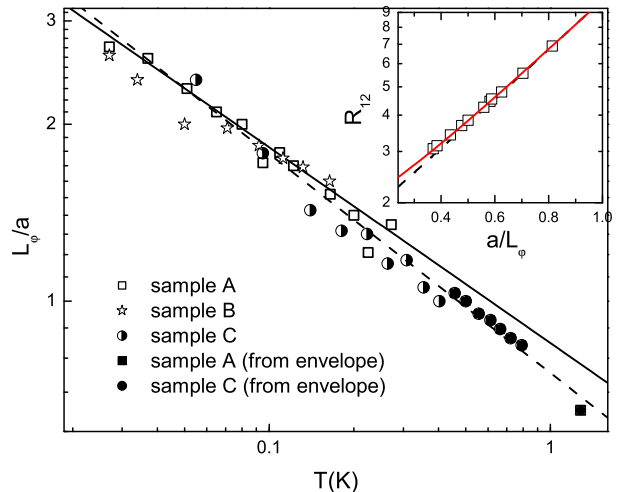


FIG. 2: inset: relation between  $L_\varphi/a$  and the ratio  $R_{12}$  of the two first harmonics on a semi-log scale. The continuous line comes from the numerical calculation of  $S(\gamma)$ . The dashed line is deduced from the expansion (2). The open circles are the experimental values of  $R_{12}$  from which  $L_\varphi$  is determined. Main panel:  $L_\varphi$  versus temperature on a log-log scale obtained from  $R_{12}$  for samples A, B and C. The fit (dashed line) yields the power law  $L_\varphi \propto T^{-0.36}$ . The dark circles and squares are obtained from the fit of the envelope. The continuous line is  $L_\varphi$  from the AAK theory (for sample A).

on the enclosed flux:  $\theta(\mathcal{C}) = 4\pi\Phi(\mathcal{C})/\phi_0$ . It is also characterised by its length  $l(\mathcal{C})$  and by a geometrical weight  $\alpha(\mathcal{C})$ . In the case of a square lattice of periodicity  $a$ , the periodically oscillating conductance can be decomposed in Fourier space as a sum of harmonics of the fundamental periodicity corresponding to  $\phi_0/2$  per elementary cell. The first terms of this expansion in  $x = e^{-2a/L_\varphi}$  read:

$$\Delta\tilde{\sigma} = -\frac{L_\varphi}{W} \left[ 2 - \frac{L_\varphi}{a} + x + \dots + \frac{x^2}{2} \cos \theta \left( 1 - \frac{3}{2}x + \dots \right) + \frac{3x^3}{8} \cos 2\theta \left( 1 - \frac{19}{12}x + \dots \right) + \frac{3x^4}{8} \cos 3\theta \left( 1 - \frac{15}{8}x + \dots \right) \right]. \quad (2)$$

Here  $\theta = 4\pi\phi/\phi_0$ , and  $\phi$  is the flux per elementary cell. The amplitude of the  $n$ th harmonics is evaluated by counting the paths enclosing  $n$  fluxes  $\phi$ . The counting is rapidly cumbersome (156 orbits are involved in the last term), but the crucial point is that the coefficient of each term depends only on the lattice geometry.

More generally, the WL correction can be obtained for all values of  $L_\varphi/a$  from the numerical computation of the determinant in Eq. (1). The numerical FFT of the computed MC yields the ratio  $R_{12}$  of the 2 first harmonics as a function of  $L_\varphi/a$  (Fig. 2a). It appears that the small orbit expansion (2) is a good approximation up to  $L_\varphi/a = 2$ . In any case the ratio of two harmonics is completely determined by  $L_\varphi/a$  and provides a method for

a direct evaluation of  $L_\varphi$  without any adjustable parameter. The square lattice is particularly appropriate for such a determination of  $L_\varphi$  due to its large harmonics content: the second harmonic is dominated by orbits of length  $6a$  instead of  $8a$  for a statistical ensemble of single rings or a necklace of identical rings, for example.

We now use this method to determine the phase coherence length of square networks etched in a 2DEG of a GaAs/AlGaAs heterostructure. The networks consist of  $10^6$  square loops of side  $a = 1 \mu\text{m}$  and nominal width  $W_0 = 0.5 \mu\text{m}$  and cover a total area of  $1 \text{mm}^2$ . A gold gate deposited  $100 \text{nm}$  above the 2DEG offers the possibility to change the number of electrons in the network. Measurements were done on three networks (A,B with gate, C without), giving the same results. Except when specified, figures show the data for sample A. We have measured the MC up to  $4.5 \text{T}$  between  $25 \text{mK}$  and  $1.3 \text{K}$ , using a standard lock-in technique (ac current of  $1 \text{nA}$  at  $30 \text{Hz}$ ). The samples were in general strongly depleted at low temperature because of the etching. The intrinsic electron density of the 2DEG,  $n_e = 4.4 \times 10^{15} \text{m}^{-2}$ , was recovered after illuminating the samples during several minutes at  $4.2 \text{K}$ . This density was determined from Shubnikov-de Haas oscillations visible above  $1 \text{T}$ . Because of depletion after etching of the 2DEG, it is difficult to estimate the real width of the wires ( $W$ ) and  $l_e$ .

At low magnetic field (Fig. 1a), the MC exhibit large AAS oscillations with a period  $12.6 \text{G}$  corresponding to a flux  $\phi_0/2$  in a square cell of area  $a^2$ . The oscillations are clearly not purely sinusoidal. At the lowest temperature,  $25 \text{mK}$ , three harmonics are visible in the Fourier spectrum of the MC (Fig. 1b). Moreover, as shown in Fig. 3 which represents the MC for a wider range of field, the oscillations disappear above  $60 \text{G}$  but the WL magnetoconductance due to the penetration of the field through the finite width of the wires constituting the network is still clearly visible. At high temperature, above  $400 \text{mK}$ , the AAS oscillations disappear even at low field. Only the positive MC remains with a smaller amplitude. In sample B, the same experiments for different gate voltages were also performed.

We first concentrate on the AAS oscillations (Fig. 1a). The Fourier spectrum of the MC exhibits a series of peaks corresponding to successive harmonics of the  $\phi_0/2$  periodicity. The finite width of the peaks (Fig. 1b) is due to the penetration of the magnetic field in the wires which damps the AAS oscillations at high field. It can be shown that this broadening does not affect the integral of the peak. A first rough analysis shows that the ratio  $R_{12}$  of integrated peaks of the two first harmonics behaves like  $R_{12} \sim \exp T^{1/3}$ . We now use the theory described above to quantitatively determine  $L_\varphi$  via the relation between  $L_\varphi/a$  and  $R_{12}$ . We deduce its temperature dependence between  $25 \text{mK}$  and  $250 \text{mK}$  as shown in Fig. 2. We find that  $L_\varphi$  follows a power law  $T^{-\eta}$ , where  $\eta = 0.36 \pm 0.05$ . The coherence length reaches almost  $3 \mu\text{m}$  at  $25 \text{mK}$  and

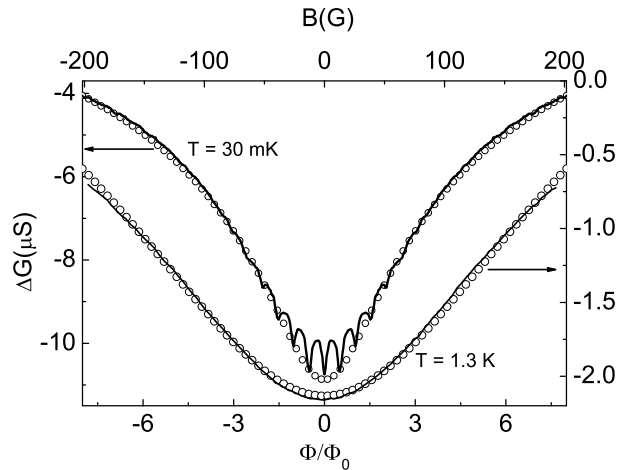


FIG. 3: High field MC, the continuous lines are the experimental data, the dots are the fits with eq. (4). Parameters of the fits are :  $l_e$  and  $W$  at  $30 \text{mK}$ , and  $L_\varphi$  and  $l_e$  at  $1.3 \text{K}$ .

there is no sign of saturation.

Once  $L_\varphi$  is determined, the sample parameters ( $W$ ,  $l_e$ ) can be deduced from the WL envelope. The magnetic field appears as an additional effective phase breaking rate for the time reversed trajectories responsible for the WL leading to an effective  $L_\varphi$  given by [16]:

$$\frac{1}{L_\varphi(\phi)^2} = \frac{1}{L_\varphi(0)^2} \left[ 1 + \frac{1}{3} \left( 2\pi \frac{\phi}{\phi_0} \frac{W_{\text{eff}} L_\varphi(0)}{a^2} \right)^2 \right] \quad (3)$$

where  $W_{\text{eff}} = W \sqrt{(3W)/(C_1 l_e)}$  is a renormalized width which appears in the WL correction for a semi ballistic wire ( $l_e \gg W$ ) due to the phenomenon of flux cancellation. The coefficient  $C_1$  depends on the specific boundary conditions. The samples under consideration are close to the case of specular boundary scattering [8] for which  $C_1 = 9.5$ . The MC envelope, given by  $\langle \Delta\sigma(\phi = 0, L_\varphi(\phi)) \rangle$ , can be analytically computed for the square lattice geometry and is given by:

$$\Delta\tilde{\sigma} = -\frac{L_\varphi}{W} \left[ \coth \frac{a}{L_\varphi} - \frac{L_\varphi}{a} + \frac{2}{\pi} \tanh \frac{a}{L_\varphi} K \left( \frac{1}{\cosh \frac{a}{L_\varphi}} \right) \right] \quad (4)$$

where  $K(x)$  is a complete elliptic integral. This expression is used in a 2-parameter ( $W$ ,  $\sigma_D$ ) fit of  $\Delta\sigma/\sigma_D = \Delta G/G_D$  where  $\sigma_D$  and  $G_D$  are the Drude conductivity and conductance. Since  $k_F$  is determined independently from Shubnikov-de Haas measurements,  $\sigma_D = \frac{e^2}{h} k_F l_e$  determines  $l_e$ . The above expression for  $W_{\text{eff}}$  can then be used to find  $W$ . For sample A/B/C,  $W = 170/270/230 \text{nm}$  and  $l_e = 220/250/360 \text{nm}$ , independent of temperature as expected. This also shows that the networks are indeed in the diffusive regime. In sample A the number of transverse channels per wire is  $M = k_F W/\pi = 9$ , and the number of effective conduct-

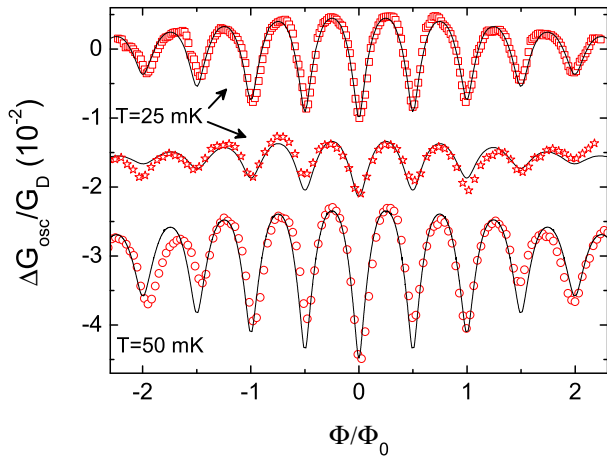


FIG. 4: Comparison between experiment (symbols) and theory (continuous line) for the oscillating part of the conductance for sample A ( $\square$ ) ( $G_D = 3.7 \times 10^{-5} \text{S}$ ), sample B ( $\star$ ) ( $G_D = 5 \times 10^{-6} \text{S}$ ) with  $V_g = -0.15 \text{V}$  corresponding to  $W = 240 \text{nm}$  and  $l_e = 170 \text{nm}$  and sample C ( $\circ$ ) ( $G_D = 2.6 \times 10^{-5}$ ) (shifted down for visibility). The only adjustable parameter is the amplitude of the oscillations.

ing channels on the scale of  $a$  is only  $M_{\text{eff}} = Ml_e/a \sim 2$ . Results for samples B and C are similar.

At higher temperature where no AAS oscillations are visible, we can nevertheless deduce  $L_\varphi$  and  $l_e$  by fitting the MC, knowing the temperature independent values of  $W$  and  $G_D$  (Fig. 3). Thus a quantitative comparison of  $L_\varphi$  with the theoretical prediction of AAK [7, 17]  $L_\varphi = \sqrt{2} \left( \frac{D^2 m^* W}{\pi k_B T} \right)^{1/3}$  written for a 2DEG wire is possible. This theory applies to diffusive metallic wires with a large number of conducting channels in a limit where e-e interactions are treated perturbatively. We find a very good agreement (see Fig. 2b) which is remarkable for two reasons: (i) we are confronted here with a small number of conducting channels where strong interaction effects could be expected, (ii) the result of AAK was not extended to network geometry. Recently it was predicted in [19] that  $L_\varphi$  extracted from the AB or AAS oscillations in a single ring of perimeter  $L$  should behave like  $L_\varphi \propto (LT)^{-1/2}$  corresponding to  $R_{12} \sim \exp L^{3/2} T^{1/2}$ . This behaviour is not observed in our experiment.

For sample B gate voltages between  $-0.3 \text{V}$  and  $0.3 \text{V}$  changed the resistance from  $30 \text{k}\Omega$  to  $400 \text{k}\Omega$ . A good filtering of the gate voltage line is needed to avoid saturation of  $L_\varphi$ . Within our experimental accuracy, we find that  $L_\varphi$  is not changed and still varies as  $T^{-1/3}$ . We estimated for each gate voltage  $W$  and  $l_e$ . When the gate voltage varies between  $0.15 \text{V}$  and  $-0.15 \text{V}$ ,  $W$  is unchanged but  $l_e$  decreases by a factor 1.5 and  $n_e$  by 30%;  $G_D$  decreases by a factor 5. This shows that the effect of the gate is mainly to disconnect bonds of the network.

As a consistency check we have computed numerically the oscillating part of the MC with formula (1) and (3)

using the value of  $W_{\text{eff}}$  determined above from the WL envelope of the MC curves. We find that this value also precisely describes the damping of the AAS oscillations, if the oscillations amplitude is multiplied by a factor ranging from 1.6 to 2 depending on the gate voltage. This can be explained by the existence of broken bonds in the network which influences envelope and oscillations differently [18]. We obtain a very good agreement between theory and experiments (Fig. 4).

In conclusion we have shown that magnetoconductance experiments in GaAs/GaAlAs networks can be described very accurately by the diagrammatic theory of quantum transport in diffusive networks. It is remarkable that this agreement is achieved in a limit where the dimensionless conductance on the scale of the period of the network,  $Ml_e/a$ , is of the order 1, and down to temperatures corresponding to  $L_\varphi \sim L_T$  i.e. close to the limit of validity of AAK theory. In contrast, metallic wires deep in the diffusive regime have a number of conducting channels of order 1000 and  $L_\varphi \gg L_T$  is always fulfilled. Moreover we extracted from the AAS oscillations the temperature dependance of the phase coherence length  $L_\varphi \propto T^{-1/3}$  that agrees with AAK theory down to 25 mK.

We thank B. Etienne for the heterojunctions, and R. Deblock and B. Reulet for fruitful discussions.

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