Discreteness and entropic fluctuations in GREM-like systems

M. Sasaki[1] and O. C. Martin[1,2]
[1] *Laboratoire de Physique Théorique et Modèles Statistiques, bât. 100, Université Paris-Sud, F–91405 Orsay, France.*
[2] *Service de Physique de l'État Condensé*
Orme des Merisiers — CEA Saclay, 91191 Gif sur Yvette Cedex, France.
(June 8, 2007)

Within generalized random energy models, we study the effects of energy discreteness and of entropy extensivity in the low temperature phase. At zero temperature, discreteness of the energy induces replica symmetry breaking, in contrast to the continuous case where the ground state is unique. However, when the ground state energy has an extensive entropy, the distribution of overlaps $P(q)$ instead tends towards a single delta function in the large volume limit. Considering now the whole frozen phase, we find that $P(q)$ varies continuously with temperature, and that state-to-state fluctuations of entropy wash out the differences between the discrete and continuous energy models.

I. INTRODUCTION

Strongly disordered systems such as spin glasses[1] have many metastable states in their low temperature frozen phase, rendering them very sensitive to perturbations. An extreme case of sensitivity arises in systems having replica symmetry breaking[2] (RSB): there the excess free-energy of the lowest excited states is $O(1)$ and so $P(q)$, the distribution of overlaps q, is non-trivial. Given such sensitivity, what properties are robust to changing the details of the microscopic Hamiltonian, and furthermore can the presence of RSB itself depend on microscopic details? Our purpose is to investigate this point in the context of tractable models of spin glasses which are of the mean field type. The main motivation for this is the question of RSB at $T = 0$ in the $\pm J$ Ising spin glass. It has been argued by Krzakala and Martin[3] that in physical systems with highly degenerate ground states there should be no replica symmetry breaking at $T = 0$ even if there is RSB at $T > 0$. Numerical investigations of this issue in the three-dimensional Ising spin glass have led to conflicting claims, either validating[4,5] this picture or on the contrary[6] suggesting that there is RSB amongst the ground states of that model. To understand better this question, Drossel and Moore[7] have investigated overlaps in the $\pm J$ model on the Migdal-Kadanoff lattice at $T = 0$ and $T > 0$, concluding that $P(q = 0)$ behaves differently with lattice size in the two cases; however that model does not have RSB at any temperature. In this work we provide a study of this question in a solvable model having RSB in its low temperature phase.

This paper begins with the random energy model[8] of Derrida and we consider the effects of having discrete energies. Because the ground state can be degenerate, $P(q)$ at $T = 0$ has a strictly positive weight at $q = 0$ while in the continuous case $P(q = 0)$ goes to zero linearly with temperature. Then we consider generalized random energy models[9] and in particular a discrete version with an infinite number of layers that can be compared to the continuous version studied by Derrida and Spohn[10]. In all these cases, the discreteness gives rise to RSB at $T = 0$. One of the unphysical features of such models is their zero entropy density at low enough temperature. Since we expect entropy fluctuations to be important for the question of replica symmetry breaking, we extend this last model so that states have random entropies. For the models considered with an infinite number of layers, we compute $P(q)$, the disorder averaged probability distribution of overlaps. We then see the effects of energy discreteness and of entropy fluctuations, in particular as $T \to 0$. Finally, we consider how replica symmetry is restored at $T = 0$ in the thermodynamic limit through entropic fluctuations, even though the energies are discrete.

II. RANDOM ENERGY MODEL

The REM[8] is a simple model of spin glasses; its partition function is

$$Z = \sum_{i=1}^{2^N} \exp\left[-\frac{E_i}{T}\right] \tag{1}$$

where N is identified with the number of spins of the system and the units are chosen so that the Boltzmann constant is 1. The energies E_i are independent identically distributed random variables of law $P_N(E)$. To make contact with physical systems, $P_N(E)$ is set to coincide with the distribution of energies of spin configurations in the d-dimensional Edwards-Anderson[11] (EA) spin glass. Such a system with N spins has 2^N energy levels but these energies are strongly correlated; the REM is obtained if one neglects these correlations. If the couplings J_{ij} of the EA model are Gaussian, then $P_N(E)$ is also Gaussian. If instead these couplings are binary, $P(J_{ij}) = [\delta(J_{ij} + 1) + \delta(J_{ij} - 1)]/2$, we are lead to a "discrete" REM where E is an integer random variable with distribution $P_N(E = -dN + 2k) = 2^{-dN}\mathrm{B}(dN, k)$.

In this expression, B is the usual binomial coefficient giving the number of ways to choose k elements out of dN. This distribution is roughly Gaussian at large N. The differences between the discrete and continuous models becomes most apparent for the extreme energies and thus in the low temperature phase where the lowest levels dominate the partition function.

Let us begin with the properties at $T = 0$. The ground state energy E_0 grows linearly with N and its variance is $O(1)$ in the large N limit[8]. The ground state is unique in the Gaussian case whereas it has a strictly positive probability of being *degenerate* in the discrete case. This has important consequences. Consider the overlap distribution $P_J(q)$ for a given disorder instance; when the ground state is non-degenerate, P_J will be a delta function at $q = 1$, while when it is degenerate, P_J will have two delta function peaks, one at $q = 0$ and one at $q = 1$. (By convention, any two distinct levels are taken to have zero overlap.) Since the degeneracy does not disappear at large N as we show below, $P(q)$, the disorder average of P_J, tends towards two delta function peaks, each of weight $O(1)$ as $N \to \infty$. Thus at $T = 0$ there is RSB in the discrete REM but none in the Gaussian REM.

Now we shall be more quantitative and determine the complete behavior of $P_J(q)$ at zero temperature. Indeed, if the ground state is g-fold degenerate for a particular disorder instance, one has

$$P_J(q) = (1 - g^{-1})\delta(q) + g^{-1}\delta(q - 1). \tag{2}$$

The problem then reduces to computing the statistics of the integer g. This can be done analytically as follows. Denote by $Q_N^{\rm gs}(n, g)$ the probability that the ground state energy is $-dN + 2n$ and its degeneracy g. In the discrete REM, we have

$$Q_N^{\rm gs}(n, g) = {\rm B}(2^N, g)\ p_N(n)^g \ \left[\sum_{k=n+1}^{dN} p_N(k)\right]^{2^N - g}, \tag{3}$$

where we have introduced $p_N(n) = P_N(E = -dN + 2n)$ to simplify the notation. Now we are interested in calculating the probability of the degeneracy

$$Q_N^{\rm deg}(g) \equiv \sum_{n=0}^{dN} Q_N^{\rm gs}(n, g). \tag{4}$$

From that we shall calculate

$$\langle g^{-1} \rangle \equiv \sum_{g=1}^{2^N} Q_N^{\rm deg}(g) g^{-1} \tag{5}$$

which according to eq. (2) gives us the disorder averaged $P(q)$ of the model.

Now let us calculate $Q_N^{\rm gs}(n, g)$ for $a(n) \equiv \frac{n}{dN} < 0.5$. We are not interested in the case $a(n) \geq 0.5$ because the probability that the ground state energy is positive is negligible (recall that E and n are related by $E = -dN + n$). Since

$$p_N(n - \Delta n) = p_N(n) \left(\frac{a(n)}{1 - a(n)}\right)^{\Delta n} \left\{1 + \mathcal{O}\left(\frac{1}{dN}\right)\right\} \tag{6}$$

holds for any finite integer Δn, the last factor in eq. (3) for $a(n) < 0.5$ can be estimated as

$$\begin{aligned} &\left[\sum_{k=n+1}^{dN} p_N(k)\right]^{2^N - g} \\ &= \left[1 - \sum_{k=0}^{n} p_N(k)\right]^{2^N - g} \\ &\approx \left[1 - \frac{[1 - a(n)]p_N(n)}{1 - 2a(n)}\right]^{2^N - g} \\ &\approx \exp\left\{-\frac{[1 - a(n)][2^N - g]p_N(n)}{1 - 2a(n)}\right\} \quad (a(n) < 0.5). \end{aligned} \tag{7}$$

By substituting this equation into eq. (3), we find

$$Q_N^{\rm gs}(n, g) \approx \frac{1}{g!}\left[p_N(n) 2^N\right]^g \exp\left\{-\frac{[1 - a(n)]2^N p_N(n)}{1 - 2a(n)}\right\}, \tag{8}$$

provided that $a(n) < 0.5$ and g is not of order $\exp(N)$.

Now let us denote by n^* the integer for which the value of $p_N(n)2^N$ is the closest to 1. Because eq. (6) is also valid for $n = n^*$, $p_N(n^* + \Delta n)2^N$ for finite Δn is expressed as

$$p_N(n^* + \Delta n)2^N = C^* \exp(\alpha^* \Delta n) \left\{1 + \mathcal{O}\left(\frac{1}{dN}\right)\right\}, \tag{9}$$

where

$$\alpha^* = \log\left(\frac{1 - a^*}{a^*}\right), \tag{10}$$

$a^* = a(n^*)$, and $C^* \equiv p_N(n^*)2^N$. Now the condition $p_N(n^*)2^N \approx 1$ determines n^* and leads to the following equation determining a^* in the large N limit:

$$(1 - d)\log(2) - d\log(1 - a^*) + a^* \log\left(\frac{1 - a^*}{a^*}\right) = 0. \tag{11}$$

The substitution of eq. (9) into eq. (8) leads us to

$$\begin{aligned} Q_N^{\rm gs}(n^* + \Delta n, g) \approx\ & \frac{1}{g!}(C^*)^g \exp(g\alpha^* \Delta n) \\ & \times \exp\left\{-\frac{[1 - a^*]C^* \exp(\alpha^* \Delta n)}{1 - 2a^*}\right\} \end{aligned} \tag{12}$$

and these two quantities become equal as $N \to \infty$.

To obtain a closed form expression for $\langle g^{-1}\rangle$, we now approximate the summation in eq. (4) by an integral and find in the large N limit

$$Q_N^{\rm deg}(g) \approx \int_{-\infty}^{\infty} {\rm d}x \frac{1}{g!}(C^*)^g \exp(g\alpha^* x) \times \exp\left\{-\frac{[1-a^*]C^* \exp(\alpha^* x)}{1-2a^*}\right\} \approx \frac{1}{g\alpha^*}\left(\frac{1-2a^*}{1-a^*}\right)^g . \quad (13)$$

This formula tells us that the probability that the ground state is degenerate is non-zero even in the limit $N \to \infty$ and that the distribution of g is well behaved, i.e., all moments are finite. From eq. (13), we finally obtain

$$\langle g^{-1}\rangle = \frac{1}{\alpha^*}\int_0^{\frac{1-2a^*}{1-a^*}} \frac{{\rm d}x}{x}\log\left(\frac{1}{1-x}\right), \quad (14)$$

where we have used

$$\sum_{g=1}^{\infty}\frac{x^g}{g^2} = \int_0^x \frac{{\rm d}t}{t}\log\left(\frac{1}{1-t}\right) \qquad (|x|\le 1). \quad (15)$$

If we apply these expressions to our model when $d = 3$, we find $a^* = 0.317$ and $\langle g^{-1}\rangle = 0.824$; the peak in the overlap distribution is thus much lower at $q = 0$ than at $q = 1$. This finishes our analysis at $T = 0$.

Consider now what happens at $T > 0$. The detailed nature of $P_N(E)$ will remain important as long as the partition function is dominated by a finite number of energy levels. This happens throughout the whole low temperature phase ($T < T_{\rm c}$) where the free-energy and $P_J(q)$ are not self-averaging. Consider in particular the T dependence of $P(q)$ at low temperature. In the Gaussian case there is a non-zero probability density to have a zero energy gap; this leads to a $P(q,T)$ that is linear in T as $T \to 0$. On the contrary, in the discrete case we have

$$P(q=0,T) = P(q=0,T=0) + O(e^{-\frac{\Delta E}{T}}), \quad (16)$$

where $\Delta E = 2$ is the energy gap of this model. Indeed this holds for each disorder instance and so also holds for the disorder average.

III. GENERALIZED RANDOM ENERGY MODEL

In the GREM[9] one considers 2^N states that are organized in a tree-like fashion, allowing for correlated energies. At the first layer of the tree, there are 2^{N_1} branches; then each such branch gives rise to 2^{N_2} branches in the second layer, etc... The nodes of the last layer are identified with states (or spin configurations), and there are 2^N of them where $N = N_1 + N_2 \ldots + N_L$. To each branch one associates a random energy. Finally, the energy of a state (leaf of the tree) is given by the sum of the energies of the L branches connecting it to the root of the tree (residing at level 0).

Just as for the REM, the low temperature phase of the GREM is sensitive to the detailed distribution of the energies. In the context of our study, we see that the properties found for the REM extend to the GREM as follows. At $T = 0$, we need consider only the ground states. At each level, there is a strictly positive probability to have degenerate lowest energies for the model having discrete energies. One thus has a strictly positive probability to find any of the possible overlaps when considering ground states only. This is to be contrasted with the Gaussian model for which $P(q) = \delta(q-1)$ when $T = 0$. Similarly, at $T > 0$, $P(q)$ will be quite sensitive to the nature of the energies as long as we stay within the spin glass phase. At very low temperatures, the weights of its peaks other than at $q = 1$ will be linear in T in the Gaussian case while they will have an exponentially small temperature dependence in the discrete case (cf. eq. 16). Finally, as one approaches the highest of the critical temperatures, many branches at each level contribute to the partition function and so from there on the detailed distribution of energies becomes irrelevant.

IV. INFINITE-LEVEL GREM

In a discrete GREM with L-layers, the ground state may be degenerate for any finite L, but what happens when $L \to \infty$? To study that limit, we set $N_i = k$, k being a fixed integer (say 1 or 2). The model is schematically shown in figure 1. There is then a fixed branching factor $K = 2^k$ at each layer, each node generating K branches. A random energy ϵ is associated with each branch of the tree. The ϵ variables are independent and drawn from the same distribution $\rho_E(\epsilon)$. The energy $E(i)$ of a state i is given by summing up the ϵ's of the branches which lie along the path connecting it to the tree's root O, e.g., in figure 1, $E(i) = \epsilon_1+\epsilon_2+\epsilon_3$ and $E(j) = \epsilon_1+\epsilon_2+\epsilon_4$. The distance d_{ij} of two states i and j is d if their first common ancestor arises on the d-th layer counted from below, e.g., in figure 1, $d_{ii} = 0$, $d_{ij} = 1$ and $d_{ik} = d_{jk} = 2$. The overlap q_{ij} is related to d_{ij} by

$$q_{ij} = 1 - d_{ij}/L. \quad (17)$$

This model has been studied in depth by Derrida and Spohn[10] (see also[12,13]); it can be viewed either as an infinite level GREM or as a directed polymer on a disordered Cayley tree. When the energy of each branch is taken from continuous distribution, the model's thermodynamics is extremely close to that of the REM. There is a critical temperature below which a finite number of states dominate the partition function, and this low temperature phase exhibits one-step RSB, while the distribution $P(q)$ tends towards a delta function at $q = 1$ as $T \to 0$.

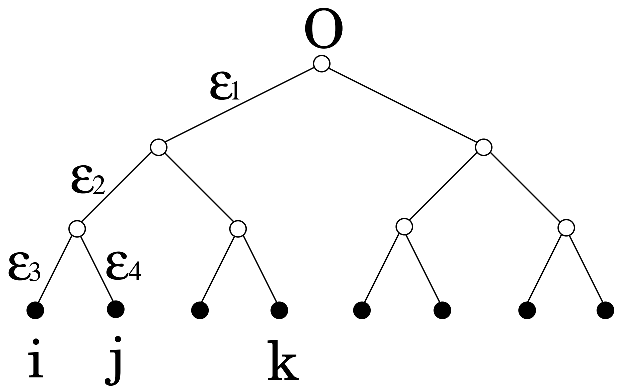

FIG. 1. Construction of the infinite-level GREM with a branching factor $K = 2$.

Since we are motivated by the question of RSB in the 3-dimensional $\pm J$ EA model, we shall investigate the behavior of $P(q)$ for the Cayley tree model when the energies on each branch are discrete. We shall take $\rho_E(\epsilon)$ to consist of a finite number of delta functions. To investigate the behavior of our model, we shall derive recursion formulae for the probability $Y(L,d)$ to find two states at a distance less or equal to d by using the same techniques as developed by Derrida and Spohn[10]. Since q and d are related by eq. (17), we can calculate $P(q)$ from $Y(L,d)$. By definition, the probability $Y(L,d)$ is given by

$$Y(L,d) \equiv \overline{\frac{1}{Z(L)^2} \sum_{ij/d_{ij} \le d} \exp[-X(i) - X(j)]}. \tag{18}$$

In this expression, $\overline{\cdots}$ represents the disorder average, $X(i) \equiv E(i)/T$, Z is the partition function of the system, and L is the number of layers. A standard integral representation for Z^{-2} leads to

$$Y(L,d) = \int_{-\infty}^{\infty} \mathrm{d}u F(L,d;u), \tag{19}$$

where

$$F(L,d;u) \equiv \overline{\exp[-2u - \mathrm{e}^{-u} Z(L)] \sum_{ij/d_{ij} \le d} \mathrm{e}^{-X(i)-X(j)}}. \tag{20}$$

Collecting terms in the sum that belong to the same sub-tree of height d, we have

$$\sum_{ij/d_{ij} \le d} \mathrm{e}^{-X(i)-X(j)} = \sum_{B_d} \exp[-2X(B_d)] z(B_d)^2. \tag{21}$$

In this expression, B_d is a general branch point in the d-th layer counted from below, $z(B)$ is the partition function of the sub-tree rooted at a branch point B, $X(B) \equiv E(B)/T$, and $E(B)$ is the energy of a branch point B given by summing up the ϵ's of the branches which lie along the path connecting B and O. Substitution of eq. (21) into eq. (20) gives

$$F(L,d;u) = \overline{\exp[-2u - \mathrm{e}^{-u} Z(L)] \sum_{B_d} \exp[-2X(B_d)] z(B_d)^2}. \tag{22}$$

From this equation, we find

$$F(d,d;u) = H_2(d;u), \tag{23}$$

where

$$H_n(d;u) \equiv \overline{\{\mathrm{e}^{-u} z(B_d)\}^n \exp[-\mathrm{e}^{-u} z(B_d)]}. \tag{24}$$

We can calculate $H_n(d;u)$ (including $H_2(d;u)$ appearing in eq. (23)) by the following recursion formulae. Now let us start from the simplest case, i.e., $n = 0$. By definition,

$$H_0(0;u) = \exp[-e^{-u}]. \tag{25}$$

Since the $z(B_{d+1})$ can be expressed as

$$z(B_{d+1}) = \sum_{B_d} \mathrm{e}^{-X(B_d)} z(B_d), \tag{26}$$

and $z(B_d)$ and $z(B'_d)$ are independent if $B_d \neq B'_d$, we obtain the recursion formula

$$\begin{aligned} H_0(d+1;u) &= \prod_{B_d} \overline{\exp[-\mathrm{e}^{-u-X(B_d)} z(B_d)]} \\ &= \tilde{H}_0(d;u)^K, \end{aligned} \tag{27}$$

where we have defined

$$\tilde{g}(u) \equiv \int \mathrm{d}\epsilon \rho_E(\epsilon) g(u + \epsilon/T) \tag{28}$$

for a general function $g(u)$. The recursion formulae for $n \neq 0$ are derived by using eqs. (25) and (27) as well as the relation

$$H_n(d;u) = \frac{\mathrm{d}^n}{\mathrm{d}u^n} H_0(d;u). \tag{29}$$

For example, the recursion formula for $H_1(d;u)$ is

$$\begin{aligned} H_1(d+1;u) &= \frac{\mathrm{d}}{\mathrm{d}u} \tilde{H}_0(d;u)^K \\ &= K \tilde{H}_1(d;u) \tilde{H}_0(d;u)^{K-1}. \end{aligned} \tag{30}$$

Finally, let us derive recursion formulae for $F(L,d;u)$. From eq. (22), we have

$$F(L+1,d;u) = \overline{\exp\left[-2u - \mathrm{e}^{-u} \sum_{B'_L} \mathrm{e}^{-X(B'_L)} z(B'_L)\right]}$$

$$\times \sum_{B_L} \sum_{B_d \in B_L} \overline{\exp[-2X(B_d)] z(B_d)^2}$$

$$= \sum_{B_L} \overline{\exp\left[-2u - 2X(B_L) - \mathrm{e}^{-u-X(B_L)} z(B_L)\right]}$$

$$\times \sum_{B_d \in B_L} \overline{\mathrm{e}^{-2[X(B_d)-X(B_L)]} z(B_d)^2}$$

$$\times \prod_{B'_L \neq B_L} \overline{\exp\left[-\mathrm{e}^{-u-X(B'_L)} z(B'_L)\right]}. \tag{31}$$

The strategy now is to perform the disorder average in two steps: first we average over sub-trees rooted at $\{B_L\}$ when fixing $\{X(B_L)\}$; then we average over $\{X(B_L)\}$. By using eq. (22) and eq. (24) with $n = 0$, the disorder average in the first step is done as

$$F(L+1, d; u) = \sum_{B_L} \overline{F(L, d; u + X(B_L))} \times \prod_{B'_L \neq B_L} \overline{H_0(L; u + X(B'_L))}. \tag{32}$$

Note that the only random variables in this equation are $\{X(B_L)\}$. The disorder average in the next step finally leads us to

$$F(L+1, d; u) = K \tilde{F}(L, d; u) \tilde{H}_0(L; u)^{K-1}. \tag{33}$$

In summary, the disorder averaged distribution of distances $Y(L, d)$ can be computed by the following procedures:

i) Calculate $H_2(d; u)$ $(=F(d, d; u))$ by evaluating numerically the recursions which are derived by applying eq. (29) to eqs. (25) and (27).

ii) Calculate $F(L, d; u)$ by using the recursion eq. (33).

iii) Compute $Y(L, d)$ by estimating numerically the integral in eq. (19).

For large number of layers we see convergence to the large L limit after introducing the (continuous) overlap $q = 1 - d/L$; this gives us the disorder average $P(q)$ or equivalently $\overline{Y(q)} = \int_0^q \overline{P(q)} \mathrm{d}q$ for the infinite tree.

Just as for the Gaussian case, the discrete model has one step RSB below a critical temperature T_c. However, the energies are discrete and we find that the degeneracy of the ground state is maintained as $L \to \infty$ and furthermore its mean saturates to a finite value. Because of this, there is RSB at $T = 0$, not only for $0 < T < T_\mathrm{c}$. When executing our recursions, the distribution of overlaps rapidly has very narrow peaks at $q = 0$ and $q = 1$ with increasing number of layers, while the probability of having overlaps between those two values goes to zero. One can thus characterize the mean probability distribution of overlaps by the weight of the delta function at $q = 0$. For that, we consider the quantity $\overline{Y(q = 0.5)}$ which gives the weight of the overlaps near $q = 0$. In

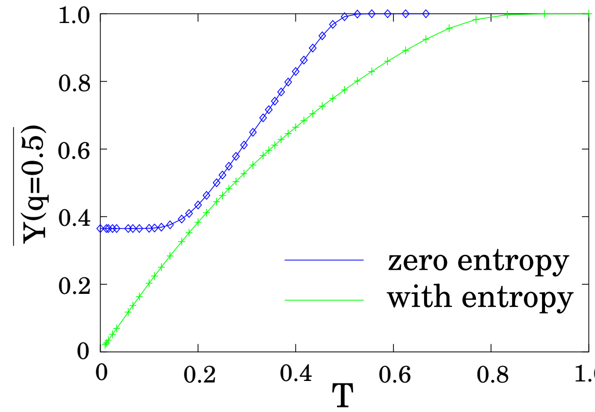

FIG. 2. $\overline{Y(q = 0.5)} = \int_0^{0.5} P(q)\mathrm{d}q$ as a function of temperature, for the infinite level GREM (top curve) and for its random entropy generalization (bottom curve). (Plots are for $K = 2$ and $L = 1000$.)

figure 2 we show the temperature dependence of that quantity (top curve). For these data, we used $K = 2$ and

$$\rho_E(\epsilon) = 0.25\delta(\epsilon) + 0.5\delta(\epsilon - 1) + 0.25\delta(\epsilon - 2). \tag{34}$$

Furthermore, we took $L = 1000$ which is big enough to represent the infinite tree limit at all T except for T close to T_c. We have checked that the curves near T_c converge as L increases, and a cusp is created at T_c in the limit $L \to \infty$, just as the continuous case[10]. But the point we want to make here concerns $T \ll T_\mathrm{c}$: the curve is very flat at low temperatures. This is as expected from the analogy with the discrete REM (cf. eq. 16), and is due to the presence of a gap above the ground state energy.

V. A MODEL WITH EXTENSIVE ENTROPY

In all the models considered so far, a finite number of the lowest energy states dominate the partition function at sufficiently low temperatures. Thus the entropy is $O(1)$ rather than $O(N)$. Clearly, in any physical system, the entropy remains extensive as long as $T > 0$. What is the possible importance of such extensivity? Krzakala and Martin[3] argued that extensive entropies will give rise to diverging entropic fluctuations, and that these fluctuations should prevent RSB at $T = 0$. To test this idea, one needs a model with extensive entropies at low temperature. There are several ways to do this in the context of GREM-like models; Cook and Derrida[14] added small loops to the tree, while Yoshino[15] replaced the random energies by random entropies. We keep the tree structure but have both energies and entropies.

We focus again on the infinite tree with a constant branching factor K. To each branch we assign a random energy ϵ *and* a random entropy σ from distributions $\rho_E(\epsilon)$ and $\rho_S(\sigma)$, respectively; then each leaf has an energy and entropy given by the sum of those terms along the path connecting that leaf to the tree's root O. In this model, both the energy and the entropy of the states become extensive. Now a crucial point is that the model with and without entropies can be mapped onto one-another when $T \neq 0$; indeed the two models are *identical* provided the distributions satisfy

$$\rho_E^*(\epsilon') = \int \mathrm{d}\epsilon \mathrm{d}\sigma \delta(\epsilon' - \epsilon + T\sigma)\rho_E(\epsilon)\rho_S(\sigma), \qquad (35)$$

where $\rho_E^*(\epsilon')$ is the distribution of the energy for the model without entropy. This condition insures that the distribution of the weight assigned to each branch ($\exp(-\epsilon/T + \sigma)$ and $\exp(-\epsilon'/T)$) is the same in the both cases. Therefore, we can utilize all of the recursions previously derived just by changing eq. (28) into

$$\tilde{g}(u) \equiv \int \mathrm{d}\epsilon \mathrm{d}\sigma \rho_E(\epsilon)\rho_S(\sigma) g(u + \epsilon/T - \sigma). \qquad (36)$$

We have investigated the distribution of overlaps in this model with entropic fluctuations. We have chosen ρ_E as before and set

$$\rho_S(\sigma) = 0.5\delta(\sigma) + 0.5\delta(\sigma - 2). \qquad (37)$$

We have also chosen the same branching factor as before, i.e., $K = 2$. The inclusion of these entropic fluctuations changes the critical temperature, but it also changes the qualitative behavior of RSB. In figure 2 we show the same quantity as before, $\overline{Y(q = 0.5)}$. We see that the entropic fluctuations destroy RSB at $T = 0$, and $P(0)$ goes to zero *linearly* with T as $T \to 0$. One may think from the figure that there is no cusp at $T = T_c$ in the model with entropy. However it can be shown from the mapping mentioned in the previous paragraph that the inclusion of entropy does *not* remove the cusp. To check this, we have verified numerically that the cusp appears slowly as L is increased and that the left derivatives of $\overline{Y(q = 0.5)}$ at $T = T_c$ tend to a non-zero limit for both models as $L \to \infty$.

We have also investigated the finite size dependence of $P(q = 0)$ at $T = 0$. From figure 3, we see that this quantity decreases as $L^{-0.5}$. This result can be understood from the following simple argument. Imagine a disorder instance in which there are only two ground states whose overlap is close to 0. The probability for an overlap to be small for such a sample is of order 1 if the difference of the entropies of the two ground states is of order 1. Recall that each ground state has an extensive entropy; furthermore, the entropy fluctuations of a ground state are necessarily of order $L^{0.5}$. Then the probability that the two states have the *same* entropy is of order $L^{-0.5}$ in the large L limit. From this we conclude that at zero temperature, $P(q = 0)$ decreases as $L^{-0.5}$ as $L \to \infty$.

It is worth pointing out that Krzakala and Martin[3] suggested that in the $3-d$ EA $\pm J$ spin glass valley-to-valley fluctuations in the entropy should grow as $L^{d_s/2}$ (d_s is the fractal dimension of surfaces of large-scale low-energy excitations) and $P(0)$ should then decrease as $L^{-d_s/2}$. A power law decay of $P(0)$ in that model has been found numerically[16,4]. The contribution of our study is to show that such a phenomenon does indeed occur beyond reasonable doubt in a particular model.

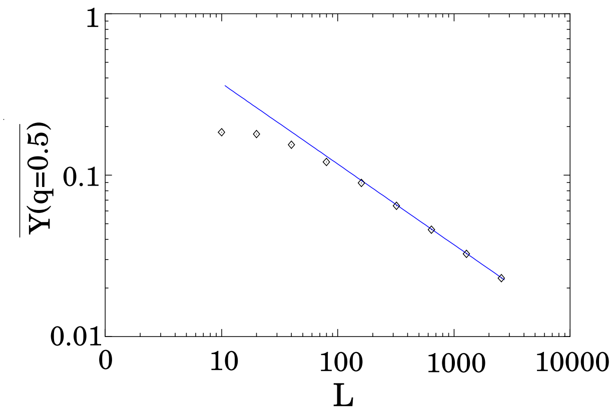

FIG. 3. $\overline{Y(q = 0.5)}$ at $T = 0$ vs. L. The parameters for these data are $K = 2$, $\rho_S(\sigma) = 0.5\delta(\sigma) + 0.5\delta(\sigma - 1)$ and eq. (34) for ρ_E. A function proportional to $L^{-0.5}$ is drawn to guide the eye.

VI. DISCUSSION

Let us compare the model with discrete energies and extensive entropies to the one where the disorder variables are continuous (be there entropy fluctuations or not). In both cases, $P(q = 0)$ goes to zero linearly with T as $T \to 0$ because the lowest *free-energy* states are non-degenerate and there is no gap. We may then summarize what we have found by saying that state-to-state entropy fluctuations of order $\sqrt{L}$ effectively remove both degeneracies and gaps. Of course at $T = 0$, the degeneracy in the energy is important, but our point is that the free-energies are still not degenerate: the contribution of different ground states to the partition function are wildly different simply because their entropy differences diverge as $\sqrt{L}$. This is the mechanism behind no RSB at zero temperature. If we consider now $T > 0$, we note that the energies of the states dominating the partition function are far above the lowest energy; the system selects the states with the minimum *free-energies*, $F = E - TS$; even though both E and S are integers, generically F is not and so any fine structure of E and S is washed out.

Our work has focused on the infinite tree model because it is tractable, but we expect the conclusions to be quite general when there is one-step RSB at $T > 0$. An open question concerns of course the case of systems having continuous RSB. One way to address this is to generalize our model so that it exhibits continuous RSB; this can be done just as for the Derrida-Spohn model[10] by making the distributions of energies depend on the level of the tree. With such a modification, the model having no entropy but discrete energies will have continuous RSB at $T = 0$, whereas if random entropies are assigned to each branch the entropic fluctuations will restore replica symmetry at $T = 0$.

One may also ask what effect would $J_{ij} = \pm 1$ have on the Sherrington-Kirkpatrick[17] (SK) model, i.e., would such discreteness give rise to RSB at $T = 0$, in contrast to what happens in the Gaussian case? The crucial property that can make RSB possible at $T = 0$ is ground state degeneracy; the gap in such a discrete SK model of N spins is $1/\sqrt{N}$ if the J_{ij}s are rescaled so that the model has a thermodynamic limit. The valley-to-valley energy differences being $O(1)$, the probability of an exact ground state degeneracy should be $O(1/\sqrt{N})$, so we do *not* expect RSB at $T = 0$ here. In fact, it seems unlikely that the discrete and continuous SK models have any differences in the thermodynamic limit because the local fields on the spins effectively become continuous in the large N limit.

Finally, let us note that the most controversial case of zero-temperature RSB arises in the $3 - d$ EA $\pm J$ spin glass; its breaking of replica symmetry at $T > 0$ is probably continuous, though even that is subject to debate. It would thus be very useful to pursue this issue further to resolve the contradictory results[16,4,6,5] obtained so far.

ACKNOWLEDGMENTS —

We thank J.-P. Bouchaud, F. Krzakala and M. Mézard for stimulating discussions. M.S. acknowledges financial support from the MENRT. The LPTMS is an Unité de Recherche de l'Université Paris XI associée au CNRS.

[1] *Spin Glasses and Random Fields*, edited by A. P. Young (World Scientific, Singapore, 1998).

[2] M. Mézard, G. Parisi, and M. A. Virasoro, *Spin-Glass Theory and Beyond*, Vol. 9 of *Lecture Notes in Physics* (World Scientific, Singapore, 1987).

[3] F. Krzakala and O. C. Martin, Europhys. Lett. **6**, 749 (2001).

[4] M. Palassini and A. P. Young, Phys. Rev. B **63**, 140408(R) (2001), cond-mat/0012164.

[5] A. Hartmann and F. Ricci-Tersenghi, (2001), cond-mat/0108307.

[6] G. Hed, A. Hartmann, and E. Domany, Europhys. Lett. **55**, 112 (2001), cond-mat/0012451.

[7] B. Drossel and M. Moore, Eur. Phys. J. B **22**, 589 (2001).

[8] B. Derrida, Phys. Rev. B **24**, 2613 (1981).

[9] B. Derrida, J. Phys. Lett. (France) **46**, 401 (1985).

[10] B. Derrida and H. Spohn, J. Stat. Phys. **51**, 817 (1988).

[11] S. F. Edwards and P. W. Anderson, J. Phys. F **5**, 965 (1975).

[12] S. N. Majumdar and P. L. Krapivsky, Phys. Rev. E **62**, 7735 (2000).

[13] D. Dean and S. Majumdar, Phys. Rev. E **64**, 046121 (2001).

[14] J. Cook and B. Derrida, Europhys. Lett **10 (3)**, 195 (1989).

[15] H. Yoshino, J. Phys. A. **30**, 1143 (1997).

[16] A. Hartmann, Eur. Phys. J. B **13**, 539 (2000), cond-mat/0107308.

[17] D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. **35**, 1792 (1975).