Fluctuations of a Nonequilibrium Interface

In a recent Letter [1] Derrida, Lebowitz, Speer, and Spohn (DLSS) analyze fluctuations of an interface between stationary states of the Toom model. Using boundary conditions that pin an interface of size L at one end, DLSS find diminished height fluctuations ($H^2 \sim L^{2\nu}$) with $\nu < \frac{1}{2}$ for various values of the ratio λ of $(+ \to -/- \to +)$ spin exchange. Based on a coarse-grained Kardar-Parisi-Zhang (KPZ) approach [2], and a heuristic argument whereby temporal fluctuations of an infinite interface ($H^2 \sim t^{2\beta}$) reappear as static fluctuations in the anchored case, DLSS identify ν with β. Although a collective variable approximation supports this identification, their numerical results are somewhat ambiguous [$\nu \approx 0.265$ in the symmetric case ($\lambda = 1$) and $\nu \approx 0.285$ for $\lambda = \frac{1}{4}$]. In this Comment, we study the Toom interface by using an equivalent tagged particle model with periodic boundary conditions and obtain numerical values for β in close agreement with the KPZ values. For $\lambda = \frac{1}{4}$ we find $\beta = 0.325 \pm 0.015$. Our result for the symmetric case is shown in Fig. 1.

In treating the symmetric case, DLSS neglect the effect of a cubic nonlinearity. Using dynamic renormalization group (RG) techniques [2], we show this term [$v_3(\partial_x h)^3$ in Eq. (13) of Ref. [1(a)]] is marginally irrelevant, leading to logarithmic corrections $H^2 \sim [t(\ln t)^{1/2}]^{1/2}$ at the linear fixed point. This RG result agrees with a coupled-mode result [3]; also, both the numerical data shown in Fig. 1 and that of DLSS for $L > 1000$ are consistent with a logarithmic correction. For the latter case, we suggest that asymptotically $H^2 \sim \{L[\ln(L/v_1)]^{1/2}\}^{1/2}$, where $v_1 = 8$ as explained below.

In the low-noise limit, Toom interface dynamics is equivalent to the dynamics of a system of hard-core particles with a vertical link representing a particle and a horizontal link a hole. The ordering of particles is preserved by using multiparticle correlated hops. Periodic boundary conditions allow a steady-state current $J = \rho/(1-\rho) - \lambda(1-\rho)/\rho$ to flow, where ρ is the density of particles. Starting with a state in which each site is occupied independently with probability ρ, the dynamics is dominated by density wave fluctuations that arise due to the motion of the initial statistical inhomogeneities through the system. A hydrodynamic argument [4] based on particle conservation determines the average velocity of density fluctuations to be $v_1 = \partial J/\partial\rho$ ($=8$ for $\lambda = 1$, $\rho = \frac{1}{2}$). A coarse-graining procedure allows us to determine higher-order gradient expansion terms $v_q(\partial_x h)^q$ in a KPZ development [1], e.g., $v_2 = \rho\,\partial^2 J/\partial\rho^2$. Numerically we monitor height fluctuations $H^2(w) = \langle [y_{n'}(t) - y_n(0) - wt/\rho]^2 \rangle$, where y_n is the position of the nth particle, $n' = n + (w - J)t$ is a sliding tag [5], and $\langle \cdots \rangle$ indicates an average over particles. The Galilean shift to remove $v_1 \partial_x h$ is accomplished by choosing $w = w_1 \equiv \rho v_1$, at which point subdiffusive spreading of the fluctuations appears, as shown in the inset.

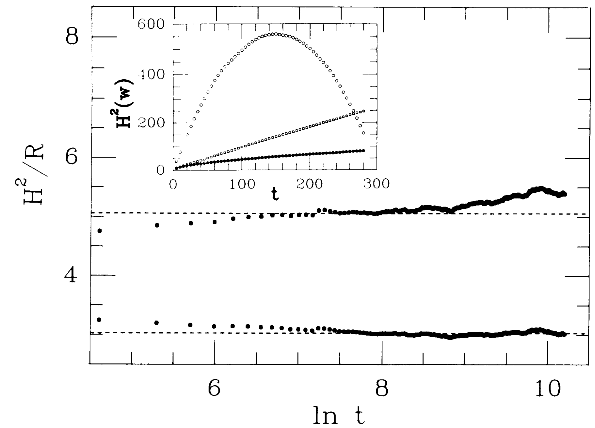

FIG. 1. Monte Carlo results for $H \equiv H(w_1)$ at the linear fixed point ($\lambda = 1$, $\rho = \frac{1}{2}$) indicate logarithmic corrections. The factor R is $[t(\ln t)^{1/2}]^{1/2}$ for the lower curve and $t^{1/2}$ for the upper one. Points are averages over 27 runs in a periodic system with $L = 240000$. Errors range from 2% (at left) to 8% (at right). Inset: The effect of varying the tag-sliding rate w in a smaller system, $L = 2400$: $w = 0$ (open circles), $w = 3.5$ (open squares), $w = w_1 = 4$ (solid circles).

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