

# Slow quench dynamics of a trapped one-dimensional Bose gas confined to an optical lattice

Jean-Sébastien Bernier,<sup>1</sup> Guillaume Roux,<sup>2</sup> and Corinna Kollath<sup>1,3</sup>

<sup>1</sup>*Centre de Physique Théorique, CNRS, École Polytechnique, 91128 Palaiseau Cedex, France.*

<sup>2</sup>*LPTMS, Université Paris-Sud, CNRS, UMR 8626, 91405 Orsay, France.*

<sup>3</sup>*Département de Physique Théorique, University of Geneva, CH-1211 Geneva, Switzerland*

We analyze the effect of a linear time-variation of the interaction strength on a trapped one-dimensional Bose gas confined to an optical lattice. The evolution of different observables such as the experimentally accessible onsite particle distribution are studied as a function of the ramp time using time-dependent exact diagonalization and density-matrix renormalization group techniques. We find that the dynamics of a trapped system typically display two regimes: for long ramp times, the dynamics are governed by density redistribution, while at short ramp times, local dynamics dominate as the evolution is identical to that of an homogeneous system. In the homogeneous limit, we also discuss the non-trivial scaling of the energy absorbed with the ramp time.

PACS numbers: 05.70.Ln, 73.43.Nq, 67.85.Hj, 02.70.-c

Manipulating many-body quantum systems by time-varying their control parameters is a practical challenge of technological importance in many areas of physics including condensed matter, quantum information, and cold atomic and molecular gases. However, our understanding of the quantum dynamics of many-particle systems and the identification of their universal dynamical features is still in its infancy [1, 2]. In recent years, it was suggested that the Kibble-Zurek mechanism [3], originally developed to describe the evolution of the early universe, could explain the dynamics of systems across quantum phase transitions. Despite a few successes, the validity of this theory to describe the evolution of all quantum systems is still not accepted. Unbiased theoretical methods, going beyond scaling arguments, are required to understand the dynamics of both homogeneous and inhomogeneous non-integrable quantum systems. In an attempt to shed some light on the evolution of bosonic systems subjected to a change of their control parameters, experiments on <sup>4</sup>He [4] and more recently on cold atoms were reported. Quenches, conducted on a trapped bosonic quantum gas loaded into an optical lattice, were performed by changing the depth of the lattice over a given time interval [5–8]. In these experiments, the out-of-equilibrium processes were investigated by considering the behavior of local observables such as the density, compressibility and onsite particle distribution. For slow to moderate quenches, two different evolution regimes which depend on the ramp time and on the initial interaction strength were observed. These two regimes are believed to be related to the local and global dynamics of the system [6, 8, 9]. How well these experiments can be used to clarify the universal dynamics of homogeneous systems has not been addressed yet.

In this work, we provide answers to this question by analyzing the response of bosons stored in a one-dimensional (1D) lattice to a slow increase of their interaction strength using the unbiased methods of exact diagonalization (ED) and density-matrix renormalization group (DMRG). In the homogeneous case, we find, in addition to the sudden quench and quasi-adiabatic behaviors observed for fast and slow ramp times respectively, that at intermediate ramp times the absorbed energy scales non-trivially with the ramp duration. In the pres-

ence of a trap, we identify two distinct regimes as a function of the ramp time. For long ramp times, the evolution is governed by density redistribution, whereas for shorter ramp times, the evolution is dominated by intrinsic local dynamics and mass transport is absent. This last response is the same as that of an homogeneous system.

We carry out our study using the 1D Bose-Hubbard model:

$$\mathcal{H} = -J \sum_j [b_{j+1}^\dagger b_j + \text{h.c.}] + \frac{U(t)}{2} \sum_j n_j(n_j - 1) - \sum_j \mu_j n_j,$$

with  $b_j^\dagger$  the operator creating a boson at site  $j$ ,  $n_j = b_j^\dagger b_j$  the density operator and  $J$  and  $U$  the hopping and interaction amplitudes. The chemical potentials  $\mu_j$  account for an external confinement. At commensurate fillings, a quantum phase transition from a gapless superfluid (SF) to a gapped Mott insulating (MI) state occurs (at  $U \approx 3.3J$  for  $n = 1$  [10]). The slow quench is performed by increasing the interaction strength linearly, i.e.  $U(t) = U_i + \frac{t}{\tau} \delta U$  with  $\tau$  the *ramp time*,  $\delta U = U_f - U_i$  the quench amplitude, and  $U_{i(f)}$  the initial (final) interaction. This can be achieved experimentally using a suitable Feshbach resonance [11]. Aspects of linear quenches have been discussed previously using various approximate methods [1, 2, 12]. Here, time evolution is computed numerically on chains of size  $L$ , using both ED with periodic boundary conditions and an onsite boson cutoff  $M \geq 7$ , and DMRG [13] with open boundary conditions and  $M = 6$ . The convergence of the DMRG results both with the number of states (a few hundreds) of the reduced space and the time-step of the Trotter-Suzuki time-evolution were checked. Denoting by  $E_{0,i/f}$  the initial and final ground-state energies and  $E_f$  the energy obtained at time  $\tau$ , we introduce the heat as the energy absorbed by the system:  $Q = E_f - E_{0,f}$ . Note that we only consider the time-evolution during the ramp and not the relaxation once the ramp is completed. Further, the derivative of the chosen ramp has a discontinuity at the initial time, by which higher modes might be excited.

*Homogeneous system* – We first aim here at understanding the intrinsic evolution of local observables by studying the homogeneous limit as many features identified in such systems

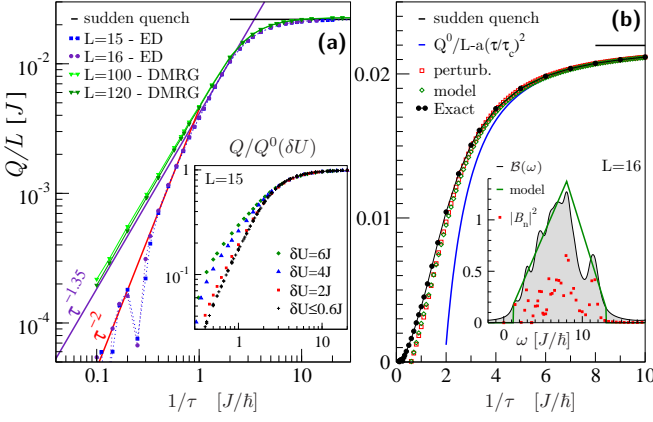


FIG. 1. (Color online)  $U_i = 2J$  and  $U_f = 3J$  at  $n = 1$ . **(a)** Scaling of the heat put into the homogeneous system vs inverse ramp time  $\tau$  showing a non-trivial exponent in the near-adiabatic regime. *Inset*: collapse of data for different  $\delta U$  (ED). **(b)** Comparison to perturbation theory ( $\tau_c \simeq 3.47\hbar/J$ ). *Inset*: Spectral function associated with perturbation theory for  $U = 2J$  (ED).

are also relevant to the trapped case. In Fig. 1(a), we show the heat produced by the quench for a given  $\delta U$  as a function of the inverse ramp time  $\tau^{-1}$ . In finite-size systems, three regimes are typically observed: at large- $\tau$  (adiabatic limit),  $Q \sim \tau^{-2}$  with oscillations [1], here associated with a finite-size gap [14]; at intermediate  $\tau$ , we find a non-trivial power-law behavior; finally, for short  $\tau$ ,  $Q$  approaches the sudden quench limit [15] quadratically.

These results can be compared with time-dependent perturbation theory in the *initial* Hamiltonian basis  $\{|n\rangle\}$  where the “ramp velocity”  $v = \delta U/\tau$  is the small parameter. To second order in  $v$ , the energy variation reads:

$$\Delta E(t) = vtB_0 - v^2 \sum_{n \neq 0} \frac{|B_n|^2}{\omega_n^3} \left[ (\omega_n t)^2 - 4 \sin^2 \left( \frac{\omega_n t}{2} \right) \right]$$

with  $\hat{B} = \sum_j n_j (n_j - 1)/2$  the perturbation operator and  $B_n = \langle n | \hat{B} | 0 \rangle$ . Numerically, we access the  $|B_n|^2$  and energies  $\omega_n = E_n - E_0$  in the low-energy regime using the Lanczos algorithm. Considering only the final time  $\tau$  and introducing the energy spectral function,  $\mathcal{B}(\omega) = \sum_n |B_n|^2 \delta(\omega - \omega_n)$ , one gets for the heat

$$Q(\tau, \delta U) = Q^0(\delta U) - \frac{\delta U^2}{\tau^2} \int_{0+}^{\infty} d\omega \frac{\mathcal{B}(\omega)}{\omega^3} \left[ (\omega\tau)^2 - 4 \sin^2 \left( \frac{\omega\tau}{2} \right) \right], \quad (1)$$

where  $Q^0(\delta U) = \delta U B_0 + E_{0,i} - E_{0,f}$  is the *exact* sudden quench expectation. Counter-intuitively, even though this result is perturbative in  $v$ , it yields the correct result in the large velocity  $v$  limit as the ramp time  $\tau$  becomes short enough in this regime. Thus, this perturbation theory provides short- $\tau$  corrections away from the sudden quench limit, which can be calculated using ground-state observables. Indeed, assuming that  $\mathcal{B}(\omega)$  has a support with an upper bound or decays exponentially, one finds that  $Q(\tau, \delta U) = Q^0(\delta U) - L \frac{\delta U^2}{J} (\tau/\tau_c)^2$

with the characteristic ramp time  $\tau_c$  given by

$$\tau_c^{-2} = \frac{J}{12L} \int_0^{\infty} d\omega \omega \mathcal{B}(\omega) = \frac{J^2}{12L} \langle 0 | B[B, K] | 0 \rangle$$

where  $K$  is the kinetic term  $\sum_j [b_{j+1}^\dagger b_j + \text{h.c.}]$ . The magnitude of  $\tau_c$  is consequently directly connected to the equilibrium three-point correlator  $B[B, K]$ . The scaling of  $\tau_c$  depends on the typical behavior of the correlator with distance. In our model, we find with negligible finite-size effects that  $\tau_c$  is an increasing function of  $U_i$ , indicating that the correlator drops off rapidly for any  $U$ . Using Eq. (1), one obtains a quantitative agreement up to intermediate velocities and close to the power-law regime (see Fig. 1(b)). Furthermore, the details of  $\mathcal{B}(\omega)$  do not alter much this short- $\tau$  regime as a truncated triangular approximation of  $\mathcal{B}(\omega)$  (“model” in Fig. 1(b)) reproduces well this regime.

We observe that the accuracy of Eq. (1) improves with decreasing  $\delta U$ . In the small  $\delta U$  limit, the quench essentially becomes a probe of the initial ground-state dynamics. To second order in  $\delta U$ , the maximum heat behaves as  $Q^0(\delta U) \simeq \delta U^2 \sum_{n>0} |B_n|^2 / \omega_n$ . Combining this result with Eq. (1), we find that  $Q(\tau, \delta U)/Q^0(\delta U) \simeq f(\tau)$  is a function of  $\tau$  only. In the Inset of Fig. 1(a), we see that curves do collapse well onto each other for  $\delta U \leq 0.6$ ; while they do not for larger  $\delta U$  as in this case heat production depends on both  $\tau$  and  $\delta U$ . Remarkably, the small  $\delta U$  limit also provides information on the large- $\tau$  (adiabatic) regimes as, in that case, the first part of the integral in Eq. (1) cancels with  $Q^0(\delta U)$ . There, we recover the results of Ref. 16. In particular, we see that for a gapped spectrum Eq. (1) reproduces the typical  $\tau^{-2}$  decay combined with some oscillating terms. Lastly, we discuss the exponent  $\eta \approx 1.35$  observed for  $U_i = 2J$  and  $\delta U = J$ . For large system sizes, this behavior is found above the sudden quench limit but below a ramp time set by the inverse finite-size gap [17]. We cannot relate this exponent to various predictions on approximated versions of the model [1], nor to the above perturbative approach. Similar non-universal behaviors were recently reported in Ref. 17. Moreover, the Inset of Fig. 1(a) shows that this exponent depends on  $\delta U$ . This may have different reasons. First, contributions of intermediate interaction values at which the system possibly possesses different low-energy physics might play a role. Second, a discontinuity in the derivative of the considered ramp form might cause higher energy excitations. Last, energy redistribution between excited states, which we found to be negligible only for small quenches, could also play a role. This exemplifies that, typically, only the small  $\delta U$  limit can display universality.

Following experiments, we turn to the discussion of local observables.  $\kappa_j = \langle n_j^2 \rangle - \langle n_j \rangle^2$  denotes the compressibility at site  $j$  while  $P_n$  is the probability of having  $n$  bosons onsite. The evolution of Fig. 2(a) starting from the SF phase typically displays the same three regimes as for the heat, with finite-size oscillations close to the adiabatic limit. Entering the MI at  $n = 1$ ,  $P_1$  increases with  $U$  while other  $P_n$  concomitantly

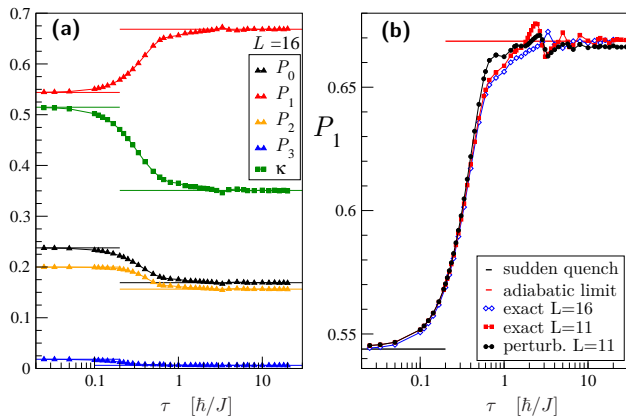


FIG. 2. (Color online) Observables for a quench from  $U_i = 2J$  to  $U_f = 4J$  (ED) **(a)** Occupation probability  $P_n$  and compressibility  $\kappa$  vs ramp time  $\tau$  in the homogeneous system. **(b)** Perturbation theory of Eq. (2) gives a quantitative prediction for  $L = 11$ . The  $L = 16$  data gives an idea of finite-size effects.

decrease. The perturbative prediction for a real symmetric observable  $A$  reads, at time  $\tau$  and to first order in  $v$ ,

$$A(\tau, \delta U) = A_0 - 2 \frac{\delta U}{\tau} \sum_{n \neq 0} \frac{\omega_n \tau - \sin(\omega_n \tau)}{\omega_n^2} A_n B_n. \quad (2)$$

Here again, it is remarkable that the simple perturbation theory provides a quantitatively good (up to a percent in this case) prediction of the exact time-evolution (see Fig. 2(b)).

*Trapped system* – In current cold atom experiments, an external parabolic trap is usually present, i.e.  $\mu_j = -V(j - \frac{L+1}{2})^2$ . As the density increases from the edges to the center, for large  $U$ , MI domains coexist with SF regions [18]. Thanks to recent advances, measurements with single-site resolution are now possible in 2D cold atoms lattice setups [7, 8], enabling one to focus on a particular domain. However, in an out-of-equilibrium situation, density gradients and parameter changes induce flows of particles. Consequently, the density redistribution will have an impact on locally measured quantities. These transport phenomena occur on timescales that depend on the velocity of excitations (typically controlled by  $J$ ) and the domain sizes.

Generally, the timescales for density redistribution are longer than the intrinsic dynamics of a local observable. We show in this section that, in the fast quench regime, an intrinsic evolution of the observables occurs while the density redistribution remains frozen. On the contrary, this redistribution dominates in the slow ramp regime. To demonstrate this point, we systematically compare the evolution of observables in the trapped cloud with its homogeneous counterpart (choosing the same initial density in the center). We consider the evolution of a trapped cloud whose majority of atoms are initially in a SF state as the system features no or only weak MI regions signaled by a trough in the compressibility. Fig. 3 displays comparisons between observables taken at the central site as a function of the ramp time, and for two different

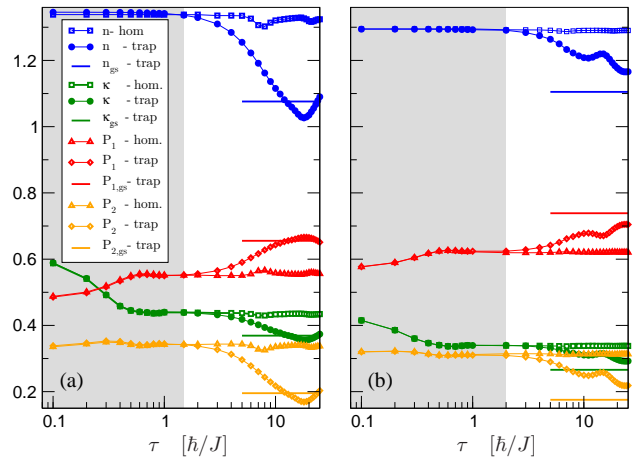


FIG. 3. (Color online). Comparing local observables at the center of the system between homogeneous and trapped ( $V = 0.006J$ ) configurations as a function of the ramp time  $\tau$  (DMRG calculations). The grey area shows the timescales over which the responses are identical. **(a)** slow change from  $U_i = 2J$  to  $U_f = 4J$ . **(b)**  $U_i = 4J$  to  $U_f = 6J$ .

interaction quenches: (a)  $U = (2 \rightarrow 4)J$ , (b)  $U = (4 \rightarrow 6)J$ .

For case (a) of Fig. 3, we see that for short ramp times the central density does not rearrange. In fact, the density remains constant for all ramp times shorter than  $\tau \approx \hbar/J$ . Meanwhile, the compressibility  $\kappa$  and probabilities  $P_{1/2}$  show well pronounced variations. Remarkably, for this range of  $\tau$ , we find all observables to be in excellent agreement with the homogeneous system predictions. This result supports our previous statement that intrinsic local dynamics dominate at short ramp times. For longer ramp times  $\tau > 2\hbar/J$ , we find a clear change in the central density, associated with the onset of particle currents [19]. Naturally, this reduction of the density modifies the particle distribution and compressibility, driving them away from the constant density behavior and close to the adiabatic expectation for the trapped cloud (with slight oscillations). On Fig. 4(a), we show the actual time evolution of the density and compressibility corresponding to a ramp time  $\tau = 20\hbar/J$ . The density configuration remains almost frozen up to time  $t = 5\hbar/J$  even though the corresponding ground state considerably differs, and begins to evolve afterward. At later times, the density profile broadens reaching a wider size than the one expected for the ground state at the corresponding  $U/J$ . At the end of the evolution process, the system is therefore in an excited state. In comparison, the compressibility distribution evolves much faster than the density. At each time step, the compressibility is relatively close to its corresponding ground state value. The difference between the time-evolved and ground-state values are most likely due to the unrelaxed density profile. Consequently, this direct time-analysis of a slow quench reinforces our previous conclusion that the intrinsic evolution of the observables and the density redistribution are characterized by two different timescales.

For case (b) of Fig. 3, the first striking feature is the even

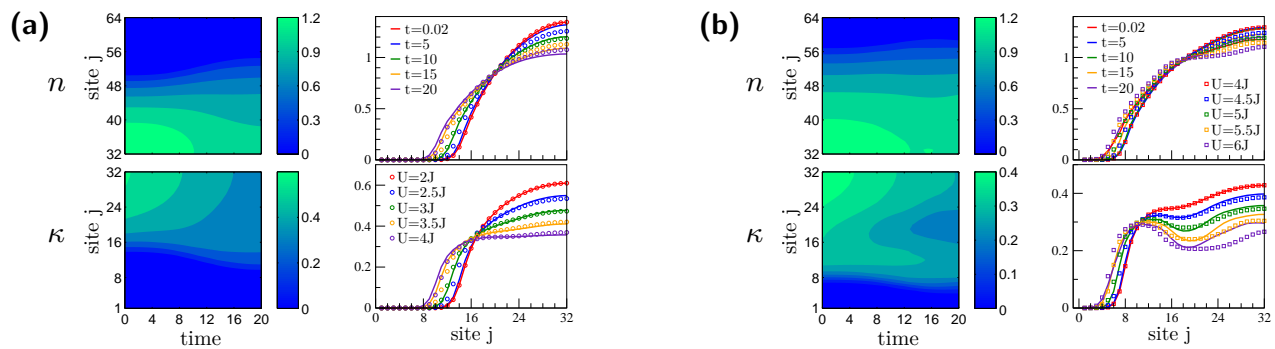


FIG. 4. Slow quenches with  $\tau = 20\hbar/J$ ,  $L = 64$  and  $V = 0.006J$ . **(a)**  $U = (2 \rightarrow 4)J$ , **(b)**  $U = (4 \rightarrow 6)J$ . Contour plots: density  $n$  (up) and compressibility  $\kappa$  (down) as a function of position and time. Graphs:  $n$  (up) and  $\kappa$  (down) vs position for various times  $t$  (full lines). Circles and squares stand for ground-state predictions with corresponding  $U(t)$ .

slower evolution of the central density with increasing ramp times when compared to the situation discussed before. Up to  $\tau \approx 2\hbar/J$ , the density remains very close to its initial value and the other local observables are identical to their corresponding homogeneous value. For longer ramp times, the central density then shows an oscillatory decrease and the other observables deviate from their constant density counterparts. However, even for the longest ramp times considered, the density has still not reached the adiabatic regime. The slowdown of the density rearrangement is attributed to the emergence of Mott “barriers” in regions where  $n_j \approx 1$ . This effect is underlined in the snapshots of Fig. 4(b). These “barriers” arise due to the rapid reduction of particle fluctuations at large  $U$  and the local reduction of transport. On Fig. 4(b), we observe a much slower redistribution of the density than for the  $U_i = (2 \rightarrow 4)J$  evolution in (a). In this case, the density profile is actually narrower than the ground state profile. The compressibility presents the same fast evolution at short time, building “barriers” before the currents can establish and precluding the redistribution from taking place.

In summary, we performed a detailed study of slow quenches in the trapped 1D Bose-Hubbard model. We identified two dynamical regimes. For short ramp times during which currents cannot develop, the intrinsic local dynamics dominates and the response is equivalent to that of an homogeneous system. Many features of this regime are well described by perturbation theory. For longer ramp times, currents do set in and the dynamics is governed by non-trivial transport phenomena. For large final interaction strengths, the global timescales are significantly enhanced by the presence of “Mott barriers”. Our results agree qualitatively well with recent experiments focusing on transport dynamics of cold atoms across the SF–MI transition [6, 8]. Indeed, based on our results, we can argue that the two experiments were probably conducted in different regimes. In Ref. 6, the slowdown of dynamics, due to the presence of Mott regions, indicates that their experiment was carried out in the regime with global dynamics. In contrast, the short timescales observed in Ref. 8

are most likely related to local dynamics.

*Note added:* During the final stages of the preparation of this manuscript, a mean-field study of the 2D version of the model appeared on arXiv [20].

*Acknowledgments* – We acknowledge discussions with P. Barmettler, G. Biroli, C. De Grandi, A. Silva and P. Werner. Support was provided by ANR (FAMOUS), DARPA-OLE, Triangle de la physique and FQRNT.

- 
- [1] J. Dziarmaga, *Advances in Physics* **59**, 1063 (2009).
  - [2] A. Polkovnikov *et al.*, arXiv:1007.5331.
  - [3] T. W. B. Kibble, *J. Phys. A* **9**, 1387 (1976). W. H. Zurek, *Nature* **317**, 505 (1985).
  - [4] M. E. Dodd *et al.*, *Phys. Rev. Lett.* **81**, 3703 (1998).
  - [5] M. Greiner *et al.*, *Nature* **419**, 51 (2002).
  - [6] C.-L. Hung *et al.*, *Phys. Rev. Lett.* **104**, 160403 (2010).
  - [7] J. F. Sherson *et al.*, *Nature* **467**, 68 (2010).
  - [8] W. S. Bakr *et al.*, *Science* **329**, 547 (2010).
  - [9] A. Rapp, S. Mandt, and A. Rosch, *Phys. Rev. Lett.* **105**, 220405 (2010).
  - [10] T. D. Kühner, S. R. White, and H. Monien, *Phys. Rev. B* **61**, 12474 (2000).
  - [11] S. Inouye *et al.*, *Nature* **392**, 151 (1998).
  - [12] C. Trefzger and K. Sengupta, *Phys. Rev. Lett.* **106**, 095702 (2011).
  - [13] A. J. Daley *et al.*, *J. Stat. Mech.*, P04005 (2004); S. R. White, and A. E. Feiguin, *Phys. Rev. Lett.* **93**, 076401 (2004).
  - [14] A finite-size gap  $\propto 1/L$  is present even in the SF phase.
  - [15] C. Kollath, A. M. Läuchli, and E. Altman, *Phys. Rev. Lett.* **98**, 180601 (2007); A. M. Läuchli and C. Kollath, *J. Stat. Mech.*, P05018 (2008); G. Roux, *Phys. Rev. A* **79**, 021608(R) (2009); G. Biroli, C. Kollath, and A. Läuchli, *Phys. Rev. Lett.* **105**, 250401 (2010); G. Roux, *Phys. Rev. A* **81**, 053604 (2010).
  - [16] M. Eckstein and M. Kollar, *New J. Phys.* **12**, 055012 (2010).
  - [17] E. Canovi *et al.*, *J. Stat. Mech.*, P03038 (2009).
  - [18] G. G. Batrouni *et al.*, *Phys. Rev. Lett.* **89**, 117203 (2002).
  - [19] The density oscillations observed in the homogeneous system at large  $\tau$  are due to boundary effects.
  - [20] S. S. Natu, K. R. A. Hazzard, and E. J. Mueller, arXiv:1009.5728; *Phys. Rev. Lett.* **106**, 125301 (2011).