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Feynman's proof of the Maxwell equations

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Feynman's proof of the Maxwell equations, discovered in 1948 but never published, is here put on record, together with some editorial comments to put the proof into its historical context.

I. THE PROOF

As I mentioned in my talk at the Feynman Memorial Session of the AAAS meeting in San Francisco,¹ Feynman showed me in October 1948 a proof of the Maxwell equations, assuming only Newton's law of motion and the commutation relation between position and velocity for a single nonrelativistic particle. In response to many enquiries, I here publish the proof in a form as close as I can come to Feynman's 1948 exposition. Unfortunately, I preserved neither Feynman's manuscript nor my original notes. What follows is a version reconstructed at some unknown time from notes which I discarded.

Assume a particle exists with position x_j ($j = 1, 2, 3$) and velocity \dot{x}_j satisfying Newton's equation

$$m\ddot{x}_j = F_j(x, \dot{x}, t), \quad (1)$$

with commutation relations

$$[x_j, x_k] = 0, \quad (2)$$

$$m[x_j, \dot{x}_k] = i\hbar \delta_{jk}. \quad (3)$$

Then there exist fields $E(x, t)$ and $H(x, t)$ satisfying the Lorentz force equation

$$F_j = E_j + \epsilon_{jkl} \dot{x}_k H_l, \quad (4)$$

and the Maxwell equations

$$\text{div } H = 0, \quad (5)$$

$$\frac{\partial H}{\partial t} + \text{curl } E = 0. \quad (6)$$

Remark: The other two Maxwell equations,

$$\text{div } E = 4\pi\rho, \quad (7)$$

$$\frac{\partial E}{\partial t} - \text{curl } H = 4\pi j, \quad (8)$$

merely define the external charge and current densities ρ and j .

Proof: Equations (1) and (3) imply

$$[x_j, F_k] + m[\dot{x}_j, \dot{x}_k] = 0. \quad (9)$$

The Jacobi identity

$$[x_l, [\dot{x}_j, \dot{x}_k]] + [\dot{x}_j, [\dot{x}_k, x_l]] + [\dot{x}_k, [x_l, \dot{x}_j]] = 0 \quad (10)$$

with (3) and (9) implies

$$[x_l, [x_j, F_k]] = 0. \quad (11)$$

Equation (9) also implies

$$[x_j, F_k] = -[x_k, F_j], \quad (12)$$

and therefore we may write

$$[x_j, F_k] = - (i\hbar/m) \epsilon_{jkl} H_l. \quad (13)$$

Equation (13) is the definition of the field H , which would in general depend on x , \dot{x} , and t . But Eq. (11) says

$$[x_l, H_m] = 0, \quad (14)$$

which means that H is a function of x and t only.

Next we satisfy (4) by assuming it to be the definition of the field E . Again, E will in general depend on x , \dot{x} , and t , but Eqs. (3), (13), and (14) imply

$$[x_m, E_j] = 0, \quad (15)$$

which says that E is a function of x and t only.

The definition (13) of H may be written

$$H_l = - (im^2/2\hbar) \epsilon_{jkl} [\dot{x}_j, \dot{x}_k] \quad (16)$$

by virtue of (9). Another application of the Jacobi identity gives

$$\epsilon_{jkl} [\dot{x}_l, [\dot{x}_j, \dot{x}_k]] = 0. \quad (17)$$

Equations (16) and (17) imply

$$[\dot{x}_l, H_l] = 0, \quad (18)$$

which is equivalent to (5). It remains to prove the second Maxwell equation (6).

Take the total derivative of Eq. (16) with respect to time. This gives

$$\frac{\partial H_l}{\partial t} + \dot{x}_m \frac{\partial H_l}{\partial x_m} = - \frac{im^2}{\hbar} \epsilon_{jkl} [\ddot{x}_j, \dot{x}_k]. \quad (19)$$

Now by (1) and (4), the right side of (19) becomes

$$\begin{aligned} & - (im/\hbar) \epsilon_{jkl} [E_j + \epsilon_{jmn} \dot{x}_m H_n, \dot{x}_k] \\ & = - (im/\hbar) (\epsilon_{jkl} [E_j, \dot{x}_k] + [\dot{x}_k H_l, \dot{x}_k] - [\dot{x}_l H_k, \dot{x}_k]) \\ & = \epsilon_{jkl} \frac{\partial E_j}{\partial x_k} + \dot{x}_k \frac{\partial H_l}{\partial x_k} - \dot{x}_l \frac{\partial H_k}{\partial x_k} \\ & \quad + (im/\hbar) H_k [\dot{x}_j, \dot{x}_k]. \end{aligned} \quad (20)$$

On the right side of Eq. (20), the last term is zero by symmetry because of (16), the third term is zero because of (5), and the second term is equal to the second term on the left of (19). The remaining terms in Eqs. (19) and (20) give

$$\frac{\partial H_l}{\partial t} = \epsilon_{jkl} \frac{\partial E_j}{\partial x_k}, \quad (21)$$

which is equivalent to (6). End of proof.

II. EDITORIAL COMMENT

When I show this proof to young physicists educated in the 1980s, their response is usually disparaging. They say the result is trivial and the proof unnecessarily complicated. It is therefore incumbent on me to explain why the result is not trivial and why Feynman chose to prove it the hard way. To understand the motivation for the proof, it is essential to put it into a historical context. The young physicists of today are as far removed from the Feynman of 1948 as Feynman was then removed from Planck and Einstein.

The argument of the young physicists is simple.² We know, they say, the commutation relation between position and momentum:

$$[x_j, p_k] = i\hbar \delta_{jk}. \quad (22)$$

If we define a vector potential A_k by

$$p_k = m\dot{x}_k + A_k, \quad (23)$$

then the two commutation relations (3) and (22) together give

$$[x_j, A_k] = 0. \quad (24)$$

Therefore, the vector potential A_k is independent of velocity, and depends only on x and t .

We also know, they say, that the momentum and velocity of a particle are related by the equations of Lagrange:

$$p_k = \frac{\partial L}{\partial \dot{x}_k}, \quad (25)$$

$$\dot{x}_k = \frac{\partial L}{\partial p_k}, \quad (26)$$

where

$$L = L(x, \dot{x}, t) \quad (27)$$

is the Lagrangian. If we integrate (25) using (23), the result is

$$L = \frac{1}{2} m \dot{x}_k \dot{x}_k + \dot{x}_k A_k + \varphi, \quad (28)$$

where φ is also independent of velocity. The scalar potential φ is defined by (28). If we now differentiate (23) using (26) and (28), the result is Newton's equation (1) with the Lorentz force (4), the fields E and H being defined by the standard expressions

$$H = \text{curl } A, \quad E = \text{grad } \varphi - \frac{\partial A}{\partial t}. \quad (29)$$

The Maxwell equations (5) and (6) follow trivially from (29). End of proof. So, the young physicists say, what is the big deal? From a modern point of view, the assumption of Feynman's commutation rule (3) implies immediately the existence of a vector potential, and as soon as you have a vector potential you also have a Maxwell field.

Feynman's point of view was quite different. In 1948 he was still doubting all the accepted dogmas of quantum mechanics. He was exploring possible alternatives to the standard theory. His motivation was to discover a new theory, not to reinvent the old one. He was well aware that, if he assumed the existence of a momentum p_k satisfying the commutation rule (22) in addition to (3), he would only recover the standard formalism of electrodynamics. That was not his purpose. His purpose was to explore as widely as possible the universe of particle dynamics. He wanted to make as few assumptions as he could. In particular, he wanted to avoid assuming the existence of momentum and Lagrangian related by (25) and (26). He chose his starting assumptions (1), (2), and (3) because they appeared to be less restrictive than the standard assumptions (22), (25), and (26). He hoped that by going along this road he might be led to new physics. He hoped to find physical models that would not be describable in terms of ordinary Lagrangians and Hamiltonians.

Feynman in 1948 was not alone in trying to build theories outside the framework of conventional physics. At that time many of the greatest physicists, including Yukawa,³ Born,⁴ and Heisenberg,⁵ were pursuing programs for the radical reform of physics. All these radical programs, including Feynman's, failed. But Feynman was the only one who thoroughly tested his program before rushing into print. His proof of the Maxwell equations was a demonstration that his program had failed. The proof showed him

that his assumptions (1), (2), and (3) were not leading to new physics. The road that he had been exploring was a dead end. From Feynman's point of view, the proof was a failure, not a success. That is why he was not interested in publishing it.

I venture to disagree with Feynman now, as I often did while he was alive. I still believe that his proof is worth publishing. It is not only a historical relic of a failed program. It also raises some new questions. The Maxwell equations are relativistically invariant, while the Newtonian assumptions (1), (2), and (3), which Feynman used for his proof, are nonrelativistic. The proof begins with assumptions invariant under Galilean transformations and ends with equations invariant under Lorentz transformations. How could this have happened? After all, it was the incompatibility between Galilean mechanics and Maxwell electrodynamics that led Einstein to special relativity in 1905. Yet here we find Galilean mechanics and Maxwell

equations coexisting peacefully. Perhaps it was lucky that Einstein had not seen Feynman's proof when he started to think about relativity.

ACKNOWLEDGMENT

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Nitrogen temperature superconducting ring experiment

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A student experiment is described for studying persistent currents in a commercially obtained ring of the "123" superconducting material at liquid-nitrogen temperature. The currents are easily detected with a standard analog Hall probe. From observations extended over a 3-week period, an upper limit on the possible resistance of one such ring was set at about $2 \times 10^{-16} \Omega$. For the rings studied, the induced current saturated at about 2 A as the applied flux change was increased. An ac technique for checking the continuity of the superconducting path around the ring is also described. These experiments provide an interesting supplement for topics in first-year electricity and magnetism. The effects are striking and easily discussed at an introductory level. For example, the current induced by turning the ring over in the Earth's field is readily seen.

I. INTRODUCTION

The superconducting ring experiment of H. Kamerlingh Onnes¹ is a landmark in the physics of the last 100 years. With the discovery^{2,3} of the new high- T_c superconductors, the experiment is easily adapted for classroom use. The "persistent current" effect is certainly the most sensitive indicator of the perfect conductivity—a fact which can be well appreciated by first-year students. It is a useful supplement to basic treatments of electromagnetism as it emphasizes fundamental principles such as Faraday induction and Lenz' law, conductivity, inductance, and the Biot-Savart law. At the same time, it is exciting, as it deals with materials and to an extent with issues currently under study around the world.

The ring experiment has the advantage of *not* requiring an extensive background in superconductivity, although for those who wish to learn more, many general references are available such as the books by Schoenberg⁴ and Tinkham.⁵ The technical literature dealing with the new materials has also been reviewed recently.⁶

Our philosophy has been to provide an approach that is as simple and generally doable as possible. The "ring" (with its drilled hole) was provided to us commercially⁷ out of the "123" ceramic ($Y_1Ba_2Cu_3O_{7-\delta}$). The ring dimensions⁷ (0.82-in. outer diameter, 0.26-in. hole diameter) were dictated by the practical requirement that it be possible to drill the hole without cracking the outside. Hence, the radial ring width was about equal to the hole diameter. In most of our experiments, the "ring current" was detected through the magnetic field it produced at a point 7.7 mm below the ring center. Since the form of the current distribution over the ring was not known, the ratio of the measured field to the total ring current could not be calculated very accurately, although it could be estimated rather well. The technique should then be described as "semiquantitative."

Section II describes qualitative observations of the ring current. These experiments are striking, easily followed at an introductory level, and can be done either as lecture demonstrations or by small student groups in the laboratory. This provides an exciting accompaniment to standard

Comment on "Feynman's proof of the Maxwell equations," by F. J. Dyson [Am. J. Phys. 58, 209-211 (1990)]

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Dyson¹ shows that Feynman was able to derive equations for the electromagnetic field and the Lorentz force from Newton's second law of motion using only the commutation relation between position and velocity. These equations for the electromagnetic field are claimed to be equivalent to Maxwell's equations, which are of course Lorentz invariant, even though Newton's law is Galilean invariant. How can this be?

In fact, Dyson only proves the two source-free Maxwell equations,

$$\text{div } \mathbf{H} = 0, \tag{1}$$

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t}. \tag{2}$$

The remaining two Maxwell equations are claimed to be definitions of charge density ρ and current density \mathbf{j} . This provides the clue to the paradox. Can there be another definition of ρ and \mathbf{j} which allows the theory to be Galilean invariant?

Le Bellac and Levy-Leblond² studied some time ago Galilean-invariant theories of electromagnetism. (Maxwell's equations are, of course, not Galilean invariant.) Le Bellac and Levy-Leblond show that a Galilean invariant theory of electromagnetism requires that one of the following two hypotheses must be dropped:

(i) the continuity equation $\text{div } \mathbf{j} = -\frac{\partial \rho}{\partial t} \neq 0,$

(ii) magnetic forces between electric currents.

So in order to keep Galilean invariance, we can still define ρ by

$$\text{div } \mathbf{E} = 4\pi\rho \tag{3}$$

as usual, but we should take as the new definition of \mathbf{j}

$$\text{curl } \mathbf{H} = 4\pi\mathbf{j} \tag{4}$$

with no displacement current. This is the version of Galilean electromagnetism called the magnetic limit in Ref. 2. In particular, only stationary currents satisfying

$$\text{div } \mathbf{j} = 0 \tag{5}$$

are allowed in this version of Galilean-invariant electromagnetism.

It is not very well known that Levy-Leblond studied Galilean-invariant quantum theories of arbitrary spin.³ He was able to show that a Galilean-invariant theory of a charged spin-1/2 particle has a gyromagnetic ratio $g = 2$. Therefore, the spin and magnetic moment of an electron are not consequences of either relativity or the Dirac equation, contrary to what is claimed in most textbooks.

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I should like to thank David Waxman for pointing out Dyson's article to me and Gabriel Barton for insisting that what I wrote should be intelligible.

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Comment on "Feynman's proof of the Maxwell equations," by F. Dyson [Am. J. Phys. 58, 209-211 (1990)]

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Freeman Dyson's statement in his recent interesting and noteworthy paper¹ that the source equations of Maxwell

$$\nabla \cdot \mathbf{E} = \tilde{\rho}/\epsilon \tag{1a}$$

and

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{c^2 \partial t} = \mu \mathbf{J}, \tag{1b}$$

"merely define the external charge and current densities, $\tilde{\rho}$ and \mathbf{J} ," I think diminishes the importance of these equations in establishing the dynamical character of the electromagnetic field.

To justify this opinion, permit me a brief summary of the foundations of classical electromagnetic theory: The relativistic Lagrangian density \mathcal{L} from which Maxwell's

equations and the ponderomotive equations of motion are derived is

$$\mathcal{L} = \lambda u_\sigma u^\sigma + \rho u_\sigma A^\sigma + \kappa F_{\sigma\tau} F^{\sigma\tau}. \quad (2)$$

Here, u_σ is the proper velocity of the matter whose invariant mass and charge densities are λ and ρ , A^σ is the vector potential of the electromagnetic field, $\kappa = -1/4\mu$, and $F_{\sigma\tau}$ is the field itself:

$$F_{\sigma\tau} = \frac{\partial A_\tau}{\partial x^\sigma} - \frac{\partial A_\sigma}{\partial x^\tau}. \quad (3)$$

The relationships connecting $F_{\sigma\tau}$ with the more familiar \mathbf{E} and \mathbf{B} are

$$E_i = cF_{i0} \text{ and } B_i = \frac{1}{2} \epsilon_{ijk} F^{jk}. \quad (4)$$

The variation of the action S , where $S = \int \mathcal{L} dx dy dz c dt$, relative to the world line of charged, massive matter produces the ponderomotive equation,

$$\lambda \frac{d^2 x^\mu}{d\tau^2} = \rho F^{\mu\nu} u_\nu, \quad (5)$$

where $F^{\mu\nu}$ is given by Eq. (3). The first set of Maxwell's equations, which Dyson addressed,

$$\nabla \cdot \mathbf{B} = 0, \quad (6a)$$

and

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \quad (6b)$$

are an identity satisfied by certain space-time derivatives of $F^{\mu\nu}$ in virtue of the definition of Eq. (3), namely,

$$\frac{\partial F_{\mu\nu}}{\partial x^\sigma} + \frac{\partial F_{\nu\sigma}}{\partial x^\mu} + \frac{\partial F_{\sigma\mu}}{\partial x^\nu} = 0. \quad (6c)$$

The variation of the action S relative to the vector field A^μ and its space-time gradients yields the second set of

Maxwell's equations, the source equations, Eqs. (1), in which $\tilde{\rho} = \rho u^0/c$.

The choice of the Lagrangian density of Eq. (2) is not by mere whim. Rather, the choice reflects an attempt to endow as much dynamical character to the field as common sense allows, all the time maintaining simplicity and, of course, relativistic covariance. The first term, $\lambda u_\sigma u^\sigma$, endows matter with inertia and is responsible for the familiar product of mass and acceleration. The second term, $\rho u_\sigma A^\sigma$, couples the field and matter in the simplest possible relativistically invariant fashion. With respect to matter, it yields the force of interaction between particle and field appearing in the ponderomotive equation, Eq. (5). Just as importantly, it suggests that the vector field A^σ may be regarded as a *dynamical variable*. We place this variable on par with *displacement* since it does not arise from the derivative of a more fundamental field.

But as a description of a Lagrangian density for the field itself, $\rho u_\sigma A^\sigma$ is incomplete because, if the field is to be endowed with a dynamical character (without which waves could not be propagated), a term corresponding to the inertial term $\lambda u_\sigma u^\sigma$ must be introduced. Clearly a derivative of the vector field, analogous to *velocity*, should appear as a scalar in the Lagrangian density to produce such an inertial effect. The space-time curl of the vector field, Eq. (3), is chosen in order to maintain gauge invariance. Thus to \mathcal{L} we add the term $\kappa F_{\mu\nu} F^{\mu\nu}$.

In this way the left sides of Maxwell's source equations, Eqs. (1), are analogous to the product of mass and acceleration while the right sides may be thought of as a force of interaction with matter.

¹F. Dyson, "Feynman's proof of the Maxwell equations," *Am. J. Phys.* 58, 209-211 (1990).

Comment on "Feynman's proof of the Maxwell equations," by Freeman J. Dyson [Am. J. Phys. 58, 209-211 (1990)]

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In his reconstruction of Feynman's derivation of the Maxwell equations starting with Newton's equations of motion for a particle in operator form and assumed commutation relations between the position and velocity of the particle, Dyson remarks that the two Maxwell equations

$$\text{div } \mathbf{E} = 4\pi\rho, \quad (1)$$

$$\partial_t \mathbf{E} - \text{curl } \mathbf{H} = 4\pi\mathbf{j}, \quad (2)$$

which are not obtained from the Feynman derivation "merely define the external charge and current densities ρ and \mathbf{j} ." Far from being a mere definition we would argue that (1) is the heart of the Maxwell equations since it is equivalent to Coulomb's law. One could equally well have

taken, as definitions of ρ and \mathbf{j}

$$\text{div}(\mathbf{E}/\sqrt{1-E^2}) = 4\pi\rho \quad (3)$$

and

$$\partial_t(\mathbf{E}/\sqrt{1-E^2}) - \text{curl } \mathbf{H} = 4\pi\mathbf{j}, \quad (4)$$

which would not yield Coulomb's law for a point source. We conclude that the reconstruction given by Dyson is not a derivation of Maxwell's equations.

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Comment on "Feynman's proof of the Maxwell equations," by Freeman J. Dyson [Am. J. Phys. 58, 209–211 (1990)]

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Concerning his reconstruction of Feynman's proof of the Maxwell equations, Dyson asks how it could have happened that the proof begins with assumptions invariant under Galilean transformations and ends with equations invariant under Lorentz transformations. The answer is that the proof deals with only two of the four Maxwell equations.

Starting from the Galilean-invariant assumptions of Newton's equation $m\ddot{x}_j = F_j(x, \dot{x}, t)$ and of the commutation relations $[x_j, x_k] = 0$ and $m[x_j, \dot{x}_k] = i\hbar \delta_{jk}$, Dyson shows how to introduce a quantity $H(x, t)$ that satisfies the Maxwell equation,

$$\text{div } H = 0. \quad (1)$$

It is easy to see that the quantity $H(x, t)$, derived solely from Galilean-invariant equations, is itself Galilean invariant and has a Galilean-invariant divergence. Thus the Maxwell equation (1) is Galilean invariant.

Dyson introduces the quantity $E(x, t)$ by way of the Lorentz-force equation,

$$F_j = E_j + \epsilon_{jkl} \dot{x}_k H_l,$$

and proceeds to show that $H(x, t)$ and $E(x, t)$ satisfy the Maxwell equation,

$$\frac{\partial H}{\partial t} + \text{curl } E = 0. \quad (2)$$

We note that E , unlike H , is not invariant under Galilean transformations $\bar{x}_i = x_i - v_i t$, $\bar{t} = t$, but transforms according to

$$\bar{E}_j = E_j + \epsilon_{jkl} v_k H_l.$$

Nevertheless, it may readily be shown that the Maxwell equation (2) is invariant under these transformations.

That the Maxwell Eqs. (1) and (2)—which are, of course, well known to be Lorentz invariant—are also Galilean invariant is to be expected, for the Galilean transformations are limiting forms of the Lorentz transformations as $c \rightarrow \infty$ and Eqs. (1) and (2) do not contain c .

But there are two other Maxwell equations, in Dyson's formulation

$$\text{div } E = 4\pi\rho \quad (3)$$

and (with correction of misprints in sign)

$$-\frac{\partial E}{\partial t} + \text{curl } H = 4\pi j. \quad (4)$$

Dyson dismisses these equations from the proof on the grounds that they merely define the external charge and current densities ρ and j . However, it may readily be seen that under Galilean transformations, for which $\bar{H}_j = H_j$, $\bar{E}_j = E_j + \epsilon_{jkl} v_k H_l$, the left-hand side of each of these equations is not invariant. If, following Dyson, we regard these equations as defining ρ and j , we can, of course, make (3) and (4) invariant by supposing ρ and j to transform in the manner required to yield this invariance. However, these transformations for ρ and j then differ radically from those obtained by letting $c \rightarrow \infty$ in the Lorentz transformations for ρ and j . Thus Eqs. (3) and (4), unlike (1) and (2), are not both Lorentz invariant and Galilean invariant.

That this is to be expected may be seen on restoring c to the equations instead of setting $c = 1$. We then have [in the system of units that gives (1) and (2)],

$$(1/c^2) \text{div } E = 4\pi\rho \quad (3')$$

and

$$-\frac{1}{c^2} \frac{\partial E}{\partial t} + \text{curl } H = 4\pi j. \quad (4')$$

On letting $c \rightarrow \infty$ we obtain equations the left-hand sides of which are invariant under Galilean transformations; the resulting Galilean invariance of ρ and j accords with that obtained by letting $c \rightarrow \infty$ in the Lorentz transformation of ρ and j .

Thus, as Dyson points out, "here we find Galilean mechanics and Maxwell equations coexisting peacefully"—but not *all* the Maxwell equations simultaneously.

Comment on "Linear regression analysis in a first physics lab," by L. Hmurcik *et al.* [Am. J. Phys. 57, 135–138 (1989)]

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The paper by Hmurcik *et al.*¹ proposes a method for teaching some elements of curve fitting and error analysis in a freshman physics lab. I have five criticisms: (1) Their Eqs. (4) and (5) (for errors in intercept and slope) are

wrong; (2) the authors make no use of the errors of measurement; (3) they train the students to do a least-squares fit (or linear regression analysis) of y on x when the errors are in x rather than in y ; (4) they give an incorrect interpre-