

Achieving a BCS transition in an atomic Fermi gas

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We consider a gas of cold fermionic atoms having two spin components with interactions characterized by their s -wave scattering length a . At positive scattering length the atoms form weakly bound bosonic molecules which can be evaporatively cooled to undergo Bose-Einstein condensation, whereas at negative scattering length BCS pairing can take place. It is shown that, by adiabatically tuning the scattering length a from positive to negative values, one may transform the molecular Bose-Einstein condensate into a highly degenerate atomic Fermi gas, with the ratio of temperature to Fermi temperature $T/T_F \sim 10^{-2}$. The corresponding critical final value of $k_F|a|$ which leads to the BCS transition is found to be about one half, where k_F is the Fermi momentum.

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Much progress has been made in the achievement of increasingly degenerate regimes of trapped atomic Fermi gases [1, 2, 3, 4, 5, 6, 7]. One of the major goals of studies of these systems is to observe a transition to a paired-fermion superfluid state. In a recent MIT experiment on a sympathetically cooled single component Fermi gas, a system with $T/T_F = 0.05$ and 3×10^7 fermions was realized [7], where T_F is the Fermi temperature. As trapped atomic gases present an essentially impurity free system, with density, temperature, and interaction strength all free parameters, they offer great opportunities for investigation of fundamental theories of superfluid states.

An important new step in experiments has been the use of Feshbach resonances, whereby one may tune the s -wave scattering length a from positive to negative over many orders of magnitude [8]. This opens up the possibility of investigating the Bose-Einstein condensate to Bardeen-Cooper-Schrieffer (BEC-BCS) crossover [9, 10, 11, 12]. The BCS limit occurs for $k_F|a| \ll 1$, where k_F is the Fermi momentum and $a < 0$. In this case the fermion pair size is much larger than the interatomic spacing. The BEC limit occurs for a positive scattering length much smaller than the interparticle separation. Then the fermions form weakly bound dimers of a size $\sim a$, and the Bose-Einstein condensation of these composite bosons may be described by the well-developed theory of trapped BEC's [14]. Recently, long-lived molecules have been created in a reversible, and therefore adiabatic fashion from a degenerate Fermi gas by tuning the scattering length [15]. This provides many possibilities for evaporatively cooling the molecules into the BEC regime. In contrast, a BCS transition requires very low temperatures not yet obtained in two-component Fermi gases.

In this Letter, we present a straightforward theoretical analysis which shows that a deeply degenerate Fermi gas may indeed be created from a molecular BEC. By adiabatically changing the scattering length from positive to negative values, the entropy is held constant, which leads to a strong decrease in temperature. This sug-

gests a novel method of cooling a Fermi gas to extremely low temperatures. One can then reach a temperature $T \sim 10^{-2}T_F$ and achieve a BCS transition.

Consider a harmonically trapped Fermi gas described by the grand canonical ensemble with two equally populated spin states. The entropy of the gas above the critical temperature is equal to the entropy of an ideal Fermi gas in a trap with mean field corrections. Omitting these corrections the grand potential Ω is [17]

$$\Omega = k_B T \int_0^\infty d\epsilon \rho(\epsilon) \ln[1 - n(\epsilon)], \quad (1)$$

with $\rho(\epsilon)$ the density of states and

$$n(\epsilon) = 1/\{\exp[\beta(\epsilon - \mu)] + 1\} \quad (2)$$

the Fermi weighting factor. For a harmonic trap, the density of states for a non-interacting two-component gas in the semi-classical limit is

$$\rho(\epsilon) = \epsilon^2/(\hbar\omega)^3 \equiv A\epsilon^2. \quad (3)$$

Substituting Eq. (3) into Eq. (1) and integrating by parts, one obtains the simplified integral $\Omega = -(A/3) \int_0^\infty d\epsilon \epsilon^3 n(\epsilon)$. In the degenerate regime, given by $\mu \gg k_B T$, the integral may be expanded as

$$\Omega \simeq -\frac{A}{3} \left[\frac{\mu^4}{4} + \frac{\pi^2(k_B T)^2 \mu^2}{2} + \frac{7\pi^4(k_B T)^4}{60} \right]. \quad (4)$$

The entropy may then be obtained from the relation [17]

$$S = -\partial_T \Omega|_{\mu, \omega}, \quad (5)$$

giving

$$S \simeq k_B \frac{A}{3} \left[\pi^2 k_B T \mu^2 + \frac{7\pi^4(k_B T)^3}{15} \right]. \quad (6)$$

A relation between the total number of particles and the chemical potential follows from the equation $N = -\partial_\mu \Omega|_{T, \Omega}$ and reads

$$N \simeq (A/3)[\mu^3 + \pi^2(k_B T)^2 \mu]. \quad (7)$$

So, to lowest order, the chemical potential is $\mu = (3N/A)^{1/3}$ and the expression for the entropy becomes

$$S = k_B N \pi^2 T / T_F + \mathcal{O}(T^3), \quad (8)$$

where $T_F \equiv (3N)^{1/3} \hbar \omega$ is the Fermi temperature of the non-interacting gas. In the case of an interacting Fermi gas, the lowest order mean field correction in the Thomas-Fermi limit leads to the appearance of an extra multiple $(1 + 64k_F a / 35\pi^2)$ in the numerator of the right hand side of Eq. (8). This gives a correction of less than 10 percent to the entropy for $k_F |a| \leq 1/2$. It may therefore be neglected in our calculations. Equation (8) is valid for $k_B T > \hbar \omega$ in an isotropic potential. Its validity extends to much lower temperatures (although much larger than the inverse density of states at the Fermi energy) for incommensurable trap frequencies.

Consider the analogous calculation for a BEC of weakly bound molecules at $a > 0$. We assume that the condensate is in the weakly interacting regime, with $\eta \equiv n_{\text{mol}}^{1/3} a_{\text{mol}} \ll 1$ where n_{mol} is the molecular density in the trap center and the scattering length of two molecules a_{mol} is related to the atomic scattering length a by $a_{\text{mol}} = 0.6a$ [18]. Note that these weakly bound molecules exhibit a remarkable collisional stability at large values of a [15, 18]. This stability originates from a strong decrease in the relaxation rate to deep bound states with increasing a , and should enable efficient evaporative cooling [18]. A small value of η may be reached in experiments by reducing the trap frequencies.

If $\eta \ll 1$, the mean field shift in the condensation temperature is $\delta T_{\text{BEC}} / T_{\text{BEC}} = -3.25\eta^{5/4}$ [14] and the quantum depletion of the condensate is negligible so that Bogoliubov theory may be used. It also implies

$$k_B T_{\text{BEC}} / E_B = 7.78\eta^{5/2} \ll 1 \quad (9)$$

where $E_B \simeq \hbar^2 / ma^2$ is the molecular binding energy. This condition, together with $T < T_{\text{BEC}}$, ensures that the molecules may be treated as weakly interacting bosons and any effect due to their thermal dissociation is negligible. The grand potential is

$$\Omega = k_B T \int_0^\infty d\epsilon \rho(\epsilon) \ln(1 - e^{-\beta\epsilon}), \quad (10)$$

where β now refers to the temperature of the Bose gas. The density of states for an interacting BEC in the Bogoliubov approximation may be calculated directly from the Bogoliubov Hamiltonian as $\rho(\epsilon) = \text{Tr}[\delta(\epsilon - \hat{H}_{\text{bog}})]$. In the Thomas-Fermi limit, which holds for a molecular condensate with chemical potential $\mu_{\text{mol}} \gg \hbar \omega$, the density of states may be approximated semi-classically by

$$\rho(\epsilon) \simeq \int \int \frac{d\vec{r} d\vec{p}}{(2\pi\hbar)^3} \delta[\epsilon - \epsilon_{\text{bog}}(\vec{p}, \vec{r})], \quad (11)$$

where the energy of Bogoliubov excitations is (see [14] and references therein)

$$\epsilon_{\text{bog}} \equiv \begin{cases} \sqrt{p^2/2m_{\text{mol}} [p^2/2m_{\text{mol}} + 2g_{\text{mol}} n_0(r)]} & |r| \leq R \\ p^2/2m_{\text{mol}} + \frac{1}{2}m_{\text{mol}}\omega^2 r^2 - \mu_{\text{mol}} & |r| > R. \end{cases} \quad (12)$$

Here $m_{\text{mol}} = 2m$ is the mass of a molecule and

$$n_0(r) = (\mu_{\text{mol}}/g_{\text{mol}})(1 - r^2/R^2) \quad (13)$$

is the Thomas-Fermi density profile, with $R \equiv (2\mu_{\text{mol}}/m_{\text{mol}}\omega^2)^{1/2}$. The coupling constant for the molecule-molecule interaction is $g_{\text{mol}} = 4\pi\hbar^2 a_{\text{mol}}/m_{\text{mol}}$. The exact calculation of Eq. (11) yields

$$\rho(\epsilon) = [\mu_{\text{mol}}^2 / (\pi\hbar^3 \omega^3)] \{ 2\sqrt{2}z \tanh^{-1}[\sqrt{2}z/(1+z)] + 4z^{3/2} - \sqrt{2}z^2 [\pi + 2 \tan^{-1}((1-z)/\sqrt{2}z)] + (1+z)^2 [\theta_0 - \sin(4\theta_0)/4] \}, \quad (14)$$

where $z \equiv \epsilon/\mu_{\text{mol}}$ is the rescaled energy and $\theta_0 \equiv \cos^{-1}(1/\sqrt{1+z})$. In the limit $k_B T \gtrsim \mu_{\text{mol}}$,

$$\rho(\epsilon) = \frac{\mu_{\text{mol}}^2}{(\hbar\omega)^3} \left[\frac{z^2}{2} + z + \mathcal{O}(z^0) \right]. \quad (15)$$

Given the density of states, the entropy can be calculated from Eq. (5) and (10). The resulting expression is

$$S = k_B \frac{N_{\text{mol}}}{\zeta(3)} \left(\frac{T}{T_{\text{BEC}}} \right)^3 G(\beta\mu_{\text{mol}}), \quad (16)$$

where ζ is the Riemann zeta function, with $\zeta(3) = 1.202\dots$, $N_{\text{mol}} = N/2$ is the number of molecules and

$$G(u) \equiv u^3 \int_0^\infty dz f(z) \left[\frac{uz}{e^{uz} - 1} - \ln(1 - e^{-uz}) \right], \quad (17)$$

with $u \equiv \beta\mu_{\text{mol}}$, and $f(z) \equiv [(\hbar\omega)^3 / \mu_{\text{mol}}^2] \rho(z\mu_{\text{mol}})$ so as to make the units explicit. Equation (17) may be integrated numerically, or explicitly in the limit of Eq. (15), in which case

$$S = k_B N_{\text{mol}} \left(\frac{T}{T_{\text{BEC}}} \right)^3 \left(\frac{2\pi^4}{45\zeta(3)} + 3 \frac{\mu_{\text{mol}}}{k_B T} \right) \quad (18)$$

This expansion is accurate to within 10% for $k_B T / \mu_{\text{mol}} \geq 1/10$. In the above expression for the entropy, the condensation temperature for a non-interacting Bose gas, $k_B T_{\text{BEC}} = \hbar\omega [N_{\text{mol}}/\zeta(3)]^{1/3}$ was used. Setting the entropy of the molecular condensate and of the Fermi gas equal one obtains the final temperature of the Fermi gas:

$$\left(\frac{T}{T_F} \right)_{\text{final}} = \frac{G(\beta\mu_{\text{mol}})}{2\pi^2 \zeta(3)} \left(\frac{T}{T_{\text{BEC}}} \right)_{\text{initial}}^3 \quad (19)$$

where β refers to the initial temperature.

In the strongly degenerate regime, evaporative cooling of a Fermi gas becomes difficult, so that until recently a

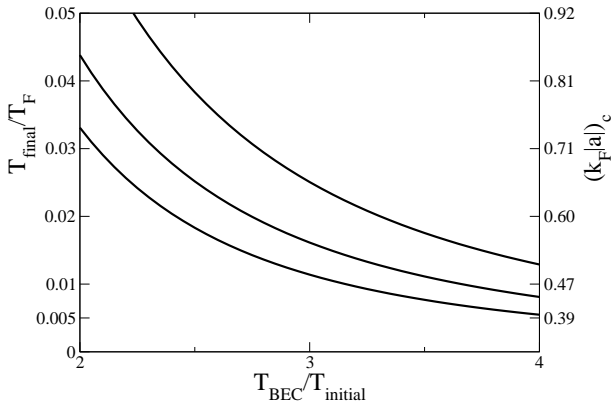


FIG. 1: Final temperature of the Fermi gas (left axis) and critical condition for a BCS transition (right axis) as a function of the initial temperature of the Bose condensed molecular gas that has been transformed to a normal attractive Fermi gas via adiabatic switching of the scattering length. The curves, from upper to lower, correspond to $\mu_{\text{mol}}/k_B T_{\text{BEC}} = 1$, $1/2$, and $1/4$, where μ_{mol} is the molecular chemical potential.

maximum degeneracy of $T/T_F \sim 0.2$ appeared to be a lower limit, preventing the observation of the BCS transition. The adiabatic tuning of a molecular condensate suggests an alternate method to cooling the fermions directly. According to Eqs. (8) and (18), the entropy is proportional to T for fermions and T^3 for bosons. The limits of evaporative cooling for a Thomas-Fermi Bose condensed gas are given by $\mu \simeq k_B T$. Thus the final degeneracy that may be obtained by adiabatic switching of an evaporatively cooled molecular BEC is given by Eq. (19), with $G(\beta\mu_{\text{mol}}) = G(1) \simeq 8.32$. Larger values of $\beta\mu_{\text{mol}}$ are in principle achievable, *e.g.* by sympathetic cooling. In Fig. 1 is shown the final temperature as a function of the initial temperature for various interaction strengths given by the ratio $\mu_{\text{mol}}/k_B T_{\text{BEC}}$.

The lowest experimentally achieved temperature of a degenerate Fermi gas to date is $T/T_F = 0.05$ [7]. This temperature may be obtained by adiabatic switching of a molecular BEC of temperature $T = 0.5 T_{\text{BEC}}$ with $\mu_{\text{mol}} \simeq k_B T$, which is routinely achieved for atomic BEC's. A temperature of $T = 0.25 T_{\text{BEC}}$ with $\mu_{\text{mol}} \simeq k_B T$, as for example achieved for an atomic Bose condensed gas in Ref. [21], would give $T/T_F = 5 \times 10^{-3}$.

This offers exciting possibilities for obtaining deeply degenerate Fermi regimes, such as observing the BCS transition in the weakly interacting regime $k_F|a| \ll 1$. The corresponding critical condition is obtained by substituting into Eq. (19) the BCS critical temperature calculated in Ref. [19]:

$$T_{\text{BCS}}/T_F = \alpha \exp(-\pi/2k_F|a|), \quad (20)$$

where $\alpha \equiv e^\gamma \pi^{-1} (2/e)^{7/3} \simeq 0.277$, γ being Euler's constant. Strictly speaking, in an isotropic harmonic potential, Eq. (20) is only valid in the limit $k_B T_{\text{BCS}} \gg \hbar\omega$.

However, for $k_F|a| = 1/2$ or $1/3$, it gives a result accurate to 30% for $k_B T_{\text{BCS}} \simeq 2\hbar\omega$ [20]. In the right axis of Fig. 1 is shown the critical value $(k_F|a|)_c$ ensuring $T_{\text{final}} = T_{\text{BCS}}$, as a function of T_{BEC}/T : $(k_F|a|)_c$ is on the order of one half, a regime already accessible by present experiments. It should be noted that our treatment is exactly valid only in the regime $k_F|a| \ll 1$, as we have used BCS theory. The values of $(k_F|a|)_c$ that we find are not extremely small compared to unity; we therefore expect corrections to both the equation of state of the normal phase and the critical temperature. Our result must therefore be considered as an estimate.

Fermi gases have been predicted to be sensitive to heating due to particle losses, much more so than Bose condensates [23]. One may ask what the limit is on the accessible final temperature set by this heating during the adiabatic tuning of a . Assume a loss rate γ_{loss} of atoms due mainly to collisions with the background gas, and let t be the duration of the adiabatic ramp. Using the derivation of Ref. [24], and assuming that $\gamma_{\text{loss}} t \ll 1$ and $(T/T_F)^2 \ll 1$, the temperature increase due to loss may be estimated as $(T/T_F)_{\text{increase}} \sim \sqrt{\gamma_{\text{loss}} t / (2\pi^2)}$. One may take $t \sim 1/[\gamma_{\text{coll}}(T/T_F)^2]$, where $\gamma_{\text{coll}} = n\sigma v_F$ is the classical collision rate involving the scattering cross-section $\sigma \simeq 4\pi a^2$. Here n is the total density, $v_F \equiv \hbar k_F/m$ is the Fermi velocity, and a factor of $(T/T_F)^2$ is included to account for the Pauli blocking for each of the two components [6]. Then, supposing $(T/T_F)_{\text{final}} \sim 10^{-2}$, one finds the condition

$$[\gamma_{\text{loss}} / (2\pi^2 \gamma_{\text{coll}})]^{1/4} \lesssim (T/T_F)_{\text{final}} \sim 10^{-2} \quad (21)$$

in order to avoid a loss-induced increase in temperature during the switching time. For the experiment of Ref. [15], $\gamma_{\text{loss}} < 10^{-2} \text{ s}^{-1}$, and Eq. (21) is satisfied for $a \gtrsim 1000a_0$ and $n \gtrsim 5 \times 10^{12} \text{ cm}^{-3}$, where a_0 is the Bohr radius.

In order to satisfy the criterion of adiabaticity, thermal equilibrium must be maintained while tuning the scattering length from positive to negative values. This requires that the thermalization rate, *i.e.*, the rate of elastic collisions, be much larger than the rate of change of a [13]. For $a > 0$, thermalization is due to collisions between non-condensed molecules. As the size of the non-condensed cloud is close to the size of the condensate for $k_B T \sim \mu_{\text{mol}}$, the effective collision rate is thus $\gamma_{\text{eff}} = \gamma_{\text{coll}}^{\text{mol}} N_T / N_{\text{mol}}$, where N_T / N_{mol} is the non-condensed fraction and $\gamma_{\text{coll}}^{\text{mol}} = n_{\text{mol}} \sigma_{\text{mol}} v_T$, with v_T the thermal velocity and $\sigma_{\text{mol}} = 8\pi a_{\text{mol}}^2$ the scattering cross section between two molecules. For $T/T_{\text{BEC}} = 1/2$, the non-condensed fraction is $(T/T_{\text{BEC}})^3 = 1/8$ [14]. Assuming $a \sim 1000a_0$ and the mass of ${}^6\text{Li}$, even for molecular densities n_{mol} as small as 10^{12} cm^{-3} , this gives a minimum thermalization time of $\sim 30 \text{ ms}$. For $a < 0$, the rate of thermalization is strongly reduced by Pauli blocking, and one has $\gamma_{\text{eff}} = \gamma_{\text{coll}}(T/T_F)^2$ [6]. At the critical value of $k_F|a| \sim 1/2$, and for a total density of $5 \times 10^{12} \text{ cm}^{-3}$,

adiabaticity requires that the change of the scattering length occurs on a time $t > 300$ ms. This time is longer than the inverse oscillation frequency of lithium atoms in typical magnetic traps, so that the change in a will not induce macroscopic oscillations in the gas.

It is thus far not possible to obtain a perfect balance of spin states in a Fermi degenerate gas [15]. It may therefore be supposed that a small fraction of fermions will remain unpaired for positive scattering lengths, due to a lack of partners, and will coexist with the molecular condensate. Fermions have an entropy proportional to T , as opposed to T^3 in the case of bosons. So even a small number of atoms could be expected to dominate the total entropy of the atom-molecule mixture at low temperatures, thereby interfering with the proposed cooling scheme. This problem can be avoided by simply removing the excess fermions from the system for $a > 0$ with a correctly tuned laser pulse, which requires that their excitation frequencies are sufficiently different from those of the molecules [22].

In conclusion, we have shown that adiabatic switching of a molecular BEC allows one to obtain a deeply degenerate Fermi gas of temperatures on the order of $T \sim 10^{-2}T_F$. This suggests a way to achieve a BCS transition in the weakly interacting regime. It is important to note that our thermodynamic approach does not require a knowledge of the equation of state of the system in the intermediate strong coupling regime where $k_F|a| \gtrsim 1$.

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Note added: Since submission of this paper, long-lived BEC's of weakly bound molecules have been observed [25], and a strongly interacting fermionic condensate in the BEC-BCS crossover regime was created [26].

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