

Experimental measurement of the persistence exponent of the planar Ising model

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Using a twisted nematic liquid crystal system exhibiting planar Ising model dynamics, we have measured the scaling exponent θ , which characterizes the time evolution $p(t) \sim t^{-\theta}$, of the probability $p(t)$ that the local order parameter has not switched its state by the time t . For 0.4 sec–200 sec following the phase quench, the system exhibits scaling behavior and, measured over this interval, $\theta=0.19 \pm 0.03$, in good agreement with theoretical analysis and numerical simulations [S1063-651X(97)51007-5]

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There has been a recent surge of interest in determining the so-called nontrivial “persistence” exponent θ [1–11] associated with the dynamics of a phase ordering system following a quench from the high-temperature phase to zero temperature. This exponent describes the asymptotic power-law decay $p(t) \sim t^{-\theta}$ of the probability that the local order parameter $\phi(\mathbf{x}, t)$ has not changed sign up to time t after the quench. For the Ising model, $p(t)$ is simply the fraction of spins that have not flipped up to time t , or, equivalently, the probability that no interface has ever crossed a given spin. This power-law decay of persistence is quite ubiquitous and appears in many other contexts, such as simple diffusion equations [12], reaction-diffusion systems [13–15], driven diffusive systems [16], and the decay of total magnetization following a quench to the critical point of spin systems [12]. While there have been considerable theoretical and numerical efforts directed towards calculating θ , few experimental measurements of this exponent have been made. A kind of persistence exponent was first introduced and measured in a breath figure experiment [1], and recently for soap bubbles [17] (in this latter example θ takes a rather trivial value, which can be inferred on simple physical grounds). In this Rapid Communication, we provide an experimental measurement of the nontrivial persistence exponent for a liquid crystal system exhibiting Ising-like behavior. We find good agreement with recent theoretical calculations.

Liquid crystals provide a useful laboratory for testing various scaling predictions of phase ordering kinetics. They have been used earlier to measure the growth of the characteristic scaling length $L(t)$, in both three-dimensional [18–21] and two-dimensional [22–26] systems. A twisted nematic system, exhibiting $T=0$ Ising-like behavior, is employed here for the measurement of θ . This system has been previously used by Orihara and co-workers [22,23] to study the behavior of the scaling length $L(t)$ and, in addition, by Mason, Pargellis, and Yurke [27] to measure the scaling exponent λ for the two-time correlation function. These studies used a liquid crystal sample placed between two glass

plates whose surfaces had been treated so that, after a thermal quench from the isotropic phase to the nematic phase, the liquid crystal organized itself into domains in which the director was forced to twist clockwise or counter-clockwise by $\pi/2$ in going from the surface of one plate to the other. The boundary between two domains of opposite twist consists of a twist disclination line. Regions of opposite twist correspond to Ising model domains in which the spins all point up or all point down. The system relaxes viscously, driven by the line tension of the disclination lines. At late stages of coarsening, the morphology of the growing domains seems to exhibit a self-similar structure in time once all length scales are rescaled by $L(t)$, where $L(t)$ represents the typical linear dimension of a growing domain and grows algebraically with time $L(t) \sim t^{1/2}$. This growth law can be simply understood by noting that the motion of the domain walls between the opposite phases is purely curvature driven [28].

So far most of the theoretical and experimental efforts have been directed towards determining the late time scaling properties of the equal-time and two-time correlation functions [18]. These functions were predicted to have scaling forms, $\langle \phi(\mathbf{0}, t) \phi(\mathbf{r}, t) \rangle \sim g[r/L(t)]$ and $\langle \phi(\mathbf{0}, t') \phi(\mathbf{r}, t) \rangle \sim [L(t)/L(t')]^{-\lambda} f[r/L(t), L(t)/L(t')]$. The autocorrelation exponent λ in two dimensions was measured experimentally only recently [27], using the liquid crystal sample described below. The experimental value of λ in $d=2$ was in good agreement with the theoretical prediction [29]. However these correlation functions do not give much information about the history of the evolution process. The simplest and most natural probe to the history seems to be the “persistence,” i.e., the probability $p(t) \sim t^{-\theta} \sim [L(t)]^{-2\theta}$ that a given spin does not flip up to time t .

This fraction of unflipped spins $p(t)$, though relatively simple to measure in numerical simulations [3,7,4,10], has been found to be nontrivial to compute theoretically. The temporal evolution of an individual spin is a non-Gaussian, non-Markovian process, and naturally, any history dependent nonlocal quantity is very hard to compute for such a process. While in $d=1$ the exact value $\theta=3/8$ has recently been determined by Derrida, Hakims, and Pasquier [6], their method unfortunately cannot be extended to higher experimentally relevant dimensions. For the $d=2$ spin flip dynamics of the

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Ising model, only numerical estimates of the exponent $\theta \approx 0.22$ [3,4,8,10] have been available to date. However, recently an approximate analytical method was developed by two of us [10] to compute θ in any dimension by exploiting a mapping to a quantum mechanics problem. For $d=1$, our approximate theory [10] yields $\theta \approx 0.35$, as compared to the exact value $\theta = 3/8$ [6], and for $d=2$, we found $\theta \approx 0.19$. This last result is also consistent with the estimate $\theta \approx 0.186$ obtained by putting $d=2$ in the large d theory of coarsening [11] and, moreover, in recent direct simulations of the time-dependent Ginzburg-Landau (TDGL) equation (model A) [30]. It is this result that we test experimentally. The data collected for this report was extracted from video tape recordings obtained in an earlier Ising system study of the λ exponent [27].

The experimental technique used for the λ measurements has been described in detail previously. Briefly, the sample cell consists of a layer of liquid crystal, 20 μm thick, sandwiched between two parallel glass microscope slides. The liquid crystal used was trans-(trans)-4-methoxy-4'-*n*-pentyl-1,1'-bicyclohexyl (Merck, CCH-501, or equivalently ZLI-3005), with an isotropic-to-nematic phase transition of 37 °C. The molecular orientation at each slide's surface was forced to be in the plane of the surface and oriented unidirectionally. The surfaces of the glass plates were prepared by first dipping them into a 0.1% by weight solution of polyvinyl alcohol and then buffing the surface in one direction with a soft cloth. The two slides were mounted orthogonally to each other, thus defining a square region of liquid crystal, about 2.5 cm on a side. The orthogonal orientation of the plates with respect to one another forces the nematic liquid crystal to phase separate into two domains, as the molecular orientation undergoes either a clockwise or counterclockwise twist of $\pi/2$ radians as one goes from one plate to the other. The circular dichroic effect was used to make the domains of opposite twist visible. A square region, 1.5 mm on a side, was observed using a Nikon E Plan 4× objective and recording the images with a high-speed color recording system (NAC, HSV-400). Images were recorded every 5 ms, each frame labeled at the top with the run number and time since the recording began.

For a well percolated system undergoing self-similar coarsening, the ratio of the area occupied by domains of one particular twist to the total area as a function of time should remain constant and have the value 0.5. This ratio, averaged over the 15 runs discussed here, was 0.375 at the time of the quench, reaching within only 0.2 sec the value of 0.576 ± 0.016 , at which it remained for the 200 sec time interval over which data was recorded. There are several possible reasons for the deviation of this ratio from the value of 0.5. The region between domains over which the director field is distorted from the pure clockwise or counterclockwise twist is comparable to the thickness of the cell, 20 μm . This makes the optical image of the boundary between domains somewhat ill defined. Hence, optical effects could cause us to preferentially choose one domain over the other when deciding whether a given point belongs to a region of clockwise or counterclockwise twist. This would be particularly serious at early times and probably accounts for the low value (0.375) of the ratio measured at early times. The ratio at late times is 15% higher than the expected value of 0.5 and may be due to

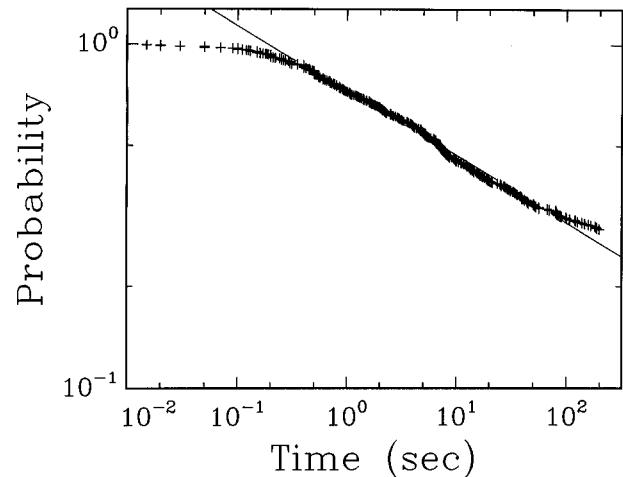


FIG. 1. Probability $p(t)$ vs time that an individual point in the system has not switched its state at least once by time t . The solid line is a least-squares fit to the data and has a slope of 0.19.

a small bias in the domain nucleation, domain growth favoring domains of one particular twist, and, at very late times, small sample statistics. For example, if alignment of the upper and lower plates deviates from perfect orthogonality, one twist direction will be favored over the other [23].

We measured, for each of 40 points (in an ordered, rectangular array) in each of 15 runs [31], the time t after the quench when the molecular orientation first switched from one twist direction to the other. The contrast was sufficient to enable us to distinguish the orientation at a particular point immediately after the phase transition. Figure 1 is a log-log plot showing the scaling of the probability $p(t)$ that an individual point has not switched its orientation by time t . A least-squares fit to the data in the range 0.4–200 sec gives a slope $\theta = 0.190 \pm 0.031$ shown by the solid line. The initial approach to scaling is most likely due to the initial nonzero size of the scale length L at the time of the quench and the identification bias resulting from the diffuse nature of the optical image of the domain boundaries. This initial scale length is on the order of the thickness of the cell (20 μm), resulting in a minimum domain size at the time of the quench. Also noticeable at late times is a “tail,” of reduced slope, in $p(t)$. The tail at late times could be due to several different effects: pinning of domain walls in the sample immediately outside of the region observed or nonorthogonal orientation of the glass plates (giving rise to a preferred molecular twist). The wiggles in the data are finite sample size effects reflecting the history of how macroscopic regions changed their orientation in particular runs.

In conclusion, our measured value of the persistence exponent $\theta = 0.190 \pm 0.031$ is in good agreement with the theoretical estimate $\theta \approx 0.19$ for an *isotropic* bidimensional spin system.

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- [31] Forty points were used in all except one run, where an imperfection in one corner pinned the domain wall. In this case, the 12 points were eliminated that were in the immediate vicinity of the site.