Non-equilibrium phase transition in a sheared granular mixture

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Abstract. - The dynamics of an impurity (or tracer particle) immersed in a dilute granular gas under uniform shear flow is investigated. A non-equilibrium phase transition is identified from an exact solution of the inelastic Boltzmann equation for a granular binary mixture in the tracer limit, where the impurity carries either a vanishing (disordered phase) or a finite (ordered phase) fraction of the total kinetic energy of the system. In the disordered phase, the granular temperature ratio (impurity "temperature" over that of the host fluid) is finite, while it diverges in the ordered phase. To correctly capture this extreme violation of energy equipartition, we show that the picture of an impurity enslaved to the host fluid is insufficient.

Under external driving, an initially mixed macroscopic granular system may segregate. This ubiquitous effect is not always desirable in applications, and among the numerous counter-intuitive properties of granular materials, it has arguably been among the most studied in the last twenty years [1,2]. Yet, it is not well understood. The reason is twofold: there is a large number of *a priori* relevant parameters involved in the description of the granular mixture; additionally, analytical results are scarce and difficult to obtain. Both shortcomings call for an improved fundamental understanding of physical situations where the different physical effects at work (size asymmetry, forcing, collisional dissipation, etc.) can be deciphered. As a consequence, seeking for theoretical progress, we simplify the problem in two respects. First, we consider the impurity limit where a tracer particle is immersed in a driven *dilute* granular gas described by the inelastic Boltzmann equation. Second, we focus on an inelastic version of the so-called Maxwell molecules [3], that can be seen as defining the kinetic theorist's Ising model. Indeed, the Boltzmann equation for elastic bodies is already notoriously difficult to deal with, and inclusion of collisional dissipation –an essential aspect of inter-grains collisions– leads to a far more complex description. The corresponding inelastic Maxwell model, that has witnessed an upsurge of interest in recent years [4–9], is inspired by Maxwell's original insight [10] that scattering processes involving collision rates independent of the relative velocity of impacting particles, define mathematically tractable approaches [11].

A valid question to raise from the outset is then that of the relevance of such a simplification. This issue should be addressed by comparison with more refined approaches. Among the few well accepted models accounting for the specifics of dissipative inter-grains collisions, the simplest is the inelastic hard sphere model [2, 12], where results pertaining to the inelastic Boltzmann equation can be compared to their Maxwell model counterpart. While the agreement concerning the Navier-Stokes transport coefficients is only qualitative [13], the situation improves significantly for sheared mixtures where the rheological properties (shear and normal stresses) agree very well with (approximate) analytical results and computer simulations for inelastic hard spheres [14]. This agreement, in a parameter space of large dimensionality, gauges and establishes the reliability of the Maxwell model to capture important effects in *sheared* granular mixtures. We come back to the relevance of our approach before our concluding section, see below, and we further note that experiments with magnetic grains also point to the usefulness of Maxwell models [15].

Our objective is to investigate the dynamics of an impurity or tracer particle (hereafter referred to as species 1) in an inelastic dilute gas (species 2), maintained in a non-equilibrium steady state by a uniform shear flow. This particular state is amenable to analytical treatment from the Boltzmann equation since it is characterized by

constant and uniform partial densities (n_i for $i = 1, 2$) as well as a uniform "granular temperature" $T(t)$ [16], while the only gradient affects the velocity profile, that is furthermore common to both species: $u_x = ay$ in a Cartesian frame [(x, y, z) in three dimensions]. This linear velocity field defines the constant shear rate a that quantifies viscous heating (shearing work). This energy injection mechanism is balanced by dissipation in inelastic collisions, the latter being parameterized by the constant coefficients of normal restitution α_{ij} for collisions between species i with j [12]. Although the total temperature $T = (n_1T_1 + n_2T_2)/(n_1 + n_2)$ (T_i is the partial temperature of species i) changes with time, the temperature ratio T_1/T_2 is time independent in the hydrodynamic regime (namely, for times much larger than the mean free time). Note that equipartition, of course, has no reason to hold in such non equilibrium conditions: $T_1/T_2 \neq 1$ [17]. A plausible expectation is that upon taking the tracer limit $c_1 \to 0$ (where $c_i = n_i/(n_1 + n_2)$ is the mole fraction of species i), the properties of the medium (or granular gas) are not affected by the presence of the tracer particles (or impurity). Let us consider the relative contribution of the tracer particles to the total energy of the system, $E_1/E = c_1T_1/T$. Out of equilibrium, one expects that E_1/E actually scales like c_1, and should therefore vanish when $c_1 \to 0$. In other words, the impurity should be enslaved to the host medium, and should not change the macroscopic properties of the system. In this letter we present an example of a violation of the above expectation: A non-equilibrium phase transition takes place, that allows to discriminate a *disordered* phase, where T_1/T in the tracer limit is finite –and hence $E_1/E = 0$–, from an *ordered* phase where E_1/E is finite –and therefore T_1/T diverges. The corresponding regions of phase space will be worked out explicitly, which will reveal that both phases may exhibit unexpected reentrant features. The apparition of an ordered phase corresponds to an extreme violation of equipartition, and has already been reported, but only in the case of an unforced system ($a = 0$) [19]. However, this work was performed at the "enslaved impurity" level, neglecting the retroaction of impurity onto the host fluid. This precludes the derivation of the order parameter. Compared to the findings of Ref. [19], we obtain explicitly the value of the transition order parameter E_1/E in the ordered phase, and show that the scenario is more complex than reported so far, with the emergence of overlooked phases. Furthermore, we investigate the fate of the previous phases, including the new ones obtained, at finite shear ($a \neq 0$). These new results are the most significant contribution of the present work.

We consider a binary mixture of grains at low density, driven by a uniform shear flow. In the Lagrangian frame moving at the linear flow velocity $\mathbf{u}$, the velocity distribution functions $f_i(\mathbf{r}, \mathbf{v}, t)$ of each species become uniform, i.e., $f_i(\mathbf{r}, \mathbf{v}, t) \equiv f_i(\mathbf{V}, t)$ where $\mathbf{V} = \mathbf{v} - \mathbf{u}$. In this frame, the set of coupled Boltzmann equations for the velocity distributions $f_i(\mathbf{V}, t)$ read [18]

$$\frac{\partial}{\partial t} f_i - aV_y \frac{\partial}{\partial V_x} f_i = \sum_j J_{ij}[\mathbf{v}|f_i, f_j], \tag{1}$$

where the Boltzmann collision operator $J_{ij}[f_i, f_j]$ is

$$J_{ij}\left[\mathbf{V}_1|f_i, f_j\right] = \frac{\omega_{ij}}{n_j\Omega_d} \int d\mathbf{V}_2 \int d\widehat{\boldsymbol{\sigma}} \left[\alpha_{ij}^{-1} f_i(\mathbf{V}_1')f_j(\mathbf{V}_2') - f_i(\mathbf{V}_1)f_j(\mathbf{V}_2)\right] . \tag{2}$$

Here, $\Omega_d = 2\pi^{d/2}/\Gamma(d/2)$ is the total solid angle in d dimensions, and $\alpha_{ij} \leq 1$ denotes the (constant) coefficient of restitution for collisions between particles of species i with j. Moreover, $\mathbf{V}_1' = \mathbf{V}_1 - \mu_{ji}\left(1 + \alpha_{ij}^{-1}\right)(\widehat{\boldsymbol{\sigma}} \cdot \mathbf{g}_{12})\widehat{\boldsymbol{\sigma}}$, $\mathbf{V}_2' = \mathbf{V}_2 + \mu_{ij}\left(1 + \alpha_{ij}^{-1}\right)(\widehat{\boldsymbol{\sigma}} \cdot \mathbf{g}_{12})\widehat{\boldsymbol{\sigma}}$, where $\mathbf{g}_{12} = \mathbf{V}_1 - \mathbf{V}_2$, $\widehat{\boldsymbol{\sigma}}$ is a unit vector directed along the centers of the two colliding spheres, and $\mu_{ij} = m_i/(m_i + m_j)$. The effective collision frequencies ω_{ij} for collisions i-j are independent of the relative velocities of the colliding particles but can depend on space and time through its dependence on densities n_i and temperature T. They can be also seen as free parameters of the model. Here, since our problem involves a delicate tracer limit, we aim at the simplest possible approach: In the "plain vanilla" Maxwell model worked out here, the collision frequencies ω_{ij} are of the form $\omega_{ij} = \nu c_j$, where ν^{-1} is an effective (constant) mean free time. The form of ω_{ij} is closer to the original model of Maxwell molecules for ordinary gas mixtures [10]. The plain vanilla Maxwell model has been previously considered by several authors [19, 20] in several problems pertaining to granular mixtures. Other choices of ω_{ij} are possible, e.g. if the goal lies in mimicking as accurately as possible inelastic hard spheres [14], but at the price of a substantial increase in the complexity of the model that prevents to get exact results.

In the uniform shear flow problem, the reduced shear rate $a^* = a/\nu$ is the relevant nonequilibrium parameter since it measures the distance from the unforced case, the much studied so-called homogeneous cooling state (HCS) [12]. Within our framework, a^* does not depend on time, so that in general a^* and α_{ij} are independent parameters. This decoupling allows us to analyze the combined effect of both control parameters on the dynamic properties of the granular mixture. This is one of the main advantages of the vanilla model used here in contrast to previous works [14]. More generally, a key feature of the Boltzmann equation for Maxwell models is that the pressure tensor P can be exactly determined in terms of the shear rate and the parameters of the mixture (the mass ratio $\mu \equiv m_1/m_2$, the mole fraction $c_1 = n_1/(n_1 + n_2)$ and the coefficients of restitution α_{ij}) [14, 21]. The knowledge of the pressure tensor allows one to determine the non-Newtonian transport properties of the mixture. The energy ratio E_1/E is also an important property that can be obtained from the pressure tensor.

We now address the tracer limit ($c_1 \to 0$) for E_1/E. After some tedious but straightforward algebra [22], an

interesting phenomenon is uncovered in this limit: while E_1/E vanishes for some parameter range –this is the naive expectation–, the possibility exists that $E_1/E \neq 0$. Two relaxation rates, $\lambda_1^{(0)}$ and $\lambda_2^{(0)}$, play a key role in delimitating the two behaviours [22]: whenever $\lambda_2^{(0)} > \lambda_1^{(0)}$, one has $E_1/E = 0$ while on the other hand, $E_1/E \neq 0$ for $\lambda_2^{(0)} < \lambda_1^{(0)}$. The corresponding expressions for $\lambda_2^{(0)}$ and $\lambda_1^{(0)}$ are [22]

$$\lambda_2^{(0)} = \frac{(1+\alpha_{22})^2}{d+2}\varphi(\widetilde{a}) - \frac{1-\alpha_{22}^2}{2d}, \tag{3}$$

$$\lambda_1^{(0)} = \frac{2\mu_{21}^2}{d+2}(1+\alpha_{12})^2\varphi\left(\frac{\widetilde{a}}{2\mu_{21}^2}\left(\frac{1+\alpha_{22}}{1+\alpha_{12}}\right)^2\right) - \frac{2}{d}\mu_{21}(1+\alpha_{12})\left[1-\frac{\mu_{21}}{2}(1+\alpha_{12})\right], \tag{4}$$

where $\widetilde{a} = 2(d+2)a^*/(1+\alpha_{22})^2$, and $\varphi(x) \equiv \frac{2}{3}\sinh^2[\frac{1}{6}\cosh^{-1}(1+\frac{27}{d}x^2)]$. In cases where $\lambda_1^{(0)} > \lambda_2^{(0)}$, the expression of E_1/E can be written as

$$\frac{E_1}{E} = \frac{D(\lambda_1^{(0)})}{\Delta_{01}(\lambda_1^{(0)})\lambda_1^{(1)} + \Delta_1(\lambda_1^{(0)})}, \tag{5}$$

where the dependence of m_1/m_2, α_{ij}, and a^* is implicitly assumed on the right-hand side. In addition, $\lambda_1^{(1)}$ is defined by the expansion $\lambda_1(a^*,c_1) \approx \lambda_1^{(0)}(a^*)+\lambda_1^{(1)}(a^*)c_1+\mathcal{O}(c_1^2)$ where λ_1 is the largest real root of a sixth-degree polynomial equation [22]. The general expressions of D, Δ_{01} and Δ_1 are too lengthy to be written down here. For the sake of illustration, we give their expressions for the case $\frac{m_1}{m_2} = \frac{1}{4}$ and $\alpha_{11} = \alpha_{22} = \alpha_{12} = \frac{1}{2}$:

$$D = -\frac{9}{25}\left[\lambda_1^{(0)} + \frac{7}{20}\right]^2\left[\lambda_1^{(0)} + \frac{76}{125}\right]^2 - \frac{18a^{*2}}{125}\left[\lambda_1^{(0)}(1+\lambda_1^{(0)}) + \frac{1331}{5000}\right], \tag{6}$$

$$\Delta_{01} = \frac{144}{125}\left(\lambda_1^{(0)} + \frac{7}{20}\right)\left\{a^{*2} - \frac{625}{48}\left(\frac{76}{125} + \lambda_1^{(0)}\right) \times \left[\frac{2581}{15625} + \lambda_1^{(0)}\left(\frac{2077}{2500} + \lambda_1^{(0)}\right)\right]\right\}, \tag{7}$$

$$\Delta_1 = \frac{153a^{*2}}{500}\left(\lambda_1^{(0)2} + \frac{223}{170}\lambda_1^{(0)} + \frac{55857}{170000}\right) - \frac{75831}{125000}\left(\lambda_1^{(0)} + \frac{7}{20}\right)\left(\lambda_1^{(0)} + \frac{76}{125}\right) \times \left[1 + 5\lambda_1^{(0)}\frac{10403 + 132100\lambda_1^{(0)}}{101108}\right]. \tag{8}$$

It must be noted that in general the expression of E_1/E does not explicitly depend on the coefficient of restitution α_{11} which is equivalent to neglecting the collisions among tracer particles themselves in the kinetic equation of f_1.

The change of behaviour (energy ratio that vanishes or not) is akin to an ordering process where the tracer is either enslaved to the host fluid ($E_1/E = 0$), or carries a finite fraction of the total kinetic energy of the system ($E_1/E \neq 0$). The latter situation, where the temperature ratio is divergent, will be subsequently referred to as the "ordered phase", characterized by an extreme breakdown of energy equipartition. Loosely speaking, the system is invariant under the transformation $c_1 \to \delta c_1$ in the disordered phase, where T_1/T reaches a finite value in the limit $c_1 \to 0$. This invariance is broken in the ordered phase, since then $T_1/T \propto c_1^{-1}$.

In light of the previous discussion, it is instructive to analyze some special cases. First, for sheared elastic gases ($a^* \neq 0$ and $\alpha_{12} = \alpha_{22} = 1$), it is easy to see that $\lambda_2^{(0)} > \lambda_1^{(0)}$ ("disordered" phase) if the mass ratio $\mu > \sqrt{2}-1 \simeq 0.414$ for any value of a^*. However, if $\mu < \sqrt{2}-1$, $\lambda_1^{(0)} > \lambda_2^{(0)}$ ("ordered" phase) for a^* larger than a critical value $a_c^*(\mu)$. We recover here the transition already found [23] for elastic Maxwell mixtures under uniform shear flow. A second interesting situation is that of the HCS ($a^* = 0$ but $\alpha_{ij} \neq 1$). In this case, there are also two different phases depending on the relative positions of $\lambda_1^{(0)}$ and $\lambda_2^{(0)}$. In particular, the transition point ($\lambda_1^{(0)} = \lambda_2^{(0)}$) –where it can be shown that the system relaxation time diverges– leads to the critical mass ratios $\mu_{\rm HCS}^{(\pm)} = (\alpha_{12} \pm \sqrt{(1+\alpha_{22}^2)/2})/(1 \mp \sqrt{(1+\alpha_{22}^2)/2}$ with $\mu_{\rm HCS}^{(-)} < \mu_{\rm HCS}^{(+)}$. The ordered phase ($\lambda_1^{(0)} > \lambda_2^{(0)}$) exists for mass ratios smaller than $\mu_{\rm HCS}^{(-)}$ or larger than $\mu_{\rm HCS}^{(+)}$. Moreover, the first ordered phase ($\mu < \mu_{\rm HCS}^{(-)}$ corresponding to light impurities) is present only when $\alpha_{12} > \sqrt{(1+\alpha_{22}^2)/2}$, and so is absent when $\alpha_{12} = \alpha_{22}$ or when $\alpha_{12} < 1/\sqrt{2}$. It therefore appears that the occurrence of this ordered phase not only requires collisional dissipation but more precisely, asymmetric dissipation. In the absence of shear ($a^* = 0$), the order parameter E_1/E can be cast in a simple form as

$$\frac{E_1}{E} = \frac{\alpha_{22}^2 - 1 + 4\mu_{21}(1+\alpha_{12})\left[1-\frac{\mu_{21}}{2}(1+\alpha_{12})\right]}{\alpha_{22}^2 - 1 + 2\mu_{21}(1-\alpha_{12}^2)}. \tag{9}$$

The existence of the second ordered phase ($\mu > \mu_{\rm HCS}^{(+)}$, heavy impurities) was already found by Ben-Naim and Krapivsky [19] in their analysis on the velocity statistics of an impurity immersed in a uniform granular fluid. It must be remarked that a similar extreme breakdown of the energy equipartition has also been reported in the HCS for inelastic hard spheres [24] since, in the ordered phase, the ratio of the mean square velocities for the impurity and fluid particles $T_1 m_2/T_2 m_1$ is finite for extremely large mass ratios ($m_1/m_2 \to \infty$).

We now turn to the general case where viscous heating and collisional dissipation are both at work. Unlike with the HCS where the light impurity ordering requires asym-

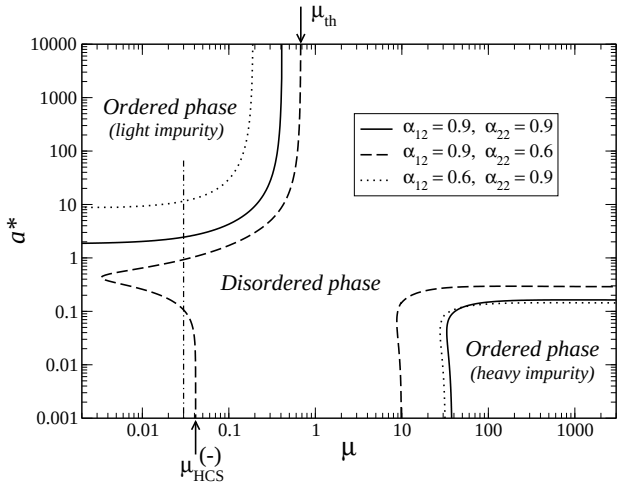

Fig. 1: Phase diagram in the reduced shear rate vs mass ratio plane, for different coefficients of restitution. The lines display the boundaries between ordered and disordered phases; the light and heavy impurity ordered phases are indicated. The vertical dot-dashed line corresponds to a cut leading to the inset of Fig. 2. The arrows shows $\mu_{\rm HCS}^{(-)}$ and $\mu_{\rm th}$ for $\alpha_{12}=0.9$ and $\alpha_{22}=0.6$. For the two other parameters sets, $\{\alpha_{12},\alpha_{22}\}=\{0.9,0.9\}$ and $\{0.6,0.9\}$, the thresholds $\mu_{\rm th}$ and $\mu_{\rm HCS}^{(+)}$ exist (not shown, for clarity) while $\mu_{\rm HCS}^{(-)}$ is not defined. Here, $d=2$, but very similar results are obtained in three dimensions.

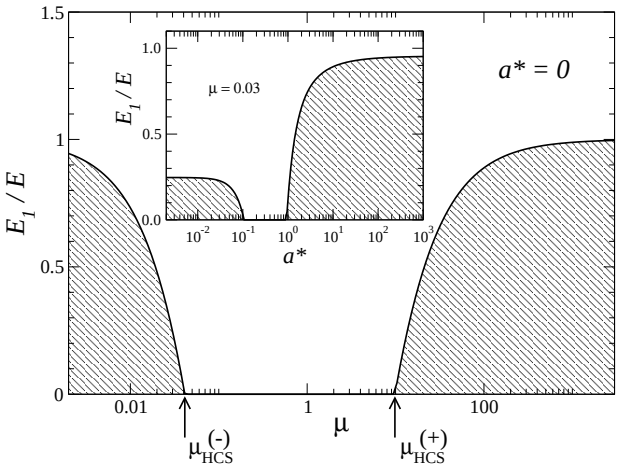

Fig. 2: Order parameter of the non equilibrium transition, as a function of mass ratio $\mu=m_1/m_2$ in the unforced case for a two-dimensional system. Here, $\alpha_{12}=0.9$ and $\alpha_{22}=0.6$ The inset shows the same quantity as a function of reduced shear rate, for $\mu=0.03<\mu_{\rm HCS}^{(-)}$. The hatched regions indicate the ordered phases (reentrant light impurity phase in the inset). The two thresholds in the unforced case are indicated by the vertical arrows ($\mu_{\rm HCS}^{(-)}\simeq 0.041$ and $\mu_{\rm HCS}^{(+)}\simeq 9.83$).

metric dissipation, we find here that adding the "shear dimension" to the phase diagram ensures the systematic existence of the light impurity ordered phase. This is illustrated in Fig. 1, see the left hand side. On the other hand, the heavy impurity order sets in at large enough mass ratio and small enough shear (lower right corner of Fig. 1). As anticipated from the study of the unforced system, the behaviour differs depending if the inequality $\alpha_{12}>\sqrt{(1+\alpha_{22}^2)/2}$ holds or not. In Fig. 1, the parameter set fulfilling the above constraint is $\alpha_{12}=0.9$ and $\alpha_{22}=0.6$. In the low shear rate limit, the figure indeed exhibits two distinct ordered phases, one at low mass ratio μ, and one in the opposite heavy intruder limit. This behaviour is fully consistent with the previously analyzed HCS scenario. Conversely, when $\alpha_{12}<\sqrt{(1+\alpha_{22}^2)/2}$, the light impurity ordered pocket is more restricted, and confined in a portion of high shear, $a^*>a_c^*$, with quite a complex dependence on the coefficients of restitution α_{22} and α_{12} (see fig. 1). In all cases, the light impurity phase can only exist provided μ does not exceed some dissipation dependent threshold, $\mu<\mu_{\rm th}$, with $\mu_{\rm th}=\sqrt{2}(1+\alpha_{12})/(1+\alpha_{22})-1$. As can be seen in Fig. 1, the order/disorder threshold shear rate diverges as $\mu\to\mu_{\rm th}$. Of course, $\mu_{\rm th}=\sqrt{2}-1$ for elastic gases, as discussed above. The phase diagram shown in Fig. 1 also indicates that the ordered phase exhibits reentrant features. Upon increasing mass ratio at fixed reduced shear rate, or conversely increasing shear rate at fixed mass ratio, the following sequence may be observed for some portion of phase space: a first transition from ordered to disordered states, followed by a reverse disorder $\to$ order transition. This is illustrated in Fig. 2, where the fraction of the total energy that is transported by the impurity is plotted against either mass ratio, or shear rate. It is apparent that, for asymptotically large shear rates, the tracer contribution to the total energy can be even larger than that of the excess component.

Before concluding, we come back to the reliability of our "plain vanilla" Maxwell approach. Although approximate, it turns out to provide a relevant framework for granular binary mixtures under uniform shear flow. This is illustrated in fig. 3 for an equimolar mixture in the steady uniform shear flow state, namely, when viscous heating and collisional cooling cancel each other and consequently, the (reduced) shear rate a^* depends on the coefficients of restitution α_{ij}. Figure 3 compares the predictions of the plain inelastic Maxwell model to the Monte Carlo results obtained within the more realistic inelastic hard sphere model. It can be seen that the plain Maxwell model prediction captures the important trend observed within a more refined framework (the discrepancies between theory and simulation are less than 15%, even for strong values of dissipation). We therefore expect that the transition found in this paper is not artefactual but is a robust feature.

In conclusion, we have analysed the tracer limit of an *exact* solution [14,21] of the Boltzmann equation for a binary granular mixture of inelastic Maxwell gases. This solution applies to *arbitrary* values of the shear rate a and the parameters of the mixture, namely, the mole fraction c_1, the

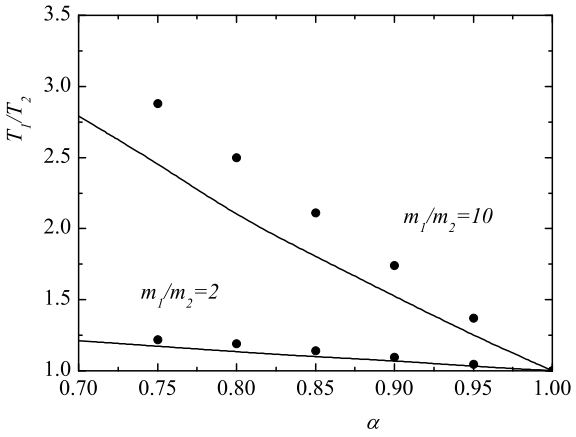

Fig. 3: Temperature ratio T_1/T_2 versus the (common) coefficient of restitution ($\alpha = \alpha_{11} = \alpha_{12} = \alpha_{22}$) in the *steady* uniform shear flow state. The predictions of the inelastic Maxwell model (continuous curve, present work) are tested against Monte Carlo data (symbols, from [25]). The parameters are $c_1 = 0.5$ (equimolar mixture) for a three dimensional system ($d = 3$). Two mass ratios are considered, corresponding to moderately heavy intruders.

mass ratio μ, and the coefficients of restitution α_{ij}. We have argued that when the granular system is driven by an externally imposed (uniform) shear, the inelastic Maxwell model correctly encodes the physics of more complex models, as evidenced by the dependence of the non-Newtonian transport properties on the numerous parameters of the mixture [14].

Within the framework of the mean-field Maxwell model, we have shown a quite *unexpected* result: the relative contribution of the tracer species (or impurity) to the total properties of the mixture does not necessarily tend to zero as $c_1 \to 0$. Consequently, the seemingly natural "enslaved impurity" picture [26] breaks down. This surprising result extends to *granular* gases some related results derived time ago for ordinary sheared mixtures [23]. The phenomenon discovered here, arising from an exact solution of the set of coupled Boltzmann equations, has been illustrated with the energy ratio E_1/E as a probe, but identical conclusions can be drawn for other properties of the system such as the (intrinsic) shear viscosity and the viscometric functions [27].

The corresponding extreme kinetic energy equipartition violation can be seen as an ordering transition, governed by the competition between two characteristic relaxation frequencies, $\lambda_1^{(0)}$ and $\lambda_2^{(0)}$. We have found two different classes of ordered phases (see the shear rate versus mass ratio diagram in fig. 1): a *light* impurity phase that exists when $\mu < \mu_{\rm HCS}^{(-)}$ for shear rates larger than a certain critical value and a *heavy* impurity phase that requires $\mu > \mu_{\rm HCS}^{(+)}$ and shear rates smaller than a certain threshold value. As fig. 2 clearly shows, both ordered phases exhibit reentrant features.

Usually, in the tracer limit, one assumes that the state of the excess component 2 (or granular gas) is not disturbed by collisions with tracer particles 1 (or impurity) and that the self-collisions among tracer particles can be also neglected [28]. The results derived in this letter show that while the second hypothesis is correct, the first expectation fails in the ordered phase. In this sense, the tracer limit must be taken with care since it presents, in the ordered phase, a similar complexity as the general problem with finite concentration. In addition, although our present description has focused on the energy ratio for a binary system, we expect that the results reported here should have interesting consequences for polydisperse mixtures, or in terms of the velocity statistics, not addressed here. Finally, an important simplification allowed by our modelisation lies in the decoupling between shear rate and dissipation, while granular gases exhibit –in the steady state– inherent coupling between inelasticity and spatial gradients. This means that a fingerprint of our scenario should be sought for experimentally in the hydrodynamic transient regime, before the steady state where collisional dissipation and viscous heating balance each other. We hope that this letter stimulates the performance of such experiments and/or computer simulations to detect the transition phenomenon reported here.

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REFERENCES

[1] See e.g. KUDROLLI A., *Rep. Prog. Phys.*, **67** (2004) 209 and references therein. For more recent contributions, see for instance, CIAMARRA M. P., CONIGLIO A. and NICODEMI M., *Phys. Rev. Lett.*, **94** (2005) 188001; SCHNAUTZ T., BRITO R., KRUELLE C. A. and REHBERG I., *Phys. Rev. Lett.*, 95 (2005) 028001; BREY J. J., RUIZ-MONTERO M. J. and MORENO F., *Phys. Rev. Lett.*, **95** (2005) 098001 (2005); SÁNCHEZ I., GUTIÉRREZ G., ZURIGUEL I. and MAZA D., *Phys. Rev. E*, **81** (2010) 062301; CLEMENT C.P., PACHECO-MARTINEZ H.A., SWIFT M.R. and KING P. J., *Europhys. Lett.*, **91** (2010) 54001.

[2] ARANSON I.S. and TSIMRING L.S., *Rev. Mod. Phys.*, **78** (2006) 641.

[3] ERNST M. H., *Phys. Rep.*, **78** (1981) 1.

[4] BALDASARRI A., MARINI BETTOLO MARCONI U. and PUGLISI A., *Europhys. Lett.*, **58** (2002) 14.

[5] ERNST M.H. and BRITO R., *Europhys. Lett.*, **58** (2002) 182.

[6] ERNST M.H., TRIZAC E. and BARRAT A., *Europhys. Lett.*, **76** (2006) 56.

[7] SANTOS A. and GARZÓ V., *J. Stat. Mech.*, (2007) P08021.
[8] ALASTUEY A. and PIASECKI J., *J. Stat. Phys.*, **139** (2010) 991.
[9] For some recent reviews see e.g. BEN-NAIM E. and KRAPIVSKY P. L., *Granular Gas Dynamics*, edited by T. Pöschel and N. Brilliantov, Vol. **624** (Lectures Notes in Physics, Springer, Berlin) 2003, p. 65; VILLANI C.,*J. Stat. Phys.*, **124** (2006) 781 (2006).
[10] MAXWELL J.C., *Phil. Trans. Roy. Soc.*, **157** (1867) 49.
[11] The kinetic theory for inelastic Maxwell models under study here should not be confused with the so-called Maxwell model (spring-damper) used as a minimal framework for the visco-elastic properties in rheology. See e.g. DROZDOV A.D., *Finite Elasticity and Viscoelasticity* (World Scientific) 1996.
[12] BRILLIANTOV N. and PÖSCHEL T., *Kinetic Theory of Granular Gases* (Oxford University Press, Oxford) 2004.
[13] SANTOS A., *Physica A*, **321** (2003) 442; GARZÓ V. and ASTILLERO A., *J. Stat. Phys.*, **118** (2005) 935.
[14] GARZÓ V., *J. Stat. Phys.*, **112** (2003) 657.
[15] KOHLSTEDT K., SNEZHKO A., SAKOZHNIKOV M.V., ARANSON I.S., OLAFSON J.S. and BEN-NAIM E., *Phys. Rev. Lett.*, **95** (2005) 068001.
[16] As routinely done in the field, we define the granular temperature kinetically, from the variance of the velocity distribution [9]. Such a quantity does not have any thermodynamic basis. For equilibrium systems only (no dissipation, no forcing) is it endowed with a thermodynamic significance.
[17] WILDMAN R.D. and PARKER D.J., *Phys. Rev. Lett.*, **88** (2002) 064301; FEITOSA K. and MENON N., *Phys. Rev. Lett.*, **88** (2002) 198301.
[18] GARZÓ V. and SANTOS A., *Kinetic Theory of Gases in Shear Flows. Nonlinear Transport* (Kluwer Academic, Dordrecht) 2003.
[19] BEN-NAIM E. and KRAPIVSKY P. L., *Eur. Phys. J. E*, **8** (2002) 507.
[20] MARINI BETTOLO MARCONI U. and PUGLISI A., *Phys. Rev. E*, **65** (2002) 051305; **66** (2002) 011301; CONSTANTINI G., MARINI BETTOLO MARCONI U. and PUGLISI A., *J. Stat. Mech.*, (2007) P08031.
[21] GARZÓ V. and TRIZAC E., *J. Non-Newtonian Fluid Mech.*, **165** (2010) 932.
[22] GARZÓ V. and TRIZAC E., in preparation.
[23] MARÍN C., SANTOS A. and GARZÓ V., *Europhys. Lett.*, **33** (1996) 599.
[24] SANTOS A. and DUFTY J. W., *Phys. Rev. Lett.*, **86** (2001) 4823.
[25] MONTANERO J. M. and GARZÓ V., *Physica A*, **310** (2002) 17.
[26] Such a point of view leads to the Boltzmann equation for the excess component and the Lorentz-Boltzmann equation for the impurity and so, the impurity cannot retroact on the host medium.
[27] YAMAKAWA H., *Modern Theory of Polymer Solutions* (Harper and Row, New York, N.Y.) 1971.
[28] RÉSIBOIS P. and DE LEENER M., *Classical Kinetic Theory* (Wiley, New York, N.Y.) 1977.