

Measuring optical tunneling times using a Hong–Ou–Mandel interferometer

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Abstract: We report a prediction for the delay measured in an optical tunneling experiment using Hong–Ou–Mandel (HOM) interference, taking into account the Goos–Hänchen shift generalized to frustrated total internal reflection situations. We precisely state assumptions under which the tunneling delay measured by an HOM interferometer can be calculated. We show that, under these assumptions, the measured delay is the group delay, and that it is apparently ‘superluminal’ for sufficiently thick air gaps. We also show how an HOM signal with multiple minima can be obtained, and that the shape of such a signal is not appreciably affected by the presence of the optical tunneling zone, thus ruling out the explanation of the anomalously short tunneling delays in terms of a reshaping of the wavepacket as it goes through the tunneling zone. Finally, we compare the predicted tunneling delay to a relevant classical delay and conclude that our predictions involve no non–causal effect.

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The experimental measurement of the delay associated with quantum tunneling, as well as its theoretical definition and physical interpretation, have been the subject of much controversy. Numerous definitions have been proposed [1], and various experiments performed [2, 3] as part of an effort to remove this ambiguity. A recurrent feature of both predictions and measurements is that quantum tunneling is associated with seemingly ‘superluminal’ delays, *i.e.* delays that appear to correspond to velocities greater than that of light in vacuum (see [4] for a detailed report on the different experiments and their interpretations).

In this work, we make use of the analogy between one-dimensional quantum tunneling and two-dimensional optical systems involving frustrated total internal reflection, in the particular case of the experiment initially proposed by Chiao *et al.* [5]. They suggested using Hong–Ou–Mandel (HOM) two-photon interference [6] to measure the optical tunneling delays in the photon counting regime, which are on the fs time scale. We present, for this experiment, an explicit derivation of both the Goos-Hänchen shift and the delay measured by HOM interference. Analytic formulae are derived for simple cases; however, our derivation, which relies on the expansion of the incident beam as a linear combination of plane waves, is more general and can be applied numerically in more complicated situations. We specifically show that the anomalously short tunneling delays cannot be explained by the suppression of the later part of the HOM signal (reshaping). Finally, we show that, in the regime for which our calculations are valid, the classical delay to which our prediction for the tunneling delay should be compared is shorter than this prediction, and thus no non-causal effect is involved in this experiment.

The optical tunneling zone consists of two identical glass prisms (index of refraction $n \approx 1.5$) facing each other, separated by a gap of air (index 1) of adjustable width d on the μm scale.

HOM interference is a two-photon interference effect: if two indistinguishable photons arrive simultaneously at the input ports of a beamsplitter, a destructive interference occurs which forbids the photons to exit through different output ports. This interference effect makes it possible to compare the optical lengths of the two arms of the interferometer, one of which is known and the other containing the tunneling zone (see Fig. 1): for equal optical delays, no photon-coincidence detections are recorded at the output ports 3 and 4 of the beamsplitter as the two photons cannot exit different ports.

The proposed experiment is to be conducted in the following way. Pairs of correlated photons are created by spontaneous parametric downconversion in a non-linear crystal [7]. They are injected into single-mode optical fibers in order for the photons to be prepared in a specific single spatial mode. One photon of each pair, the ‘unknown’ photon, travels through the tunneling zone, hitting the glass-to-air interface with an angle greater than the critical refraction angle $\theta_c = \arcsin \frac{1}{n}$, and thus having a small probability to undergo optical tunneling. The second, ‘reference’, photon, travels through an adjustable delay line. They are subsequently brought together at a symmetrical beamsplitter at which HOM interference occurs (Fig. 1). First, the two prisms will be brought together ($d = 0$) and the length of the ‘reference’ arm will be set to the value leading to HOM interference. We will then give d a non-zero value. The delay between the two photons is measured as the increase in the delay line length necessary to recover the destructive interference.

We characterise the tunneling zone by its amplitude transmission coefficient t_G , which we have derived for an s -polarized plane wave. This coefficient can be calculated for an evanescent wave with incidence angle θ ($\theta > \theta_c$) in a similar way as for a Fabry–Pérot cavity with a

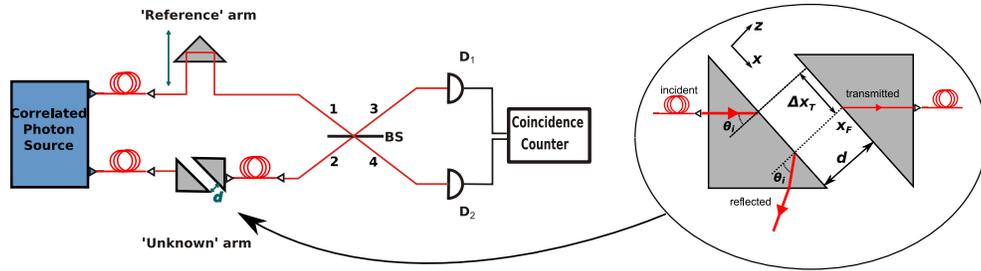


Fig. 1. Left: experimental setup. Inset: schematics of the tunneling zone and the incident, reflected and transmitted beams, taking the generalized Goos–Hänchen shift into account. Note that the width of the air gap, the size of the prisms, and the waist of the beam are not to scale.

propagating wave:

$$t_G = \left(\cosh(d/z_0) - i \sinh(d/z_0) \frac{1 + n^2 \cos(2\theta)}{2n\kappa \cos \theta} \right)^{-1} \quad (1)$$

where $z_0 = \frac{1}{2\pi} \frac{\lambda}{\kappa}$, $\kappa = \sqrt{n^2 \sin^2 \theta - 1}$, and λ is the wavelength in vacuum of the incident plane wave.

We consider the case of a monochromatic Gaussian incident beam, whose wavelength λ_i is in the visible range and whose waist w_0 is on the mm scale, impinging on the air gap with incidence angle θ_i . The evanescent nature of the field inside the gap yields relevant effects only for values of the gap width d which are greater than the characteristic attenuation length z_0 of these evanescent waves. Additionally, we assume $w_0 \gg d/\kappa$, which ensures that the amplitude attenuation of a plane wave crossing the air gap with incidence angle $\theta \approx \theta_i$ is approximately constant over the angular dispersion $\Delta\theta = \frac{\lambda}{w_0}$ of the Gaussian beam. This last assumption, which corresponds to a realistic experimental situation, is essential, and all of the results subsequently presented in this work rely on it. In particular, it is under this assumption that we show that the tunneling delay corresponds to the group delay, and that for sufficiently large gap widths this group delay, which does not depend on w_0 , becomes apparently ‘superluminal’. We are then able to explain why such a short delay is not associated with any non-causal effect. We have not investigated the regime for which this assumption does not hold. Indeed, a transverse distortion of the incident beam as it crosses the tunneling zone would make it much harder to define or measure the tunneling delay.

Our prediction for the optical tunneling delay is qualitatively modified by a spatial shift Δx_T , along the second glass–to–air interface, of the centroid of the transmitted beam with respect to that of the incident beam [8] (*cf.* Fig. 1). This effect is due to the evanescent nature of the waves inside the air gap. It is known as the Goos–Hänchen (GH) shift in total internal reflection situations [9], and it can be generalized to include frustrated total internal reflection (FTIR).

We consider the incident Gaussian beam as a sum of plane waves and apply the transmission coefficient $t_G(\mathbf{k})$ to each plane-wave component. The resulting integral can be evaluated, in the paraxial approximation, using the method of stationary phase, which yields the following analytical expression: $\Delta x_T = -\frac{\lambda_i}{2\pi n \cos \theta_i} \frac{1}{d} \frac{d(\arg t_G)}{d\theta} \Big|_{\theta=\theta_i}$. Incidentally, the relative phase between the transmission and reflection coefficients is constant, therefore the reflected beam is also shifted, by the same amount. The magnitude of this shift is plotted as a function of the gap

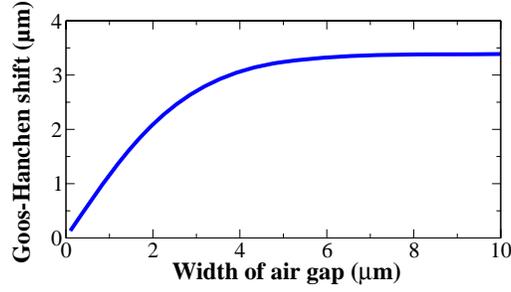


Fig. 2. Magnitude of the Goos-Hänchen shift on the transmitted beam Δx_T as a function of the gap width d , for incident wavelength $\lambda_i = 702$ nm in vacuum, incidence angle $\theta_i = 41.9^\circ$, and prism glass index $n = 1.5$.

width in Fig. 2. For large values of d , this shift saturates to $\frac{\lambda_i}{2\pi n \cos \theta_i} \frac{1}{\sqrt{1 - \frac{1}{n^2 \sin^2 \theta_i}}}$ which is the value predicted for the shift in the reflected beam in a non-frustrated situation [9].

This generalized GH shift is of the order of the wavelength λ_i , *i.e.* on the μm scale. Realistic values for the width d of the air gap, chosen such that intensity transmission through the tunneling zone is finite (albeit small), are of the same magnitude. Consequently, the GH shift has to be taken into account to predict the delay measured using the HOM interferometer.

We now calculate the tunneling delay measured in this experiment, assuming the quantized electromagnetic field produced by the correlated photon source is given by $|\psi\rangle = M|\text{vac}\rangle_1|\text{vac}\rangle_2 + \eta \sum_u e^{-u^2/4\sigma^2} |\omega_i + u\rangle_1 |\omega_i - u\rangle_2$, where $\omega_i = 2\pi \frac{c}{\lambda_i}$, σ characterises the spectral width of the source, $\omega_i + u$ and $\omega_i - u$ are the frequencies of the two photons of a given correlated photon pair, and η is the efficiency of the parametric downconversion process [10]. The positive-frequency field operators at entrance ports 1 and 2 of the beamsplitter can be expressed as

$$\begin{cases} \mathbf{E}_1^{(+)} &= \sum_{\omega} a_1(\omega) e^{-i\omega(t-\tau_1)} \\ \mathbf{E}_2^{(+)} &= \sum_{\omega} t_G(\omega) a_2(\omega) e^{-i\omega(t-\tau_2)} e^{ik_x x_F} \end{cases} \quad (2)$$

where x_F is the position on the (second) air-glass interface corresponding to the position of the fiber-collimator on the output side of the second cube (*cf.* Fig. 1), and $c\tau = c\tau_1 - c\tau_2$ is the difference in optical lengths between the ‘unknown’ and ‘reference’ arms of the interferometer, excluding the actual air gap. Considering the ideal case of a symmetrical beamsplitter, the field operators at exit ports 3 and 4 are given by $\mathbf{E}_{3,4}^{(+)} = \mathbf{E}_1^{(+)} \pm \mathbf{E}_2^{(+)}$. The photon-coincidence signal $S_0(t_0) = \int dt \langle \psi | \mathbf{E}_3^{(-)}(t_0) \mathbf{E}_4^{(-)}(t_0 + t) \mathbf{E}_4^{(+)}(t_0 + t) \mathbf{E}_3^{(+)}(t_0) | \psi \rangle$, integrated over the resolving time of the detectors, reduces to:

$$S = \int_{-\infty}^{+\infty} du e^{-u^2/2\sigma^2} [|t_G(\omega_i + u)|^2 - \text{Re}(t_G(\omega_i + u) t_G^*(\omega_i - u) e^{-2iu\tau})] . \quad (3)$$

We have numerically calculated the photon-coincidence signal directly from Eq. 3 (*cf.* the single HOM dips in Fig. 4). Such a numerical calculation is possible in the general case. Furthermore, if the spectral bandwidth σ of the source is sufficiently narrow, the preceding integral can be calculated using the approximation of stationary phase. We thus obtain:

$$S_0 \propto |t_G(\omega_i)|^2 \left(1 - e^{-2\sigma^2(\tau-\mu)^2} \right) \quad (4)$$

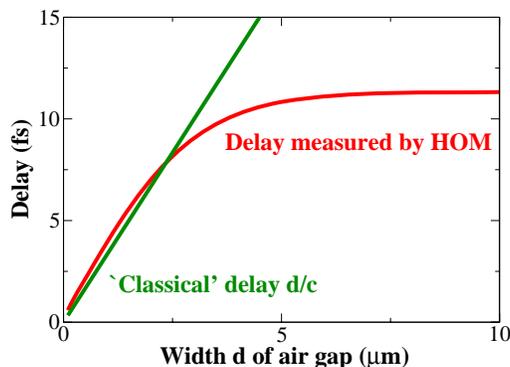


Fig. 3. Predicted delay μ as a function of the gap width d , calculated for incident wavelength $\lambda_i = 702$ nm in vacuum, incidence angle $\theta_i = 41.9^\circ$, and prism glass index $n = 1.5$.

where $\mu = \left. \frac{\partial \arg t}{\partial \omega} \right|_{\omega_i, \theta_i} + \frac{x_F}{c} n \sin \theta_i$. The value of μ is the displacement, on the plot of $S_0(\tau)$, of the minimum of the HOM dip with respect to its position when there is no air gap ($t_G = 1$). Therefore, it can be interpreted as a measurement of the delay of the photons crossing the gap. The expression for this delay depends on x_F , however it is only meaningful if the collecting fiber–collimator is positioned at the point where the transmitted beam is expected to exit the second cube, taking the generalized Goos–Hänchen shift into account: $x_F = \Delta x_T$. One thus obtains the following prediction for the tunneling delay measured in this experiment:

$$\mu = \left. \frac{\partial(\arg t_G)}{\partial \omega} \right|_{\omega=\omega_i} - \frac{1}{\omega_i} \tan \theta_i \left. \frac{\partial \arg t_G}{\partial \theta_i} \right|_{\theta=\theta_i}. \quad (5)$$

This expression for μ involves the derivative of the gap transmission function with respect to frequency, and therefore corresponds to the group–delay. Our calculation thus provides theoretical grounds for Steinberg et al.’s experimental measurement, using an HOM interferometer, of the delay due to tunneling through a 1D photonic band–gap barrier, which they found to agree with the group–delay prediction [3].

The dependence of μ on the thickness d of the air gap is represented in Fig. 3, where it is compared to d/c , the shortest possible delay required to cross the air gap at the speed of light. Our prediction for the delay is greater than d/c for small values of d ; however, it becomes smaller than d/c , and therefore apparently superluminal, for greater values of d . It saturates to a finite value for large values of d [8]. This behaviour is qualitatively similar to Hartman’s prediction for the one–dimensional tunneling time of a massive particle [11].

The method we present can be applied to realistic wavepackets with no restriction on their bandwidth σ , and it allows the calculation of more elaborate two–photon coincidence signals than the single HOM dip. This allows us to investigate whether or not the shape of the coincidence signal is appreciably distorted by the tunneling zone. Were the coincidence signal to undergo a significant reshaping as it crossed the air gap, one might argue that the predicted superluminal delay of the wavepacket peak was simply an artifact of this reshaping.

We will first consider the tunneling of a realistic wavepacket numerically. As is shown in Fig. 4 (curves with single minima), tunneling slightly decreases the HOM contrast $(S_{\max} - S_{\min})/S_{\max}$, and leaves the shape of the dip intact. This suggests that tunneling does affect the shape of the wavepacket, making the photon that has tunneled “different” from the one that has not. To understand if such a change has the same nature as “reshaping” (where the trailing edge

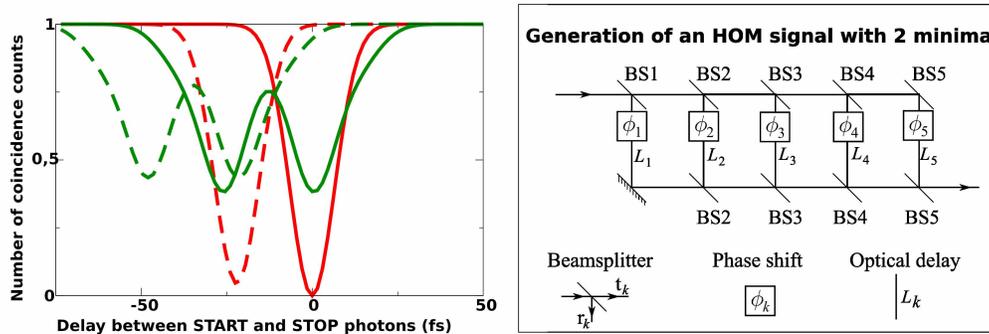


Fig. 4. Left: Coincidence signals with one (red) and two (green) minima, calculated for $d = 10\mu\text{m}$, $\lambda_i = 702\text{nm}$, $\sigma = 20\text{nm}$, $t_W(\omega_i + u) = 0.4 + 0.6\cos(\tau_0 u) + 0.7\sin(\tau_0 u/2)$, and $\tau_0 = 2/\sigma$, both without the tunneling zone (solid lines) — adding the delay d/c (shortest possible delay required to cross the gap) — and with the tunneling zone (dashed lines). The numbers of coincidence counts are normalized to 1 in the non-interfering region; for large values of τ , the ratio between the numbers of counts with and without the air gap is $\approx |t_G(\omega_i, d)|^2 = 10^{-6}$. The signals presented here are the exact numerical results obtained from Eq. 3, hence the contrasts of the incident and transmitted signals are not exactly the same, as they would have been in the stationary phase approximation. Right: Combination of beamsplitters and delay lines that can be used to produce the transmission function t_W . The transmission and reflection coefficients t_k and r_k characterising the beamsplitters, the phase shifts ϕ_k , and the optical delays L_k are chosen such that the interference between the five paths of the optical circuit yields an overall transmission which is proportional to t_W .

of the incident wavepacket is suppressed compared to the leading edge as it undergoes optical tunneling [4]), we study the more complex case of a photon-coincidence signal which has two minima when there is no tunneling zone. We suggest to generate such a signal by adding, to one arm of the correlated photon source, a combination of beamsplitters and delay-lines whose overall transmission function is $t_W(\omega_i + u) = (a + b\cos(\tau_0 u) + c\sin(\frac{\tau_0}{2} u)) e^{i\frac{\tau_0}{2} u}$ (cf. Fig. 4). The corresponding photon-coincidence signal can be numerically calculated from Eq. 3 in which $t_G(\omega)$ is replaced by $t_G(\omega) \cdot t_W(\omega)$. We have performed this calculation both without the tunneling zone ($t_G = 1$) and with the tunneling zone (t_G given by Eq. 1). These signals are represented in Fig. 4. Both exhibit two dips, whose contrast and relative separation in time are affected by the presence of the air gap in the same two-fold way as in the case of a wavepacket with a single peak: the air gap causes (i) an important drop in the transmitted intensity (which is an exponentially decaying function of d), and (ii) a time delay, which corresponds to μ as calculated previously for the single-dipped coincidence signal (cf. Eq. 5 and Fig. 3).

The shape of the photon-coincidence signal not being affected by the presence of the air gap indicates that the shape of a wavepacket having undergone FTIR depends on the whole wavepacket incident on the tunneling zone. Indeed, if the tunneling zone only allowed the transmission of the earlier part of the incident wavepacket and rejected the later part, a photon-coincidence signal which has two minima in the absence of the tunneling zone would have only one in the presence of the tunneling zone. Figure 4 shows this is not the case: the interference signal which has two HOM dips in the absence of the tunneling zone retains two dips in the presence of the tunneling zone. This result confirms that it is not possible to explain the anomalously short tunneling delays in terms of a reshaping of the HOM signal [4]. Our calculation is valid regardless of the temporal width of the pulse. Specifically, it does not require the tunneling to be in a quasi-static regime as defined by Winful [4]: indeed, in the case of Fig. 4, the

temporal width of the coincidence signal is comparable to the width of the air gap, yet there is no change in the shape of the HOM signal.

If the peak intensity transmitted through the tunneling barrier were much greater than the intensity in the leading edge of an initially identical wavepacket propagating at the speed of light [4], one could claim that a non-causal behaviour were observed. Increasing the width of the gap causes the global intensity attenuation to increase, but it does not affect the range of frequencies of the waves which undergo tunneling (stop-band of the tunneling barrier). In the case of FTIR, the global transmitted intensity decays as e^{-2d/z_0} with increasing and large gap widths d , whereas the intensity that arrives at a time d/c before the peak of the pulse decreases as $e^{-2\sigma^2 d^2/c^2}$. For very large gap widths, the preceding criterion would be satisfied, thus apparently making it possible to observe a non-causal effect. This is of course not the case, and this erroneous conclusion reveals the limit of the analogy between true one-dimensional tunneling and FTIR: in the large gap-width regime, the two-dimensional nature of FTIR must be taken into account.

A key feature of FTIR, which is related to the fact that there is no real propagation of light in the gap, is that Fermat's principle cannot be applied because no path linking the input wavefront to the output one achieves a local minimum of the propagation time. Indeed, as is shown in Fig. 5, the optical path length of the ray-optics trajectory displaced a distance h away from the centroid of the incident Gaussian beam is smaller than the one corresponding to the centroid by an amount of the order of κh for $h \gg d$. The upper part of the beam thus contributes to the field at x_F earlier than the part of the beam in the vicinity of the centroid does. Consequently, the spatial width of the beam results in an effective temporal width $\kappa \frac{w_0}{c}$.

One of the assumptions for the calculation of the group delay presented above (which exhibits a saturation for large gap widths) is that the spatial shape of the transmitted beam is not modified by the barrier traversal, which translates as $w_0 \gg d/\kappa$. Therefore, the effective temporal width $\kappa \frac{w_0}{c}$ must be much larger than d/c for our calculation to be valid. The preceding condition replaces, in the case of 2D FTIR, the quasi-static condition for 1D tunneling [4].

The upper part of the incident beam that is farther away from the centroid than $h_0 \sim d/\kappa$, following a virtual classical trajectory as represented on Fig. 5, contributes to the field at x_F earlier than the calculated tunneling time. In the regime for which our calculations are valid ($d > z_0$ and $w_0 \gg d/\kappa$), the total contribution to the intensity of the $h \gtrsim h_0$ part of the incident beam is of the order of $e^{-2d^2/(\kappa^2 w_0^2)}$. This fraction of the incident intensity is larger than the attenuation (of the order of e^{-2d/z_0}) due to the air gap. Therefore, no non-causal effect can be observed in this FTIR experiment and the anomalously short tunneling times can be considered to be due to the contribution of the spatial 'wings' of the incident Gaussian beam.

We have reported a prediction for the delay due to optical tunneling as measured by an HOM interferometer. We have performed exact numerical calculations of the HOM interference signals with and without the tunneling zone, taking into account the Goos-Hänchen shift generalized to frustrated total internal reflection situations. We have derived an analytical expression for the tunneling delay measured in this particular experiment, which corresponds to the group-delay prediction. We have also shown that this delay is not due to a reshaping of the wavepacket as it crosses the tunneling zone, and that it exhibits a saturation with increasing gap widths. We did see that the contrast of the HOM dip decreases when the stationary phase approximation is dropped, indicating that FTIR tunneling distorts the wavepacket. Our analysis of the tunneling time in terms of the group delay relies on the assumption that the incident Gaussian beam has a sufficiently large waist. Under this assumption, we have shown that the relevant classical delay to which our prediction for the optical tunneling delay must be compared is shorter than this prediction, thus ruling out superluminal effects in this experiment.

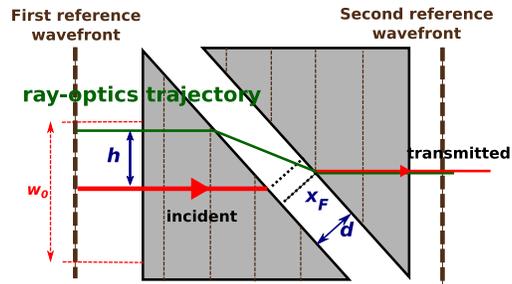


Fig. 5. Ray optics trajectory between a reference wavefront before the first prism and another wavefront in the second prism, which is shorter than our prediction for the tunneling delay. If the waist w_0 of the incident beam is large enough, the distance h of this trajectory to the centroid is such that the corresponding intensity is comparable to that of the transmitted beam.

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